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12-1. A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

$v_2 = 30 \text{ km/h} = 8.33 \text{ m/s}$

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$(8.33)^2 = 0 + 2a_c(20-0)$

$a_c = 1.74 \text{ m/s}^2$ **Ans**

$v_2 = v_1 + a_c t$

$8.33 = 0 + 1.74(t)$

$t = 4.80 \text{ s}$ **Ans**

12-2. A car starts from rest and reaches a speed of 80 ft/s after traveling 500 ft along a straight road. Determine its constant acceleration and the time of travel.

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$(80)^2 = 0 + 2a_c(500-0)$

$a_c = 6.40 \text{ ft/s}^2$ **Ans**

$v_2 = v_1 + a_c t$

$80 = 0 + 6.4(t)$

$t = 12.5 \text{ s}$ **Ans**

12-3. A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (18)^2 + 2(32.2)(50-0)$

$v_2 = 59.532 = 59.5 \text{ ft/s}$ **Ans**

$v_2 = v_1 + a_c t$

$59.532 = 18 + 32.2(t)$

$t = 1.29 \text{ s}$ **Ans**

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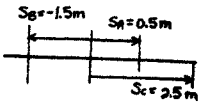
***12-4.** A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.



Total displacement $(s_C - s_A) = 2$ m

Total distance traveled $(0.5 + 1.5 + 1.5 + 2.5) = 6$ m

Total time traveled $(2 + 4) = 6$ s

$$v_{avg} = \frac{2}{6} = 0.333 \text{ m/s}$$ Ans

$$v_{sp} = \frac{6}{6} = 1 \text{ m/s}$$ Ans

12-5. Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

$$v = v_1 + a_c t$$

$$120 = 70 + 6000(t)$$

$$t = 8.33(10^{-3}) \text{ hr} = 30 \text{ s}$$ Ans

$$v^2 = v_1^2 + 2a_c(s - s_1)$$

$$(120)^2 = 70^2 + 2(6000)(s - 0)$$

$$s = 0.792 \text{ km} = 792 \text{ m}$$ Ans

12-6. A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

Prob. 12–6

$$v = 60(1 - e^{-t})$$

$$\int_0^t ds = \int v\,dt = \int_0^3 60(1 - e^{-t})\,dt$$

$$s = 60(t + e^{-t})\Big|_0^3$$

$$s = 123 \text{ ft}$$ Ans

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$$a = 60e^{-3} = 2.99 \text{ ft/s}^2$$ Ans

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12-7. The position of a particle along a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \le t \le 10$ s.

$s = t^3 - 9t^2 + 15t$

$v = \dfrac{ds}{dt} = 3t^2 - 18t + 15$

$a = \dfrac{dv}{dt} = 6t - 18$

a_{max} occurs at $t = 10$ s,

$a_{max} = 6(10) - 18 = 42$ ft/s^2 **Ans**

v_{max} occurs when $t = 10$ s

$v_{max} = 3(10)^2 - 18(10) + 15 = 135$ ft/s **Ans**

***12-8.** From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

$(+\downarrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

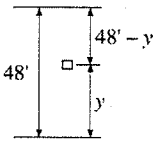
$80.7^2 = 0 + 2(32.2)(s - 0)$

$s = 101.13$ ft

$\#$ of floors $= \dfrac{101.13}{12} = 8.43$

The car must be dropped from the 9th floor. **Ans**

12-9. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s, decelerate at 0.3 ft/s, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.



$+\uparrow v^2 = v_1^2 + 2a_c(s - s_1)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)((48 - y) - 0)$

$0 = 1.2y - 0.6(48 - y)$

$y = 16.0$ ft, $v_{max} = 4.382$ ft/s < 8 ft/s

$+\uparrow v = v_1 + a_c t$

$4.382 = 0 + 0.6t_1$

$t_1 = 7.303$ s

$0 = 4.382 - 0.3t_2$

$t_2 = 14.61$ s

$t = t_1 + t_2 = 21.9$ s **Ans**

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12-10. A particle travels in a straight line such that for a short time $2\,s \le t \le 6\,s$ its motion is described by $v = (4/a)$ ft/s, where a is in ft/s^2. If $v = 6$ ft/s when $t = 2$ s, determine the particle's acceleration when $t = 3$ s.

$$a = \frac{dv}{dt} = \frac{4}{v}$$

$$\int_6^v v\,dv = \int_2^t 4\,dt$$

$$\frac{1}{2}v^2 - 18 = 4t - 8$$

$$v^2 = 8t + 20$$

At $t = 3$ s, choosing the positive root

$$v = 6.63 \text{ ft/s}$$

$$a = \frac{4}{6.63} = 0.603 \text{ ft/s}^2 \qquad \text{Ans}$$

12-11. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ m/s^2, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$$\int_2^v dv = \int_0^t (2t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

When $t = 6$ s,

$$v = 32 \text{ m/s} \qquad \text{Ans}$$

$$s = 67 \text{ m} \qquad \text{Ans}$$

Since $v \neq 0$ then

$$d = 67 - 1 = 66 \text{ m} \qquad \text{Ans}$$

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*12-12. When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})$ m/s^2, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.

Prob. 12–12

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}(v^5 - 32)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s} \qquad \text{Ans}$$

$$a\,ds = v\,dv$$

$$ds = \frac{v\,dv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Big|_2^{3.925}$$

$$= 9.98 \text{ m} \qquad \text{Ans}$$

12-13. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t =$ 6 s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

Position : The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \text{Ans}$$

Total Distance Traveled : The velocity of the particle can be determined by applying Eq. 12–1.

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

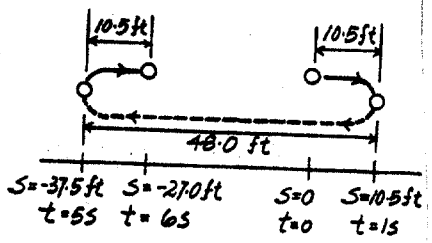
$$4.50t^2 - 27.0t + 22.5 = 0$$
$$t = 1 \text{ s} \quad \text{and} \quad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$
$$s|_{t=1s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$
$$s|_{t=5s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \text{Ans}$$



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12-14. The position of a particle on a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

$s = t^3 - 9t^2 + 15t$

$v = \frac{ds}{dt} = 3t^2 - 18t + 15$

$v = 0$ when $t = 1$ s and $t = 5$ s

$t = 0,\ s = 0$

$t = 1$ s, $s = 7$ ft

$t = 5$ s, $s = -25$ ft

$t = 6$ s, $s = -18$ ft **Ans**

$s_T = 7 + 7 + 25 + (25 - 18) = 46$ ft **Ans**

12-15. A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$ m when $t = 0$.

$\frac{ds}{dt} = \frac{5}{4+s}$

$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$

$4s + 0.5s^2 - 32.5 = 5t$

When $t = 6$ s,

$s^2 + 8s - 125 = 0$

Solving for the positive root

$s = 7.87$ m **Ans**

***12-16.** A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

$v = \frac{5}{4+s}$

$v\, dv = a\, ds$

$dv = \frac{-5\, ds}{(4+s)^2}$

$\frac{5}{(4+s)}\left(\frac{-5\, ds}{(4+s)^2}\right) = a\, ds$

$a = \frac{-25}{(4+s)^3}$

When $s = 2$ m,

$a = -0.116$ m/s^2 **Ans**

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12-17. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Velocity: The velocity of particles A and B can be determine using Eq. 12-2.

$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \quad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determine using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

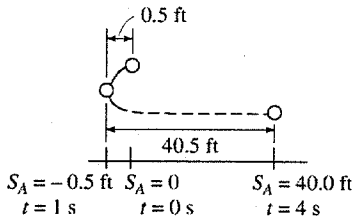
$$\int_0^{s_B} ds_B = \int_0^1 (4t^3 - 8t)dt$$

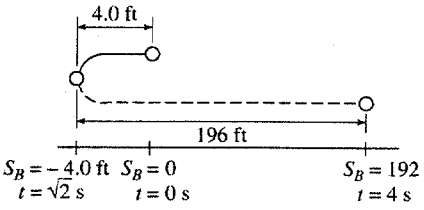
$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1s} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4s} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$





Particle A has traveled

$d_A = 2(0.5) + 40.0 = 41.0$ ft **Ans**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

$d_B = 2(4) + 192 = 200$ ft **Ans**

At $t = 4$ s. the distance between A and B is

$\Delta s_{AB} = 192 - 40 = 152$ ft **Ans**

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12-18. A car starts from rest and moves along a straight line with an acceleration of $a = (3s^{-1/3})\ \text{m/s}^2$, where s is in meters. Determine the car's acceleration when $t = 4$ s.

$$a = 3s^{-\frac{1}{3}}$$

$$a\,ds = v\,dv$$

$$\int_0^s 3s^{-\frac{1}{3}}\,ds = \int_0^v v\,dv$$

$$\frac{3}{2}(3)s^{\frac{2}{3}} = \frac{1}{2}v^2$$

$$v = 3s^{\frac{1}{3}}$$

$$\frac{ds}{dt} = 3s^{\frac{1}{3}}$$

$$\int_0^s s^{-\frac{1}{3}}\,ds = \int_0^t 3\,dt$$

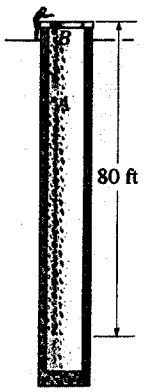
$$\frac{3}{2}s^{\frac{2}{3}} = 3t$$

$$s = (2t)^{\frac{3}{2}}$$

$$s|_{t=4} = (2(4))^{\frac{3}{2}} = 22.62 = 22.6\ \text{m}$$

$$a|_{t=4} = 3(22.62)^{-\frac{1}{3}} = 1.06\ \text{m/s}^2$$ **Ans**

12-19. A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.



$$+\downarrow s = s_1 + v_1 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s_A = 64.4\ \text{ft}$$

$$s_B = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1\ \text{ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3\ \text{ft}$$

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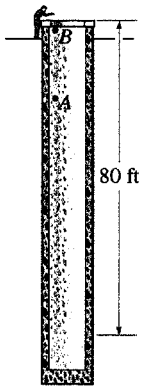
*12-20. A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the time interval between the instant A strikes the water and the instant B strikes the water. Also, at what speed do they strike the water?



B is dropped one second after A, so that

$\Delta t = 1$ s Ans

$+\downarrow s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$80 = 0 + 0 + \frac{1}{2}(32.2)(t^2)$

$t = 2.2291$ s

$+\downarrow v = v_0 + a_c t$

$v = 0 + 32.2(2.2291)$

$v = 71.8$ ft/s Ans

Also,

$v^2 = v_0^2 + 2a_c s$

$v^2 = 0^2 + 2(32.2)(80)$

$v = 71.8$ ft/s Ans

12-21. A particle travels in a straight line with accelerated motion such that $a = -ks$, where s is the distance from the starting point and k is a proportionality constant which is to be determined. For $s = 2$ ft the velocity is 4 ft/s, and for $s = 3.5$ ft the velocity is 10 ft/s. What is s when $v = 0$?

$a = -ks$

$a\,ds = v\,dv$

$-k\int_2^s s\,ds = \int_4^v v\,dv$

$-k(\frac{s^2}{2} - \frac{(2)^2}{2}) = (\frac{v^2}{2} - \frac{(4)^2}{2})$

$-k(\frac{s^2}{2} - 2) = \frac{v^2}{2} - 8$

Set $s = 3.5$ ft, $v = 10$ ft/s,

Then $k = -10.18 = -10.2$ s^{-2} Ans

When $v = 0$

$10.18(\frac{s^2}{2} - 2) = -8$

$s = 1.56$ ft Ans

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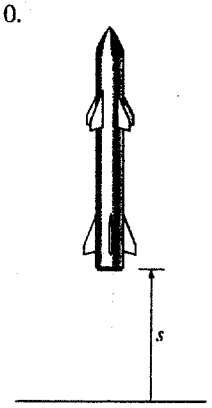
12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$$a\,ds = v\,dv$$

$$\int_0^s (6+0.02\,s)\,ds = \int_0^v v\,dv$$

$$6s + 0.01s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12s + 0.02s^2}$$

When $s = 2000$ m,

$$v = 322 \text{ m/s} \qquad \textbf{Ans}$$

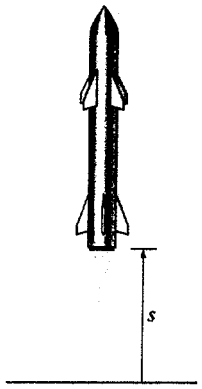
$$\int_0^s \frac{ds}{\sqrt{12s + 0.02s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}} \ln\left(\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right)\Bigg|_0^s = t$$

Set $s = 2000$ m

$$t = 19.3 \text{ s} \qquad \textbf{Ans}$$

12-23. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$$a\,ds = v\,dv$$

$$\int_0^s (6+0.02\,s)\,ds = \int_0^v v\,dv$$

$$6s + 0.01s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12s + 0.02s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12s + 0.02s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}} \ln\left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$$t = 5.62 \text{ s} \qquad \textbf{Ans}$$

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***12-24.** At $t = 0$ bullet A is fired vertically with an initial (muzzle) velocity of 450 m/s. When $t = 3$ s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occur?

$$+\uparrow s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 450t + \frac{1}{2}(-9.81)t^2$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 600(t-3) + \frac{1}{2}(-9.81)(t-3)^2$$

Require $s_A = s_B$

$$450t - 4.905t^2 = 600t - 1800 - 4.905t^2 + 29.43t - 44.145$$

$t = 10.3$ s **Ans**

$s_A = s_B = 4.11$ km **Ans**

***12-25.** A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

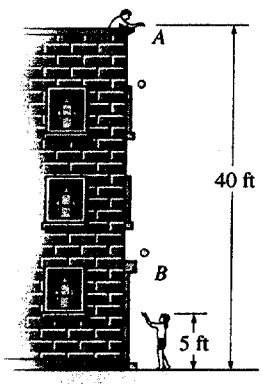
$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$v = 1.29$ m/s **Ans**

12-26. Ball A is released from rest at a height of 40 ft at the same time that a second ball B is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball B was thrown upward.



For ball # 1 :

$$(+\downarrow) s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$20 = 0 + 0 + \tfrac{1}{2}(32.2)t^2$$

$$t = 1.1146 \text{ s}$$

For ball # 2 :

$$(+\uparrow) s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$15 = 0 + v_B(1.1146) + \tfrac{1}{2}(-32.2)(1.1146)^2$$

$v_B = 31.4$ ft/s **Ans**

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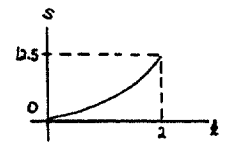
■12-27. A projectile, initially at the origin, moves vertically downward along a straight-line path through a fluid medium such that its velocity is defined as $v = 3(8e^{-t} + t)^{1/2}$ m/s, where t is in seconds. Plot the position s of the projectile during the first 2 s. Use the Runge-Kutta method to evaluate s with incremental values of $h = 0.25$ s.

$v = 3(8e^{-t} + t)^{1/2}$

$s_0 = 0$ at $t = 0$



Using the Runge–Kutta method:

s	t
0	0
2.01	0.25
3.83	0.50
5.49	0.75
7.03	1.00
8.48	1.25
9.87	1.50
11.2	1.75
12.5	2.00

***12-28.** The acceleration of a particle along a straight line is defined by $a = (2t - 9)$ m/s^2, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

$a = 2t - 9$

$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$

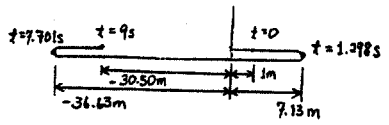
$v - 10 = t^2 - 9t$

$v = t^2 - 9t + 10$

$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$

$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$

$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$



Note when $v = 0$ at $t^2 - 9t + 10 = 0$

$t = 1.298$ s and $t = 7.701$ s

When $t = 1.298$ s, $s = 7.13$ m

When $t = 7.701$ s, $s = -36.63$ m

When $t = 9$ s, $s = -30.50$ m

(a) $s = -30.5$ m Ans

(b) $s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$s_{Tot} = 56.0$ m Ans

(c) $v = 10$ m/s Ans

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12-29. A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s, determine the distance it travels before it stops.

$$a = \frac{dv}{dt} = -1.5v^{\frac{1}{2}}$$

$$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$$

$$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$$

$$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$$

$$v = (2 - 0.75t)^2 \text{ m/s} \quad (1)$$

$$\int_0^s ds = \int_0^t (2 - 0.75t)^2\,dt = \int_0^t \left(4 - 3t + 0.5625t^2\right) dt$$

$$s = 4t - 1.5t^2 + 0.1875t^3 \quad (2)$$

From Eq. (1), the particle will stop when

$$0 = (2 - 0.75t)^2$$

$$t = 2.667 \text{ s}$$

$$s|_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56 \text{ m} \quad \textbf{Ans}$$

12-30. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29 \text{ m/s} \quad \textbf{Ans}$$

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12-31. Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

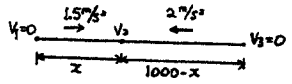
Using formulas of constant acceleration:

$v_2 = 1.5\,t_1$

$x = \frac{1}{2}(1.5)(t_1^2)$

$0 = v_2 - 2\,t_2$

$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$



Combining equations:

$t_1 = 1.33\,t_2; \quad v_2 = 2\,t_2$

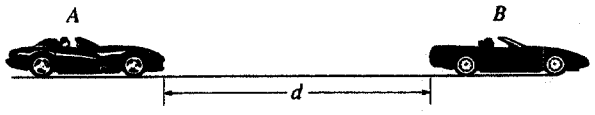
$x = 1.33\,t_2^2$

$1000 - 1.33\,t_2^2 = 2\,t_2^2 - t_2^2$

$t_2 = 20.702\text{ s} \quad ; t_1 = 27.603\text{ s}$

$t = t_1 + t_2 = 48.3\text{ s}$ **Ans**

***12-32.** When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



Motion of car A :

$v = v_0 + a_c t$

$0 = v v_A - a_A t \qquad t = \frac{v_A}{a_A}$

$v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = v_A^2 + 2(-a_A)(s_A - 0)$

$s_A = \frac{v_A^2}{2a_A}$

Motion of car B :

$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$

The distance between cars A and B is

$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$ **Ans**

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12-33. If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude,* determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \to \infty$).

(a) $$a = \frac{dv}{dt} = 9.81\left[1 - v^2\left(10^{-4}\right)\right]$$

$$\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = \int_0^t 9.81\left(10^{-4}\right) dt \qquad (1)$$

$$\frac{1}{100}\tanh^{-1}\left(\frac{v}{100}\right)\Big|_0^v = 9.81\left(10^{-4}\right)t$$

$$\tanh^{-1}\left(\frac{v}{100}\right) = \left(9.81\left(10^{-2}\right)t\right) \qquad (2)$$

$$v = 100\tanh\left(9.81\left(10^{-2}\right)(5)\right)$$

$$= 100\tanh(0.4905) = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

(b) From Eq. (2), with $t \to \infty$,

$$v = 100\tanh\infty = 100 \text{ m/s} \qquad \textbf{Ans}$$

Also note that Eq. (1) can be written as

$$10^4\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = 9.81t$$

$$10^4\left[\left(\frac{1}{2(10^2)}\right)\ln\left(\frac{10^2 + v}{10^2 - v}\right)\right]_0^v = 9.81t$$

$$50\left[\ln\left(\frac{100 + v}{100 - v}\right) - \ln 1\right] = 9.81t$$

When $t = 5$ s,

$$\frac{100 + v}{100 - v} = e^{0.981} = 2.667$$

$$100 + v = 266.7 - 2.667v$$

$$v = \frac{166.7}{3.667} = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

12-34. As a body is projected to a high altitude above the earth's *surface,* the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81$ m/s^2 and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \to \infty$.

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2}\Big|_v^0 = \frac{g_0R^2}{R + y}\Big|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s} \qquad \textbf{Ans}$$

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12-35. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-34), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12-34.

From Prob. 12–34,

$$(+\uparrow) \quad g = -g_0 \frac{R^2}{(R+y)^2}$$

Since $g\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

When $y_0 = 500$ km, $y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans}$$

***12-36.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v < v_f$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)(v_f^2 - v^2)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f} \ln\left(\frac{v_f + v}{v_f - v}\right)\Big|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g} \ln\left(\frac{v_f + v}{v_f - v}\right) \qquad \textbf{Ans}$$

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12-37. An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the s–t, v–t, and a–t graphs that describe the motion.

$v_1 = 0$

$$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1\text{ h})\,5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 237.6\text{ ft/s}$$

$$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$$

$$a_c = 5.64538\text{ ft/s}^2$$

$$v_2 = v_1 + a_c t$$

$$237.6 = 0 + 5.64538\,t$$

$$t = 42.09 = 42.1\text{ s}$$

$$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1\text{ h})\,5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 322.67\text{ ft/s}$$

$$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$$

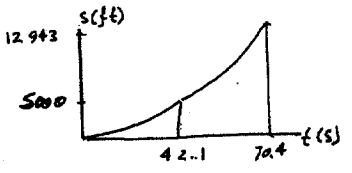
$$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$$

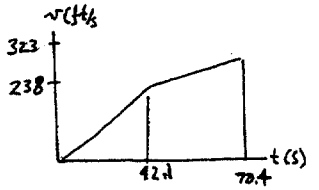
$$s = 12\,943.34\text{ ft}$$

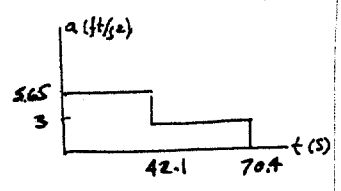
$$v_3 = v_2 + a_c t$$

$$322.67 = 237.6 + 3\,t$$

$$t = 28.4\text{ s}$$







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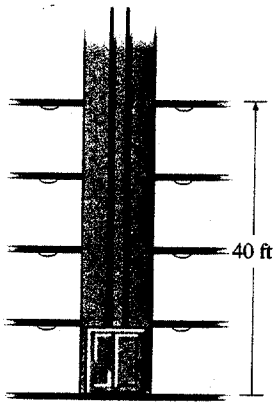
12-38. The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the *a–t*, *v–t*, and *s–t* graphs for the motion.



$+\uparrow v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5t_1$

$+\uparrow v_3 = v_1 + a_c t$

$0 = v_{max} - 2t_2$

Thus

$t_1 = 0.4\, t_2$

$+\uparrow s_2 = s_1 + v_1 t + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\, t_1^2$

$+\uparrow 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\, t_2^2$

$+\uparrow v^2 = v_1^2 + 2a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10\, h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\, h = 160 - 4h$

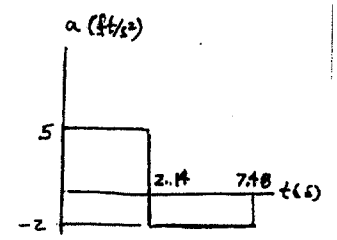
$h = 11.429$ ft

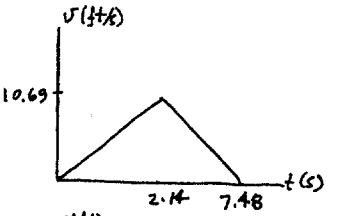
$v_{max} = 10.69$ ft/s

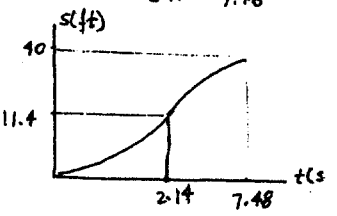
$t_1 = 2.138$ s

$t_2 = 5.345$ s

$t = t_1 + t_2 = 7.48$ s **Ans**







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12-39. A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v-t graph for the motion.

Total Distance Traveled : The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12 – 5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12 – 4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad [1]$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is travelling at a constant velocity of $v = 0.5t'$ (Eq. [1]). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = v t_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

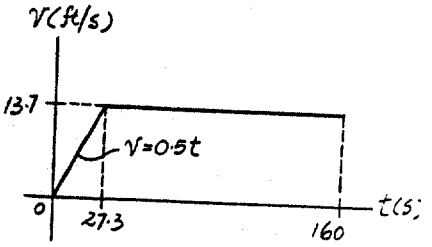
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

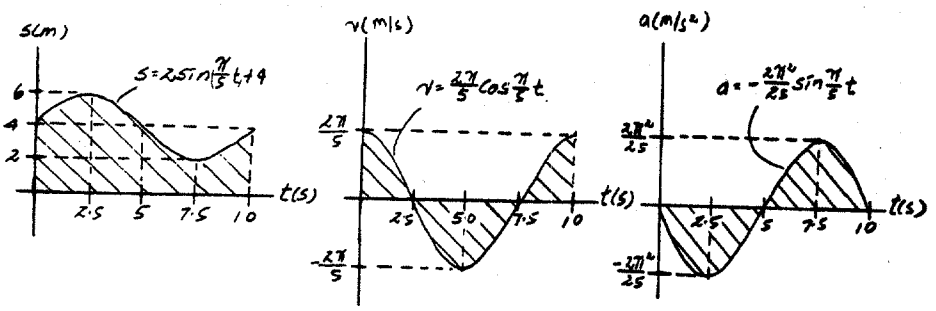
Chose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans}$$

v – t Graph : The equation for the velocity is given by Eq. [1]. When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.



***12-40.** If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 10$ s.



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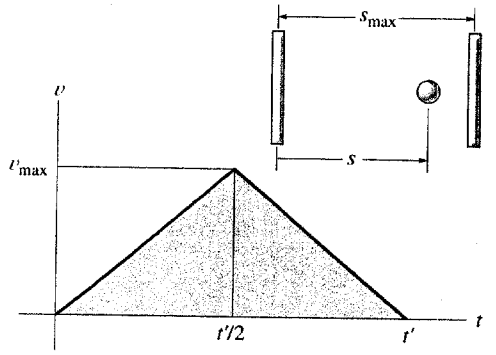
12-41. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



$a_c = 4$ m/s^2

$\frac{s}{2} = 100$ mm $= 0.1$ m

$v^2 = v_0^2 + 2a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442$ m/s **Ans**

$v = v_0 + a_c t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721$ s **Ans**

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

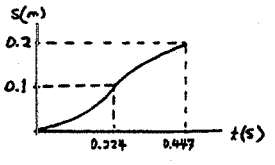
$s = 0.1$ m

$\int_{0.894}^{v} dv = -\int_{0.2235}^{t} 4\,dt$

$v = -4t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

$s = -2t^2 + 1.788t - 0.2$



When $t = 0.447$ s,

$s = 0.2$ m

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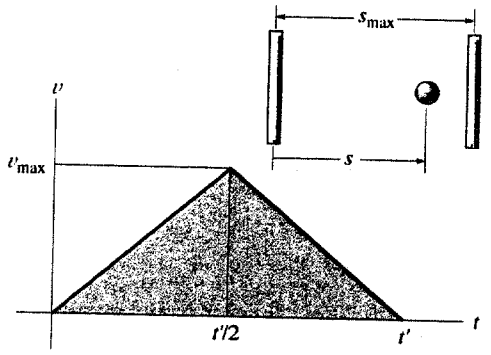
12-42. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



For $0 < t < 0.1$ s

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

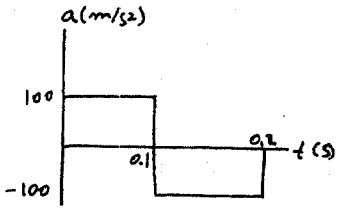
$ds = v\,dt$

$\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m



For $0.1\text{ s} < t < 0.2$ s

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

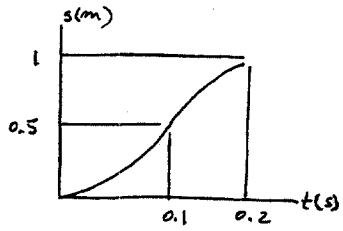
$\int_{0.5}^s ds = \int_{0.1}^t (-100\,t + 20)dt$

$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m



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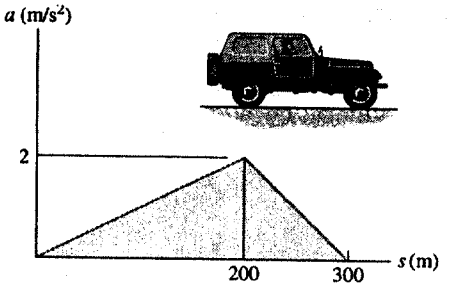
12-43. The a-s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v-s graph. At $s = 0$, $v = 0$.



***a*–*s* Graph :** The function of acceleration a in terms of s for the interval $0\text{ m} \le s < 200\text{ m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s)\ \text{m/s}^2$$

For the interval $200\text{ m} < s \le 300\text{ m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s+6)\ \text{m/s}^2$$

***v*–*s* Graph :** The function of velocity v in terms of s can be obtained by applying $v\,dv = a\,ds$. For the interval $\mathbf{0\text{ m} \le s < 200\text{ m}}$,

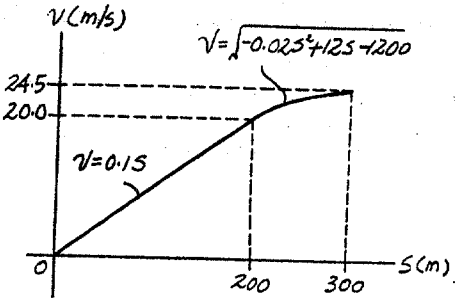
$$v\,dv = a\,ds$$
$$\int_0^v v\,dv = \int_0^s 0.01s\,ds$$
$$v = (0.1s)\ \text{m/s}$$

At $s = 200$ m, $\quad v = 0.100(200) = 20.0$ m/s

For the interval $\mathbf{200\text{ m} < s \le 300\text{ m}}$,

$$v\,dv = a\,ds$$
$$\int_{20.0\,\text{m/s}}^{v} v\,dv = \int_{200\,\text{m}}^{s} (-0.02s+6)\,ds$$
$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right)\ \text{m/s}$$

At $s = 300$ m, $\quad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5$ m/s



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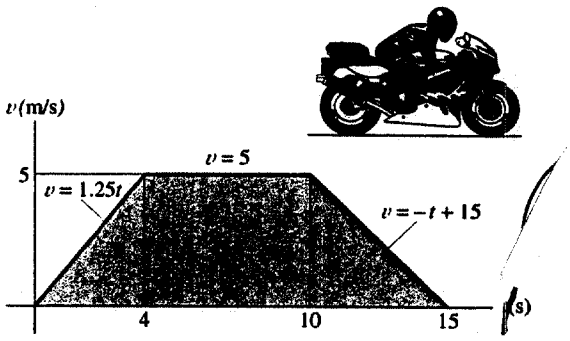
***12-44.** A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



At $t = 8$ s

$$a = \frac{dv}{dt} = 0 \qquad \text{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8-4)(5) = 30$$

$$s = 30 \text{ m} \qquad \text{Ans}$$

At $t = 12$ s

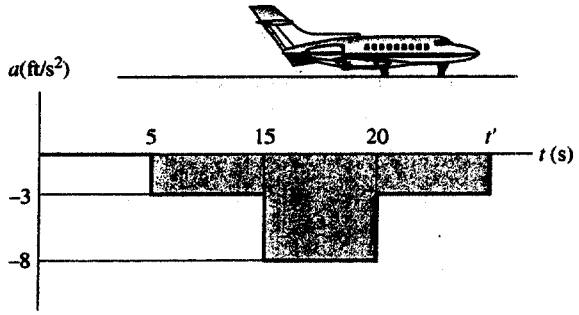
$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2 \qquad \text{Ans}$$

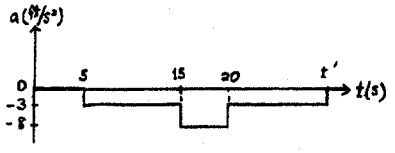
$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10-4)(5) + \frac{1}{2}(15-10)(5) - \frac{1}{2}(\tfrac{3}{5})(5)(\tfrac{3}{5})(5)$$

$$s = 48 \text{ m} \qquad \text{Ans}$$

12-45. An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



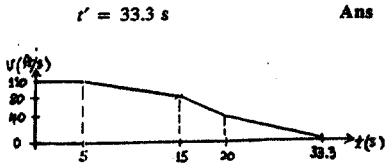


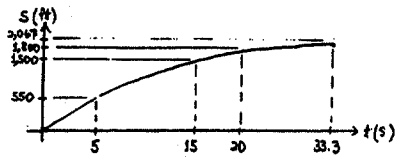
$v_0 = 110$ ft/s

$$\Delta v = \int a\,dt$$

$$0 - 110 = -3(15-5) - 8(20-15) - 3(t' - 20)$$

$$t' = 33.3 \text{ s} \qquad \text{Ans}$$





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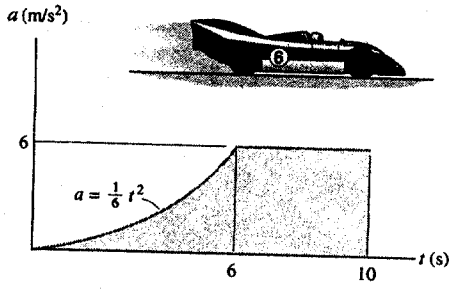
12-46. A race cars starting from rest travels along a straight road and for 10 s has the acceleration shown. Construct the v-t graph that describes the motion and find the distance traveled in 10 s.



***v*–*t* Graph :** The velocity function in terms of time t can be obtained by applying formula $a = \frac{dv}{dt}$. For time interval $\mathbf{0\ s \le t < 6\ s}$,

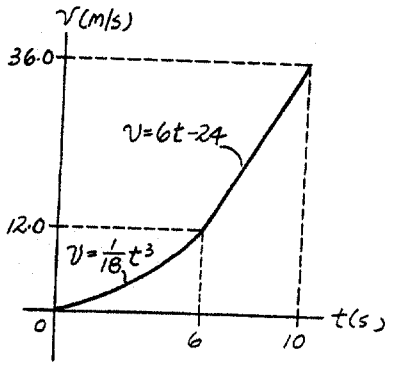
$$dv = adt$$
$$\int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$
$$v = \left(\frac{1}{18}t^3\right) \text{m/s}$$

At $t = 6$ s, $\quad v = \frac{1}{18}(6^3) = 12.0$ m/s

For time interval $\mathbf{6\ s < t \le 10\ s}$,

$$dv = adt$$
$$\int_{12.0\text{m/s}}^v dv = \int_{6s}^t 6dt$$
$$v = (6t - 24) \text{m/s}$$

At $t = 10$ s, $\quad v = 6(10) - 24 = 36.0$ m/s



Position : The position in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $\mathbf{0\ s \le t < 6\ s}$,

$$ds = vdt$$
$$\int_0^s ds = \int_0^t \frac{1}{18}t^3 dt$$
$$s = \left(\frac{1}{72}t^4\right) \text{m}$$

When $t = 6$ s, $\quad s = \frac{1}{72}(6^4) = 18.0$ m

For time interval $\mathbf{6\ s < t \le 10\ s}$,

$$ds = vdt$$
$$\int_{18.0\text{m}}^s dv = \int_{6s}^t (6t - 24)\, dt$$
$$s = \left(3t^2 - 24t + 54\right) \text{m}$$

When $t = 10$ s, $\quad s = 3(10^2) - 24(10) + 54 = 114$ m **Ans**

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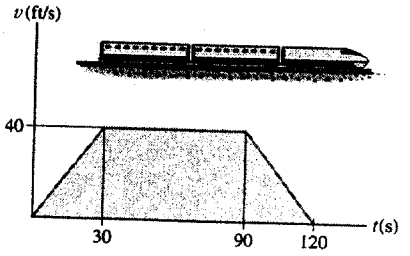
12-47. The v–t graph for the motion of a train as it moves from station A to station B is shown. Draw the a–t graph and determine the average speed and the distance between the stations.



For $0 \le t < 30$ s $\quad a = \dfrac{\Delta v}{\Delta t} = \dfrac{40}{30} = 1.33 \text{ ft/s}^2$

For $30 < t < 90$ s $\quad a = \dfrac{\Delta v}{\Delta t} = 0$

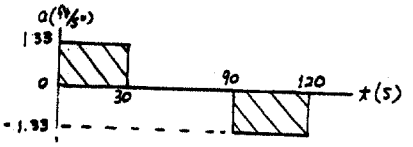
For $90 < t < 120$ s $\quad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0-40}{120-90} = -1.33 \text{ ft/s}^2$

$$\Delta s = \int v\, dt$$

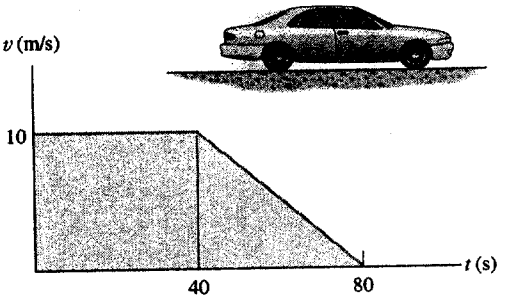
$$s - 0 = \frac{1}{2}(40)(30) + 40(90-30) + \frac{1}{2}(40)(120-90)$$

$$s = 3600 \text{ ft} \quad \textbf{Ans}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{3600}{120} = 30 \text{ ft/s} \quad \textbf{Ans}$$



***12-48.** The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops (t = 80 s). Construct the a-t graph.



Distance Traveled : The total distance traveled can be obtained by computing the area under the $v - t$ graph.

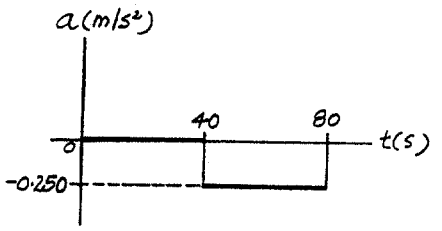
$$s = 10(40) + \frac{1}{2}(10)(80-40) = 600 \text{ m} \quad \textbf{Ans}$$

a–t Graph : The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $\mathbf{0 \text{ s} \le t < 40 \text{ s}}$,

$$a = \frac{dv}{dt} = 0$$

For time interval $\mathbf{40 \text{ s} < t \le 80 \text{ s}}$, $\dfrac{v-10}{t-40} = \dfrac{0-10}{80-40}$, $v = \left(-\dfrac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



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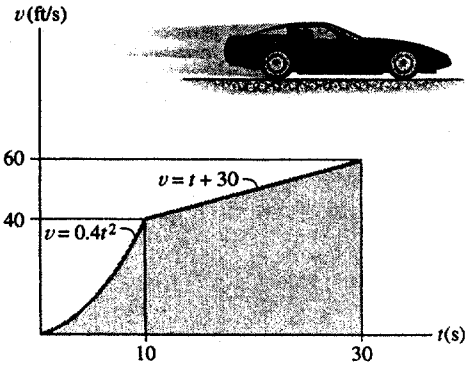
12-49. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s :

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

At $t = 10$ s :

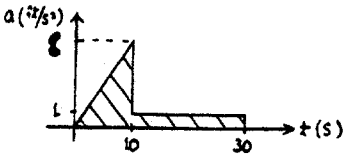
$$a = 8 \text{ ft/s}^2$$

For $10 < t \leq 30$ s :

$$v = t + 30$$

$$a = \frac{dv}{dt} = 1$$

$$a_{max} = 8 \text{ ft/s}^2 \quad \textbf{Ans}$$



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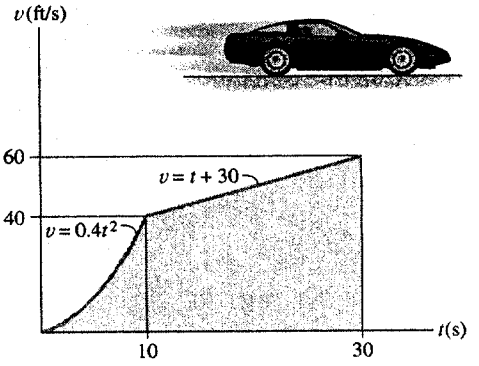
12-50. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^{s} ds = \int_{10}^{t} (t + 30)\,dt$$

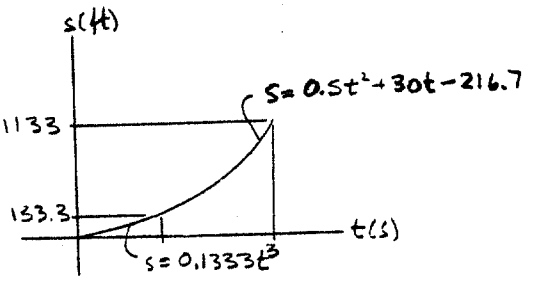
$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s} \quad \textbf{Ans}$$

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft} \quad \textbf{Ans}$$



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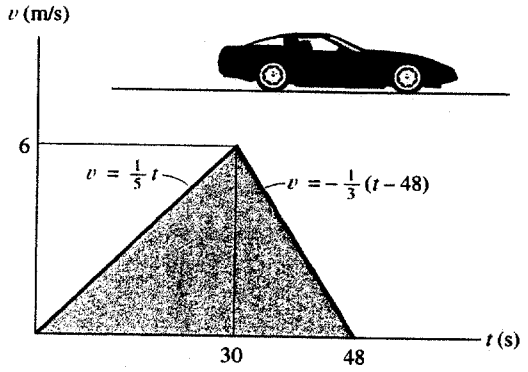
12-51. A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



$0 \le t < 30$

$v = \frac{1}{5}t$

$a = \frac{dv}{dt} = \frac{1}{5}$

$\int_0^s ds = \int_0^t \frac{1}{5} t\, dt$

$s = \frac{1}{10}t^2$

When $t = 30$ s, $s = 90$ m

$v = -\frac{1}{3}(t-48)$

$a = \frac{dv}{dt} = -\frac{1}{3}$

$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t-48)\, dt$

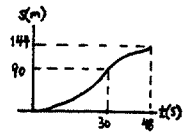
$s = -\frac{1}{6}t^2 + 16t - 240$

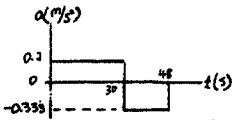
When $t = 48$ s, $s = 144$ m **Ans**

Also, from $v-t$ graph

$\Delta s = \int v\, dt$

$s - 0 = \frac{1}{2}(6)(48) = 144$ m **Ans**





***12-52.** A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

For package :

$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s_2 - s_0)$

$v^2 = (4)^2 + 2(-32.2)(-100 - 0)$

$v = 80.35$ ft/s $\downarrow$

$(+\uparrow)$ $v = v_0 + a_c t$

$-80.35 = 4 + (-32.2)t$

$t = 2.620$ s

For elevator :

$(+\uparrow)$ $s_2 = s_0 + vt$

$s = 100 + 4(2.620)$

$s = 110$ ft **Ans**

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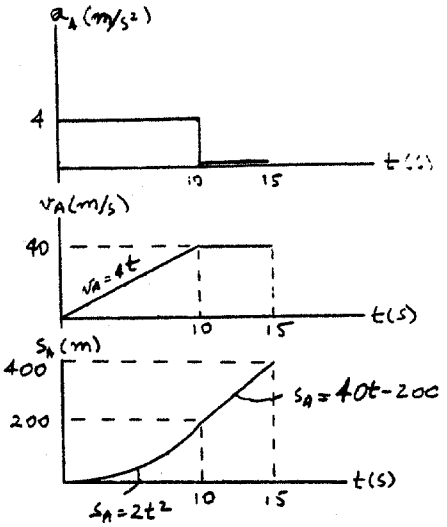
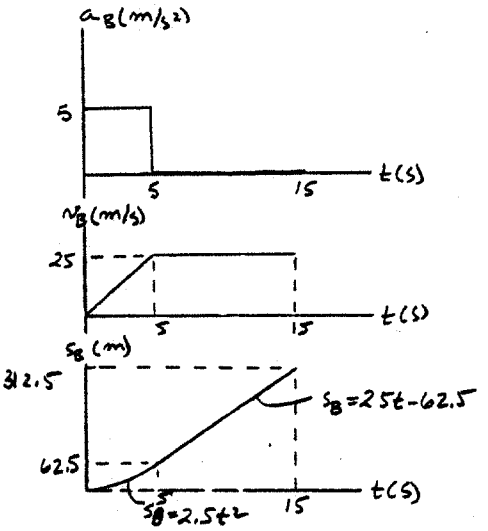
12-53. Two cars start from rest side by side and travel along a straight road. Car A accelerates at 4 m/s^2 for 10 s and then maintains a constant speed. Car B accelerates at 5 m/s^2 until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?





Car A:

$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $v_A = 40$ m/s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

At $t = 10$ s, $s_A = 200$ m

$t > 10$ s, $ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$s_A = 40t - 200$

At $t = 15$ s, $s_A = 400$ m

Car B:

$v = v_0 + a_c t$

$v_B = 0 + 5t$

When $v_B = 25$ m/s, $t = \frac{25}{5} = 5$ s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$

At $t = 5$ s, $s_B = 62.5$ m

$t > 5$ s, $ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_{5}^{t} 25\,dt$$

$s_B - 62.5 = 25t - 125$

$s_B = 25t - 62.5$

When $t = 15$ s, $s_B = 312.5$

Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m Ans

Car A is ahead of car B.

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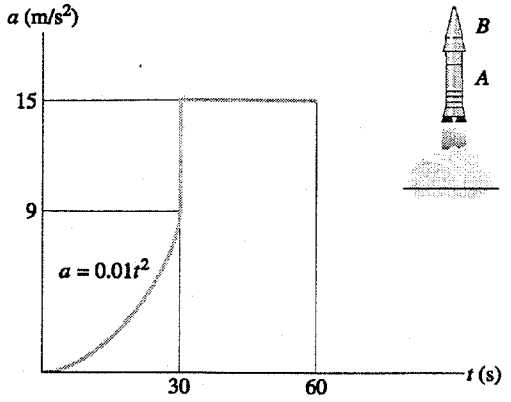
12-54. A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



For $0 \le t < 30$ s

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v = 0.00333t^3$$

When $t = 30$ s, $v = 90$ m/s

$$\int_0^s ds = \int_0^t 0.00333t^3 dt$$

$$s = 0.000833t^4$$

When $t = 30$ s, $s = 675$ m

For 30 s $\le t < 60$ s

$$\int_{90}^v dv = \int_{30}^t 15 dt$$

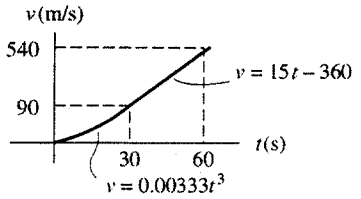
$$v = 15t - 360$$

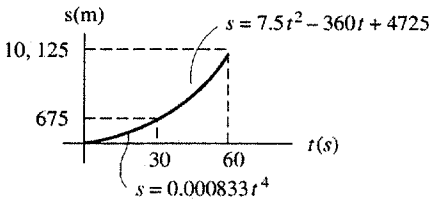
When $t = 60$ s, $v = 540$ m/s

$$\int_{675}^s ds = \int_{30}^t (15t - 360) dt$$

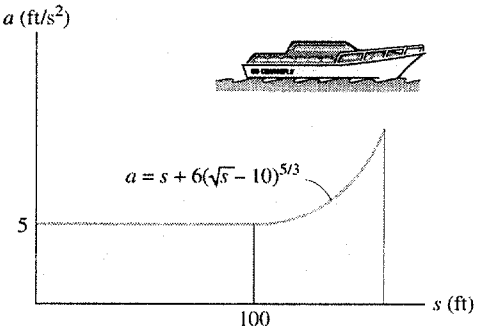
$$s = 7.5t^2 - 360t + 4725$$

When $t = 60$ s, $s = 10\,125$ m





■12-55. The $a-s$ graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



Velocity: The velocity v in terms of s can be obtained by applying $v\,dv = a\,ds$. For the interval **0 ft ≤ s < 100 ft**,

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$v = (\sqrt{10s})\ \text{ft/s}$$

At $s = 75$ ft, $v = \sqrt{10(75)} = 27.4$ ft/s **Ans**

At $s = 100$ ft, $v = \sqrt{10(100)} = 31.62$ ft/s **Ans**

For the interval **100 ft < s ≤ 125 ft**,

$$v\,dv = a\,ds$$

$$\int_{31.62\ \text{ft/s}}^v v\,dv = \int_{100\ \text{ft}}^{125\ \text{ft}} [s + 6(\sqrt{s} - 10)^{5/3}]\,ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\frac{v^2}{2}\Big|_{31.62\ \text{ft/s}}^{v} = 2888.53$$

$$v = 82.3\ \text{ft/s}$$ **Ans**

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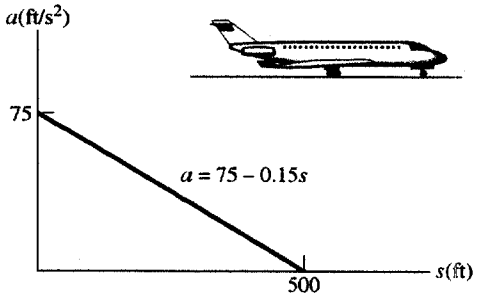
***12-56.** The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 200 ft. Also, how much time is required for it to travel 200 ft?



$a = 75 - 0.15s$

$\int_0^v v\,dv = \int_0^s (75 - 0.15s)\,ds$

$v = \sqrt{150\,s - 0.15\,s^2}$

At $s = 200$ ft

$v = \sqrt{150(200) - 0.15(200)^2} = 155$ ft/s **Ans**

$v = \frac{ds}{dt}$

$\int_0^t dt = \int_0^{200} \frac{ds}{\sqrt{150\,s - 0.15\,s^2}}$

$t = 2.582 \sin^{-1}\left(\frac{0.3\,s - 150}{150}\right)\Big|_0^{200} = 2.39$ s **Ans**

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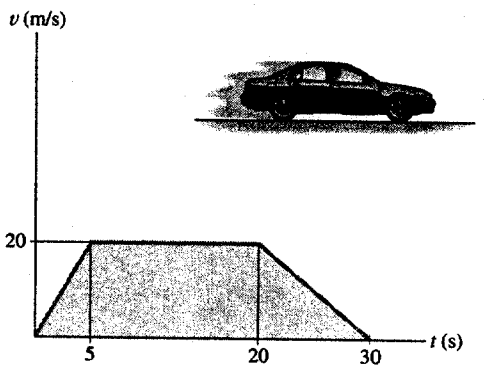
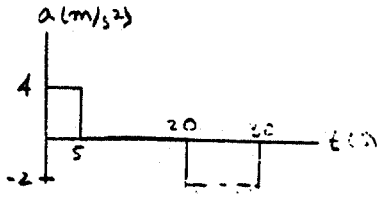
12-57. The v–t graph of a car while traveling along a road is shown. Draw the s–t and a–t graphs for the motion.



$0 \le t \le 5$ $\quad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4 \text{ m/s}^2$

$5 \le t \le 20$ $\quad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20-20}{20-5} = 0 \text{ m/s}^2$

$20 \le t \le 30$ $\quad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0-20}{30-20} = -2 \text{ m/s}^2$

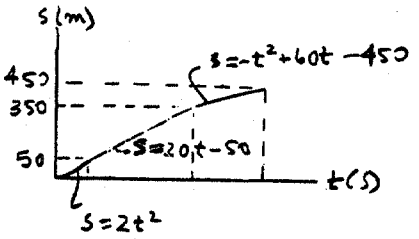


From the $v-t$ graph at $t_1 = 5\,s$, $t_2 = 20\,s$, and $t_3 = 30$ s,

$s_1 = A_1 = \dfrac{1}{2}(5)(20) = 50 \text{ m}$

$s_2 = A_1 + A_2 = 50 + 20(20-5) = 350 \text{ m}$

$s_3 = A_1 + A_2 + A_3 = 350 + \dfrac{1}{2}(30-20)(20) = 450 \text{ m}$



The equations defining the portions of the $s-t$ graph are

$0 \le t \le 5$ s $\quad v = 4t; \quad ds = v\,dt; \quad \int_0^s ds = \int_0^t 4t\,dt; \quad s = 2t^2$

$5 \le t \le 20$ s $\quad v = 20; \quad ds = v\,dt; \quad \int_{50}^s ds = \int_5^t 20\,dt; \quad s = 20t - 50$

$20 \le t \le 30$ s $\quad v = 2(30-t); \quad ds = v\,dt; \quad \int_{350}^s ds = \int_{20}^t 2(30-t)\,dt; \quad s = -t^2 + 60t - 450$

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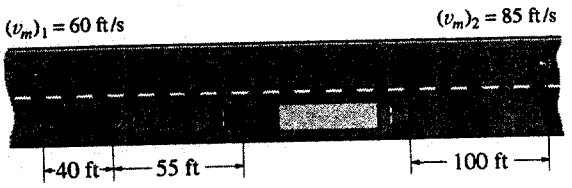
12-58. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the v–t and s–t graphs for the motorcycle during this time.



Motorcycle :
Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Distance traveled,

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel : $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft

Motorcycle :

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t$$

Truck :

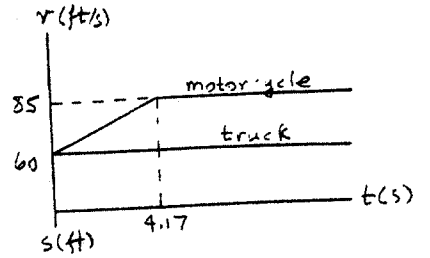
$$s = 0 + 60t$$

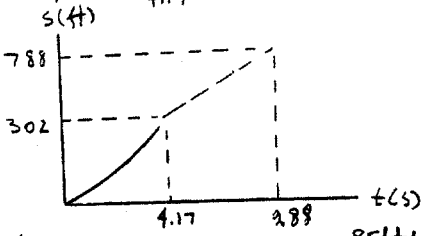
Thus $t = 5.717$ s

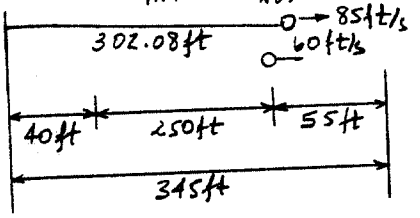
$t = 4.167 + 5.717 = 9.88$ s **Ans**

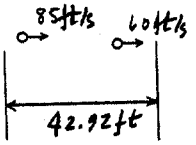
Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$









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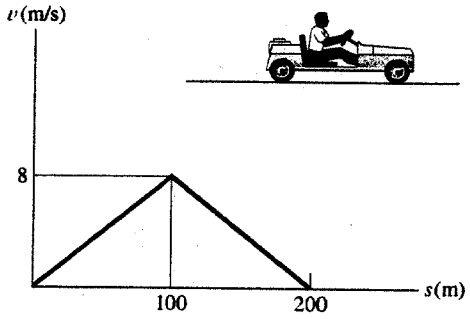
12-59. The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



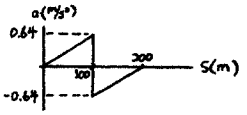
For $0 \le s < 100$

$v = 0.08\,s, \quad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32$ m/s^2 **Ans**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32$ m/s^2 **Ans**

Also,

$v\,dv = a\,ds$

$a = v\left(\frac{dv}{ds}\right)$

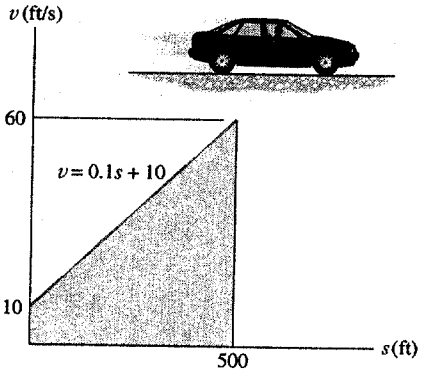
At $s = 50$ m,

$a = 4\left(\frac{8}{100}\right) = 0.32$ m/s^2 **Ans**

At $s = 150$ m,

$a = 4\left(\frac{-8}{100}\right) = -0.32$ m/s^2 **Ans**

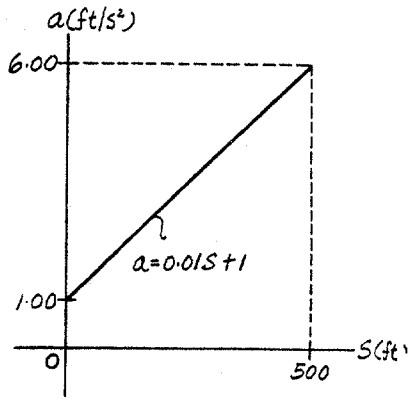
***12-60.** The v-s graph for the car is given for the first 500 ft of its motion. Construct the a-s graph for $0 \le s \le 500$ ft. How long does it take to travel the 500-ft distance? The car starts at $s = 0$ when $t = 0$.



***a*–*s* Graph :** The acceleration a in terms of s can be obtained by applying $v\,dv = a\,ds$.

$$a = v\frac{dv}{ds} = (0.1s + 10)(0.1) = (0.01s + 1)\ \text{ft/s}^2$$

At $s = 0$ and $s = 500$ ft, $a = 0.01(0) + 1 = 1.00$ ft/s^2 and $a = 0.01(500) + 1 = 6.00$ ft/s^2, respectively.



***Position* :** The position s in terms of time t can be obtained by applying $v = \frac{ds}{dt}$.

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.1s + 10}$$

$$t = 10\ln(0.01s + 1)$$

When $s = 500$ ft, $\quad t = 10\ln[0.01(500) + 1] = 17.9$ s **Ans**

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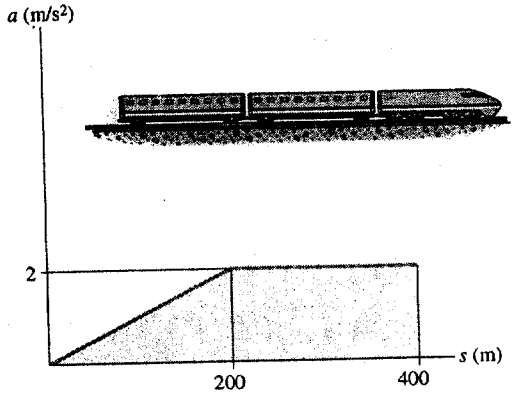
12-61. The a–s graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the v–s graph. $v = 0$ at $s = 0$.



$0 \le s \le 200$: $\quad a = \frac{1}{100}s$

$$a\,ds = v\,dv$$

$$\int_0^s \frac{1}{100}s\,ds = \int_0^v v\,dv$$

$$\frac{1}{200}s^2 = \frac{1}{2}v^2$$

$$v = 0.1s$$

At $s = 200$, $\quad v = 20$ m/s

$200 \le s \le 400$: $\quad a = 2$

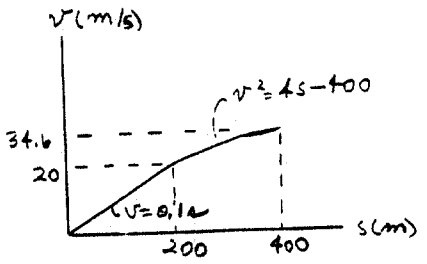
$$a\,ds = v\,dv$$

$$\int_{200}^s 2\,ds = \int_{20}^v v\,dv$$

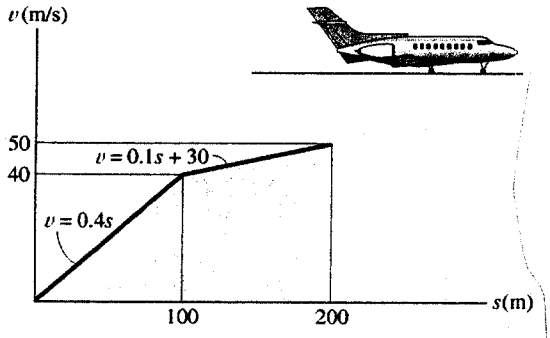
$$2(s - 200) = \frac{1}{2}(v^2 - 400)$$

$$v^2 = 4s - 400$$

At $s = 400$ m, $\quad v = \sqrt{4(400) - 400} = 34.6$ m/s



12-62. The v–s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100$ m and $s = 150$ m. Draw the a–s graph.



For $0 \le s < 100$ m

$$a\,ds = v\,dv$$

$$a\,ds = 0.4s\,(0.4\,ds)$$

$$a = 0.16s$$

At $s = 100$ m, $\quad a = 16$ m/s² **Ans**

For 100 m $< s \le 200$ m

$$a\,ds = v\,dv$$

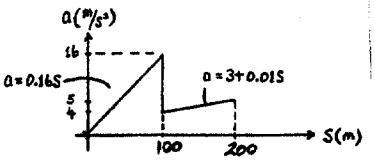
$$a\,ds = (30 + 0.1s)(0.1\,ds)$$

$$a = 3 + 0.01s$$

At $s = 150$ m, $\quad a = 4.5$ m/s² **Ans**

At $s = 100$ m, $\quad a = 4$ m/s²

At $s = 200$ m, $\quad a = 5$ m/s²



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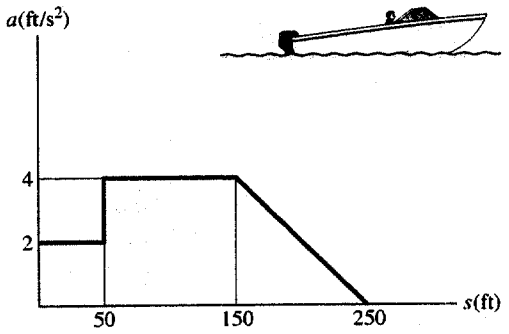
12-63. Starting from rest at $s = 0$, a boat travels in a straight line with an acceleration as shown by the a–s graph. Determine the boat's speed when $s = 40$, 90, and 200 ft.



Since $\int a\,ds = \int_0^v v\,dv$

$\int a\,ds = \frac{1}{2}v^2$

$v = \sqrt{2\int a\,ds}$

$\int a\,ds$ = area under $a-s$ graph.

For $s = 40$ ft

$v = \sqrt{2(2)(40)} = 12.7$ ft/s Ans

For $s = 90$ ft

$v = \sqrt{2[2(50) + 4(40)]} = 22.8$ ft/s Ans

For $s = 200$ ft

$v = \sqrt{2[2(50) + 4(100) + \frac{1}{2}(50)(4+2)]}$

$v = 36.1$ ft/s Ans

Also,

For $0 \le s < 50$ ft

$a = 2$, $v\,dv = a\,ds$

$\int_0^s 2\,ds = \int_0^v v\,dv$

$v = \sqrt{4s}$

When $s = 40$ ft, $v = \sqrt{4(40)} = 12.7$ ft/s Ans

When $s = 50$ ft, $v = \sqrt{4(50)} = 14.14$ ft/s

For 50 ft $< s < 150$ ft

$a = 4$,

$\int_{50}^s 4\,ds = \int_{14.14}^v v\,dv$

$v = \sqrt{8s - 200}$

When $s = 90$ ft, $v = \sqrt{8(90) - 200} = 22.8$ ft/s Ans

When $s = 150$ ft, $v = \sqrt{8(90) - 200} = 31.62$ ft/s

For 150 ft $< s < 250$ ft

$a = -\frac{4}{100}s + 10$

$\int_{150}^s (-\frac{1}{25}s + 10)\,ds = \int_{31.62}^v v\,dv$

$v = (-\frac{1}{25}s^2 + 20s - 1100)^{1/2}$

When $s = 200$ ft

$v = [-\frac{1}{25}(200)^2 + 20(200) - 1100]^{1/2}$

$v = 36.1$ ft/s Ans

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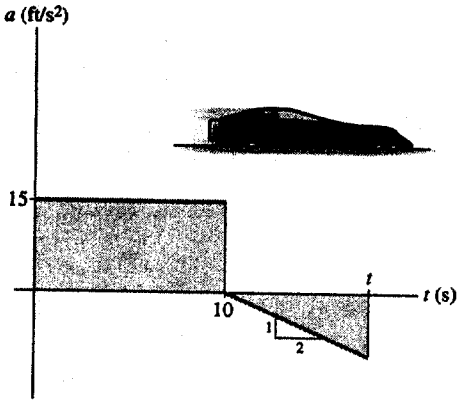
***12-64.** The test car starts from rest and is subjected to a constant acceleration of $a_c = 15\ \text{ft/s}^2$ for $0 \le t < 10\ \text{s}$. The brakes are then applied, which causes a deceleration at the rate shown until the car stops. Determine the car's maximum speed and the time t when it stops.



$v_{max} = A_1 = (15)(10) = 150\ \text{ft/s}$ **Ans**

From the graph, for $t > 10\ \text{s}$, $a = -\frac{1}{2}(t - 10)$

$$dv = a\,dt$$

$$\int_{150}^{v} dv = \int_{10}^{t} -\frac{1}{2}(t - 10)\,dt$$

$$v - 150 = -\frac{1}{2}\left[\frac{1}{2}t^2 - 10t\right]_{10}^{t}$$

$$v = 150 - \frac{1}{4}t^2 + 5t + \frac{1}{4}(10)^2 - \frac{1}{2}(10)^2$$

$$= -\frac{1}{4}t^2 + 5t + 125$$

When the car stops, $v = 0 = -\frac{1}{4}t^2 + 5t + 125$ (1)

Solving for the positive root,

$t = 34.5\ \text{s}$ **Ans**

Using the $a - t$ graph, we can obtain the same result by requiring

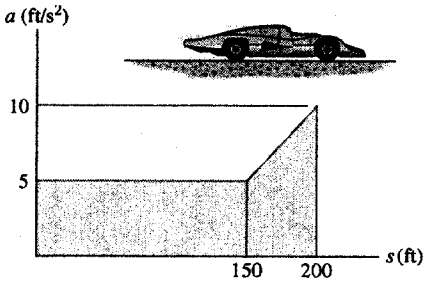
$$A_1 + A_2 = (15)(10) + \frac{1}{2}(a)(t - 10) = 0$$

$$150 + \frac{1}{2}\left[-\frac{1}{2}(t - 10)\right](t - 10) = 0$$

$$-\frac{1}{4}t^2 + 5t + 125 = 0$$

Which is the same as Eq. (1).

12-65. The a-s graph for a race car moving along a straight track has been experimentally determined. If the car starts from rest at $s = 0$, determine its speed when $s = 50$ ft. 150 ft, and 200 ft, respectively.



Velocity : The velocity v in terms of s can be obtained by applying $v\,dv = a\,ds$. For the interval $0\ \text{ft} \le s < 150\ \text{ft}$,

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$v = \left(\sqrt{10s}\right)\ \text{m/s}$$

At $s = 50$ ft, $v = \sqrt{10(50)} = 22.4\ \text{ft/s}$ **Ans**

At $s = 150$ ft, $v = \sqrt{10(150)} = 38.7\ \text{ft/s}$ **Ans**

For the interval $150\ \text{ft} < s \le 200\ \text{ft}$, $\frac{a - 5}{s - 150} = \frac{10 - 5}{200 - 150}$, $a = \left(\frac{1}{10}s - 10\right)\ \text{ft/s}^2$.

$$v\,dv = a\,ds$$

$$\int_{38.7\,\text{ft/s}}^{v} v\,dv = \int_{150\,\text{ft}}^{s} \left(\frac{1}{10}s - 10\right)ds$$

$$v = \left(\sqrt{\frac{1}{10}s^2 - 20s + 2250}\right)\ \text{ft/s}$$

At $s = 200$ ft, $v = \sqrt{\frac{1}{10}(200^2) - 20(200) + 2250} = 47.4\ \text{ft/s}$ **Ans**

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12-66. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12\,t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

Velocity : The velocity express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$d\mathbf{v} = \mathbf{a}\,dt$$

$$\int_0^{\mathbf{v}} d\mathbf{v} = \int_0^t \left(6t\mathbf{i} + 12t^2\mathbf{k}\right) dt$$

$$\mathbf{v} = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\ \text{ft/s}$$

Position : The position express in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_{\mathbf{r}_1}^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(3t^2\mathbf{i} + 4t^3\mathbf{k}\right) dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathbf{r} = \left\{\left(t^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(t^4 + 5\right)\mathbf{k}\right\}\ \text{ft}$$

When $t = 1$ s, $\mathbf{r} = \left(1^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(1^4 + 5\right)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.
The coordinates of the particle are

$$(4,\ 2,\ 6)\ \text{ft} \qquad \textbf{Ans}$$

12-67. The velocity of a particle is given by $v = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

Acceleration : The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}$ m/s^2. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans}$$

Position : The position express in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7,\ 16.0,\ 14.0)\ \text{ft} \qquad \textbf{Ans}$$

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■*12-68. A particle is traveling with a velocity of $\mathbf{v} = \{3\sqrt{t}e^{-0.2t}\mathbf{i} + 4e^{-0.8t^2}\mathbf{j}\}$ m/s, where t is in seconds. Determine the magnitude of the particle's displacement from $t = 0$ to $t = 3$ s. Use Simpson's rule with $n = 100$ to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = 2$ s?

$$ds = v\,dt$$

$$\Delta s_x = \int_0^3 3\sqrt{t}\,e^{-0.2t}\,dt = 7.341$$

$$\Delta s_y = \int_0^3 4\,e^{-0.8t^2}\,dt = 3.963$$

Thus,

$$\Delta s = \sqrt{(7.341)^2 + (3.963)^2} = 8.34 \text{ m} \quad \textbf{Ans}$$

$$a_x = \dot{v}_x = 3\left(\frac{1}{2}\right)t^{-\frac{1}{2}}e^{-0.2t} + 3\sqrt{t}\,e^{-0.2t}(-0.2)\Big|_{t=2} = 0.1422$$

$$a_y = \dot{v}_y = 4\,e^{-0.8t^2}(-0.8)(2t)\Big|_{t=2} = -0.5218$$

$$a = \sqrt{(0.1422)^2 + (-0.5218)^2} = 0.541 \text{ m/s}^2 \quad \textbf{Ans}$$

12-69. The position of a particle is defined by $r = \{5(\cos 2t)\,\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity : The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10\sin 2t\mathbf{i} + 8\cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10\sin 2(1)\mathbf{i} + 8\cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \quad \textbf{Ans}$$

Acceleration : The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20\cos 2t\mathbf{i} - 16\sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20\cos 2(1)\mathbf{i} - 16\sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \quad \textbf{Ans}$$

Travelling Path : Here, $x = 5\cos 2t$ and $y = 4\sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad [1]$$

$$\frac{y^2}{16} = \sin^2 2t \qquad [2]$$

Adding Eqs[1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \textbf{ (Equation of an Ellipse) } (\textit{Q.E.D.})$$

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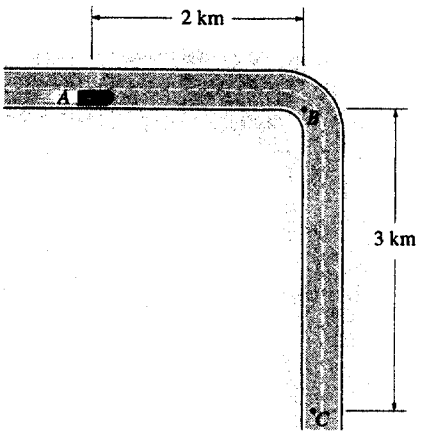
12-70. The car travels from A to B, and then from B to C, as shown in the figure. Determine the magnitude of the displacement of the car and the distance traveled.

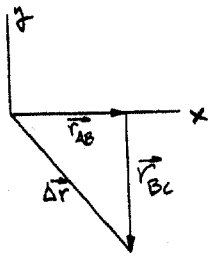


Displacement: $\Delta \mathbf{r} = \{2\mathbf{i} - 3\mathbf{j}\}$ km

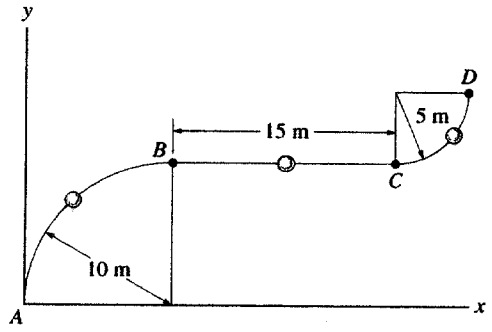
$\Delta r = \sqrt{2^2 + 3^2} = 3.61$ km **Ans**

Distance traveled:

$d = 2 + 3 = 5$ km **Ans**



12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2+4+3} = 4.28 \text{ m/s}$$ **Ans**

***12-72.** A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes. and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km}$$ **Ans**

and the magnitude of the displacement is

$$\Delta r = \sqrt{2^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km}$$ **Ans**

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23$ min $= 1380$ s. The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{3.606(10^3)}{1380} = 2.61 \text{ m/s}$$ **Ans**

and the average speed is

$$(v_{sp})_{avg} = \frac{s}{\Delta t} = \frac{9(10^3)}{1380} = 6.52 \text{ m/s}$$ **Ans**

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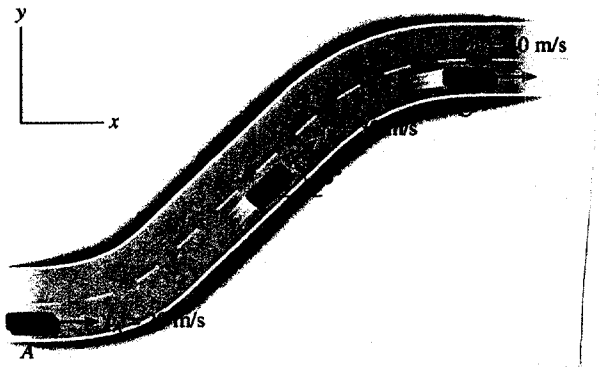
12-73. A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



$\mathbf{v}_A = 20\,\mathbf{i}$

$\mathbf{v}_B = 21.21\,\mathbf{i} + 21.21\,\mathbf{j}$

$\mathbf{v}_C = 40\,\mathbf{i}$

$$\mathbf{a}_{AB} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{21.21\,\mathbf{i} + 21.21\mathbf{j} - 20\,\mathbf{i}}{3}$$

$\mathbf{a}_{AB} = \{0.404\,\mathbf{i} + 7.07\,\mathbf{j}\}\ \text{m/s}^2$ **Ans**

$$\mathbf{a}_{AC} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$\mathbf{a}_{AC} = \{2.50\,\mathbf{i}\}\ \text{m/s}^2$ **Ans**

12-74. A particle moves along the curve $y = e^{2x}$ such that its velocity has a constant magnitude of $v = 4$ ft/s. Determine the x and y components of velocity when the particle is at $y = 5$ ft.

Velocity : Taking the first derivative of the path $y = e^{2x}$, we have

$$\dot{y} = 2e^{2x}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = 2e^{2x}v_x \qquad [2]$$

Here, $v = 4$ ft/s. Then

$$v^2 = v_x^2 + v_y^2$$

$$v_x^2 + v_y^2 = 16 \qquad [3]$$

Solving Eqs. [2] and [3] yields

$$v_x = 4\sqrt{\frac{1}{1+4e^{4x}}} \quad \text{and} \quad v_y = 8\sqrt{\frac{e^{4x}}{1+4e^{4x}}}$$

At $y = 5$ ft, $5 = e^{2x}$, $x = 0.8047$ ft. Thus,

$$v_x = 4\sqrt{\frac{1}{1+4e^{4(0.8047)}}} = 0.398 \text{ ft/s}$$ **Ans**

$$v_y = 8\sqrt{\frac{e^{4(0.8047)}}{1+4e^{4(0.8047)}}} = 3.98 \text{ ft/s}$$ **Ans**

12-75. The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration.

$y^2 = 4kx$

$2yv_y = 4kv_x$

$2v_y^2 + 2ya_y = 4ka_x$

$v_y = ct$

$a_y = c$ **Ans**

$2(ct)^2 + 2yc = 4ka_x$

$$a_x = \frac{c}{2k}\left(y + ct^2\right)$$ **Ans**

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***12-76.** A particle is moving along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

Velocity : Taking the first derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\dot{y} = \dot{x} - \frac{1}{400}(2x\dot{x})$$

$$\dot{y} = \dot{x} - \frac{x}{200}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq.[1] becomes

$$v_y = v_x - \frac{x}{200}v_x \qquad [2]$$

Here, $v_x = 2$ ft/s at $x = 20$ ft. Then, From Eq.[2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \qquad \text{Ans}$$

Acceleration : Taking the second derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}(\dot{x}^2 + x\ddot{x}) \qquad [3]$$

However, $\ddot{x} = a_x$ and $\ddot{x} = a_y$. Thus, Eq.[3] becomes

$$a_y = a_x - \frac{1}{200}(v_x^2 + xa_x) \qquad [4]$$

Since $v_x = 2$ ft/s is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq.[4]

$$a_y = 0 - \frac{1}{200}[2^2 + 20(0)] = -0.020 \text{ ft/s}$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \qquad \text{Ans}$$

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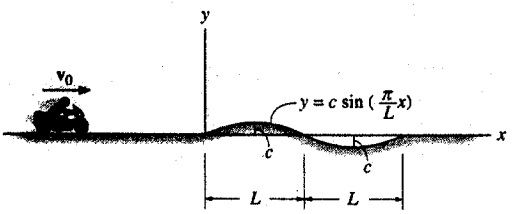
12-77. The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



$$y = c\sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \quad \textbf{Ans}$$

$$v_y = \frac{v_0\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \quad \textbf{Ans}$$

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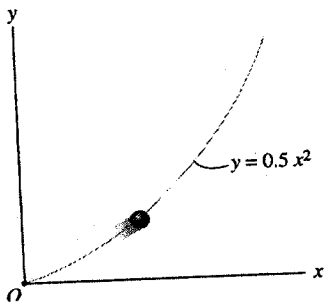
12-78. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0, y = 0$.



Position : The x position of the particle can be obtained by applying the $v_x = \frac{dx}{dt}$.

$$dx = v_x\,dt$$
$$\int_0^x dx = \int_0^t 5t\,dt$$
$$x = \left(2.50t^2\right)\ \text{ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50-0)^2 + (3.125-0)^2} = 4.00\ \text{ft}$$ **Ans**

Acceleration : Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x}$$ [1]

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. [1] becomes

$$a_y = v_x^2 + xa_x$$ [2]

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s, $a_x = \frac{dv_x}{dt} = 5$ ft/s and $x = 2.50$ ft. Then, from Eq. [2]

$$a_y = 5^2 + 2.50(5) = 37.5\ \text{ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\ \text{ft/s}^2$$ **Ans**

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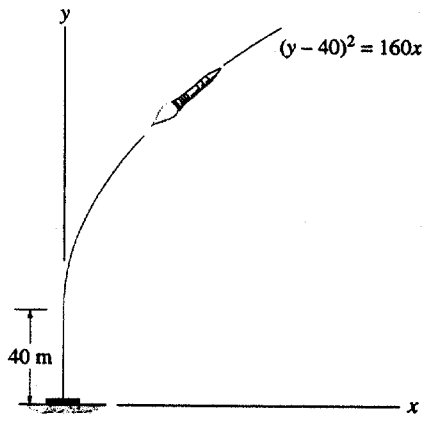
12-79. When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



$v_y = 180$ m/s

$(y - 40)^2 = 160x$

$2(y-40)v_y = 160v_x$ (1)

$2(80-40)(180) = 160v_x$

$v_x = 90$ m/s

$v = \sqrt{90^2 + 180^2} = 201$ m/s **Ans**

$a_y = \frac{dv_y}{dt} = 0$

From Eq. 1,

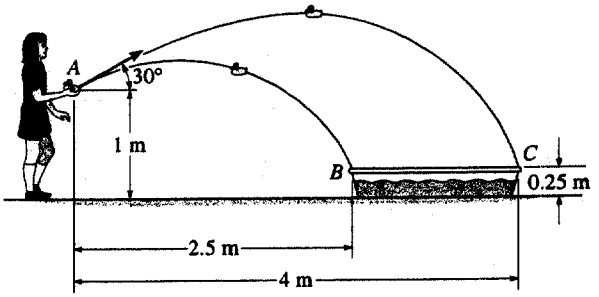
$2v_y^2 + 2(y-40)a_y = 160a_x$

$2(180)^2 + 0 = 160a_x$

$a_x = 405$ m/s^2

$a = 405$ m/s^2 **Ans**

***12-80.** The girl always throws the toys at an angle of 30° from point A as shown. Determine the time between throws so that both toys strike the edges of the pool B and C at the same instant. With what speed must she throw each toy?



To strike B:

$(\overset{+}{\rightarrow}) s = s_0 + v_0 t$

$2.5 = 0 + v_A \cos 30° t$

$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.6687$ s

$(v_A)_B = 4.32$ m/s **Ans**

To strike C:

$(\overset{+}{\rightarrow}) s = s_0 + v_0 t$

$4 = 0 + v_A \cos 30° t$

$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.790$ s

$(v_A)_C = 5.85$ m/s **Ans**

Time between throws:

$\Delta t = 0.790$ s $- 0.6687$ s $= 0.121$ s **Ans**

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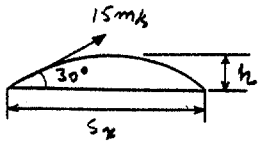
12-81. The nozzle of a garden hose discharges water at the rate of 15 m/s. If the nozzle is held at ground level and directed $\theta = 30°$ from the ground, determine the maximum height reached by the water and the horizontal distance from the nozzle to where the water strikes the ground.



$(v_0)_x = 15\cos 30° = 12.99$ m/s

$(v_0)_y = 15\sin 30° = 7.5$ m/s

Maximum height :

$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (7.5)^2 + 2(-9.81)(h - 0)$

$h = 2.87$ m **Ans**

Time of travel to top of path :

$(+\uparrow)$ $v = v_0 + a_c t$

$0 = 7.5 + (-9.81)t$

$t = 0.7645$ s

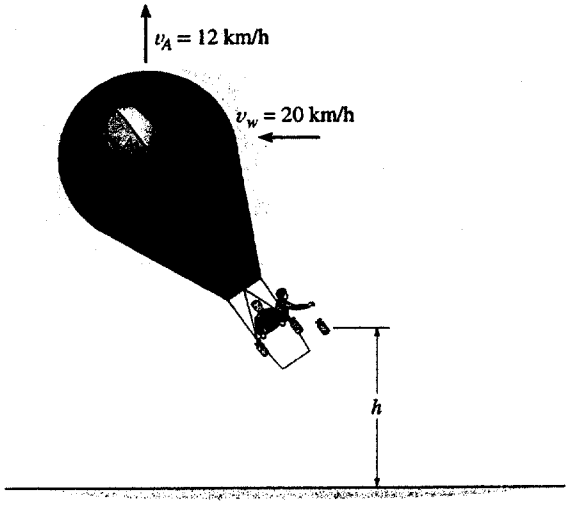
Total time along path

$t = 2(0.7645) = 1.529$ s

Range

$s_x = v_x t = (12.99)(1.529) = 19.9$ m **Ans**

12-82. The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?



$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (3.33)^2 + 2(-9.81)(-50 - 0)$

$v_y = 31.50$ m/s

$(+\uparrow)$ $v = v_0 + a_c t$

$-31.50 = 3.33 - 9.81t$

$t = 3.55$ s **Ans**

$v_x = 20$ km/h $= 5.556$ m/s

$v = \sqrt{(31.50)^2 + (5.556)^2} = 32.0$ m/s **Ans**

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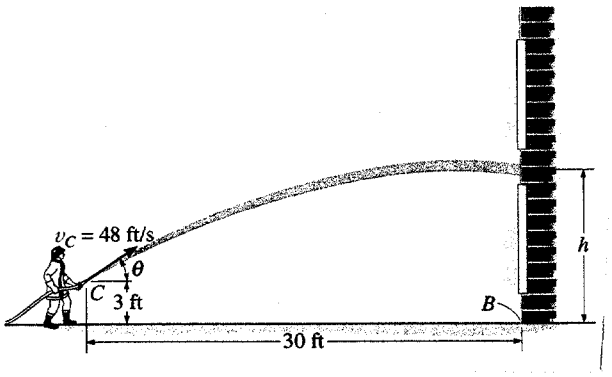
12-83. Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is $v_C = 48$ ft/s.



$(+\uparrow)\ v = v_0 + a_c\, t$

$0 = 48 \sin\theta - 32.2\, t$

$(\overset{+}{\rightarrow})\ s = s_0 + v_0\, t$

$30 = 0 + 48\,(\cos\theta)(t)$

$48 \sin\theta = 32.2 \dfrac{30}{48 \cos\theta}$

$\sin\theta\cos\theta = 0.41927$

$\sin 2\theta = 0.83854$

$\theta = 28.5°$

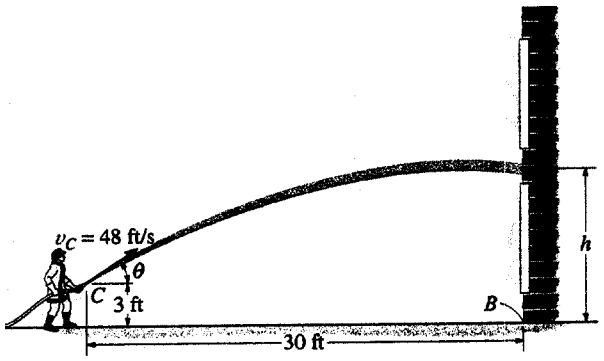
$t = 0.7111$ s

$(+\uparrow)\ s = s_0 + v_0\, t + \dfrac{1}{2} a_c\, t^2$

$h - 3 = 0 + 48 \sin 28.5°\,(0.7111) + \dfrac{1}{2}(-32.2)(0.7111)^2$

$h = 11.1$ ft **Ans**

■***12-84.** Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is $v_C = 48$ ft/s.



$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$30 = 0 + 48 \cos\theta\, t$

$t = \dfrac{30}{48\cos\theta}$

$(+\uparrow)\ s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$

$0 = 3 + 48\sin\theta\, t + \dfrac{1}{2}(-32.2)t^2$

$0 = 3 + \dfrac{48 \sin\theta\,(30)}{48\cos\theta} - 16.1\left(\dfrac{30}{48\cos\theta}\right)^2$

$0 = 3\cos^2\theta + 30\sin\theta\cos\theta - 6.2891$

$3\cos^2\theta + 15 \sin 2\theta = 6.2891$

Solving

$\theta = 6.41°$ **Ans**

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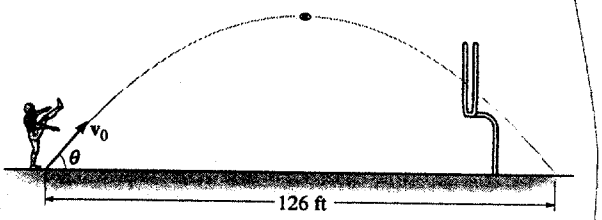
12-85. From a videotape, it was observed that a pro football player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



$$\left(\overset{+}{\rightarrow}\right) \quad s = s_0 + v_0 t$$

$$126 = 0 + (v_0)_x (3.6)$$

$$(v_0)_x = 35 \text{ ft/s}$$

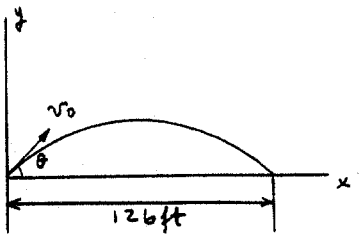
$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

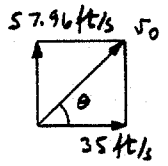
$$0 = 0 + (v_0)_y (3.6) + \frac{1}{2}(-32.2)(3.6)^2$$

$$(v_0)_y = 57.96 \text{ ft/s}$$

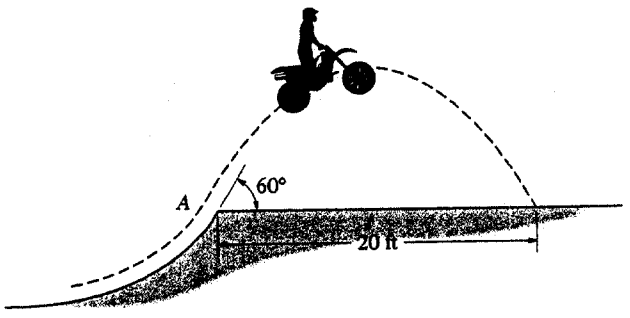
$$v_0 = \sqrt{(35)^2 + (57.96)^2} = 67.7 \text{ ft/s} \quad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{57.96}{35}\right) = 58.9° \quad \textbf{Ans}$$





12-86. During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 20 ft away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$20 = 0 + v_A \cos 60° t$$

$$(+\uparrow) \quad s = s_0 + v_0 + \frac{1}{2} a_c t^2$$

$$0 = 0 + v_A \sin 60° t + \frac{1}{2}(-32.2) t^2$$

Solving

$$t = 1.4668 \text{ s}$$

$$v_A = 27.3 \text{ ft/s} \quad \textbf{Ans}$$

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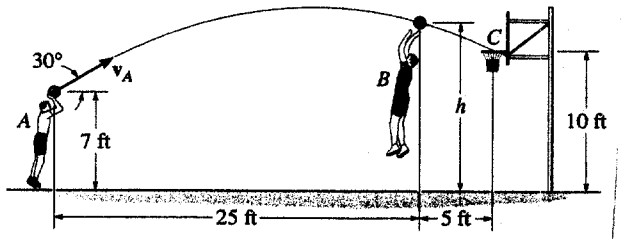
12-87. Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$30 = 0 + v_A \cos 30° \, t_{AC}$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$10 = 7 + v_A \sin 30° \, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$25 = 0 + 36.73 \cos 30° \, t_{AB}$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

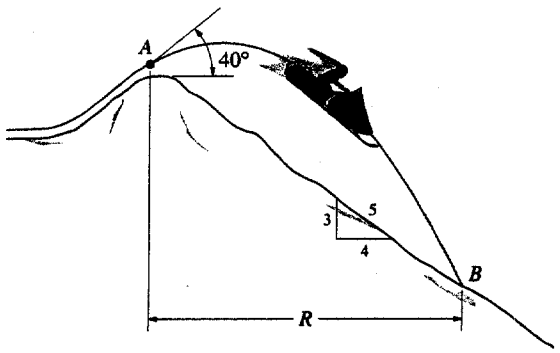
$h = 7 + 36.73 \sin 30° \, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$

Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans**

***12-88.** The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



$(\overset{+}{\rightarrow})$ $s_B = s_A + v_A t$

$R = 0 + 10 \cos 40° \, t$

$(+\uparrow)$ $s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

$-R(\frac{3}{4}) = 0 + 10 \sin 40° \, t - \frac{1}{2}(9.81) t^2$

Solving:

$R = 19.0$ m **Ans**

$t = 2.48$ s **Ans**

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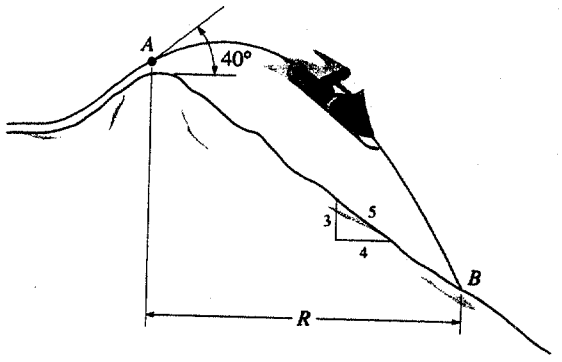
12-89. The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the speed at which it strikes the ground at B and its maximum acceleration along the trajectory AB.



At all times $a = g = -9.81\ \text{m/s}^2$ **Ans**

From Prob. 12–98:

$R = 19.0\ \text{m}$

$t = 2.48\ \text{s}$

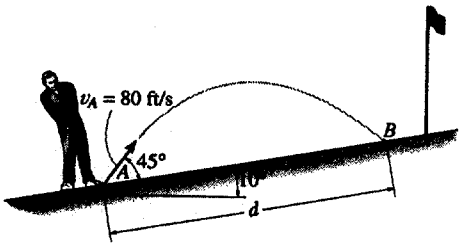
$(v_B)_x = (v_A)_x = 10\cos 40° = 7.6604\ \text{m/s}$

$(+\uparrow)$ $(v_B)_y = (v_A)_y + a_c\, t$

$(v_B)_y = 10\sin 40° - 9.81\,(2.48) = -17.901\ \text{m/s}$

$v_B = \sqrt{(7.6604)^2 + (-17.901)^2} = 19.5\ \text{m/s}$ **Ans**

12-90. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



Horizontal Motion : The horizontal component of velocity is $(v_0)_x = 80\cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d\cos 10°$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$d\cos 10° = 0 + 45.89t \qquad [1]$$

Vertical Motion : The vertical component of initial velocity is $(v_0)_y = 80\sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d\sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$d\sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)\, t^2 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$d = 166\ \text{ft} \qquad \textbf{Ans}$$
$$t = 3.568\ \text{s}$$

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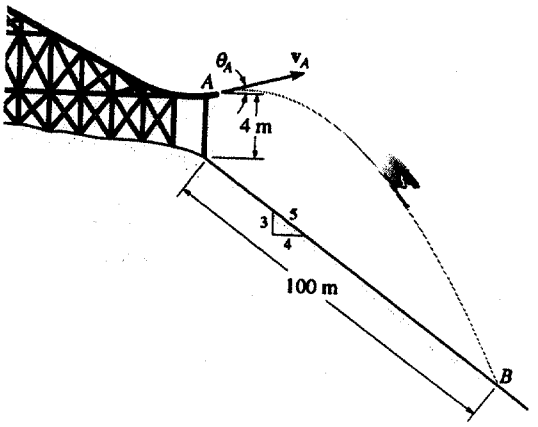
12-91. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



$$\left(\overset{+}{\rightarrow}\right) \quad s = v_0 t$$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

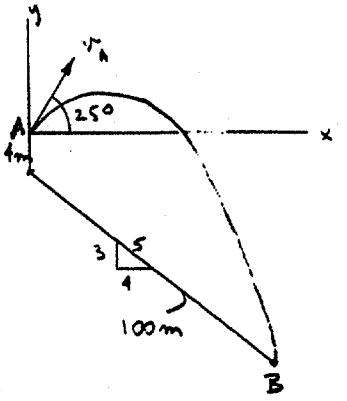
$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81) t_{AB}^2$$

Solving,

$v_A = 19.4$ m/s **Ans**

$t_{AB} = 4.54$ s **Ans**



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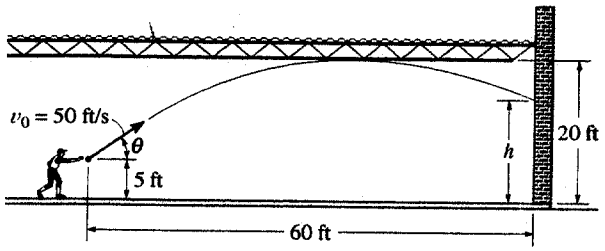
***12-92.** The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50$ ft/s. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



$v_x = 50\cos\theta$

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$x = 0 + 50\cos\theta\ t$ (1)

$(+\uparrow)\ v = v_0 + a_c t$

$v_y = 50\sin\theta - 32.2\,t$ (2)

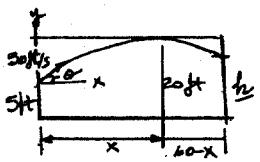
$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$y = 0 + 50\sin\theta\ t - 16.1\,t^2$ (3)

$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (50\sin\theta)^2 + 2(-32.2)(s - 0)$

$v_y^2 = 2500\sin^2\theta - 64.4\,s$ (4)



Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500\sin^2\theta - 64.4(15)$

$\theta = 38.433° = 38.4°$ Ans

From Eq. (2)

$0 = 50\sin 38.433° - 32.2\,t$

$t = 0.9652$ s

From Eq.(1)

$x = 50\cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

From Eq. (3)

$y = 50\sin 38.433°(1.53193) - 16.1(1.53193)^2$

$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft Ans

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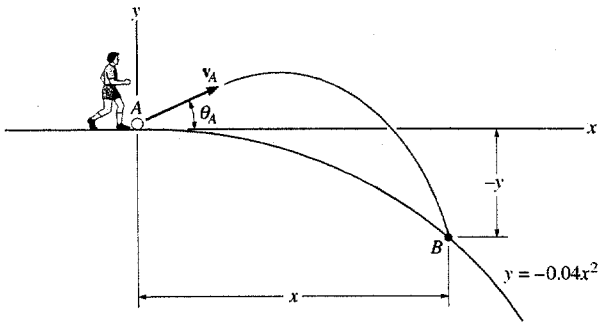
12-93. The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



$(v_A)_x = 80 \cos 30° = 69.28$ ft/s

$(v_A)_y = 80 \sin 30° = 40$ ft/s

$(\stackrel{+}{\rightarrow}) \; s = s_0 + v_0 t$

$$x = 0 + 69.28\,t \qquad (1)$$

$(+\uparrow) \; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-y = 0 + 40\,t + \frac{1}{2}(-32.2)t^2 \qquad (2)$$

$y = -0.04\,x^2$

From Eqs. (1) and (2):

$-y = 0.5774\,x - 0.003354\,x^2$

$0.04\,x^2 = 0.5774x - 0.003354\,x^2$

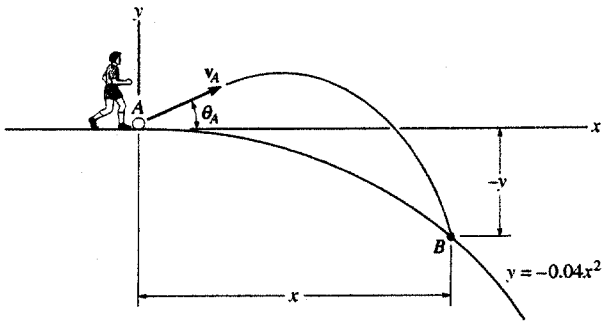
$0.04335x^2 = 0.5774x$

$x = 13.3$ ft **Ans**

Thus

$y = -0.04(13.3)^2 = -7.09$ ft **Ans**

12-94. The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



$(\stackrel{+}{\rightarrow}) \; s = s_0 + v_0 t$

$15 = 0 + v_A \cos 30°\, t$

$(+\uparrow) \; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans**

$t = 1.047$ s

$(\stackrel{+}{\rightarrow}) \; (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow) \; v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans**

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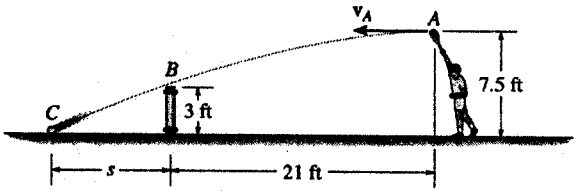
12-95. Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



Vertical Motion : The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion : The horizontal component of velocity is $(v_0)_x = v_A$. For the ball travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans}$$

For the ball travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 + s = 0 + 39.72(0.6825)$$

$$s = 6.11 \text{ ft} \qquad \textbf{Ans}$$

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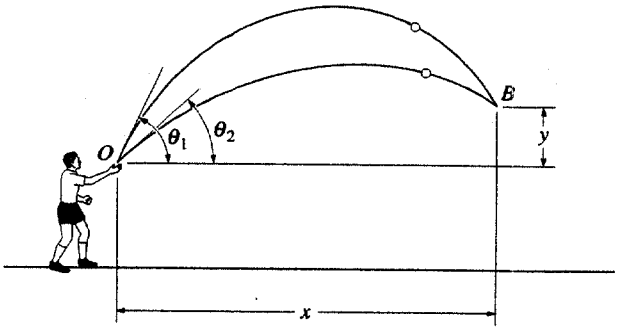
***12-96.** A boy at O throws a ball in the air with a speed v_O at an angle θ_1. If he then throws another ball at the same speed v_O at an angle $\theta_2 < \theta_1$, determine the time between the throws so the balls collide in mid air at B.



Time of flight

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$$x_1 = 0 + v_0 \cos\theta_1\ t_1 \qquad (1)$$

$$x_2 = 0 + v_0 \cos\theta_2\ t_2 \qquad (2)$$

Thus,
$x_1 = x_2$

$$\Delta t = t_1 - t_2 = \frac{x_1}{v_0}\left(\frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_1 \cos\theta_2}\right) \qquad (3)$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$y = 0 + (v_0 \sin\theta_1)(t_1) - \frac{1}{2} g\, t_1^2 \qquad (4)$$

Use Eq. (1)

$$y = x_1 \tan\theta_1 - \frac{1}{2} g \frac{x_1^2}{v_0^2 \cos^2\theta_1}$$

In the same way :

$$y = x_2 \tan\theta_2 - \frac{1}{2} g \frac{x_2^2}{v_0^2 \cos^2\theta_2}$$

Equating and solving for $x_1 = x_2 = x$

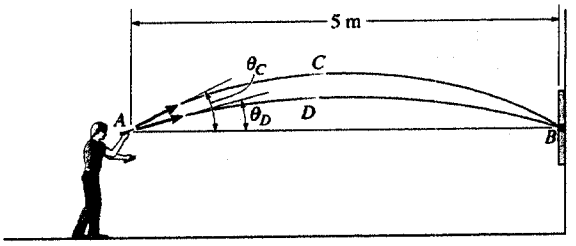
$$x = \frac{2v_0^2}{g}\left[\frac{(\cos^2\theta_1 \cos^2\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos^2\theta_2 - \cos^2\theta_1)}\right]$$

Substituting into Eq. (3) yields

$$\Delta t = \frac{2v_0}{g}\left[\frac{(\cos\theta_1 \cos\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos\theta_2 + \cos\theta_1)}\right]$$

$$\Delta t = \frac{2v_0}{g}\left[\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2 + \cos\theta_1}\right] \qquad \text{Ans}$$

12-97. The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at $\theta_C (> \theta_D)$, then the second dart is thrown at θ_D.



$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$$5 = 0 + (10 \cos\theta)\, t \qquad (1)$$

$(+\uparrow)\ v = v_0 + a_c t$

$$-10 \sin\theta = 10 \sin\theta - 9.81\, t$$

$$t = \frac{2(10 \sin\theta)}{9.81} = 2.039 \sin\theta$$

From Eq. (1),

$$5 = 20.39 \sin\theta \cos\theta$$

Since $\sin 2\theta = 2 \sin\theta \cos\theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7°$ Ans

$\theta_C = 75.3°$ Ans

From Eq.(1) : $t_D = 0.517$ s

$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s Ans

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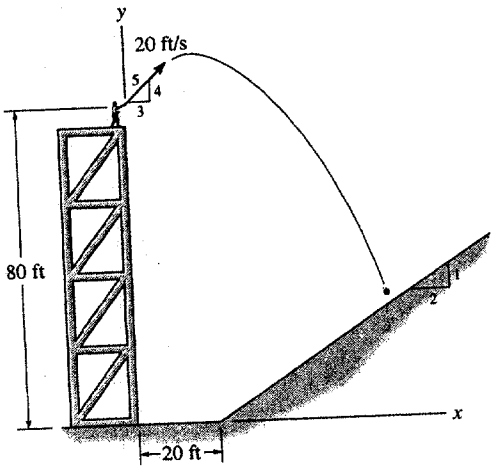
12-98. The ball is thrown from the tower with a velocity of 20 ft/s as shown. Determine the x and y coordinates to where the ball strikes the slope. Also, determine the speed at which the ball hits the ground.



Assume ball hits slope.

$\left(\stackrel{+}{\rightarrow}\right)$ $s = s_0 + v_0 t$

$$x = 0 + \frac{3}{5}(20)t = 12t$$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 80 + \frac{4}{5}(20)t + \frac{1}{2}(-32.2)t^2 = 80 + 16t - 16.1t^2$$

Equation of slope: $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 20)$$

$$y = 0.5x - 10$$

Thus,

$$80 + 16t - 16.1t^2 = 0.5(12t) - 10$$

$$16.1t^2 - 10t - 90 = 0$$

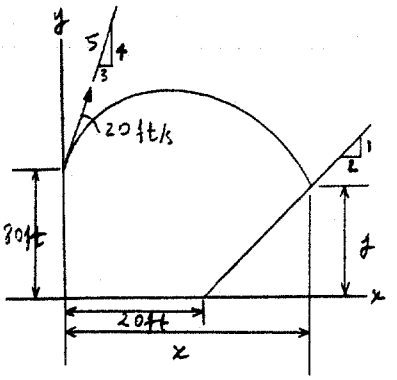
Choosing the positive root:

$t = 2.6952$ s

$x = 12(2.6952) = 32.3$ ft **Ans**

Since 32.3 ft > 20 ft, assumption is valid.

$y = 80 + 16(2.6952) - 16.1(2.6952)^2 = 6.17$ ft **Ans**



$\left(\stackrel{+}{\rightarrow}\right)$ $v_x = (v_0)_x = \frac{3}{5}(20) = 12$ ft/s

$(+\uparrow)$ $v_y = (v_0)_y + a_c t = \frac{4}{5}(20) + (-32.2)(2.6952) = -70.785$ ft/s

$v = \sqrt{(12)^2 + (-70.785)^2} = 71.8$ ft/s **Ans**

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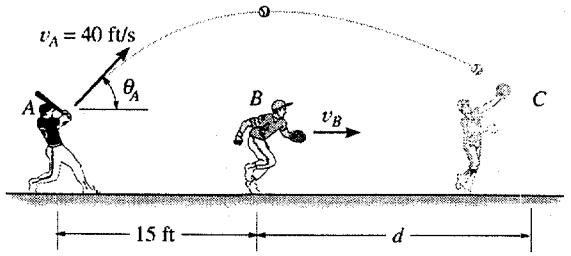
12-99. The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

$$d = R - 15 = 43.03 - 15 = 28.0 \text{ ft} \quad \textbf{Ans}$$

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 40 \cos 60° = 20.0 \text{ ft/s} \quad \textbf{Ans}$$

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***12-100.** A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

$v = 16$ m/s

$a_t = 8$ m/s^2

$r = 50$ m

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2$$ Ans

12-101. A car moves along a circular track of radius 250 ft such that its speed for a short period of time $0 \le t \le 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

$v = 3(t + t^2)$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 3$ s, $a_t = 3 + 6(3) = 21$ ft/s^2

$$a_n = \frac{[3(3+3^2)]^2}{250} = 5.18 \text{ ft/s}^2$$

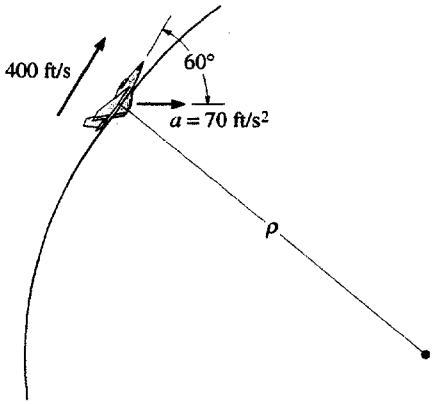
$$a = \sqrt{(21)^2 + (5.18)^2} = 21.6 \text{ ft/s}^2$$ Ans

$$\int ds = \int_0^3 3(t + t^2)\, dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$$

$\Delta s = 40.5$ ft Ans

12-102. At a given instant the jet plane has a speed of 400 ft/s and an acceleration of 70 ft/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed and the radius of curvature ρ of the path.



$$a_t = 70\cos 60° = 35.0 \text{ ft/s}^2$$ Ans

$$a_n = \frac{(400)^2}{\rho} = 70\sin 60°$$

$\rho = 2.64(10^3)$ ft Ans

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12-103. A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2 \qquad \textbf{Ans}$$

***12-104.** A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

$$a_t = 2 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2 \qquad \textbf{Ans}$$

■**12-105.** Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)\,dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$$t = 4.147 \text{ s}$$

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s} \qquad \textbf{Ans}$$

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2 \qquad \textbf{Ans}$$

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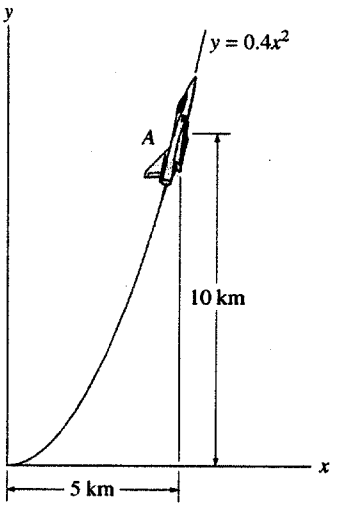
12-106. The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s^2. Determine the magnitude of acceleration of the plane when it is at point A.



$y = 0.4x^2$

$\frac{dy}{dx} = 0.8x\Big|_{x=5\text{ km}} = 4$

$\frac{d^2y}{dx^2} = 0.8$

$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62\text{ km}$

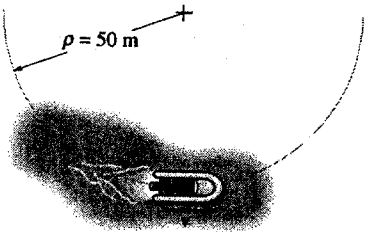
$a_t = 0.8\text{ m/s}^2$

$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\text{ km/s}^2$

$a_n = 0.457\text{ m/s}^2$

$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\text{ m/s}^2$ **Ans**

12-107. Starting from rest, motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.8t)$ m/s, where t is in seconds. Determine the magnitudes of the boats velocity and acceleration when it has traveled 20 m.



Velocity : The time for which the boat to travel 20 m must be determined first.

$$ds = v\,dt$$
$$\int_0^{20\text{m}} ds = \int_0^t 0.8t\,dt$$
$$t = 7.071\text{ s}$$

The magnitude of the boat's velocity is

$$v = 0.8(7.071) = 5.657\text{ m/s} = 5.66\text{ m/s}$$ **Ans**

Acceleration : The tangential acceleration is

$$a_t = \dot{v} = 0.8\text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{5.657^2}{50} = 0.640\text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.8^2 + 0.640^2} = 1.02\text{ m/s}^2$$ **Ans**

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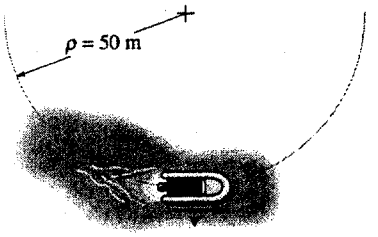
***12-108.** Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boats velocity and acceleration at the instant $t = 3$ s.



Velocity : When $t = 3$ s, the boat travels at a speed of

$$v = 0.2\left(3^2\right) = 1.80 \text{ m/s} \qquad \textbf{Ans}$$

Acceleration : The tangential acceleration is $a_t = \dot{v} = (0.4t)$ m/s^2. When $t = 3$ s,

$$a_t = 0.4(3) = 1.20 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{1.80^2}{50} = 0.0648 \text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.20^2 + 0.06048^2} = 1.20 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-109. A car moves along a circular track of radius 250 ft, and its speed for a short period of time $0 \le t \le 2$ s is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s?

$$v = 3\left(t + t^2\right)$$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 2$ s,

$$a_t = 3 + 6(2) = 15 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{\left[3(2 + 2^2)\right]^2}{250} = 1.296 \text{ ft/s}^2$$

$$a = \sqrt{(15)^2 + (1.296)^2} = 15.1 \text{ ft/s}^2 \qquad \textbf{Ans}$$

$$ds = v\,dt$$

$$\int ds = \int_0^2 3\left(t + t^2\right) dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3 \Big]_0^2$$

$$\Delta s = 14 \text{ ft} \qquad \textbf{Ans}$$

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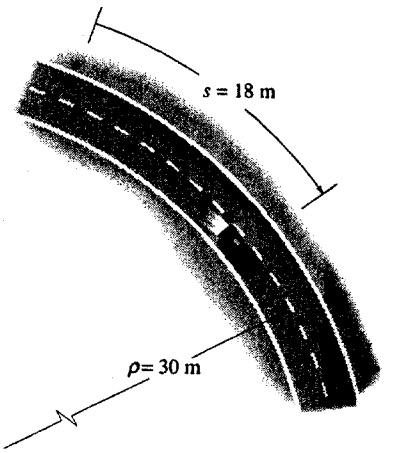
•12-110. The car travels along the curved path such that its speed is increased by $\dot{v} = (0.5e^t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



$$\int_0^v dv = \int_0^t 0.5e^t dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving, $t = 3.7064$ s

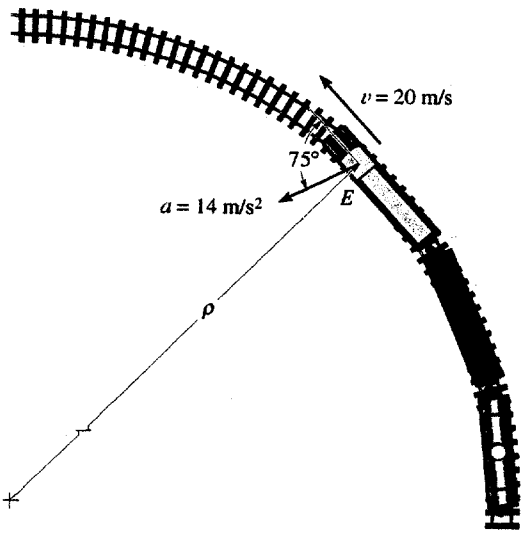
$$v = 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s}$$ **Ans**

$$a_t = \dot{v} = 0.5e^t|_{t=3.7064\text{ s}} = 20.35 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2$$ **Ans**

12-111. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



$$a_t = 14\cos 75° = 3.62 \text{ m/s}^2$$ **Ans**

$$a_n = 14\sin 75°$$

$$a_n = \frac{(20)^2}{\rho}$$

$$\rho = 29.6 \text{ m}$$ **Ans**

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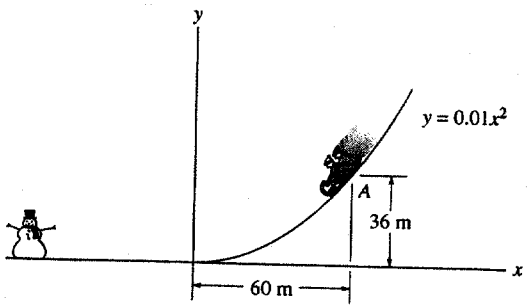
***12-112.** A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $\dot{v}_A = 3$ m/s^2.



Acceleration : The radius of curvature of the path at point A must be determined first. Here, $\frac{dy}{dx} = 0.02x$ and $\frac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1+(0.02x)^2]^{3/2}}{|0.02|}\right|_{x=60\text{m}} = 190.57 \text{ m}$$

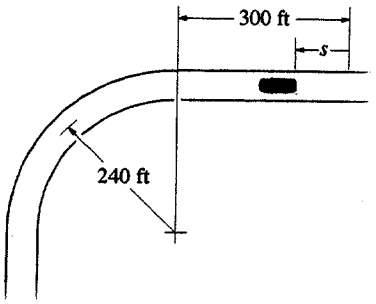
To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247 \text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-113. The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



$a_t = 0.05t^2$

$\int_0^v dv = \int_0^t 0.05t^2\, dt$

$v = 0.0167t^3$

$\int_0^s ds = \int_0^t 0.0167t^3\, dt$

$s = 4.167(10^{-3})t^4$

When $t = 18$ s, $s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18)^3 = 97.2$ ft/s **Ans**

$a_n = \frac{(97.2)^2}{240} = 39.37$ ft/s^2

$a_t = 0.05(18)^2 = 16.2$ ft/s^2

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6$ ft/s^2 **Ans**

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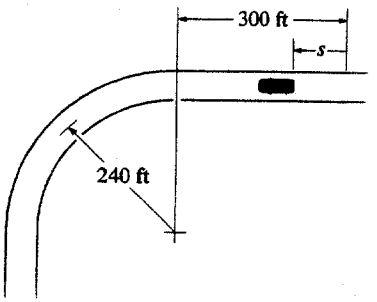
12-114. The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



The car is on the curved path.

$a_t = 0.05\,t^2$

$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$

$v = 0.0167\,t^3$

$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$

$s = 4.167(10^{-3})\,t^4$

$550 = 4.167(10^{-3})\,t^4$

$t = 19.06$ s

So that

$v = 0.0167\,(19.06)^3 = 115.4$

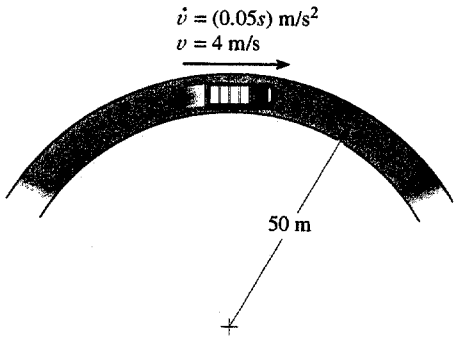
$v = 115$ ft/s **Ans**

$a_n = \frac{(115.4)^2}{240} = 55.48$ ft/s^2

$a_t = 0.05(19.06)^2 = 18.16$ ft/s^2

$a = \sqrt{(55.48)^2 + (18.16)^2} = 58.4$ ft/s^2 **Ans**

12-115. The truck travels in a circular path havng a radius of 50 m at a speed of 4 = m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



$v\,dv = a_t\,ds$

$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$

$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$

$v = 4.583 = 4.58$ m/s **Ans**

$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420$ m/s^2

$a_t = 0.05(10) = 0.5$ m/s^2

$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653$ m/s^2 **Ans**

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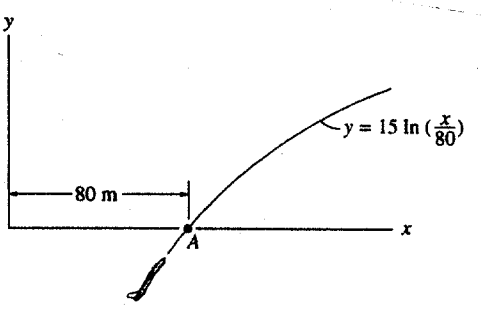
***12-116.** The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point A ($y = 0$).



$$y = 15\ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x=80\text{ m}} = -0.002344$$

$$\rho\Big|_{x=80\text{ m}} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=80\text{ m}}$$

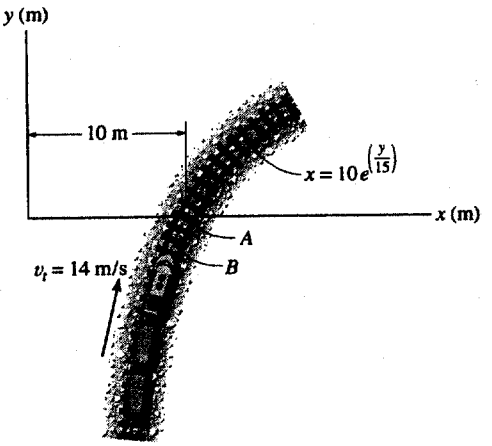
$$= \frac{\left[1+(0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 26.9\text{ m/s}^2 \qquad \textbf{Ans}$$

12-117. A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A ($y = 0$).



$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15\ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

At $x = 10$,

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06\text{ m}$$

$$a_t = \frac{dv}{dt} = 0$$

$$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02\text{ m/s}^2 \qquad \textbf{Ans}$$

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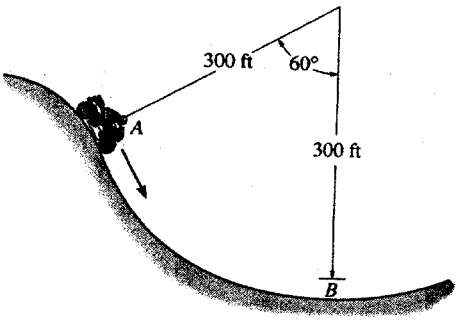
12-118. When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)$ ft/s^2, where t is in seconds. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



$\int_0^v dv = \int_0^t 0.3t\,dt$

$v = 0.15t^2$

$\int_0^s ds = \int_0^t 0.15t^2\,dt$

$s = 0.05t^3$

When $s = \frac{\pi}{3}(300)$ ft, $\frac{\pi}{3}(300) = 0.05t^3$ $t = 18.453$ s

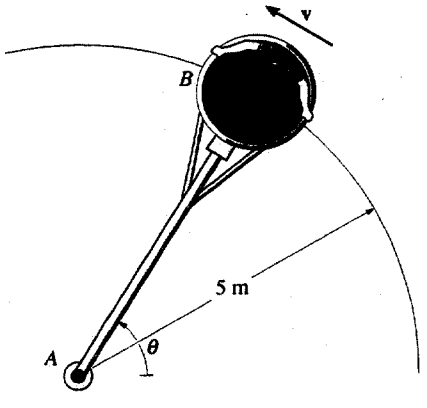
$v = 0.15(18.453)^2 = 51.08$ ft/s $= 51.1$ ft/s **Ans**

$a_t = \dot{v} = 0.3t|_{t=18.453\,s} = 5.536$ ft/s^2

$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696$ ft/s^2

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3$ ft/s^2 **Ans**

■12-119. The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)$ m/s^2, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



$\frac{dv_B}{dt} = 0.5e^t$

$\int_0^{v_B} dv_B = \int_0^t 0.5e^t\,dt$

$v_B = 0.5(e^t - 1)$

$\int_0^{s_B} ds_B = \int_0^t 0.5(e^t - 1)\,dt$

$s_B = 0.5(e^t - t)|_0^t = 0.5(e^t - t - 1)$

At $\theta = 30°$,

$s_B = (\frac{30°}{180°}\pi)(5) = 2.618$ m

Thus,

$6.236 = (e^t - t)$

Solving by trial and error,

$t = 2.123$ s

Thus,

$v_B = 0.5(e^{2.123} - 1) = 3.678 = 3.68$ m/s **Ans**

$(a_B)_t = \dot{v}_B = 0.5(e^{2.123}) = 4.178$ m/s^2

$(a_B)_n = \frac{v_B^2}{5} = \frac{(3.678)^2}{5} = 2.706$ m/s^2

$a_B = \sqrt{(4.178)^2 + (2.706)^2} = 4.98$ m/s^2 **Ans**

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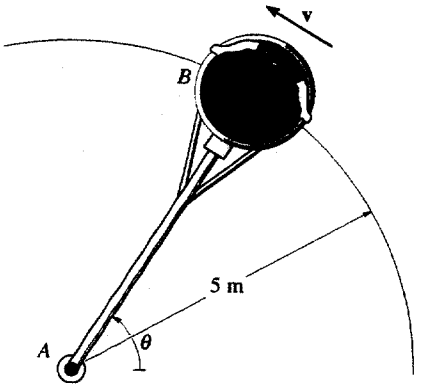
***12-120.** The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)$ m/s^2, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car. Also, through what angle θ has it traveled?



$$\int_0^v dv = \int_0^t 0.5\,e^t\,dt$$

$$v = 0.5\,e^t\Big|_0^t = 0.5\,(e^t - 1)$$

$t = 2$ s,

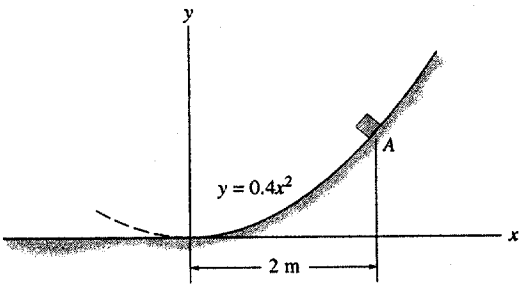
$$v = 0.5(e^2 - 1) = 3.1945 = 3.19 \text{ m/s} \qquad \textbf{Ans}$$

$$a_t = 0.5\,e^2 = 3.6945 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(3.1945)^2}{5} = 2.041 \text{ m/s}^2$$

$$a = \sqrt{(3.6945)^2 + (2.041)^2} = 4.22 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-121. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v_B = 8$ m/s and the increase in speed is $dv_B/dt = 4$ m/s^2. Determine the magnitude of the acceleration of the box at this instant.



$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = \left.0.8x\right|_{x=2\text{ m}} = 1.6$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396 \text{ m}$$

$$a_n = \frac{v_B^{\,2}}{\rho} = \frac{8^2}{8.396} = 7.622 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61 \text{ m/s}^2 \qquad \textbf{Ans}$$

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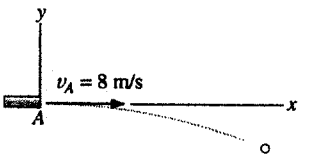
12-122. The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



$v_x = 8$ m/s

$\left(\overset{+}{\rightarrow}\right)$ $s = v_0 t$

$x = 8t$

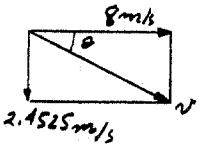
$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

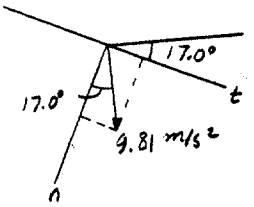
$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$

$y = -0.0766x^2$ (Parabola) **Ans**





$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

$a_x = 0$ $a_y = 9.81$ m/s²

$a_n = 9.81 \cos 17.04° = 9.38$ m/s² **Ans**

$a_t = 9.81 \sin 17.04° = 2.88$ m/s² **Ans**

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12-123. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

Velocity : Here, $\mathbf{r} = \{(2t+t^2)\mathbf{i} + t^2\mathbf{j}\}$ m. To determine the velocity $\mathbf{v}$, apply Eq. 12 – 7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2+2t)\mathbf{i} + 2t\mathbf{j}\} \text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2+2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ = 7.21 m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21 \text{ m/s} \qquad \textbf{Ans}$$

The velocity $\mathbf{v}$ makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.

Acceleration : To determine the acceleration $\mathbf{a}$, apply Eq. 12 - 9.

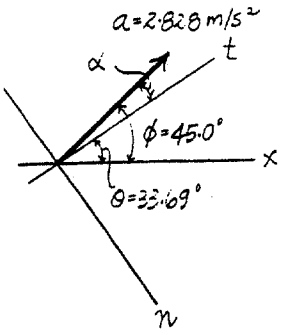
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\} \text{ m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828 \text{ m/s}^2$$

The acceleration $\mathbf{a}$ makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis.
From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555 \text{ m/s}^2 \qquad \textbf{Ans}$$
$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77 \text{ m/s}^2 \qquad \textbf{Ans}$$



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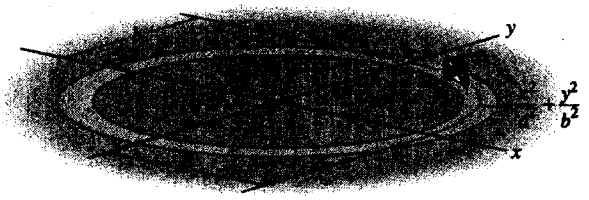
***12-124.** The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.



Acceleration : Defferentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad [1]$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \to a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >>> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \to \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}$ $= \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$. Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}(a^3) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_{max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{max} = (a_n)_{max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans}$$

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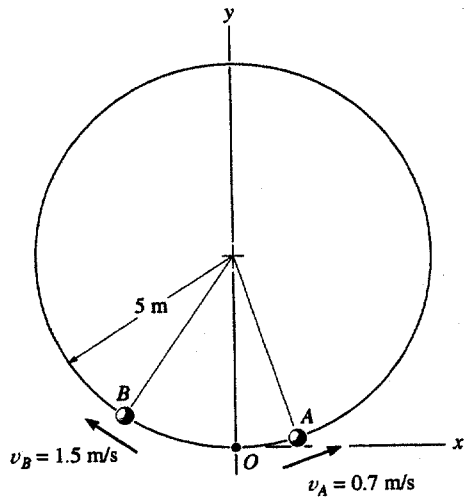
12-125. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine in $t = 2$ s, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.

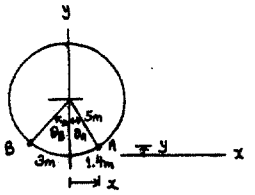


(a) $s_A = 0.7(2) = 1.40$ m Ans

$s_B = 1.5(2) = 3$ m Ans

(b) $\theta_A = \frac{1.40}{5} = 0.280$ rad. $= 16.04°$

$\theta_B = \frac{3}{5} = 0.600$ rad. $= 34.38°$



For A

$x = 5 \sin 16.04° = 1.382 = 1.38$ m

$y = 5(1 - \cos 16.04°) = 0.1947 = 0.195$ m

$\mathbf{r}_A = \{1.38\,\mathbf{i} + 0.195\,\mathbf{j}\}$ m Ans

For B

$x = -5 \sin 34.38° = -2.823 = -2.82$ m

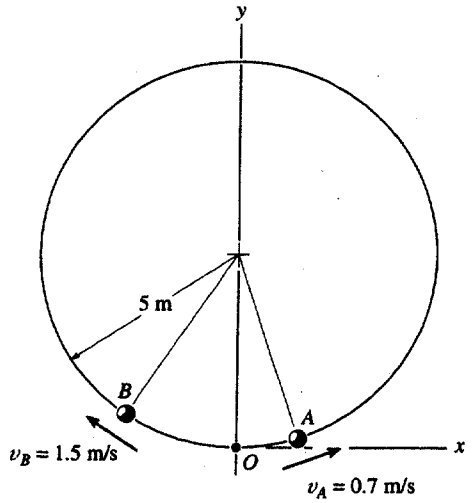
$y = 5(1 - \cos 34.38°) = 0.8734 = 0.873$ m

$\mathbf{r}_B = \{-2.82\,\mathbf{i} + 0.873\,\mathbf{j}\}$ m Ans

(c) $\Delta \mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\,\mathbf{i} + 0.678\,\mathbf{j}\}$ m

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26$ m Ans

12-126. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



$s_t = 2\pi(5) = 31.4159$ m

$s_A = 0.7t$

$s_B = 1.5t$

Require

$s_A + s_B = 31.4159$

$0.7t + 1.5t = 31.4159$

$t = 14.28$ s $= 14.3$ s Ans

$a_B = \frac{v_B^2}{\rho} = \frac{(1.5)^2}{5} = 0.45$ m/s^2 Ans

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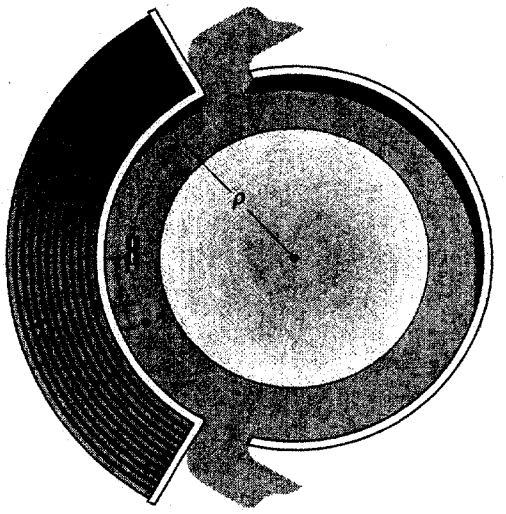
12-127. The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



$a_t = 0.4s = \frac{v\,dv}{ds}$

$a\,ds = v\,dv$

$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$

$\frac{0.4s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_{15}^{v}$

$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$

$v^2 = 0.4s^2 + 225$

$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$

$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$

$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$

$\ln\left(s + \sqrt{s^2 + 562.5}\right)\Big|_0^s = 0.632\,456t$

$\ln\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$

At $s = 20$ m,

$t = 1.21$ s **Ans**

***12-128.** A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at 2 ft/s^2. Determine the time needed for his acceleration to become 4 ft/s^2.

$a = \sqrt{a_n^2 + a_t^2}$

$a_t = 2$

$v = v_0 + a_c t$

$v = 0 + 2t$

$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$

$4 = \sqrt{(2)^2 + (\frac{(2t)^2}{8})^2}$

$16 = 4 + \frac{16\,t^4}{64}$

$t = 2.63$ s **Ans**

12-129. A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

$y = a + bx + cx^2$

$\dot{y} = b\dot{x} + 2c\,x\dot{x}$

$\ddot{y} = b\ddot{x} + 2c(\dot{x})^2 + 2c\,x\ddot{x}$

When $x = 0$, $\dot{y} = b\dot{x}$

$v_0^2 = \dot{x}^2 + b^2\dot{x}^2$

$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}}$ **Ans**

$v_y = \frac{v_0 b}{\sqrt{1 + b^2}}$ **Ans**

$a_n = \frac{v_0^2}{\rho}$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$

$\frac{dy}{dx} = b + 2c\,x$

$\frac{d^2y}{dx^2} = 2c$

At $x = 0$, $\rho = \frac{(1 + b^2)^{3/2}}{2c}$

$a_n = \frac{2c\,v_0^2}{(1 + b^2)^{3/2}}$ **Ans**

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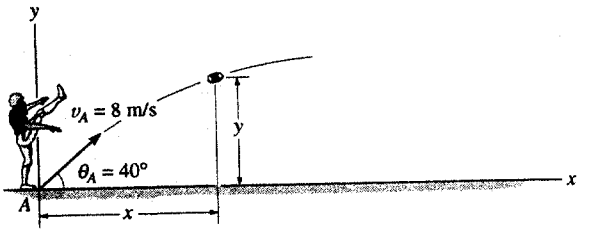
■**12-130.** The ball is kicked with an initial speed v_A = 8 m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the ball's velocity and the normal and tangential components of its acceleration when $t = 0.25$ s.



Horizontal Motion : The horizontal component of velocity is $(v_0)_x$ = 8cos 40° = 6.128 m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$
$$x = 0 + 6.128t \qquad [1]$$

Vertical Motion : The vertical component of initial velocity is $(v_0)_y$ = 8sin 40° = 5.143 m/s . The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$
$$y = 0 + 5.143t + \frac{1}{2}(-9.81)(t^2) \qquad [2]$$

Eliminate t from Eqs[1] and [2], we have

$$y = \{0.8391x - 0.1306x^2\} \text{ m} = \{0.839x - 0.131x^2\} \text{ m} \qquad \textbf{Ans}$$

The tangent of the path makes an angle $\theta = \tan^{-1}\frac{3.644}{4} = 42.33°$ with the x axis.

Acceleration : When $t = 0.25$ s, from Eq.[1], $x = 0 + 6.128(0.25) =$ 1.532 m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1}0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = a\sin\alpha = 9.81\sin 23.70° = 3.94 \text{ m/s}^2 \qquad \textbf{Ans}$$
$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98 \text{ m/s}^2 \qquad \textbf{Ans}$$

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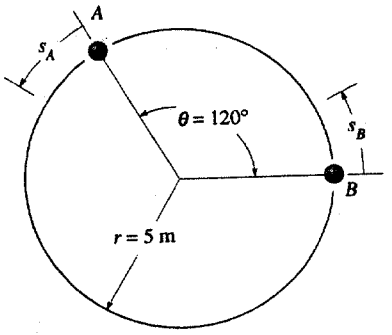
■**12-131.** Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A is increased by $\dot{v}_A = (4s_A)$ m/s^2, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



Distance Traveled : Initially the distance between two particles is $d_0 = \rho\theta$ $= 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. When $t = 1$ s, particle B travels a distance of $s_B = 8(1)$ $= 8$ m. The distance traveled by particle A can be obtained as follows

$$v_A\,dv_A = a_A\,ds_A$$
$$\int_{8\,\text{m/s}}^{v_A} v_A\,dv_A = \int_0^{s_A} 4s_A\,ds_A$$
$$v_A = 2\sqrt{s_A^2 + 16} \qquad [1]$$

$$dt = \frac{ds_A}{v_A}$$
$$\int_0^{1\,\text{s}} dt = \int_0^{s_A} \frac{ds_A}{2\sqrt{s_A^2 + 16}}$$
$$1 = \frac{1}{2}\sinh^{-1}\left(\frac{s_A}{4}\right)$$
$$s_A = 14.51\ \text{m}$$

Thus, the distance between two cyclists after $t = 1$ s is

$$d = d_0 + s_A - s_B = 10.47 + 14.51 - 8 = 17.0\ \text{m} \qquad \textbf{Ans}$$

Acceleration : The tangential acceleration for cyclist A and B when $t = 1$ s is $(a_t)_A = 4s_A = 4(14.51) = 58.03$ m/s^2 and $(a_t)_B = 0$ (particle B travels at constant speed), respectively. When $t = 1$ s, from Eq. [1], $v_A = 2\sqrt{14.51^2 + 16}$ $= 30.10$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{30.10^2}{5} = 181.17\ \text{m/s}^2$$
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80\ \text{m/s}^2$$

The magnitudes of the acceleration for particle A and B are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{58.03^2 + 181.17^2} = 190\ \text{m/s}^2 \qquad \textbf{Ans}$$
$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 12.80^2} = 12.8\ \text{m/s}^2 \qquad \textbf{Ans}$$

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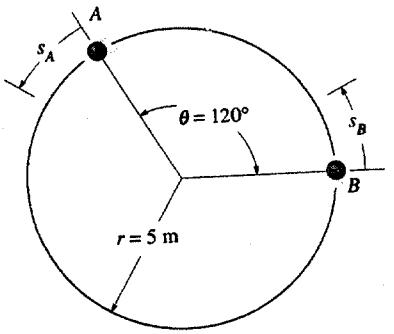
***12-132.** Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increased by $\dot{v}_B = 4$ m/s^2, and at the same instant A has an increase in speed $\dot{v}_A = 0.8t$ m/s^2, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Distance Traveled : Initially the distance between the two particles is $d_0 = \rho\theta$ $= 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)t^2 = \left(8t + 2t^2\right) \text{ m} \qquad [1]$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\,\text{m/s}}^{v_A} dv_A = \int_0^t 0.8t\,dt$$

$$v_A = \left(0.4t^2 + 8\right) \text{ m/s} \qquad [2]$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right) dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $t = 2.5074$ s $= 2.51$ s **Ans**

Note : If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6$ s > 2.51 s.

Acceleration : The tangential acceleration for particle A and B When $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8(2.5074) = 2.006$ m/s^2 and $(a_t)_B = 4$ m/s^2, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11 \text{ m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01 \text{ m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2 \text{ m/s}^2 \quad \textbf{Ans}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1 \text{ m/s}^2 \quad \textbf{Ans}$$

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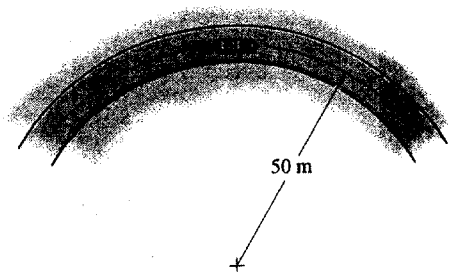
12-133. The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity : The speed v in terms of position s can be obtained by applying $v dv = a ds$.

$$v dv = a ds$$

$$\int_{4\,m/s}^{v} v dv = \int_{0}^{s} 0.05 s ds$$

$$v = \left(\sqrt{0.05s^2 + 16}\right)\ \text{m/s}$$

At $s = 10$ m, $\quad v = \sqrt{0.05(10^2) + 16} = 4.583\ \text{m/s} = 4.58\ \text{m/s}$ **Ans**

Acceleration : The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05(10) = 0.500\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653\ \text{m/s}^2$$ **Ans**

■12-134. A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

$$v = 60(1 - e^{-t^2})$$

$$a_t = \frac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\, t e^{-t^2}$$

$$a_t|_{t=2} = 120(2)e^{-4} = 4.3958$$

$$v|_{t=2} = 60(1 - e^{-4}) = 58.9011$$

$$a_n = \frac{(58.9011)^2}{100} = 34.693$$

$$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0\ \text{m/s}^2$$ **Ans**

$$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\, dt$$

$$s = 67.1\ \text{ft}$$ **Ans**

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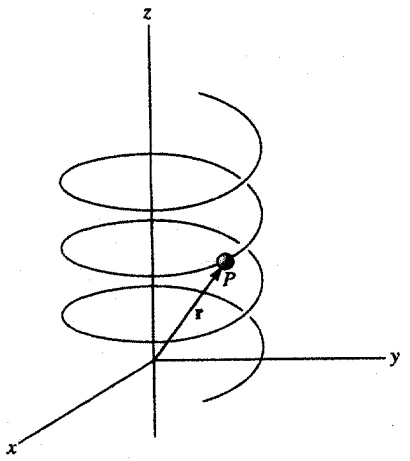
12-135. A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$

When $t = 8$ s,

$\mathbf{v}_P = -0.2\sin(0.8\text{rad})\mathbf{i} + 0.15\cos(0.8\text{rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = -0.02\cos(0.8\text{rad})\mathbf{i} - 0.015\sin(0.8\text{rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$

Since the binormal vector is perpendicular to the plane containing the $n-t$ axis, and $\mathbf{a}_P$ and $\mathbf{v}_P$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.143\,47 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$

$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_P \times \mathbf{v}_P$.
For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans**

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***12-136.** The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(r\ddot{\theta} + r\dot{\theta} + 2\dot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\dot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\ddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans}$$

12-137. If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2e^{-t}$

$\dot{\theta} = -2e^{-t} = -0.27067$

$\ddot{\theta} = 2e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97$ m/s^2 **Ans**

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12-138. A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

When $\theta = \frac{\pi}{6}$ rad, $\frac{\pi}{6} = \cos 2t \quad t = 0.5099$ s

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\Big|_{t=0.5099\ s} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\Big|_{t=0.5099\ s} = -2.0944 \text{ rad/s}^2$$

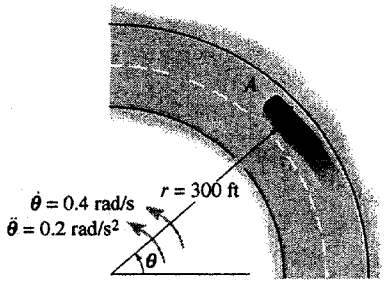
$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2 \quad \textbf{Ans}$$

12-139. A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



Velocity : Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \quad \textbf{Ans}$$

Acceleration : Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \quad \textbf{Ans}$$

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***12-140.** If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

$r = 2\cos t \qquad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$

$\theta = \frac{t}{2} \qquad \dot{\theta} = \frac{1}{2} \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = -2\sin t$ **Ans**

$v_\theta = r\dot{\theta} = 2\cos t\left(\frac{1}{2}\right) = \cos t$ **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - 2\cos t\left(\frac{1}{2}\right)^2 = -\frac{5}{2}\cos t$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\frac{1}{2}\right) = -2\sin t$ **Ans**

12-141. If a particle's position is described by the polar coordinates $r = (2\sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and tangential components of its velocity and acceleration when $t = 1$ s.

When $t = 1$ s,

$\theta = 4t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2\sin 2\theta = 1.9787$

$\dot{r} = 4\cos 2\theta\,\dot{\theta} = -2.3280$

$\ddot{r} = -8\sin 2\theta(\dot{\theta})^2 + 8\cos 2\theta\,\ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33$ m/s **Ans**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6$ m/s^2 **Ans**

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12-142. A particle is moving along a circular path having a 400-mm radius. Its position as a function of time is given by $\theta = (2t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

$r = 400$ mm $\quad \dot{r} = 0 \quad \ddot{r} = 0$

$\theta = 2t^2 \quad \dot{\theta} = 4t \quad \ddot{\theta} = 4$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(4t)^2 = -6400t^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(4) + 0 = 1600$

When $\theta = 30° = 30°\left(\frac{\pi}{180°}\right) = 0.5236$ rad, then,

$0.5236 = 2t^2, \quad t = 0.5117$ s

Hence,

$a = \sqrt{(-6400(0.5117)^2)^2 + (1600)^2} = 2316.76 \text{ mm/s}^2 = 2.32 \text{ m/s}^2$ **Ans**

12-143. A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

$\mathbf{r} = 2t\mathbf{i} + 4t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$

$\theta = \tan^{-1}(\frac{16}{4}) = 75.964°$

$v = \sqrt{(2)^2 + (16)^2} = 16.1245$ ft/s

$\phi = \tan^{-1}(\frac{16}{2}) = 82.875°$

$a = 8$ ft/s^2

$\phi - \theta = 6.9112°$

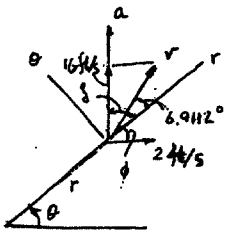
$v_r = 16.1245\cos 6.9112° = 16.0$ ft/s **Ans**

$v_\theta = 16.1245\sin 6.9112° = 1.94$ ft/s **Ans**

$\delta = 90° - \theta = 14.036°$

$a_r = 8\cos 14.036° = 7.76$ ft/s^2 **Ans**

$a_n = 8\sin 14.036° = 1.94$ ft/s^2 **Ans**



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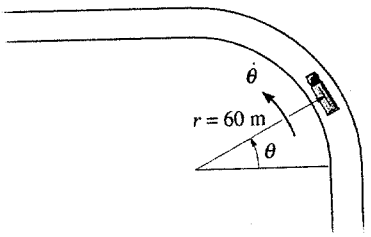
***12-144.** A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\dot{\theta} = 60\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$= 0 - 60(0.333)^2$

$= -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 60\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\ddot{\theta}$

$\ddot{\theta} = 0$

Thus,

$a_\theta = 0$

$a = a_r = 6.67$ m/s^2 **Ans**

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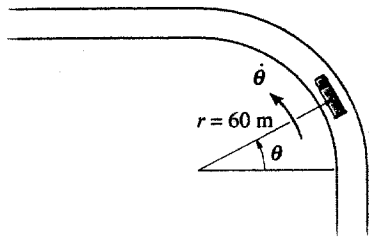
12-145. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2. Determine the truck's radial and transverse components of acceleration.



$r = 60$

$a_t = 3$ m/s^2

$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67$ m/s^2

$a_r = -a_n = -6.67$ m/s^2 **Ans**

$a_\theta = a_t = 3$ m/s^2 **Ans**

12-146. A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine is in degrees, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

$r = 6$ in., $\dot{r} = 0$, $\ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3\cos 3t$

$\ddot{\theta} = -9\sin 3t$

At $\theta = 30°$,

$\frac{30°}{180°}\pi = \sin 3t$

$t = 10.525$ s

Thus,

$\dot{\theta} = 2.5559$ rad/s

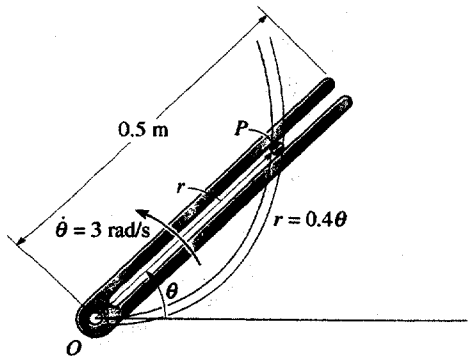
$\ddot{\theta} = -4.7124$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3$ in./s^2 **Ans**

12-147. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

At $\theta = \frac{\pi}{3}$, $\quad r = 0.4189$

$\dot{r} = 0.4(3) = 1.20$

$\ddot{r} = 0.4(0) = 0$

$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans**

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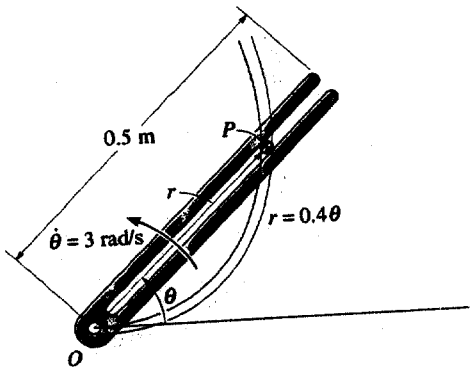
***12-148.** Solve Prob. 12-147 if the slotted link has an angular acceleration $\ddot{\theta} = 8$ rad/s^2 when $\dot{\theta} = 3$ rad/s at $\theta = \pi/3$ rad.



$\dot{\theta} = 3$ rad/s $\quad r = 0.4\theta$

$\dot{r} = 0.4\dot{\theta}$

$\ddot{r} = 0.4\ddot{\theta}$

$\theta = \frac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

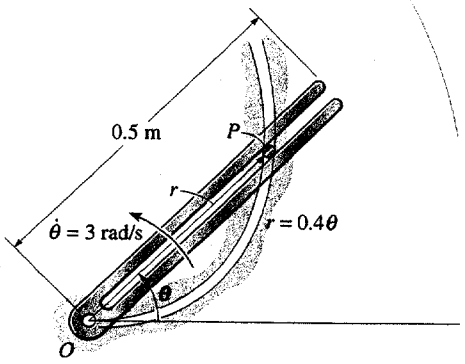
$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6$ m/s^2 **Ans**

12-149. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



$r = 0.4\theta$

$\dot{r} = 0.4\dot{\theta}$

$\ddot{r} = 0.4\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$\theta = \frac{0.5}{0.4} = 1.25$ rad

$\dot{r} = 1.20$

$\ddot{r} = 0$

$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.5(3) = 1.50$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans**

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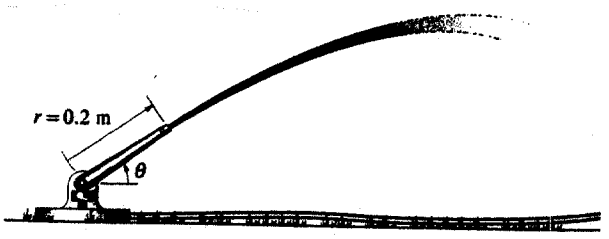
***12-152.** At the instant shown, the watersprinkler is rotating with an angular speed $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 0.2$ m.



$r = 0.2$

$\dot{r} = 3 \qquad \dot{\theta} = 2$

$\ddot{r} = 0 \qquad \ddot{\theta} = 3$

$v_r = 3$

$v_\theta = 0.2(2) = 0.4$

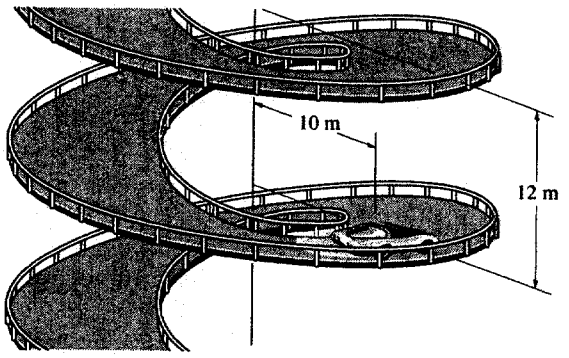
$v = \sqrt{(3)^2 + (0.4)^2} = 3.03$ m/s **Ans**

$a_r = 0 - (0.2)(2)^2 = -0.80$ m/s^2

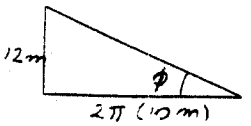
$a_\theta = 0.2(3) + 2(3)(2) = 12.6$ m/s^2

$a = \sqrt{(-0.80)^2 + (12.6)^2} = 12.6$ m/s^2 **Ans**

12-153. The automobile is traveling from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81°$



$v = 1.5$ m/s

$v_r = 0$

$v_\theta = 1.5\cos 10.81° = 1.473$ m/s

$v_z = -1.5\sin 10.81° = -0.2814$ m/s

Since

$r = 10 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_\theta = r\dot{\theta} = 1.473 \qquad \dot{\theta} = \frac{1.473}{10} = 0.1473$

Since $\theta = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(0.1473)^2 = -0.217$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(0) + 2(0)(0.1473) = 0$

$a_z = \ddot{z} = 0$

$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217$ m/s^2 **Ans**

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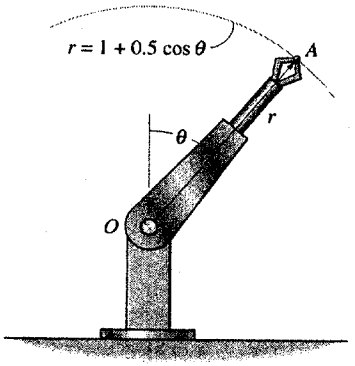
12-154. Because of telescopic action, the end of the industrial robotic arm extends along the path of the limaçon $r = (1 + 0.5\cos\theta)$ m. At the instant $\theta = \pi/4$, the arm has an angular rotation $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.25$ rad/s^2. Determine the radial and transverse components of the velocity and acceleration of the object A held in its grip at this instant.



$\theta = \frac{\pi}{4} = 45°$

$\theta = 0.6$

$\ddot{\theta} = 0.25$

$r = 1 + 0.5\cos\theta$

$\dot{r} = -0.5\sin\theta\dot{\theta}$

$\ddot{r} = -0.5\left(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right)$

At $\theta = 45°$

$r = 1.354 \quad \dot{r} = -0.2121 \quad \ddot{r} = -0.2157$

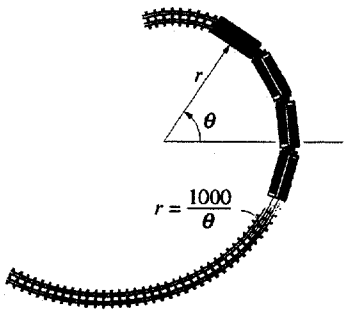
$v_r = \dot{r} = -0.212$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2157 - (1.354)(0.6)^2 = -0.703$ m/s^2 **Ans**

$v_\theta = r\dot{\theta} = 1.354(0.6) = 0.812$ m/s **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.354(0.25) + 2(-0.2121)(0.6) = 0.0838$ m/s^2 **Ans**

12-155. For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



$r = \frac{1000}{\theta}$

$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$

Since

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$

$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$

Thus,

$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$

At $\theta = \frac{9\pi}{4}$

$\dot{\theta} = 0.140$

$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$

$v_r = \dot{r} = -2.80$ m/s **Ans**

$v_\theta = r\dot{\theta} = \frac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans**

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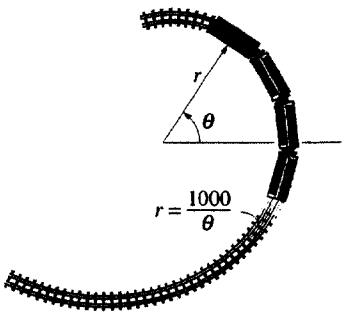
***12-156.** For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$r = \frac{1000}{\theta}$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \frac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

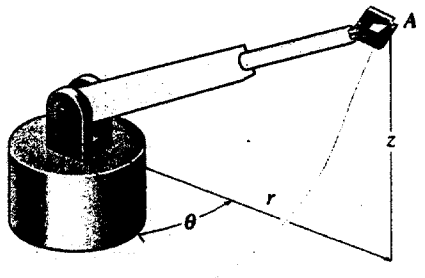
$v_r = \dot{r} = -4.00$ m/s **Ans**

$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans**

12-157. The arm of the robot has a fixed length so that $r = 3$ ft and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



$\theta = 0.5t$ $\quad r = 3$ $\quad z = 3\sin 2t$

$\dot{\theta} = 0.5$ $\quad \dot{r} = 0$ $\quad \dot{z} = 6\cos 2t$

$\ddot{\theta} = 0$ $\quad \ddot{r} = 0$ $\quad \ddot{z} = -12\sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95$ ft/s **Ans**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44$ ft/s^2 **Ans**

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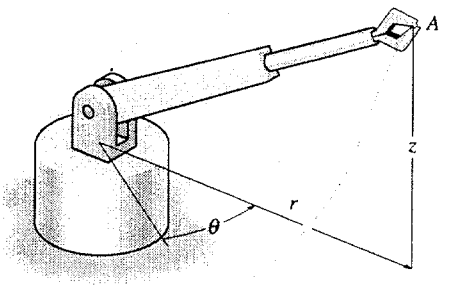
12-158. For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



$\theta = 0.5t$ rad $\quad r = 3$ ft $\quad z = 4t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\quad \dot{r} = 1.5$ ft/s $\quad \dot{z} = 8t$ ft/s

$\ddot{\theta} = 0 \quad \ddot{r} = 0 \quad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \quad r = 3 \quad z = 36$

$\dot{\theta} = 0.5 \quad \dot{r} = 1.5 \quad \dot{z} = 24$

$\ddot{\theta} = 0 \quad \ddot{r} = 0 \quad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans**

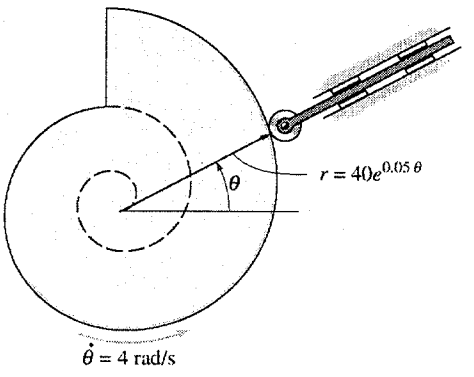
$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans**

12-159. The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



$r = 40e^{0.05\theta} \quad r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\theta}\dot{\theta} \quad \dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(4) = 8.21$ mm/s **Ans**

$\ddot{r} = 0.1e^{0.05\theta}(\dot{\theta})^2 + 2e^{0.05\theta}\ddot{\theta}$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 0 = 1.64$ mm/s^2 **Ans**

$\theta = \frac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

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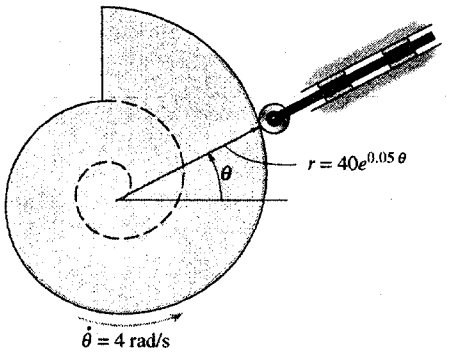
***12-160.** Solve Prob. 12-159, if the cam has an angular acceleration of $\ddot{\theta} = 2$ rad/s^2 when its angular velocity is $\dot{\theta} = 4$ rad/s at $\theta = 30°$.



$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \frac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(4) = 8.2122$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(2) = 5.749$

$v_r = \dot{r} = 8.2122$

$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$

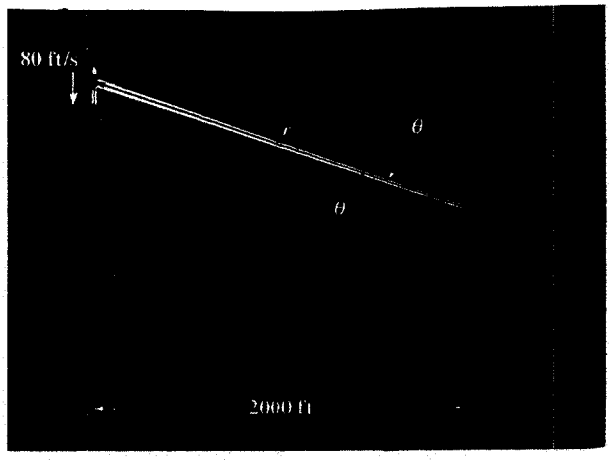
$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164$ mm/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 5.749 - 41.0610(4)^2 = -651.2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 41.0610(2) + 2(8.2122)(4) = 147.8197$

$a = \sqrt{(-651.2)^2 + (147.8197)^2} = 668$ mm/s^2 **Ans**

12-161. The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed of 80 ft/s. Determine the angular rate of rotation of the light when the automobile is $r = 3000$ ft from the boat.



$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\ \dot{\theta}$

At $r = 3000$ ft, $\theta = 41.8103°$

$\dot{r} = -3354.102\ \dot{\theta}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(80)^2 = [(-3354.102)^2 + (3000)^2](\dot{\theta})^2$

$\dot{\theta} = 0.0177778 = 0.0178$ rad/s **Ans**

12-162. If the car in Prob. 12–161 is accelerating at 15 ft/s^2 at the instant $r = 3000$ ft, determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.

$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\ \dot{\theta}$

At $r = 3000$ ft, $\theta = 41.8103°$

$\dot{r} = -3354.102\ \dot{\theta}$

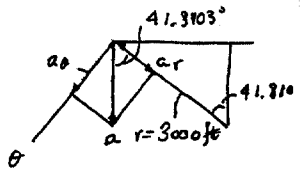
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_\theta = 3000\ddot{\theta} + 2(-3354.102)(0.0177778)^2$

Since $a_\theta = 15 \sin 41.8103° = 10$ m/s

Then,

$\ddot{\theta} = 0.00404$ rad/s^2 **Ans**



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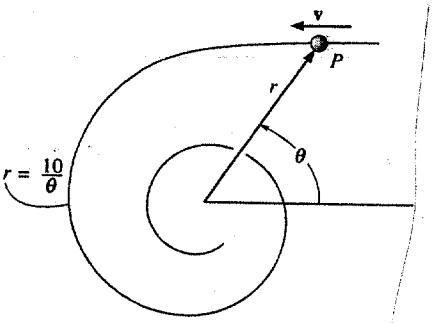
12-163. A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine the magnitudes v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\left(1+\theta^2\right)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1+\theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = -\frac{20}{\sqrt{1+\theta^2}} \quad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = \frac{20\theta}{\sqrt{1+\theta^2}} \quad \textbf{Ans}$$

When $\theta = 1$ rad,

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s} \quad \textbf{Ans}$$

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s} \quad \textbf{Ans}$$

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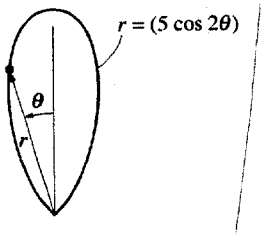
***12-164.** A particle travels along the portion of the "four-leaf rose" defined by the equation $r = (5 \cos 2\theta)$ m. If the angular velocity of the radial coordinate line is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = 30°$. When $t = 0$, $\theta = 0$.



$\dot{\theta} = 3t^2$

$\ddot{\theta} = 6t$

$\int_0^\theta d\theta = \int_0^t 3t^2\, dt$

$\theta = t^3$

At $\theta = 30° = \dfrac{\pi}{6}$

$t = (\pi/6)^{1/3} = 0.806$

$\dot{\theta} = 1.95$

$\ddot{\theta} = 4.84$

$r = 5 \cos 2\theta$

$\dot{r} = -10 \sin 2\theta\, \dot{\theta}$

$\ddot{r} = -10(2\cos 2\theta(\dot{\theta})^2 + \sin 2\theta\, \ddot{\theta})$

At $\theta = 30°$

$r = 2.5$

$\dot{r} = -16.88$

$\ddot{r} = -79.87$

$v_r = \dot{r} = -16.9$ m/s **Ans**

$v_\theta = r\dot{\theta} = 2.5(1.95) = 4.87$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -79.86 - 2.5(1.95)^2 = -89.4$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(4.84) + 2(-16.88)(1.95) = -53.7$ m/s^2 **Ans**

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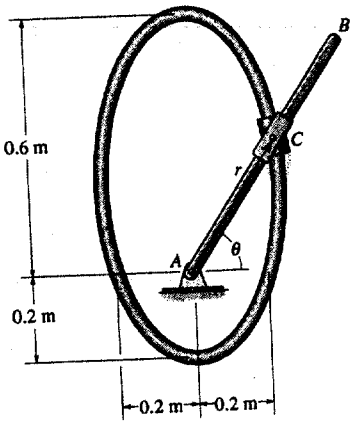
■12-165. The double collar C is pin connected such that one collar slides over a fixed rod and the other slides over a rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |0.4 \sin \theta + 0.2|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0°$. Use Simpson's rule to determine θ at $t = 1$ s.



$\dot{\theta} = e^{0.5t^2}\big|_{t=1\,s} = 1.649$ rad/s

$\ddot{\theta} = e^{0.5t^2}(t)\big|_{t=1\,s} = 1.649$ rad/s^2

$\theta = \int_0^1 e^{0.5t^2}\,dt = 1.195$ rad $= 68.47°$

$r = 0.4\sin\theta + 0.2$

$\dot{r} = 0.4\cos\theta\dot{\theta}$

$\ddot{r} = -0.4\sin\theta\dot{\theta}^2 + 0.4\cos\theta\ddot{\theta}$

At $t = 1$ s,

$r = 0.5721$

$\dot{r} = 0.2421$

$\ddot{r} = -0.7694$

$v_r = \dot{r} = 0.242$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.5721(1.649) = 0.943$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.7694 - 0.5721(1.649)^2 = -2.32$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5721(1.649) + 2(0.2421)(1.649) = 1.74$ m/s^2 **Ans**

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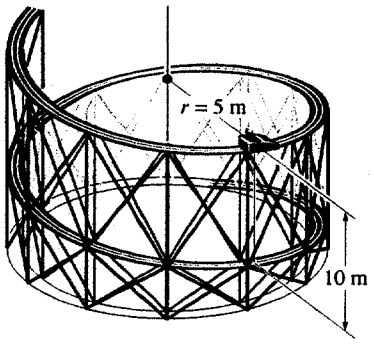
12-166. The roller coaster is traveling down along the spiral ramp with a constant speed $v = 6$ m/s. If the track descends a distance of 10 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the roller coaster's acceleration as it moves along the track, $r = 5$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}[10/2\pi(5)] = 17.66°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine θ and $\dot{z}$.



$\phi = 17.66°$

$v = 6$ m/s

$v_z = -6 \sin 17.66° = -1.820$ m/s

$v_\theta = 6 \cos 17.66° = 5.717$ m/s

Since $r = 5$

$\dot{r} = 0$

$\ddot{r} = 0$

$r\dot{\theta} = v_\theta = 5.717$

$\dot{\theta} = \frac{5.717}{5} = 1.143$

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$0 = 2\dot{r}\ddot{r} + 2(r\dot{\theta})(\dot{r}\dot{\theta} + r\ddot{\theta})$

$\ddot{\theta} = 0$

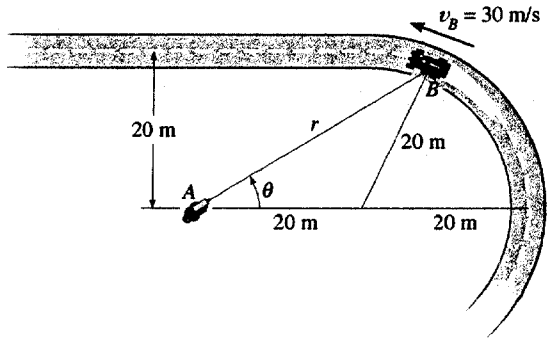
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1.143)^2 = -6.537$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = \dot{v}_z = 0$

$a = 6.54$ m/s^2 **Ans**

12-167. A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



$r = 2(20\cos\theta) = 40\cos\theta$

$\dot{r} = -40\sin\theta\,\dot{\theta}$

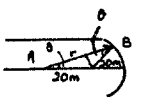
$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$

$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$

$\dot{\theta} = \frac{30}{40} = 0.75$ rad/s **Ans**



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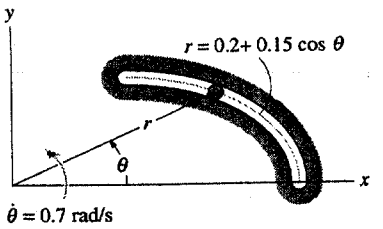
***12-168.** The pin follows the path described by the equation $r = (0.2 + 0.15\cos\theta)$ m. At the instant $\theta = 30°$, $\dot{\theta} = 0.7$ rad/s and $\ddot{\theta} = 0.5$ rad/s^2. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.



$r = 0.2 + 0.15\cos\theta = 0.2 + 0.15\cos 30° = 0.3299$ m

$\dot{r} = -0.15\sin\theta\dot{\theta} = -0.15\sin 30°(0.7) = -0.0525$ m/s

$\ddot{r} = -0.15\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right] = -0.15\left[\cos 30°(0.7)^2 + \sin 30°(0.5)\right] = -0.10115$ m/s^2

$v_r = \dot{r} = -0.0525$ m/s

$v_\theta = r\dot{\theta} = 0.3299(0.7) = 0.2309$ m/s

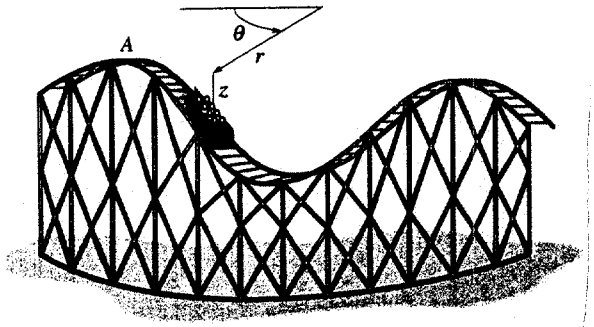
$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-0.0525)^2 + (0.2309)^2} = 0.237$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.10115 - 0.3299(0.7)^2 = -0.2628$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3299(0.5) + 2(-0.0525)(0.7) = 0.09145$ m/s^2

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-0.2628)^2 + (0.09145)^2} = 0.278$ m/s^2 **Ans**

12-169. For a short time the position of the roller-coaster car along its path is defined by the equations $r = 25$ m, $\theta = (0.3t)$ rad, and $z = (-8\cos\theta)$ m, where t is in seconds. Determine the magnitude of the car's velocity and acceleration when $t = 4$ s.



$r = 25$ m $\quad \theta = 0.3t|_{t=4\text{ s}} = 1.2$ rad

$\dot{r} = 0 \quad \dot{\theta} = 0.3$ rad/s

$\ddot{r} = 0 \quad \ddot{\theta} = 0$

$z = -8\cos\theta \quad \dot{z} = 8\sin\theta\dot{\theta}|_{\theta=1.2\text{ rad}} = 2.2369$ m/s

$\ddot{z} = 8\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]|_{\theta=1.2\text{ rad}} = 0.2609$ m/s^2

$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 25(0.3) = 7.5$ m/s $\quad v_z = \dot{z} = 2.2369$ m/s

$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2} = \sqrt{0^2 + 7.5^2 + 2.2369^2} = 7.83$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 25(0.3)^2 = -2.25$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 25(0) + 2(0)(0.3) = 0$

$a_z = \ddot{z} = 0.2609$ m/s^2

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-2.25)^2 + 0^2 + 0.2609^2} = 2.27$ m/s^2 **Ans**

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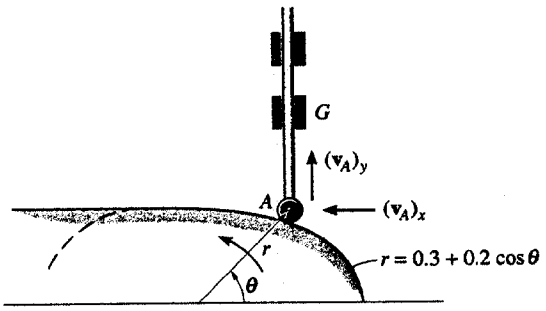
12-170. The mechanism of a machine is constructed so that the roller at A follows the surface of the cam described by the equation $r = (0.3 + 0.2\cos\theta)$ m. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the roller's velocity and acceleration when $\theta = 30°$. Neglect the size of the roller. Also compute the velocity components $(\mathbf{v}_A)_x$ and $(\mathbf{v}_A)_y$ of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides translate horizontally to the left.



$\dot{\theta} = 0.5$

$\ddot{\theta} = 0$

$r = (0.3 + 0.2\cos\theta)$

$\dot{r} = -0.2\sin\theta\,\dot{\theta}$

$\ddot{r} = -0.2(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta})$

At $\theta = 30°$

$r = 0.473$

$\dot{r} = -0.05$

$\ddot{r} = -0.0433$

$v_r = \dot{r} = -0.05$

$v_\theta = r\dot{\theta} = 0.473(0.5) = 0.237$

$v = \sqrt{(-0.05)^2 + (0.237)^2} = 0.242$ m/s **Ans**

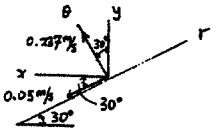
$a_r = \ddot{r} - r\dot{\theta}^2 = -0.0433 - 0.473(0.5)^2$

$a_r = -0.162$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.05)(0.5)$

$a_\theta = -0.05$ m/s^2

$a = \sqrt{(-0.162)^2 + (-0.5)^2} = 0.169$ m/s^2 **Ans**



$(\overset{+}{\leftarrow})$ $v_x = 0.05\cos 30° + 0.237\sin 30°$

$v_x = 0.162$ m/s **Ans**

$(+\uparrow)$ $v_y = -0.05\sin 30° + 0.237\cos 30°$

$v_y = 0.180$ m/s **Ans**

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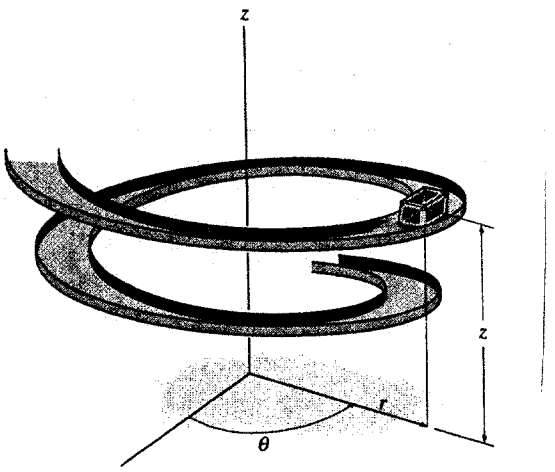
12-171. The crate slides down the section of the spiral ramp such that $r = (0.5z)$ ft and $z = (100 - 0.1t^2)$ ft, where t is in seconds. If the rate of rotation about the z axis is $\dot{\theta} = 0.04\pi t$ rad/s, determine the magnitudes of the velocity and acceleration of the crate at the instant $z = 10$ ft.



$r = 0.5z$

$z = 100 - 0.1t^2$

Thus,

$r = 50 - 0.05t^2$

$\dot{r} = -0.1t$

$\ddot{r} = -0.1$

$\dot{\theta} = 0.04\pi\, t \text{ rad/s} = 0.12566t \text{ rad/s}$

$\ddot{\theta} = 0.12566$

$\dot{z} = -0.2t$

$\ddot{z} = -0.2$

At $z = 10$ ft,

$10 = 100 - 0.1t^2$

$t = 30$ s

$r = 50 - 0.05(30)^2 = 5$

$\dot{r} = -0.1(30) = -3$

$\ddot{r} = -0.1$

$\dot{\theta} = 0.12566(30) = 3.76991$

$\ddot{\theta} = 0.12566$

$\dot{z} = -0.2(30) = -6$

$\ddot{z} = -0.2$

$v_r = \dot{r} = -3$

$v_\theta = r\dot{\theta} = 5(3.76991) = 18.850$

$v_z = \dot{z} = -6$

$v = \sqrt{(-3)^2 + (18.850)^2 + (-6)^2} = 20.0$ ft/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1 - 5(3.76991)^2 = -71.16$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(0.12566) + 2(-3)(3.76991) = -21.99$

$a_z = \ddot{z} = -0.2$

$a = \sqrt{(-71.16)^2 + (-21.99)^2 + (-0.2)^2} = 74.5$ ft/s^2 **Ans**

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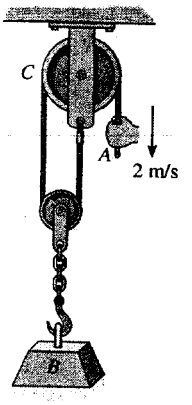
***12-172.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

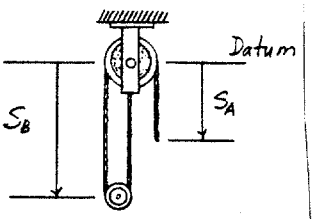


Position - Coordinate Equation : Datum is established at fixed pulley C. The position of point A and block B with respect to datum are s_A and s_B, respectively.

$$2s_B + s_A = l$$

Time Derivative : Taking the time derivative of the above equation yields

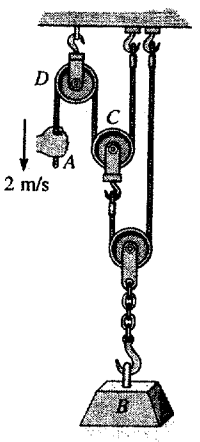
$$2v_B + v_A = 0 \qquad [1]$$



Since $v_A = 5$ m/s, from Eq.[1]

$$(+\downarrow) \qquad 2v_B + 2 = 0$$
$$v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

12-173. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



Position - Coordinate Equation : Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B and s_C, respectively. Since the system consists of two cords, two position - coordinate equations can be derived.

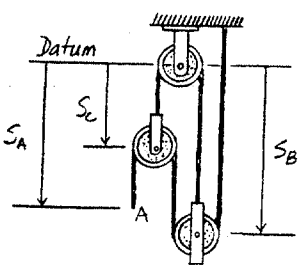
$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad [1]$$
$$s_B + s_C = l_2 \qquad [2]$$

Eliminating s_C from Eqs.[1] and [2] yields

$$s_A + 4s_B = l_1 + 2l_2$$

Time Derivative : Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad [3]$$



Since $v_A = 2$ m/s, from Eq.[3]

$$(+\downarrow) \qquad 2 + 4v_B = 0$$
$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

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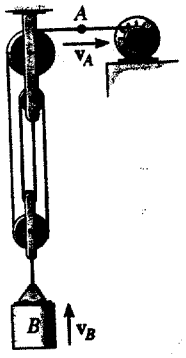
12-174. Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load at B 15 ft in 5 s.



$$v_B = \frac{-15}{5} = -3 \text{ ft/s} = 3 \text{ ft/s} \uparrow$$

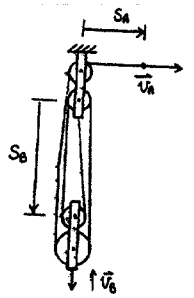
$$4s_B + s_A = l$$

$$4\Delta s_B = -\Delta s_A$$

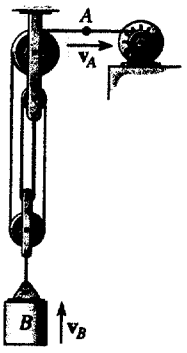
$$4v_B = -v_A$$

$$4(-3) = -v_A$$

$$v_A = 12 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$



12-175. Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

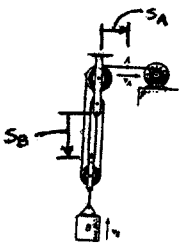
$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$

$$(+\downarrow) \quad v_B = (v_B)_0 + a_B t$$

$$-8 = 0 - (0.05)(t)$$

$$t = 160 \text{ s} \qquad \textbf{Ans}$$



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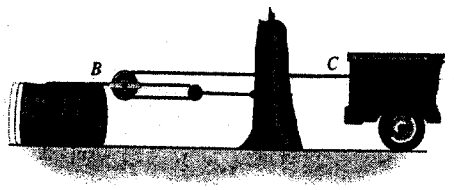
***12-176.** Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



$2s_B + (s_B - s_C) = l$

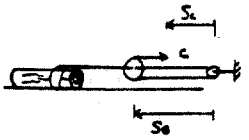
$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

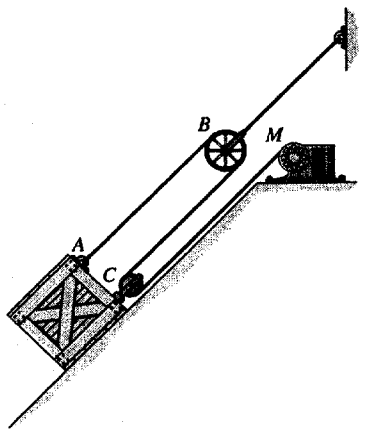
Since $\Delta s_C = -4$, then

$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans**



12-177. The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of 4 ft/s.



Position - Coordinate Equation : Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.

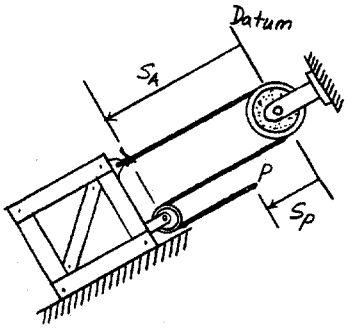
$$2s_A + (s_A - s_P) = l$$
$$3s_A - s_P = 0$$

Time Derivative : Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad [1]$$

Since $v_A = 4$ ft/s, from Eq.[1]

$$(+) \qquad 3(4) - v_P = 0$$
$$v_P = 12 \text{ ft/s} \qquad \textbf{Ans}$$



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12-178. Determine the displacement of the block at B if A is pulled down 4 ft.

$2s_A + 2s_C = l_1$

$\Delta s_A = -\Delta s_C$

$s_B - s_C + s_B = l_2$

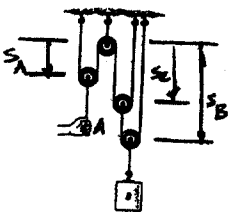
$2\Delta s_B = \Delta s_C$

Thus,

$2\Delta s_B = -\Delta s_A$

$2\Delta s_B = -4$

$\Delta s_B = -2\text{ ft} = 2\text{ ft}\uparrow$ **Ans**



12-179. The cable at B is pulled downwards at 4 ft/s, and is slowing at 2 ft/s^2. Determine the velocity and acceleration of block A at this instant.

$2s_A + (h - s_C) = l$

$2v_A = v_C$

$s_C + (s_C - s_B) = l$

$2v_C = v_B$

$v_B = 4v_A$

$a_B = 4a_A$

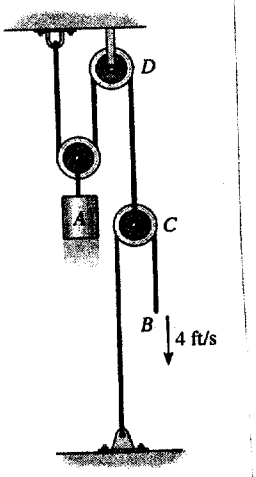
Thus,

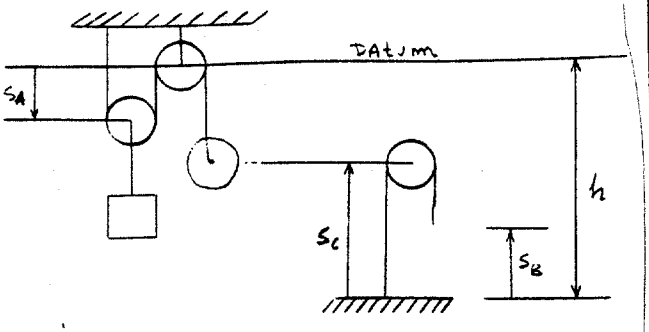
$-4 = 4v_A$

$v_A = -1\text{ ft/s} = 1\text{ ft/s}\uparrow$ **Ans**

$2 = 4a_A$

$a_A = 0.5\text{ ft/s} = 0.5\text{ ft/s}^2\downarrow$ **Ans**





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***12-180.** The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger P is pushed downward with a speed of 4 ft/s, determine the speed of the load at A.

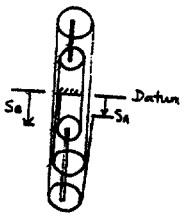
$$5s_B + (s_B - s_A) = l$$

$$6s_B - s_A = l$$

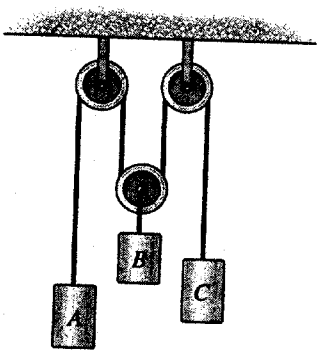
$$6v_B - v_A = 0$$

$$6(4) = v_A$$

$$v_A = 24 \text{ ft/s} \qquad \textbf{Ans}$$



12-181. If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B.

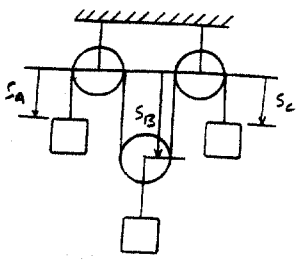


$$s_A + 2s_B + s_C = l$$

$$v_A + 2v_B + v_C = 0$$

$$4 + 2v_B - 2 = 0$$

$$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow \qquad \textbf{Ans}$$



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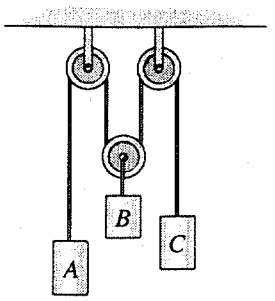
12-182. If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the relative velocity of block B with respect to C.



$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

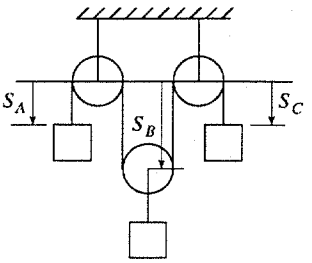
$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$

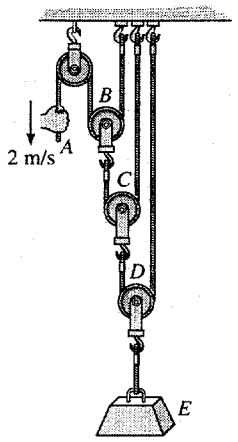
$v_B = v_C + v_{B/C}$

$[12 \uparrow] = [18 \downarrow] + [v_{B/C} \uparrow]$

$v_{B/C} = 30 \text{ ft/s} \uparrow$ **Ans**



12-183. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.



Position-Coordinate Equation: Datum is established at fixed pulley. The position of point A, pulley B and C and block E with respect to datum are s_A, s_B, s_C and s_E, respectively. Since the system consists of three cords, three position–coordinate equations can be developed.

$2s_B + s_A = l_1$ [1]

$s_C + (s_C - s_B) = l_2$ [2]

$s_E + (s_E - s_C) = l_3$ [3]

Eliminating s_C and s_B from Eqs. [1], [2] and [3], we have

$s_A + 8s_E = l_1 + 2l_2 + 4l_3$

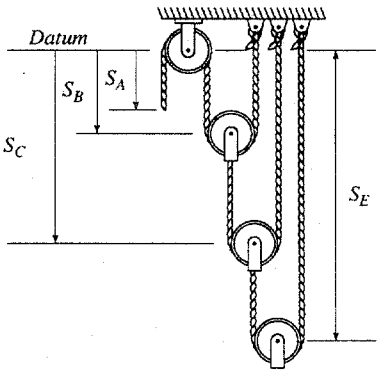
Time Derivative: Taking the time derivative of the above equation yields

$v_A + 8v_E = 0$ [4]

Since $v_A = 2$ m/s, from Eq. [3]

$(+\downarrow)$ $2 + 8v_E = 0$

$v_E = -0.250 \text{ m/s} = 0.250 \text{ m/s} \uparrow$ **Ans**



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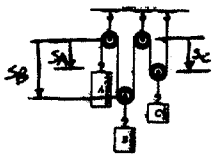
***12-184.** If block A of the pulley system is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s, determine the speed of block B.

$$s_A + 2s_B + 2s_C = l$$

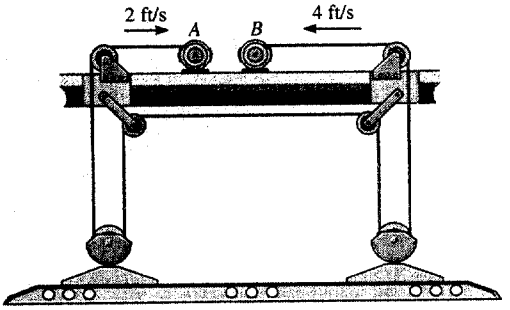
$$v_A + 2v_B + 2v_C = 0$$

$$4 + 2v_B + 2(-2) = 0$$

$$v_B = 0 \qquad \textbf{Ans}$$



12-185. The crane is used to hoist the load. If the motors at A and B are drawing in the cable at a speed of 2 ft/s and 4 ft/s, respectively, determine the speed of the load.



Position - Coordinate Equation : Datum is established as shown. The position of point A and B and load C with respect to datum are s_A, s_B and s_C, respectively.

$$4s_C + s_A + s_B + 2h = l$$

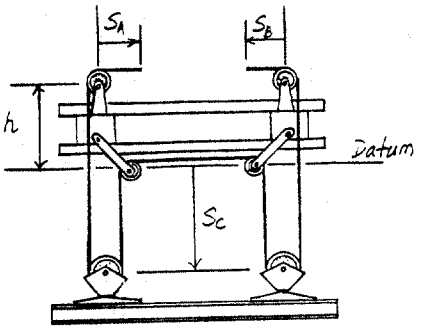
Time Derivative : Since h is a constant. Taking the time derivative of the above equation yields

$$4v_C + v_A + v_B = 0 \qquad [1]$$

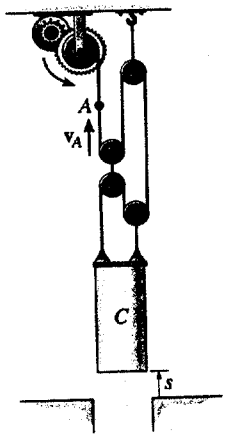
Since $v_A = 2$ ft/s and $v_B = 4$ ft/s, from Eq. [1]

$$4v_C + 2 + 4 = 0$$

$$v_C = -1.50 \text{ ft/s} = 1.50 \text{ ft/s} \uparrow \qquad \textbf{Ans}$$



12-186. The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.

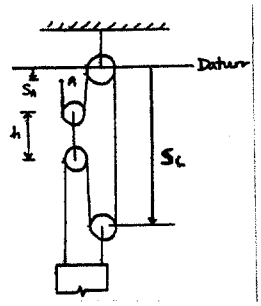


$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3s_C - 2h - s_A$$

$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow \qquad \textbf{Ans}$$



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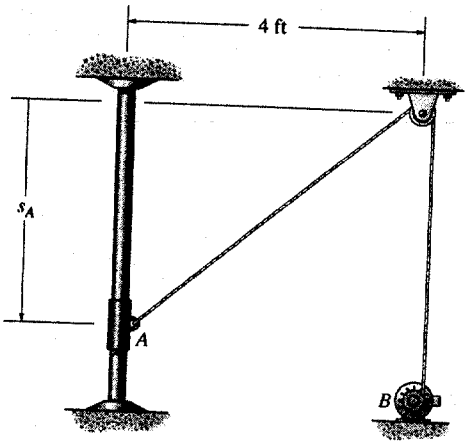
12-187. The motion of the collar at A is controlled by a motor at B such that when the collar is at $s_A = 3$ ft it is moving upwards at 2 ft/s and slowing down at 1 ft/s^2. Determine the velocity and acceleration of the cable as it is drawn into the motor B at this instant.



$$\sqrt{s_A^2 + 4^2} + s_B = l$$

$$\frac{1}{2}\left(s_A^2 + 16\right)^{-\frac{1}{2}}(2s_A)\dot{s}_A + \dot{s}_B = 0$$

$$\dot{s}_B = -s_A \dot{s}_A \left(s_A^2 + 16\right)^{-\frac{1}{2}}$$

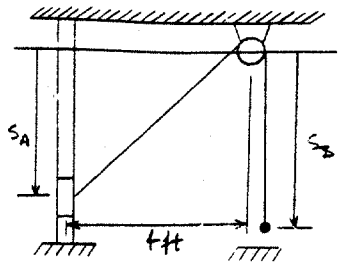
$$\ddot{s}_B = -\left[\left(\dot{s}_A\right)^2 \left(s_A^2 + 16\right)^{-\frac{1}{2}} + s_A \ddot{s}_A \left(s_A^2 + 16\right)^{-\frac{1}{2}} + s_A \dot{s}_A \left(-\frac{1}{2}\right)\left(s_A^2 + 16\right)^{-\frac{3}{2}}\left(2s_A \dot{s}_A\right)\right]$$

$$\ddot{s}_B = \frac{\left(s_A \dot{s}_A\right)^2}{\left(s_A^2 + 16\right)^{\frac{3}{2}}} - \frac{\left(\dot{s}_A\right)^2 + s_A \ddot{s}_A}{\left(s_A^2 + 16\right)^{\frac{1}{2}}}$$

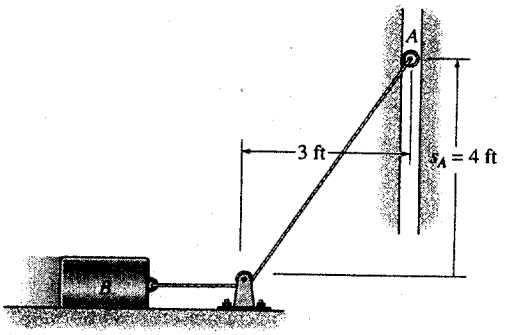
Evaluating these equations :

$$\dot{s}_B = -3(-2)\left((3)^2 + 16\right)^{-\frac{1}{2}} = 1.20 \text{ ft/s} \downarrow \quad \textbf{Ans}$$

$$\ddot{s}_B = \frac{((3)(-2))^2}{((3)^2 + 16)^{\frac{3}{2}}} - \frac{(-2)^2 + 3(1)}{((3)^2 + 16)^{\frac{1}{2}}} = -1.11 \text{ ft/s}^2 = 1.11 \text{ ft/s}^2 \uparrow \quad \textbf{Ans}$$



***12-188.** The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4$ ft/s^2 when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2 \dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

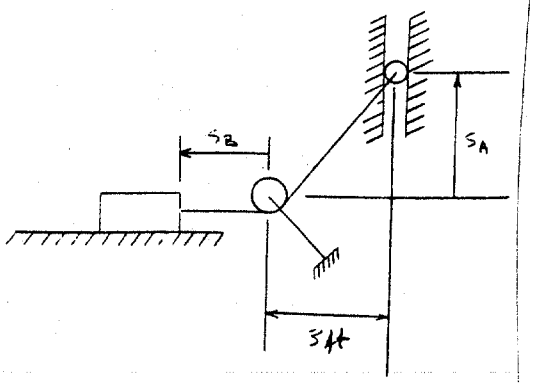
At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4$ ft/s^2

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

$$v_B = -2.4 \text{ ft/s} = 2.40 \text{ ft/s} \rightarrow \quad \textbf{Ans}$$

$$\ddot{s}_B - \left(\frac{1}{5}\right)^3 (4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$$a_B = -3.85 \text{ ft/s}^2 = 3.85 \text{ ft/s}^2 \rightarrow \quad \textbf{Ans}$$



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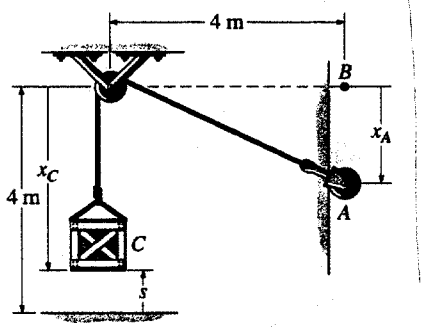
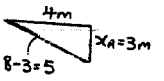
12-189. The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



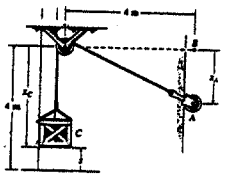
$$x_C + \sqrt{x_A^2 + (4)^2} = l$$



$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$

When $s = 1$ m, $l = 8$ m,



$x_C = 3$ m

$x_A = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$

Thus,

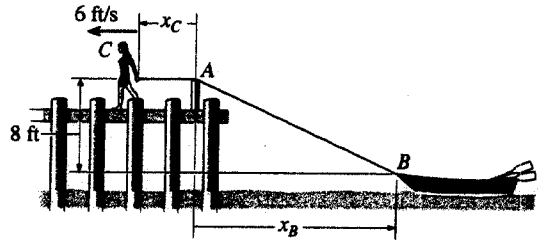
$$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$$

$$v_C = -1.2 \text{ m/s} = 1.2 \text{ m/s} \uparrow \quad \text{Ans}$$

$$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$$

$$a_C = -0.512 \text{ m/s}^2 = 0.512 \text{ m/s}^2 \uparrow \quad \text{Ans}$$

12-190. The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2x_B\dot{x}_B + \dot{x}_C = 0 \qquad (1)$$

$\dot{x}_C = 6$ ft/s

When $AB = 50$ ft,

$x_B = \sqrt{(50)^2 - (8)^2} = 49.356$ ft

From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \quad \text{Ans}$$

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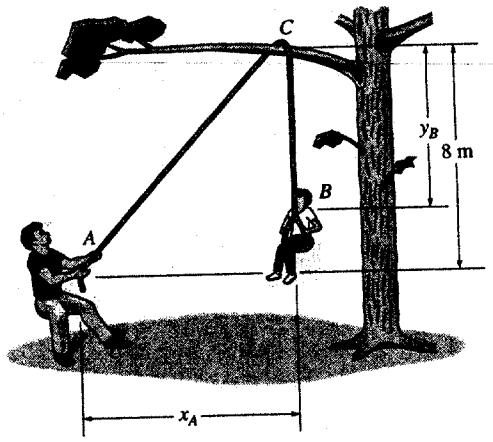
12-191. The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0, y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



Position - Coordinate Equation : Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$
$$16 = \sqrt{x_A^2 + 8^2} + y_B$$
$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad [1]$$

Time Derivative : Taking the time derivative of Eq. [1] where $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}} \frac{dx_A}{dt}$$
$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}} v_A \qquad [2]$$

At the instant $y_B = 4$ m, from Eq. [1], $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A [s_A - (s_0)_A]$$
$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$
$$v_A = 1.891\ \text{m/s}$$

Substitute the above results into Eq. [2] yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\ \uparrow \qquad \textbf{Ans}$$

Note : The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

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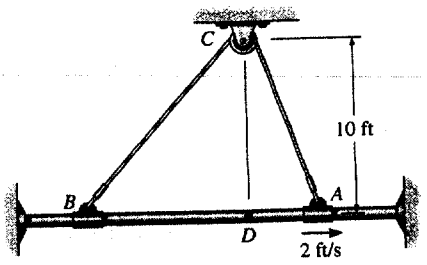
***12-192.** Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.

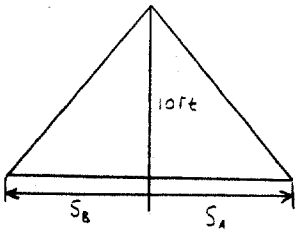


$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}\left(100 + s_B^2\right)^{-\frac{1}{2}}\left(2 s_B \dot{s}_B\right) + \frac{1}{2}\left(100 + s_A^2\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$\dot{s}_B = -\left(\frac{s_A \dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$



At $s_A = 4$,

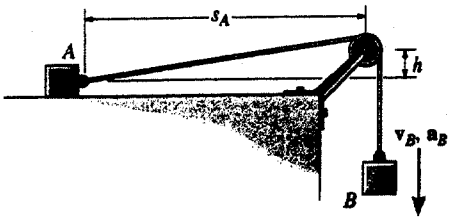
$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

$$s_B = 23.163 \text{ ft}$$

Thus,

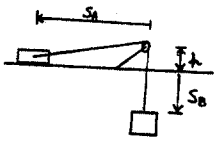
$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow \quad \textbf{Ans}$$

12-193. If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



$$l = s_B + \sqrt{s_A^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\, 2 s_A \dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\dot{s}_B (s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} \quad \textbf{Ans}$$

$$a_A = \dot{v}_A = -\dot{v}_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B\left(\frac{1}{2}\right)\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left(1 + \left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2} \quad \textbf{Ans}$$

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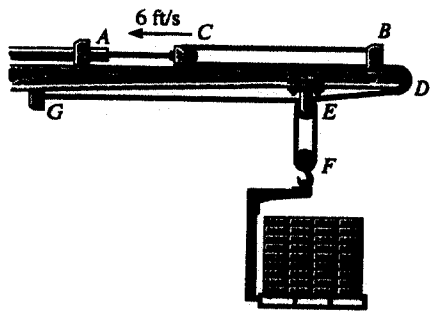
12-194. Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.

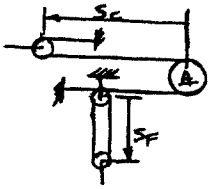


$$2s_C + 2s_F = l$$

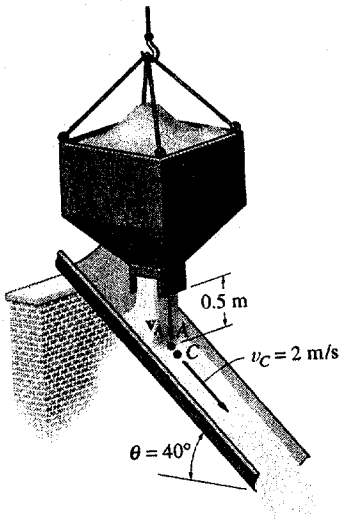
$$2\Delta s_C = -2\Delta s_F$$

$$\Delta s_C = -\Delta s_F$$

$$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft}$$ **Ans**



12-195. Sand falls from rest 0.5 m vertically onto a chute. If the sand then slides with a velocity of $v_C = 2$ m/s down the chute, determine the relative velocity of the sand just falling on the chute at A with respect to the sand sliding down the chute. The chute is inclined at an angle of 40° with the horizontal.



$$(+\uparrow) \quad v_A^2 = (v_A)_1^2 + 2a_c(s_A - s_{A1})$$

$$v_A^2 = 0 + 2(-9.81)(0.5 - 0)$$

$$v_A = -3.1321\text{ m/s}$$

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-3.1321\mathbf{j} = 2\cos 40°\mathbf{i} - 2\sin 40°\mathbf{j} + (v_{A/C})_x\mathbf{i} + (v_{A/C})_y\mathbf{j}$$

$$0 = 2\cos 40° + (v_{A/C})_x$$

$$-3.1321 = -2\sin 40° + (v_{A/C})_y$$

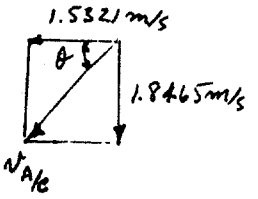
Solving,

$$(v_{A/C})_x = -1.5321$$

$$(v_{A/C})_y = -1.8465$$

$$v_{A/C} = \sqrt{(-1.5321)^2 + (-1.8465)^2} = 2.40\text{ m/s}$$ **Ans**

$$\theta = \tan^{-1}\left(\frac{1.8465}{1.5321}\right) = 50.3°$$ **Ans**



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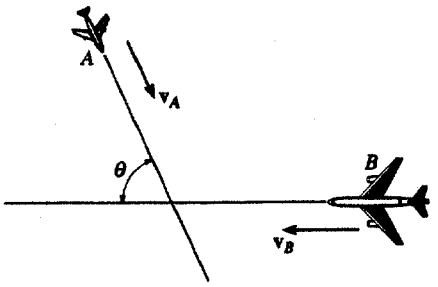
***12-196.** Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[500 \leftarrow] = [600 \overset{75°}{\nabla 0}] + \mathbf{v}_{B/A}$$

$$(\overset{+}{\leftarrow}) \quad 500 = -600\cos 75° + (v_{B/A})_x$$

$$(v_{B/A})_x = 655.29 \leftarrow$$

$$(+\uparrow) \quad 0 = -600\sin 75° + (v_{B/A})_y$$

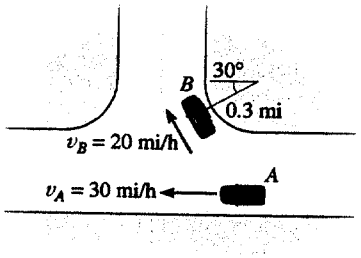
$$(v_{B/A})_y = 579.56 \uparrow$$

$$(v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$$

$$v_{A/B} = 875 \text{ km/h} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{579.56}{655.29}\right) = 41.5° \measuredangle \qquad \text{Ans}$$

12-197. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$20 \overset{30°}{\searrow} = 30 + (v_{B/A})_x + (v_{B/A})_y$$
$$\leftarrow \quad \rightarrow \quad \uparrow$$

$$(\overset{+}{\rightarrow}) \quad -20\sin 30° = -30 + (v_{B/A})_x$$

$$(+\uparrow) \quad 20\cos 30° = (v_{B/A})_y$$

Solving

$$(v_{B/A})_x = 20 \rightarrow$$

$$(v_{B/A})_y = 17.32 \uparrow$$

$$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9° \measuredangle \qquad \text{Ans}$$

$$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$1200 \overset{30°}{\searrow} + \overset{1333.3}{\underset{30°}{\measuredangle 0}} = 0 + (a_{B/A})_x + (a_{B/A})_y$$
$$\rightarrow \quad \uparrow$$

$$(\overset{+}{\rightarrow}) \quad -1200\sin 30° + 1333.3\cos 30° = (a_{B/A})_x$$

$$(+\uparrow) \quad 1200\cos 30° + 1333.3\sin 30° = (a_{B/A})_y$$

Solving

$$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$$

$$a_{B/A} = \sqrt{(554.7)^2 + 1705.9)^2} = 1.79(10^3) \text{ mi/h}^2 \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0° \measuredangle \qquad \text{Ans}$$

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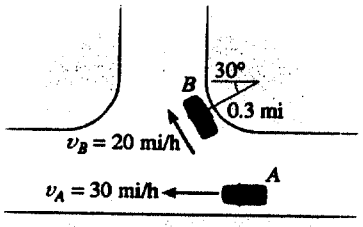
12-198. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If A is increasing its speed at 400 mi/h^2 whereas the speed of B is decreasing at 800 mi/h^2, determine the velocity and acceleration of B with respect to A.



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\left[\frac{20^2}{0.3} = 1333.3\right]_{\measuredangle\theta} + [800]_{30°\,\searrow} = [400]_{\leftarrow} + [(a_{B/A})_x]_{\rightarrow} + [(a_{B/A})_y]_{\uparrow}$$

$$(\overset{+}{\rightarrow}) \quad 1333.3\cos 30° + 800\sin 30° = -400 + (a_{B/A})_x$$

$$(a_{B/A})_x = 1954.7 \rightarrow$$

$$(+\uparrow) \quad 1333.3\sin 30° - 800\cos 30° = (a_{B/A})_y$$

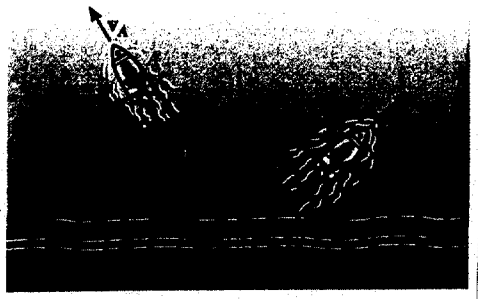
$$(a_{B/A})_y = -26.154 = 26.154 \downarrow$$

$$(a_{B/A}) = \sqrt{(1954.7)^2 + (26.154)^2}$$

$$a_{B/A} = 1955 \text{ mi/h}^2 \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{26.154}{1954.7}\right) = 0.767° \; \measuredangle_B \qquad \text{Ans}$$

12-199. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$ ft/s and $v_B = 15$ ft/s, determine the speed of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20\sin 30°\mathbf{i} + 20\cos 30°\mathbf{j} = 15\cos 45°\mathbf{i} + 15\sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

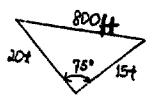
$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} - 6.714\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (-6.714)^2} = 21.7 \text{ ft/s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0° \; \measuredangle \qquad \text{Ans}$$

$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75°$$

$$t = 36.9 \text{ s} \qquad \text{Ans}$$



Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9 \text{ s} \qquad \text{Ans}$$

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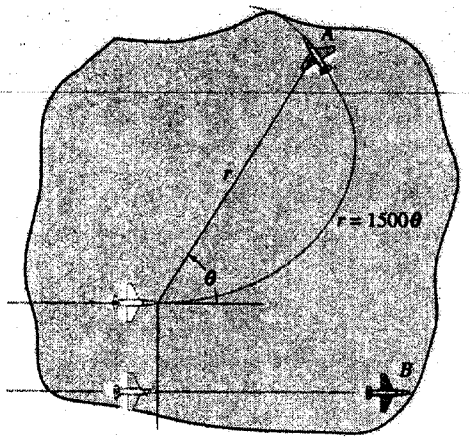
***12-200.** Two planes A and B are flying side by side at a constant speed of 900 km/h. Maintaining this speed, plane A begins to travel along the spiral path $r = (1500\theta)$ km, where θ is in radians, whereas plane B continues to fly in a straight line. Determine the speed of plane A with respect to plane B when $r = 750$ km.



Relative Velocity : At $r = 750$ km, $750 = 1500\theta$, $\theta = 0.5$ rad $= 28.64°$. Here, $\tan\psi = \frac{r}{dr/d\theta} = \frac{1500\theta}{1500} = \theta = 0.5$, $\psi = 26.57°$ and $\theta + \psi = 55.21°$. Applying Eq. 12 – 34 gives

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$
$$900\cos 55.21°\mathbf{i} + 900\sin 55.21°\mathbf{j} = 900\mathbf{i} + \mathbf{v}_{A/B}$$
$$\mathbf{v}_{A/B} = \{-386.52\mathbf{i} + 739.15\mathbf{j}\} \text{ km/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-386.52)^2 + 739.15^2} = 834 \text{ km/h} \qquad \textbf{Ans}$$

The direction of the relative velocity is the same as the drection of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{739.15}{386.52} = 62.4° \qquad \textbf{Ans}$$

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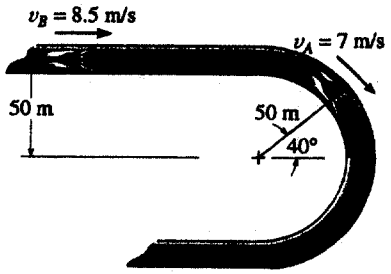
12-201. At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing his speed at 0.5 m/s^2. The bicyclist at B is traveling at 8.5 m/s along the straight-a-way and increasing his speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[7 \underset{40°}{\nwarrow}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$$

$$(\overset{+}{\rightarrow}) \quad 7\sin 40° = 8.5 + (v_{A/B})_x$$

$$(+\downarrow) \quad 7\cos 40° = (v_{A/B})_y$$

Thus,

$$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$$

$$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$$

$$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$$

$$v_{A/B} = 6.69 \text{ m/s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{5.36}{4.00}\right) = 53.3° \measuredangle \qquad \text{Ans}$$

$$(a_A)_n = \frac{7^2}{50} = 0.980 \text{ m/s}^2$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$[0.980] \underset{40°}{\measuredangle} + [0.5] \underset{40°}{\nwarrow} = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$$

$$(\overset{+}{\rightarrow}) \quad -0.980\cos 40° + 0.5\sin 40° = 0.7 + (a_{A/B})_x$$

$$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$$

$$(+\downarrow) \quad 0.980\sin 40° + 0.5\cos 40° = (a_{A/B})_y$$

$$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$$

$$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$$

$$a_{A/B} = 1.52 \text{ m/s}^2 \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{1.013}{1.129}\right) = 41.9° \measuredangle \qquad \text{Ans}$$

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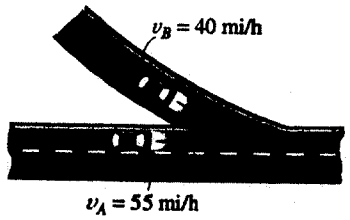
12-202. At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



$$\mathbf{v}_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\ \text{mi/h}$$

$$\mathbf{v}_A = \{-55\mathbf{i}\}\ \text{mi/h}$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\ \text{mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\ \text{mi/h} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5°\ \measuredangle\theta \qquad \textbf{Ans}$$

$$(a_B)_n = \frac{v_A^2}{\rho} = \frac{40^2}{0.5} = 3200\ \text{mi/h}^2 \qquad (a_B)_t = 1200\ \text{mi/h}^2$$

$$\mathbf{a}_B = (3200\cos 60° - 1200\cos 30°)\mathbf{i} + (3200\sin 60° + 1200\sin 30°)\mathbf{j}$$
$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\ \text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

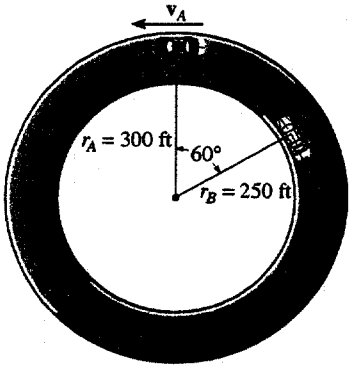
$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\ \text{mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\ \text{mi/h}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6°\ \measuredangle\theta \qquad \textbf{Ans}$$

12-203. Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105\sin 30°\,\mathbf{i} + 105\cos 30°\,\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\}\ \text{ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4\ \text{ft/s} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6°\ \measuredangle\theta \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25\cos 60°\mathbf{i} - 25\sin 60°\mathbf{j} - 44.1\sin 60°\mathbf{i} - 44.1\cos 60°\mathbf{j} + \mathbf{a}_{A/B}$$

$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\}\ \text{ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8\ \text{ft/s}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4°\ \measuredangle\theta \qquad \textbf{Ans}$$

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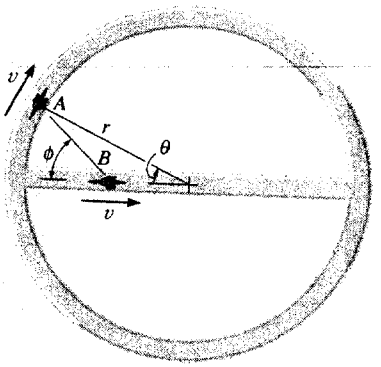
***12-204.** The two cyclists A and B travel at the same constant speed v. Determine the speed of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



$\mathbf{v}_A = v\sin\theta\,\mathbf{i} + v\cos\theta\,\mathbf{j} \qquad \mathbf{v}_B = v\,\mathbf{i}$

$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$

$= (v\sin\theta\,\mathbf{i} + v\cos\theta\,\mathbf{j}) - v\,\mathbf{i}$

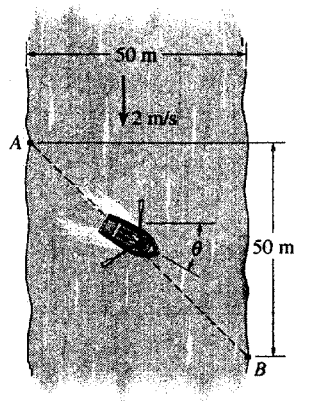
$= (v\sin\theta - v)\mathbf{i} + v\cos\theta\,\mathbf{j}$

$v_{A/B} = \sqrt{(v\sin\theta - v)^2 + (v\cos\theta)^2}$

$= \sqrt{2v^2 - 2v^2\sin\theta}$

$= v\sqrt{2(1-\sin\theta)}$ **Ans**

12-205. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



Relative Velocity :

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$
$$v_b\sin 45°\,\mathbf{i} - v_b\cos 45°\,\mathbf{j} = -2\mathbf{j} + 5\cos\theta\,\mathbf{i} - 5\sin\theta\,\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b\sin 45° = 5\cos\theta \qquad [1]$$
$$-v_b\cos 45° = -2 - 5\sin\theta \qquad [2]$$

Solving Eqs.[1] and [2] yields

$$\theta = 28.57°$$
$$v_b = 6.210\text{ m/s} = 6.21\text{ m/s} \qquad \textbf{Ans}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4\text{ s} \qquad \textbf{Ans}$$

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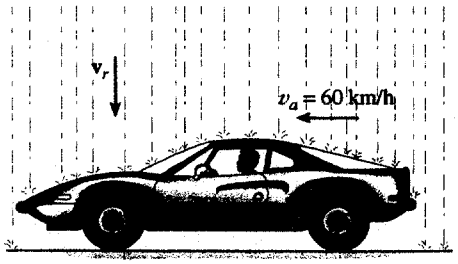
12-206. A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity v_r of the rain if it is assumed to fall vertically.



$$\mathbf{v}_r = \mathbf{v}_a + \mathbf{v}_{r/a}$$

$$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$$

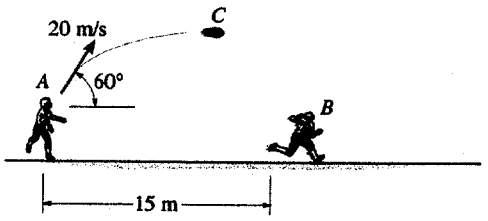
$$(\overset{+}{\rightarrow}) \quad 0 = -60 + v_{r/a}\cos 30°$$

$$(+\uparrow) \quad -v_r = 0 - v_{r/a}\sin 30°$$

$$v_{r/a} = 69.3 \text{ km/h}$$

$$v_r = 34.6 \text{ km/h} \qquad \text{Ans}$$

12-207. At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



Ball:

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$s_C = 0 + 20\cos 60° t$$

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$-20\sin 60° = 20\sin 60° - 9.81 t$$

$$t = 3.53 \text{ s}$$

$$s_C = 35.31 \text{ m}$$

Player B:

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$s_B = 0 + v_B t$$

Require,

$$35.31 = 15 + v_B(3.53)$$

$$v_B = 5.75 \text{ m/s} \qquad \text{Ans}$$

At the time of the catch

$$(v_C)_x = 20\cos 60° = 10 \text{ m/s} \rightarrow$$

$$(v_C)_y = 20\sin 60° = 17.32 \text{ m/s} \downarrow$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$10\mathbf{i} - 17.32\mathbf{j} = 5.75\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$$

$$(\overset{+}{\rightarrow}) \quad 10 = 5.75 + (v_{C/B})_x$$

$$(+\uparrow) \quad -17.32 = (v_{C/B})_y$$

$$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$$

$$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$$

$$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{17.32}{4.25}\right) = 76.2° \ \measuredangle \qquad \text{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

$$-9.81\mathbf{j} = 0 + \mathbf{a}_{C/B}$$

$$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow \qquad \text{Ans}$$

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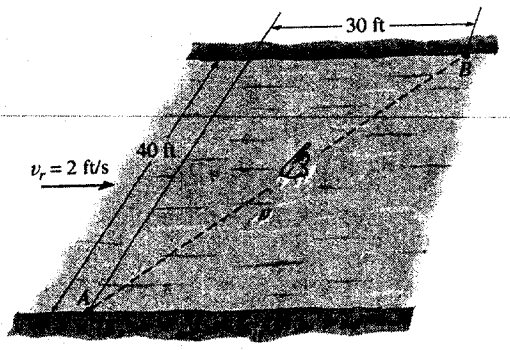
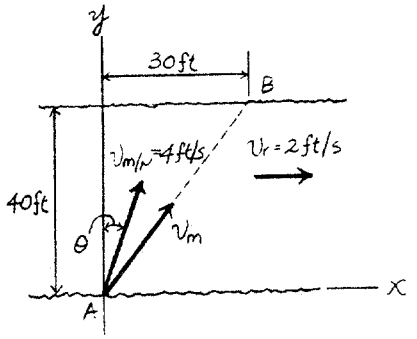
***12-208.** A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself towards point B to reach this point. Why?



Relative Velocity :

$$\mathbf{v}_m = \mathbf{v}_r + \mathbf{v}_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the **i** and **j** components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad [1]$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 13.29°$$

$$v_b = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans}$$

In oder for the man to reached point B, the man has to direct himself at an angle θ = 13.3° with y axis.

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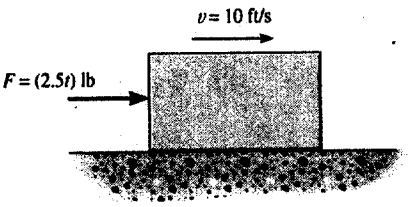
13-1. The moon has a mass of $73.5(10^{21})$ kg, and the earth has a mass of $5.98(10^{24})$ kg. If their centers are $384(10^6)$ m apart, determine the gravitational attractive force between the two bodies.

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = 6.673(10^{-11})\left[\frac{73.5(10^{21})(5.98(10^{24}))}{(384(10^6))^2}\right]$$

$$F = 199(10^{18})\ \text{N} \qquad \textbf{Ans}$$

13-2. The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where t is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.



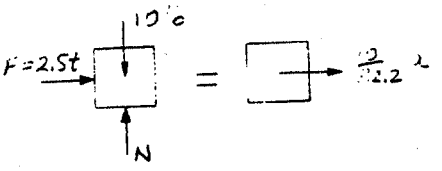
$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \quad 2.5t = \left(\frac{10}{32.2}\right)a$$

$$a = 8.05t$$

$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t 8.05t\,dt$$

$$v = 4.025t^2 + 10$$



When $t = 3$ s,

$$v = 46.2\ \text{ft/s} \qquad \textbf{Ans}$$

$$ds = v\,dt$$

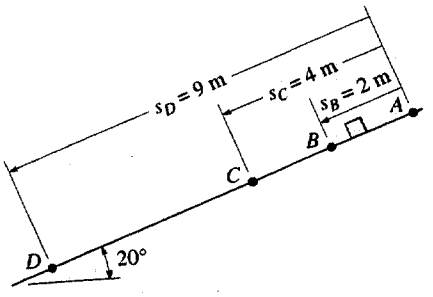
$$\int_0^s ds = \int_0^t (4.025t^2 + 10)\,dt$$

$$s = 1.3417t^3 + 10t$$

When $t = 3$ s,

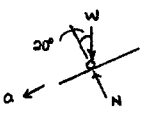
$$s = 66.2\ \text{ft} \qquad \textbf{Ans}$$

13-3. By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., $s \propto t^2$, by determining the time t_B, t_C, and t_D needed for a block of mass m to slide from rest at A to points B, C, and D, respectively. Neglect the effects of friction.



$$W\sin 20° = \frac{W}{g}a$$

$$a = 9.81(\sin 20°) = 3.355\ \text{m/s}^2$$



$$s = \frac{1}{2}at^2$$

s	t	
2 m	1.09 s	Ans
4 m	1.54 s	Ans
9 m	2.32 s	Ans

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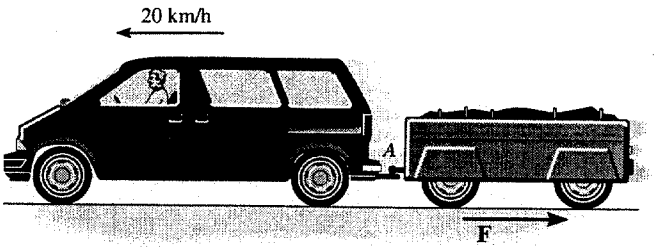
***13-4.** The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



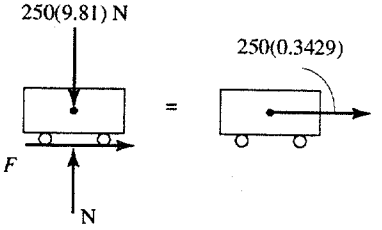
$$20 \text{ km/h} = \frac{20(10^3)}{3600} = 5.556 \text{ m/s}$$

$$(\overset{+}{\leftarrow}) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

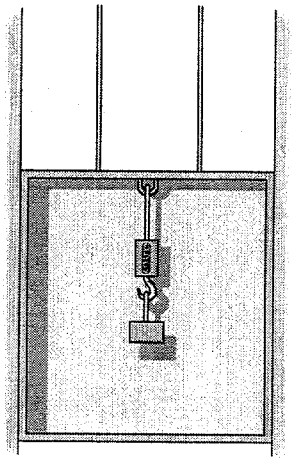
$$0 = 5.556^2 + 2(a)(45 - 0)$$

$$a = -0.3429 \text{ m/s}^2 = 0.3429 \text{ m/s}^2 \rightarrow$$

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad F = 250(0.3429) = 85.7 \text{ N} \quad \textbf{Ans}$$



13-5. A block having a mass of 2 kg is placed on a spring scale located in an elevator that is moving downward. If the scale reading, which measures the force in the spring, is 20 N, determine the acceleration of the elevator. Neglect the mass of the scale.

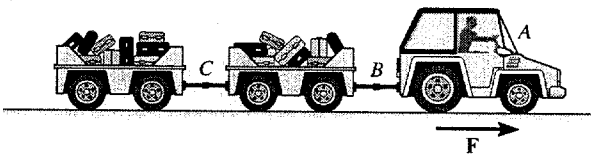


$$+\uparrow \sum F_y = ma_y; \quad 20 - 2(9.81) = 2a$$

$$a = 0.19 \text{ m/s}^2 \uparrow \qquad \textbf{Ans}$$

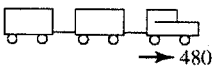
The elevator is slowing down.

13-6. The baggage truck A has a mass of 800 kg and is used to pull the two cars, each with mass 300 kg. If the tractive force F on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.



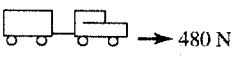
$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 480 = [800 + 2(300)]a$$

$$a = 0.3429 = 0.343 \text{ m/s}^2 \quad \textbf{Ans}$$



$$\overset{+}{\rightarrow} \sum F_x = ma_x: \quad 480 = (800 + 300)a$$

$$a = 0.436 \text{ m/s}^2 \qquad \textbf{Ans}$$



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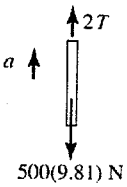
13-7. The 500-kg fuel assembly for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that $s = 0$ and $v = 0$ when $t = 0$, and $s = 2.5$ m when $t = 1.5$ s. Determine the tension in the cable at A during the motion.

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$2.5 = 0 + 0 + \frac{1}{2}(a)(1.5)^2$

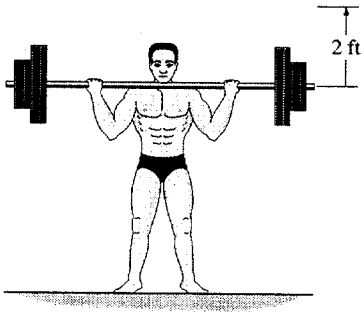


$a = 2.222\ \text{m/s}^2$

$+\uparrow \sum F_y = ma_y;\quad 2T - 500(9.81) = 500(2.222)$

$T = 3008\ \text{N} = 3.01\ \text{kN}$ **Ans**

***13-8.** The man weighs 180 lb and supports the barbells which have a weight of 100 lb. If he lifts them 2 ft in the air in 1.5 s starting from rest, determine the reaction of *both* of his feet on the ground during the lift. Assume the motion is with uniform acceleration.



$+\uparrow \sum F_y = ma_y;\quad F - 100 - 180 = \frac{100}{32.2} a$

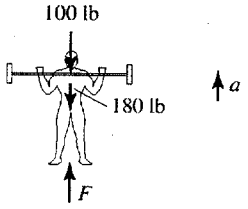
$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$2 = 0 + 0 + \frac{1}{2} a (1.5)^2$

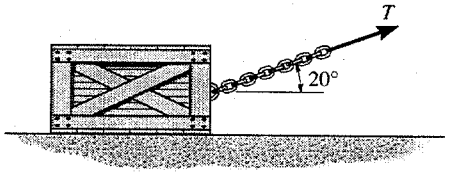
$a = 1.778\ \text{ft/s}^2$

Thus,

$F = 285.52 = 286\ \text{lb}$ **Ans**



13-9. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **T** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



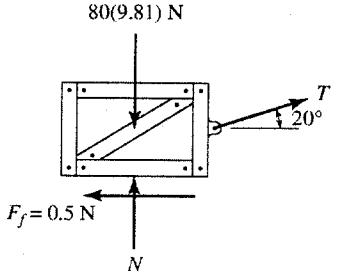
Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a)

$+\uparrow \sum F_y = 0;\quad N + P \sin 20° - 80(9.81) = 0$ [1]

$\overset{+}{\rightarrow} \sum F_x = 0;\quad T \cos 20° - 0.5N = 0$ [2]

Solving Eqs. [1] and [2] yields

$T = 353.29\ \text{N}\qquad N = 663.97\ \text{N}$



(a)

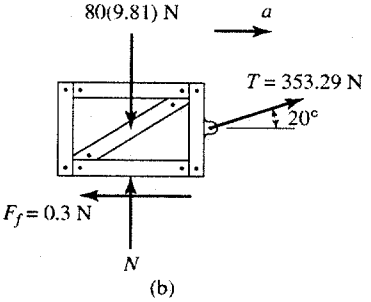
Equation of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$+\uparrow \sum F_y = ma_y;\quad N - 80(9.81) + 353.29 \sin 20° = 80(0)$

$N = 663.97\ \text{N}$

$\overset{+}{\rightarrow} \sum F_x = ma_x;\quad 353.29 \cos 20° - 0.3(663.97) = 80a$

$a = 1.66\ \text{m/s}^2$ **Ans**



(b)

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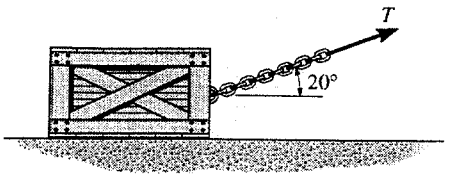
13-10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$ and the coefficient of kinetic friction is $\mu_k = 0.3$ and the towing force is $T = (90t^2)$ N, where t is in seconds.



Equations of Equilibrium: At $t = 2$ s, $T = 90(2^2) = 360$ N. From FBD(a)

$+\uparrow \sum F_y = 0;$ $\quad N + 360\sin 20° - 80(9.81) = 0$ $\quad N = 661.67$ N

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad 360\cos 20° - F_f = 0$ $\quad F_f = 338.29$ N

Since $F_f > (F_f)_{max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.

Equation of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

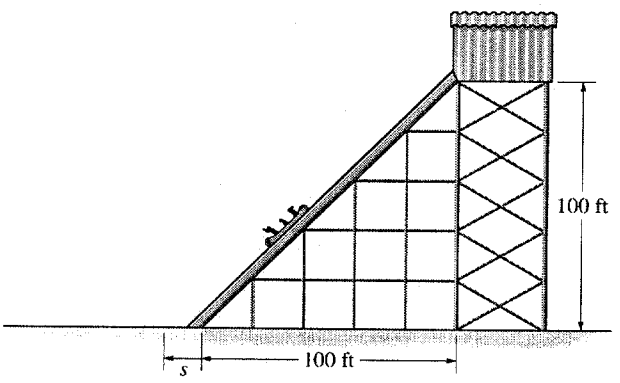
$+\uparrow \sum F_y = ma_y;$ $\quad N - 80(9.81) + 360\sin 20° = 80(0)$

$N = 661.67$ N

$\overset{+}{\rightarrow} \sum F_x = ma_x;$ $\quad 360\cos 20° - 0.3(661.67) = 80a$

$a = 1.75$ m/s^2 **Ans**

13-11. The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_r = 30$ lb, and in the pool for a short distance $F_r = 80$ lb, determine how fast the sled is traveling when $s = 5$ ft.



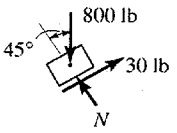
$+\swarrow \sum F_x = ma_x;$ $\quad 800\sin 45° - 30 = \dfrac{800}{32.2}a$

$a = 21.561$ ft/s^2

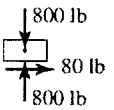
$v_1^2 = v_0^2 + 2a_c(s - s_0)$

$v_1^2 = 0 + 2(21.561)(100\sqrt{(2 - 0)})$

$v_1 = 78.093$ ft/s



$\overset{+}{\leftarrow} \sum F_x = ma_x;$ $\quad -80 = \dfrac{800}{32.2}a$



$a = -3.22$ ft/s^2

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (78.093)^2 + 2(-3.22)(5 - 0)$

$v_2 = 77.9$ ft/s **Ans**

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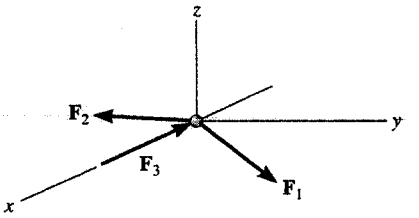
***13-12.** The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



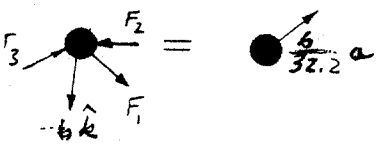
$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components :

$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \qquad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \qquad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $v = 0$, $t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \qquad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \qquad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$



Since $ds = v\,dt$, integrating from $s = 0$, $t = 0$ yields

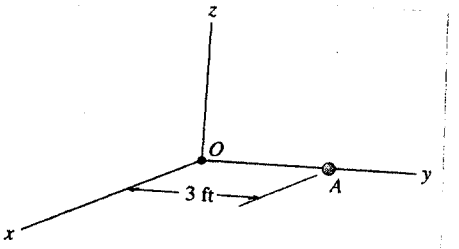
$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \qquad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \qquad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$

When $t = 2$ s then, $s_x = 14.31$ ft, $s_y = 35.78$ ft $s_z = -89.44$ ft

Thus,

$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft} \quad \textbf{Ans}$$

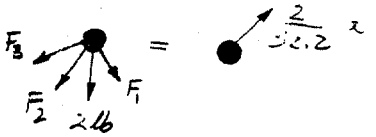
13-13. The 2-lb particle A is acted upon by its weight and the force system $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{3\mathbf{i} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{1\mathbf{i} - t^2\mathbf{j} - 2\mathbf{k}\}$ lb, where t is in seconds. Determine the distance the particle is from the origin 3 s after it has been released from rest.



$$\Sigma F_x = ma_x; \quad 2 + 3 + 1 = \left(\frac{2}{32.2}\right)a_x$$

$$\Sigma F_y = ma_y; \quad 6 - t^2 = \left(\frac{2}{32.2}\right)a_y$$

$$\Sigma F_z = ma_z; \quad -2 - 1 - 2 - 2 = \left(\frac{2}{32.2}\right)a_z$$



Since $dv = a\,dt$, integrating from $v = 0$ when $t = 0$,

$$\left(\frac{2}{32.2}\right)v_x = 6t \qquad \left(\frac{2}{32.2}\right)v_y = 6t - \frac{t^3}{3} \qquad \left(\frac{2}{32.2}\right)v_z = -7t$$

Since $ds = v\,dt$, integrating from $s_x = s_z = 0$, $s_y = 3$ ft when $t = 0$,

$$\left(\frac{2}{32.2}\right)s_x = 3t^2 \qquad \left(\frac{2}{32.2}\right)s_y = 3t^2 - \frac{t^4}{12} + 0.186\,34 \qquad \left(\frac{2}{32.2}\right)s_z = -\frac{7}{2}t^2$$

When $t = 3$ s, $s_x = 434.70$ ft $s_y = 329.025$ ft $s_z = -507.15$ ft

$$s = \sqrt{(434.70)^2 + (329.25)^2 + (-507.15)^2} = 745 \text{ ft} \quad \textbf{Ans}$$

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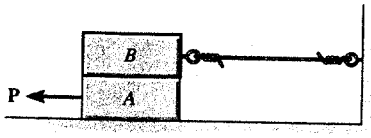
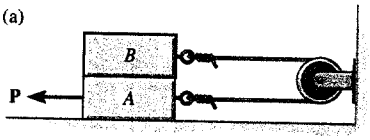
13-14. Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)

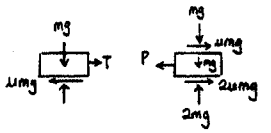


(b)

(a) **Block A :**

$$\stackrel{+}{\leftarrow} \Sigma F_x = m a_x; \quad P - 3\mu mg = m a_A$$

$$a_A = \frac{P}{m} - 3\mu g \qquad \text{Ans}$$



(b) $s_B + s_A = l$

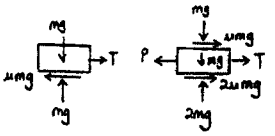
$$a_A = -a_B \qquad (1)$$

Block A :

$$\stackrel{+}{\leftarrow} \Sigma F_x = m a_x; \quad P - T - 3\mu mg = m a_A \qquad (2)$$

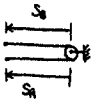
Block B :

$$\stackrel{+}{\leftarrow} \Sigma F_x = m a_x; \quad \mu mg - T = m a_B \qquad (3)$$



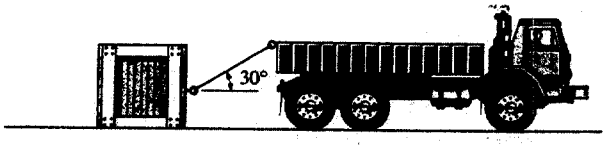
Subtract Eq.(3) from Eq.(2) :

$$P - 4\mu mg = m(a_A - a_B)$$



Use Eq.(1); $\quad a_A = \frac{P}{2m} - 2\mu g \qquad$ Ans

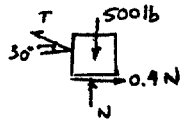
13-15. The driver attempts to tow the crate using a rope that has a tensile strength of 200 lb. If the crate is originally at rest and has a weight of 500 lb, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



Equilibrium : In order to slide the crate, the towing force must overcome static friction.

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \quad -T\cos 30° + 0.4N = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad N + T\sin 30° - 500 = 0 \qquad [2]$$



Solving Eqs.[1] and [2] yields :

$T = 187.6$ lb $\quad N = 406.2$ lb

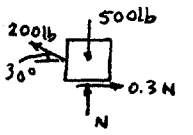
Since $T < 200$ lb, the cord will not break at the moment the crate slides.

After the crate begins to slide, the kinetic friction is used for the calculation.

$$+\uparrow \Sigma F_y = ma_y; \quad N + 200\sin 30° - 500 = 0 \qquad N = 400 \text{ lb}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \quad 200\cos 30° - 0.3(400) = \frac{500}{32.2}a$$

$$a = 3.43 \text{ ft/s}^2 \qquad \text{Ans}$$



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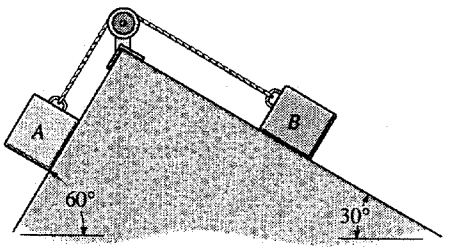
***13-16.** The double inclined plane supports two blocks A and B, each having a weight of 10 lb. If the coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.1$, determine the acceleration of each block.



Equation of Motion: Since blocks A and B are sliding along the plane, the friction forces developed between the blocks and the plane are $(F_f)_A = \mu_k N_A = 0.1 N_A$ and $(F_f)_B = \mu_k N_B = 0.1 N_B$. Here, $a_A = a_B = a$. Applying Eq. 13-7 to FBD(a), we have

$$\nwarrow + \sum F_{y'} = ma_{y'}; \quad N_A - 10\cos 60° = \left(\frac{10}{32.2}\right)(0) \quad N_A = 5.00 \text{ lb}$$

$$\nearrow + \sum F_{x'} = ma_{x'}; \quad T + 0.1(5.00) - 10\sin 60° = -\left(\frac{10}{32.2}\right)a \quad [1]$$

From FBD(b),

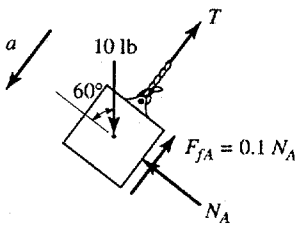
$$\nearrow + \sum F_{y'} = ma_{y'}; \quad N_B - 10\cos 30° = \left(\frac{10}{32.2}\right)(0) \quad N_B = 8.660 \text{ lb}$$

$$\nwarrow + \sum F_{x'} = ma_{x'}; \quad T - 0.1(8.660) - 10\sin 30° = \left(\frac{10}{32.2}\right)a \quad [2]$$

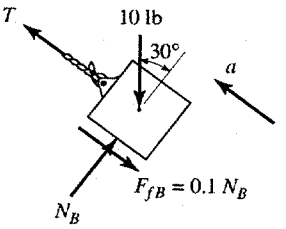
Solving Eqs. [1] and [2] yields

$a = 3.69 \text{ ft/s}^2$ **Ans**

$T = 7.013$ lb

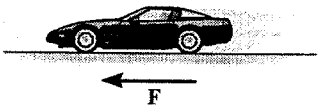


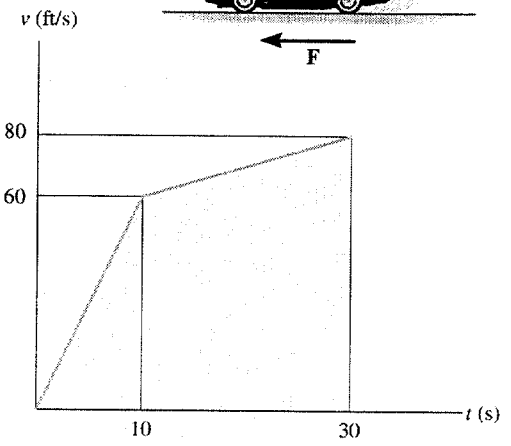
(a)



(b)

13-17. The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.





Kinematics: For $\mathbf{0} \le t < \mathbf{10}$ **s**, $v = \frac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $\mathbf{10} < t \le \mathbf{30}$ **s**, $\frac{v - 60}{t - 10} = \frac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$

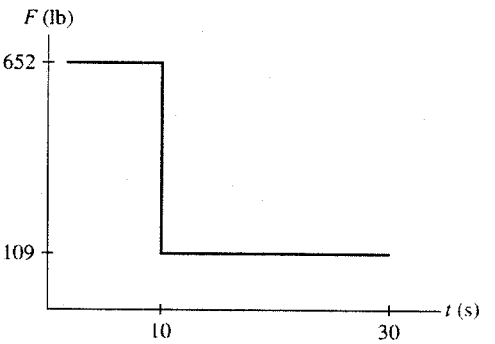
Equation of Motion:

For $\mathbf{0} \le t < \mathbf{10}$ **s**

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb} \quad \textbf{Ans}$$

For $\mathbf{10} < t \le \mathbf{30}$ **s**

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb} \quad \textbf{Ans}$$



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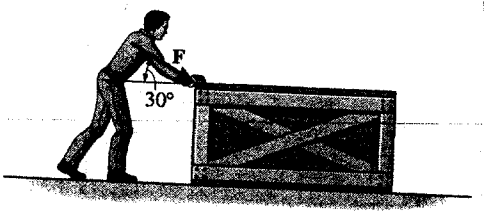
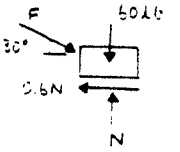
13-18. The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the static coefficient of friction is $\mu_s = 0.6$ and the kinetic coefficient of friction is $\mu_k = 0.3$.



Force to produce motion :

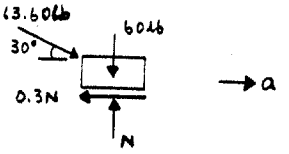


$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F\cos 30° - 0.6N = 0$

$+\uparrow \Sigma F_y = 0; \quad N - 60 - F\sin 30° = 0$

$N = 91.80 \text{ lb} \quad F = 63.60 \text{ lb}$

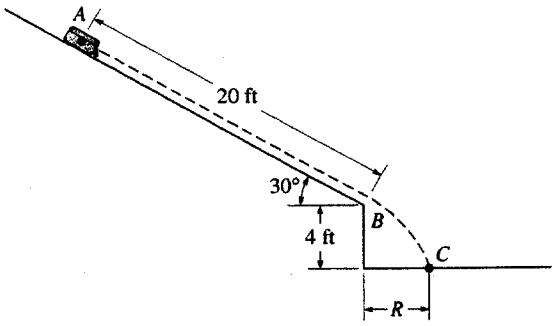
Since $N = 91.80$ lb,



$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 63.60\cos 30° - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$

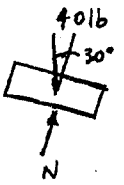
$a = 14.8 \text{ ft/s}^2$ **Ans**

13-19. A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?



$+\searrow \Sigma F_x = m a_x; \quad 40\sin 30° = \frac{40}{32.2}a$

$a = 16.1 \text{ ft/s}^2$



$(+\searrow) v^2 = v_0^2 + 2a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38 \text{ ft/s}$

$(+\searrow) v = v_0 + a_c t;$

$25.38 = 0 + 16.1\, t_{AB}$

$t_{AB} = 1.576 \text{ s}$

$(\overset{+}{\rightarrow}) s_x = (s_x)_0 + (v_x)_0 t$

$R = 0 + 25.38\cos 30° (t_{BC})$

$(+\downarrow) s_y = (s_y)_0 + (v_y)_0 t + \frac{1}{2}a_c t^2$

$4 = 0 + 25.38\sin 30° t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2413 \text{ s}$

$R = 5.30 \text{ ft}$ **Ans**

Total time $= t_{AB} + t_{BC} = 1.82 \text{ s}$ **Ans**

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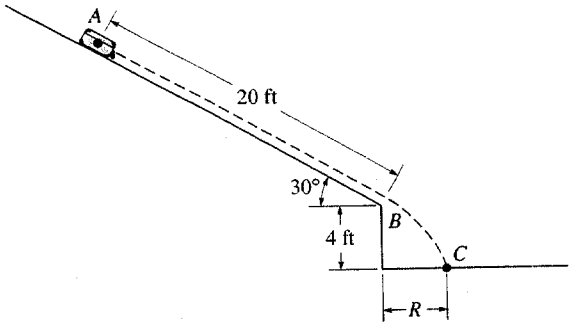
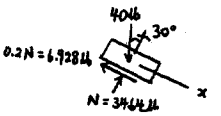
***13-20.** Solve Prob. 13–19 if the suitcase has an initial velocity down the ramp of $v_A = 10\,\text{ft/s}$ and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



$+\nwarrow \Sigma F_x = m a_x;\quad 40\sin 30^\circ - 6.928 = \frac{40}{32.2}a$

$a = 10.52\ \text{ft/s}^2$



$(+\nwarrow)\ v^2 = v_0^2 + 2a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82\ \text{ft/s}$

$(+\nwarrow)\ v = v_0 + a_c t;$

$22.82 = 10 + 10.52\, t_{AB}$

$t_{AB} = 1.219\ \text{s}$

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\, t$

$R = 0 + 22.82\cos 30^\circ\,(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\, t + \frac{1}{2}a_c t^2$

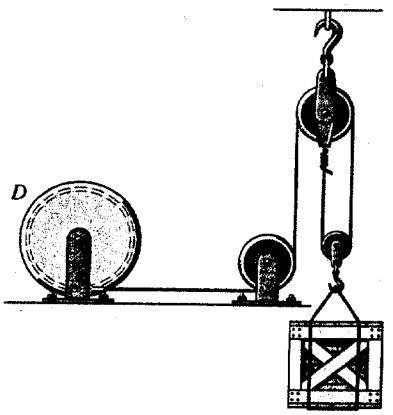
$4 = 0 + 22.82\sin 30^\circ\, t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2572\ \text{s}$

$R = 5.08\ \text{ft}$ **Ans**

Total time $= t_{AB} + t_{BC} = 1.48\ \text{s}$ **Ans**

13-21. The winding drum D is drawing in the cable at an accelerated rate of $5\,\text{m/s}^2$. Determine the cable tension if the suspended crate has a mass of 800 kg.

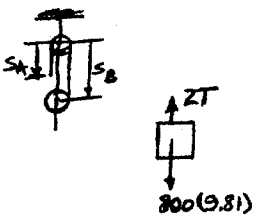


$s_A + 2s_B = l$

$a_A = -2a_B$

$5 = -2a_B$

$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2\uparrow$



$+\uparrow \Sigma F_y = ma_y;\quad 2T - 800(9.81) = 800(2.5)$

$T = 4924\ \text{N} = 4.92\ \text{kN}$ **Ans**

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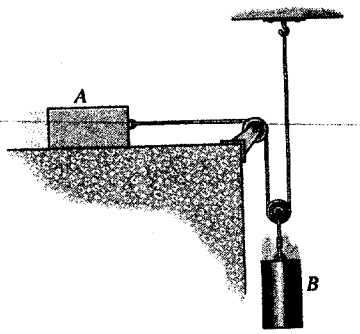
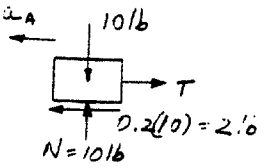
13-22. The 10-lb block A is traveling to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.



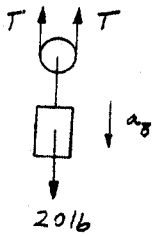
Block A :

$\stackrel{+}{\leftarrow} \Sigma F_x = ma_x; \quad -T + 2 = \left(\frac{10}{32.2}\right)a_A \quad (1)$



Weight B :

$+\downarrow \Sigma F_y = ma_y; \quad 20 - 2T = \left(\frac{20}{32.2}\right)a_B \quad (2)$



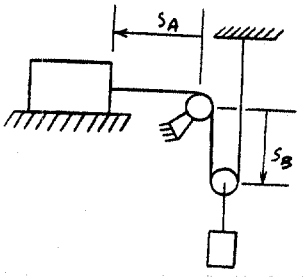
Kinematics :

$s_A + 2s_B = l$

$a_A = -2a_B \quad (3)$

Solving Eq.s (1) – (3) :

$a_A = -17.173 \text{ ft/s}^2 \quad a_B = 8.587 \text{ ft/s}^2 \quad T = 7.33 \text{ lb}$

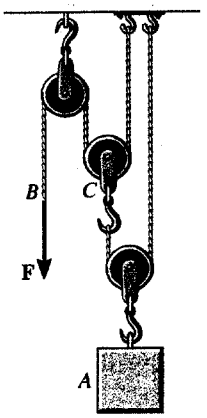


$v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = (2)^2 + 2(17.173)(4 - 0)$

$v = 11.9$ ft/s **Ans**

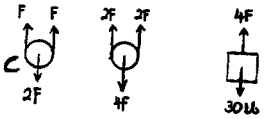
13-23. A force $F = 15$ lb is applied to the cord. Determine how high the 30-lb block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.



$+\uparrow \Sigma F_y = ma_y; \quad -30 + 4F = \frac{30}{32.2}a_A$

$F = 15$ lb

$a_A = 32.2 \text{ ft/s}^2$



$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 0 + \frac{1}{2}(32.2)(2)^2$

$s = 64.4$ ft **Ans**

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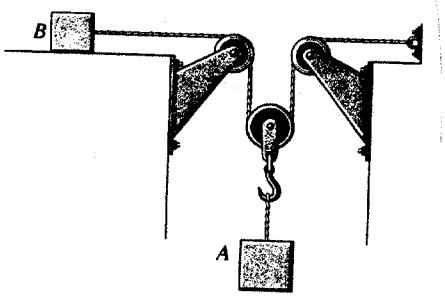
***13-24.** At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



Block A :

$$+\downarrow \Sigma F_y = ma_y; \quad 10 - 2T = \frac{10}{32.2}a_A$$

Block B :

$$\stackrel{+}{\leftarrow} \Sigma F_x = ma_x; \quad -T + 0.2(4) = \frac{4}{32.2}a_B$$

$$2s_A + s_B = l$$

$$2a_A = -a_B$$

Solving:

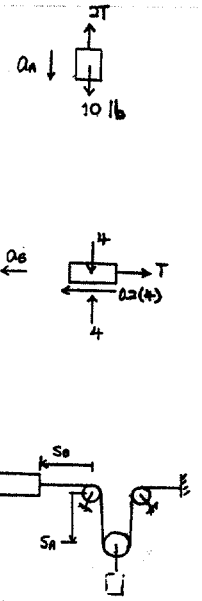
$T = 3.38$ lb

$a_A = 10.403$ ft/s^2

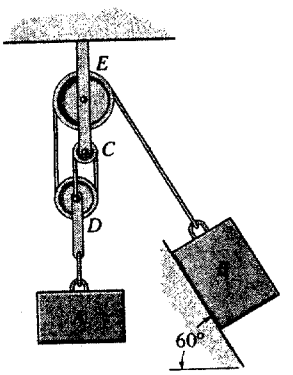
$a_B = -20.81$ ft/s^2

$$(+\downarrow)\ v_A = (v_A)_0 + a_A t$$

$$v_A = 6 + 10.403(2) = 26.8 \text{ ft/s}$$ **Ans**



13-25. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



Kinematic : Applying equation $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$, we have

$$(+) \quad 0.75 = 0 + 0 + \frac{1}{2}a_B(2^2) \qquad a_B = 0.375 \text{ m/s}^2$$

Establish the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \qquad [1]$$

From Eq. [1],

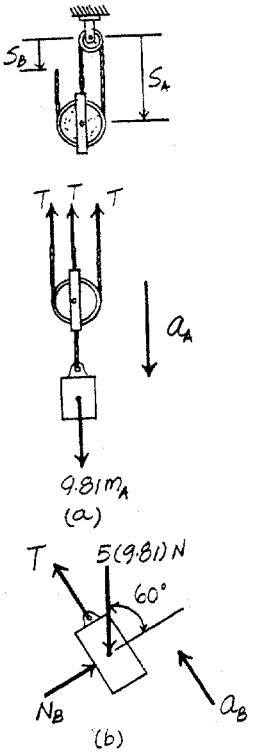
$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

$$+\Sigma F_{y'} = ma_{y'}; \quad T - 5(9.81)\sin 60° = 5(0.375)$$
$$T = 44.35 \text{ N}$$

From FBD(a),

$$+\uparrow \Sigma F_y = ma_y; \quad 3(44.35) - 9.81m_A = m_A(-0.125)$$
$$m_A = 13.7 \text{ kg}$$ **Ans**



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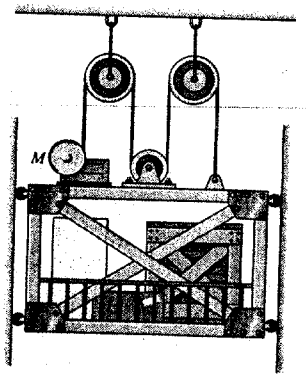
13-26. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by using the track and wheels mounted along its sides. Starting from rest, in $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* Determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys and cables.



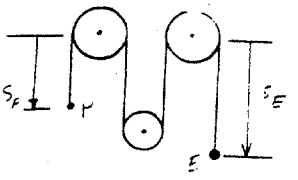
$3s_E + s_P = l$

$3v_E = -v_P$

$(+\downarrow)$ $v_P = v_E + v_{P/E}$

$-3v_E = v_E + 6$

$v_E = -\frac{6}{4} = -1.5 \text{ m/s} = 1.5 \text{ m/s} \uparrow$



$(+\uparrow)$ $v = v_0 + a_c t$

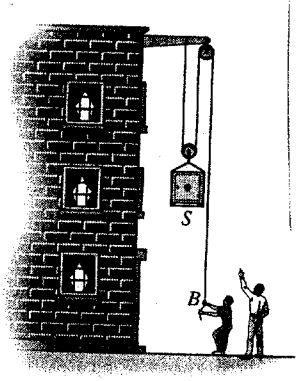
$1.5 = 0 + a_E(2)$

$a_E = 0.75 \text{ m/s}^2 \uparrow$ **Ans**

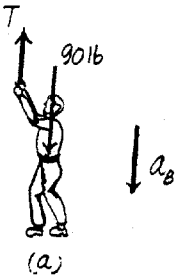
$+\uparrow \Sigma F_y = ma_y$; $4T - 500(9.81) = 500(0.75)$

$T = 1320 \text{ N} = 1.32 \text{ kN}$ **Ans**

13-27. The safe S has a weight of 200 lb and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight 90 lb, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.



Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(a),

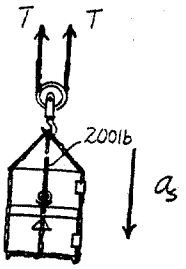


(a)

$+\uparrow \Sigma F_y = ma_y$; $T - 90 = -\left(\frac{90}{32.2}\right)a_B$ [1]

From FBD(b),

$+\uparrow \Sigma F_y = ma_y$; $2T - 200 = -\left(\frac{200}{32.2}\right)a_S$ [2]



(b)

Kinematic : Establish the position - coordinate equation, we have

$2s_S + s_B = l$

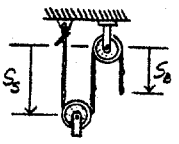
Taking time derivative twice yields

$(+\downarrow)$ $2a_S + a_B = 0$ [3]

Solving Eqs. [1], [2] and [3] yields

$a_B = -2.30 \text{ ft/s}^2 = 2.30 \text{ ft/s}^2 \uparrow$ **Ans**

$a_S = 1.15 \text{ ft/s}^2 \downarrow$ $T = 96.43 \text{ lb}$



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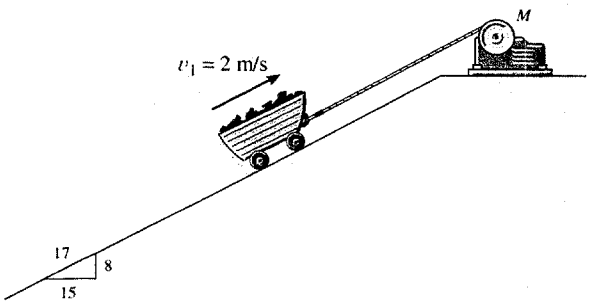
***13-28.** The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.

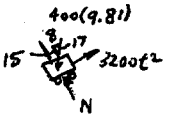


$+\nearrow \Sigma F_s = ma_s$; $3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a$ $a = 8t^2 - 4.616$

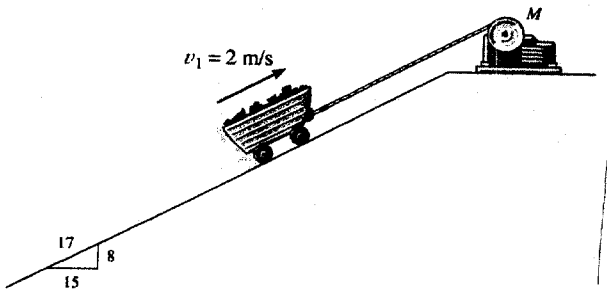
$dv = a\,dt$

$\int_2^v dv = \int_0^2 (8t^2 - 4.616)\,dt$

$v = 14.1$ m/s **Ans**



13-29. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



$+\nearrow \Sigma F_s = ma_s$; $3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a$ $a = 8t^2 - 4.616$

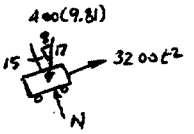
$dv = a\,dt$

$\int_2^v dv = \int_0^t (8t^2 - 4.616)\,dt$

$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$

$\int_0^s ds = \int_0^2 (2.667t^3 - 4.616t + 2)\,dt$

$s = 5.43$ m **Ans**



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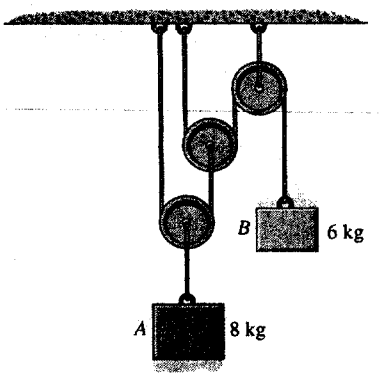
13-30. Determine the tension developed in the cords attached to each block and the accelerations of the blocks. Neglect the mass of the pulleys and cords.



Equation of Motion : From FBD(a),

$$+\uparrow \Sigma F_y = ma_y; \qquad T_A - 8(9.81) = -8a_A \qquad [1]$$

From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad T_B - 6(9.81) = -6a_B \qquad [2]$$

From FBD(c),

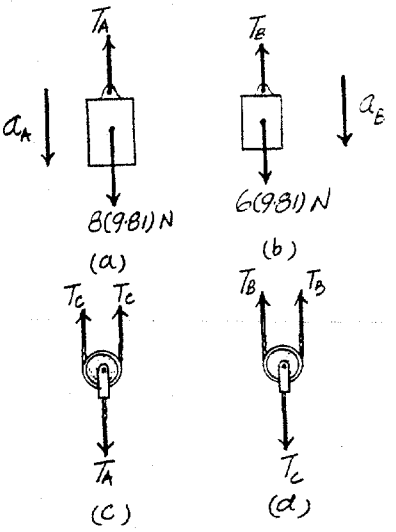
$$+\uparrow \Sigma F_y = 0; \qquad 2T_C - T_A = 0 \qquad [3]$$

From FBD(d),

$$+\uparrow \Sigma F_y = 0; \qquad 2T_B - T_C = 0 \qquad [4]$$

Eliminate T_C from Eqs. [3] and [4] yields

$$T_A = 4T_B \qquad [5]$$



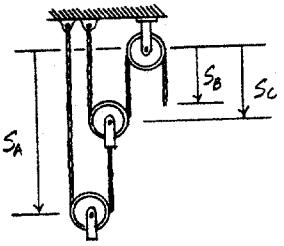
Kinematic : Establish the position-coordinate equation, we have

$$s_A + (s_A - s_C) = l_1 \qquad [6]$$

$$2s_C + s_B = l_2 \qquad [7]$$

Eliminate s_C from Eqs. [6] and [7] yields

$$4s_A + s_B = 2l_1 + l_2 \qquad [8]$$



Taking time derivative twice for Eq. [8] yields

$$(+\downarrow) \qquad 4a_A + a_B = 0 \qquad [9]$$

Solving Eqs. [1], [2], [5] and [9] yields

$a_A = -1.51 \text{ m/s}^2 = 1.51 \text{ m/s}^2 \uparrow$ $\quad a_B = 6.04 \text{ m/s}^2 \downarrow$ **Ans**

$T_A = 90.6 \text{ N}$ $\quad T_B = 22.6 \text{ N}$ **Ans**

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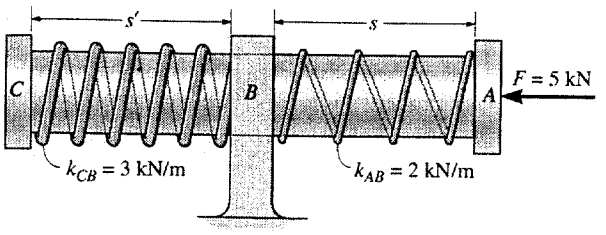
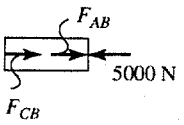
13-31. The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is originally at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



$F_{CB} = k_{CB}x = 3000x \quad F_{AB} = k_{AB}x = 2000x$



$\overset{+}{\leftarrow} \sum F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

$2500 - 2500x = a$

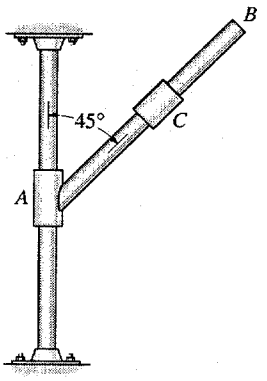
$a\,dx = v\,dv$

$$\int_0^{0.2} (2500 - 2500x)dx = \int_0^v v\,dv$$

$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

$v = 30$ m/s **Ans**

***13-32.** The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to a downward acceleration of 2 m/s^2. In all cases, the collar moves in the plane.



(a)

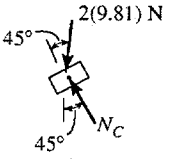
$+\swarrow \sum F_{x'} = ma_{x'}; \quad 2(9.81)\sin 45° = 2a_C \quad a_C = 6.94$ m/s^2 **Ans**

(b) From part (a) $\mathbf{a}_{C/A} = 6.94$ m/s^2

$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$ Where $\mathbf{a}_A = 0$

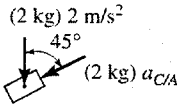
$= 6.94$ m/s^2 **Ans**

(c)



$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$

$= \underset{\nearrow}{2} + a_{C/A} \swarrow \quad [1]$



$+\swarrow \sum F_{x'} = ma_{x'}; \quad 2(9.81)\sin 45° = 2(2\cos 45° + a_{C/A})$

$a_{C/A} = 5.5225$ m/s^2 $\swarrow$

From Eq. [1]

$\mathbf{a}_C = \underset{\downarrow}{2} + \underset{\swarrow}{5.5225} = \underset{\leftarrow}{3.905} + \underset{\downarrow}{5.905}$

$a_C = \sqrt{3.905^2 + 5.905^2} = 7.08$ m/s^2 **Ans**

$\theta = \tan^{-1}\dfrac{5.905}{3.905} = 56.5°$ $\theta\swarrow$ **Ans**

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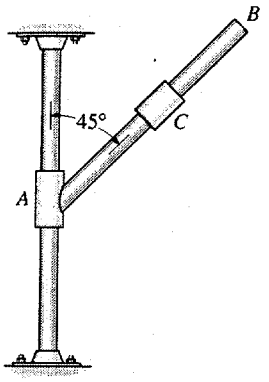
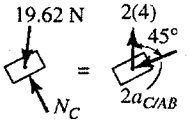
13-33. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration of 4 m/s^2.





$\overset{+}{\leftarrow}\sum F_x = ma_x;\quad N\sin 45° = 2a_{C/AB}\sin 45°$

$N = 2a_{C/AB}$

$+\uparrow\sum F_y = ma_y;\quad N\cos 45° - 19.62 = 2(4) - 2a_{C/AB}\cos 45°$

$a_{C/AB} = 9.76514$

$\mathbf{a}_C = \mathbf{a}_{AB} + \mathbf{a}_{C/AB}$

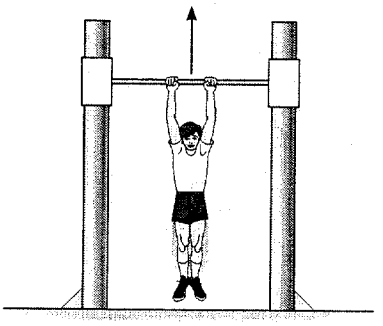
$(a_C)_x = 0 + 9.76514\sin 45° = 6.905 \leftarrow$

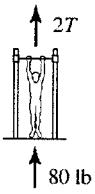
$(a_C)_y = 4 - 9.76514\cos 45° = 2.905 \downarrow$

$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49\ \text{m/s}^2$ **Ans**

$\theta = \tan^{-1}\left(\frac{2.905}{6.905}\right) = 22.8°$ **Ans**

13-34. The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of $v = (4t^2)$ ft/s, where t is in seconds.





a) $T = 40$ lb **Ans**

b) $v = 4t^2$

$a = 8t$

$+\uparrow\sum F_y = ma_y;\quad 2T - 80 = \frac{80}{32.2}(8t)$

At $t = 2$ s,

$= 59.9$ lb **Ans**

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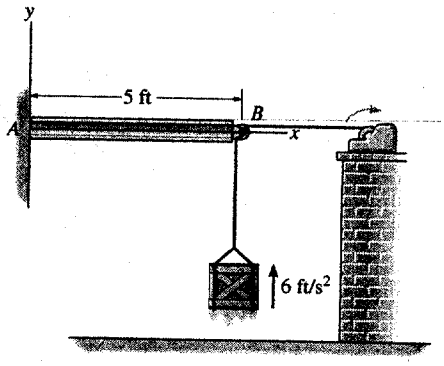
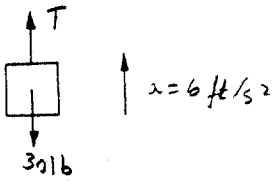
13-35. The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



Crate :

$+\uparrow \Sigma F_y = ma_y;\quad T - 30 = \left(\frac{30}{32.2}\right)(6) \quad T = 35.59 \text{ lb}$

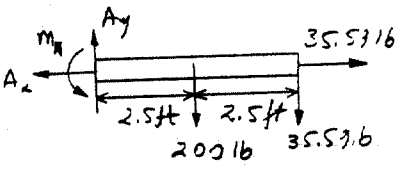


Beam :

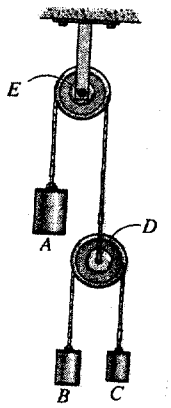
$\rightarrow \Sigma F_x = 0;\quad -A_x + 35.59 = 0 \quad A_x = 35.6 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 200 - 35.59 = 0 \quad A_y = 236 \text{ lb}$ **Ans**

$(+\Sigma M_A = 0;\quad M_A - 200(2.5) - (35.59)(5) = 0 \quad M_A = 678 \text{ lb} \cdot \text{ft}$ **Ans**



***13-36.** If cylinders B and C have a mass of 15 kg and 10 kg, respectively, determine the required mass of A so that it does not move when all the cylinders are released. Neglect the mass of the pulleys and the cords.



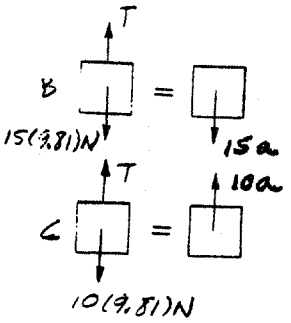
Point D does not move.

For B :

$+\downarrow \Sigma F_y = ma_y;\quad 15(9.81) - T = 15a$

For C :

$+\uparrow \Sigma F_y = ma_y;\quad -10(9.81) + T = 10a$



Solving,

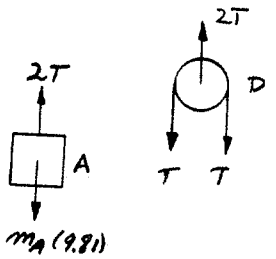
$a = 1.962 \text{ m/s}^2 \quad T = 117.72 \text{ N}$

By statics :

For A :

$+\uparrow \Sigma F_y = ma_y;\quad 2T - m_A(9.81) = 0$

$$m_A = \frac{2(117.72)}{9.81} = 24 \text{ kg}$$ **Ans**



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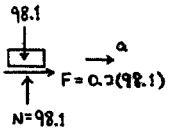
13-37. The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.

$\overset{+}{\rightarrow} \Sigma F_x = m a_x;$ $\quad 0.2(98.1) = 10\,a$

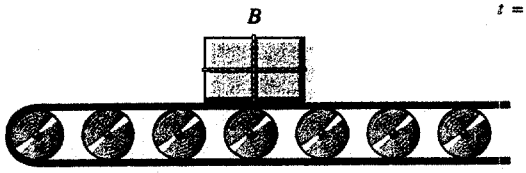
$a = 1.962 \text{ m/s}^2$



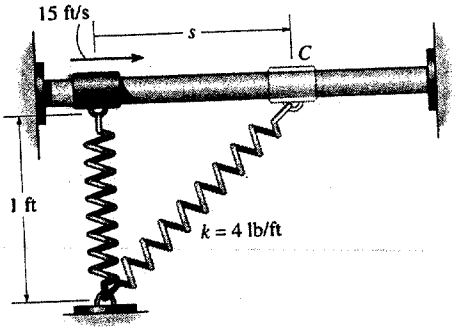
$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$4 = 0 + 1.962\,t$

$t = 2.04 \text{ s}$ **Ans**



13-38. The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



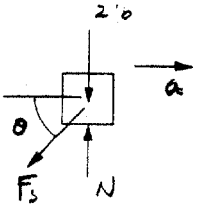
$F_s = kx; \quad F_s = 4\left(\sqrt{1+s^2} - 1\right)$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad -4\left(\sqrt{1+s^2} - 1\right)\left(\frac{s}{\sqrt{1+s^2}}\right) = \left(\frac{2}{32.2}\right)\left(v\frac{dv}{ds}\right)$

$-\int_0^1 \left(4s\,ds - \frac{4s\,ds}{\sqrt{1+s^2}}\right) = \int_{15}^{v}\left(\frac{2}{32.2}\right) v\,dv$

$-\left[2s^2 - 4\sqrt{1+s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$

$v = 14.6 \text{ ft/s}$ **Ans**



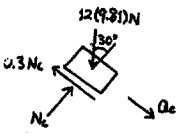
13-39. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.

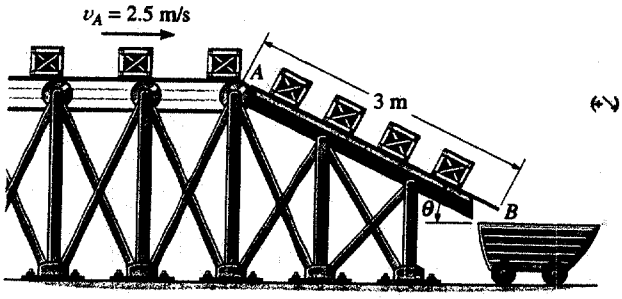
$\nearrow + \Sigma F_y = ma_y; \quad N_C - 12(9.81)\cos 30° = 0$

$N_C = 101.95 \text{ N}$

$+\nwarrow \Sigma F_x = ma_x; \quad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$a_C = 2.356 \text{ m/s}^2$





$(+\nwarrow) \quad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$

$v_B^2 = (2.5)^2 + 2(2.356)(3-0)$

$v_B = 4.5152 = 4.52 \text{ m/s}$ **Ans**

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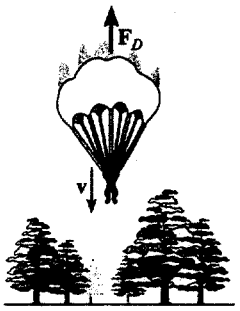
***13-40.** A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \to \infty$.



$$+\downarrow \Sigma F_z = m a_z; \quad mg - kv^2 = m\frac{dv}{dt}$$

$$m\int_0^v \frac{m\,dv}{(mg-kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k}-v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}$$

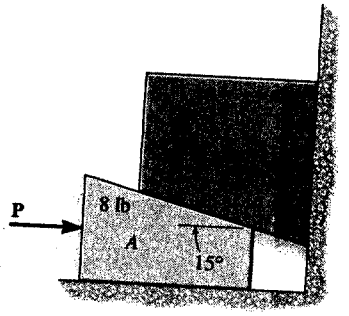
$$e^{2t\sqrt{\frac{gk}{m}}} = \frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}$$

$$\sqrt{\frac{mg}{k}}e^{2t\sqrt{\frac{gk}{m}}} - v\,e^{2t\sqrt{\frac{gk}{m}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{gk}{m}}}-1}{e^{2t\sqrt{\frac{gk}{m}}}+1}\right] \qquad \textbf{Ans}$$

When $t \to \infty$ $\quad v_t = \sqrt{\frac{mg}{k}}$ **Ans**

13-41. If a horizontal force $P = 12$ lb is applied to block A determine the acceleration of block B. Neglect friction.



Block *A*:

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 12 - N_B \sin 15° = \left(\frac{8}{32.2}\right)a_A \qquad (1)$$

Block *B*:

$$+\uparrow \Sigma F_y = ma_y; \quad N_B \cos 15° - 15 = \left(\frac{15}{32.2}\right)a_B \qquad (2)$$

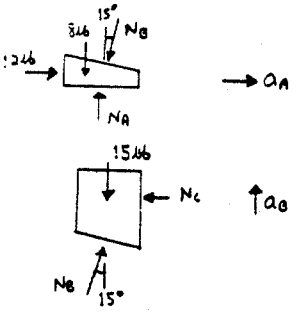
$$s_B = s_A \tan 15°$$

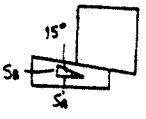
$$a_B = a_A \tan 15° \qquad (3)$$

Solving Eqs. (1)–(3),

$a_A = 28.3$ ft/s^2 $\quad N_B = 19.2$ lb

$a_B = 7.59$ ft/s^2 **Ans**





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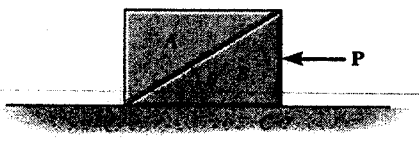
13-42. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not move relative to B. All surfaces are smooth.



Require

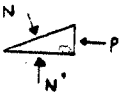
$a_A = a_B = a$

Block A :

$+\uparrow \Sigma F_y = 0; \quad N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad N\sin\theta = ma$

$a = g\tan\theta$



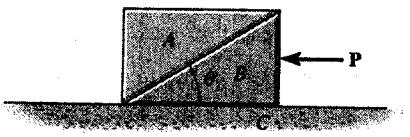
Block B :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

$P = 2mg\tan\theta$ **Ans**

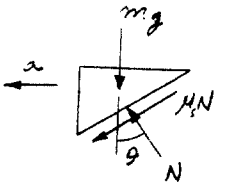
13-43. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not slip up B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.



Require

$a_A = a_B = a$

Block A :



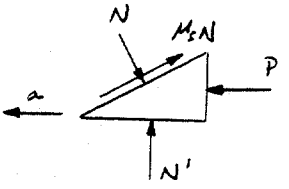
$+\uparrow \Sigma F_y = 0; \quad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B :



$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans**

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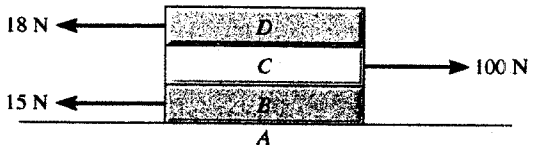
***13-44.** Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



Plates B, C and D

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 100 - 15 - 18 - F = 0$

$F = 67 \text{ N}$

$F_{max} = 0.3(294.3) = 88.3 \text{ N} > 67 \text{ N}$

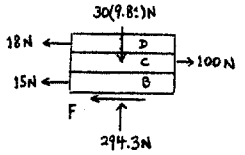


Plate B will not slip.

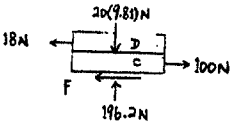
$a_B = 0$ Ans

Plates D and C

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 100 - 18 - F = 0$

$F = 82 \text{ N}$

$F_{max} = 0.3(196.2) = 58.86 \text{ N} < 82 \text{ N}$

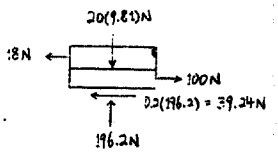


Slipping between B and C.

Assume no slipping between D and C,

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad 100 - 39.24 - 18 = 20 a_x$

$a_x = 2.138 \text{ m/s}^2 \rightarrow$

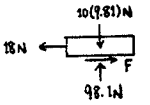


Check slipping between D and C.

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad F - 18 = 10(2.138)$

$F = 39.38 \text{ N}$

$F_{max} = 0.3(98.1) = 29.43 \text{ N} < 39.38 \text{ N}$



Slipping between D and C.

Plate C:

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad 100 - 39.24 - 19.62 = 10 a_C$

$a_C = 4.11 \text{ m/s}^2 \rightarrow$ Ans

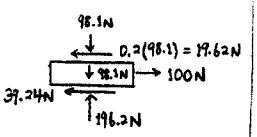
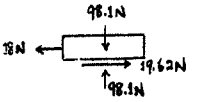


Plate D:

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad 19.62 - 18 = 10 a_D$

$a_D = 0.162 \text{ m/s}^2 \rightarrow$ Ans



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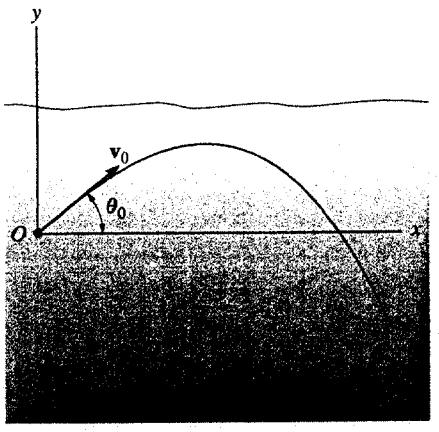
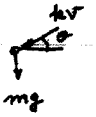
13-45. A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity v_0 as shown. If the liquid develops a frictional or drag resistance on the projectile which is proportional to its velocity, i.e., $F = kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?





$$\stackrel{+}{\rightarrow} \Sigma F_x = m a_x; \qquad -k v \cos\theta = m a_x$$

$$+\uparrow \Sigma F_y = m a_y; \qquad -m g - k v \sin\theta = m a_y$$

or

$$-k\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$-m g - k\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

Integrating yields

$$\ln \dot{x} = \frac{-k}{m}t + C_1$$

$$\ln(\dot{y} + \frac{mg}{k}) = \frac{k}{m}t + C_2$$

When $t = 0$, $\dot{x} = v_0\cos\theta_0$, $\dot{y} = v_0\sin\theta_0$,

$$\dot{x} = v_0\cos\theta_0\, e^{-(k/m)t}$$

$$\dot{y} = -\frac{m g}{k} + (v_0\sin\theta_0 + \frac{m g}{k})e^{-(k/m)t}$$

Integrating again,

$$x = \frac{-m v_0}{k}\cos\theta_0 e^{-(k/m)t} + C_3$$

$$y = -\frac{mg}{k}t - (v_0\sin\theta_0 + \frac{mg}{k})(\frac{m}{k})e^{-(k/m)t}$$

When $t = 0$, $x = y = 0$, thus

$$x = \frac{m v_0}{k}\cos\theta_0(1 - e^{-(k/m)t}) \qquad \text{Ans}$$

$$y = -\frac{m g t}{k} + \frac{m}{k}(v_0\sin\theta_0 + \frac{mg}{k})(1 - e^{-(k/m)t}) \qquad \text{Ans}$$

As $t \to \infty$

$$x_{max} = \frac{m v_0\cos\theta_0}{k} \qquad \text{Ans}$$

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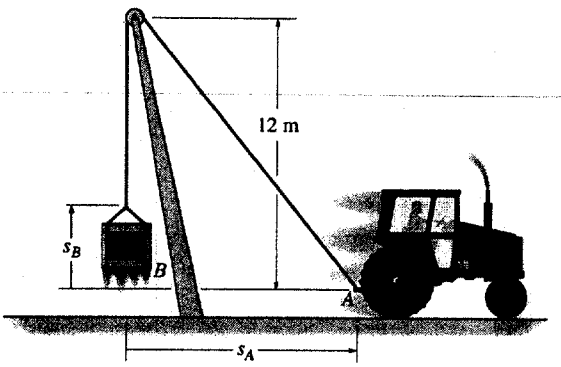
■ **13-46.** The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



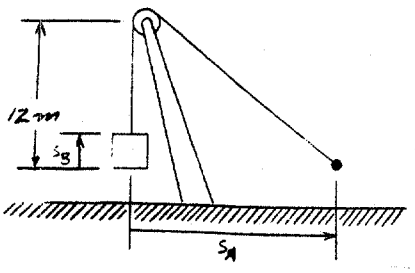
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

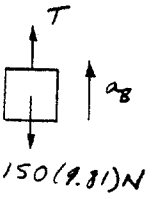
$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

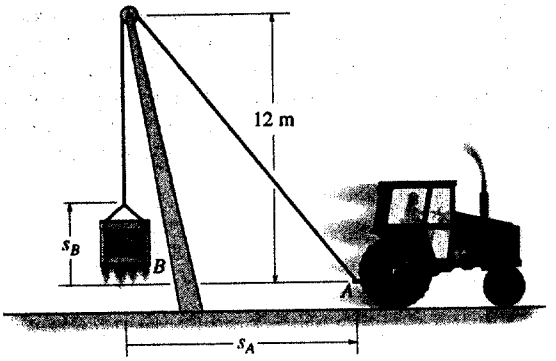




$+\uparrow \Sigma F_y = ma_y$; $\quad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans**

13-47. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



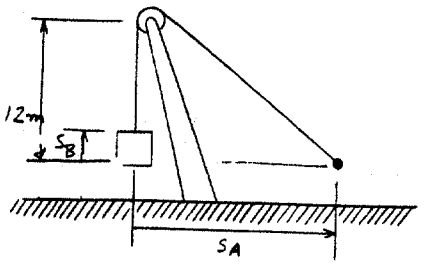
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

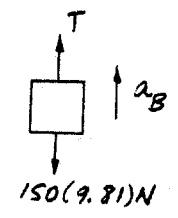
$$-s_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$





$+\uparrow \Sigma F_y = ma_y$; $\quad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans**

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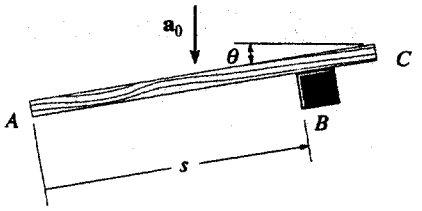
***13-48.** The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



$+\nearrow \Sigma F_s = m a_s; \quad 0 = m a_B \sin\phi$

$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$

$\mathbf{a}_B = \mathbf{a}_0 + \mathbf{a}_{B/AC}$

$+\nearrow \quad a_B \sin\phi = -a_0 \sin\theta + a_{B/AC}$

Thus, $\quad 0 = m(-a_0 \sin\theta + a_{B/AC})$

$a_{B/AC} = a_0 \sin\theta$

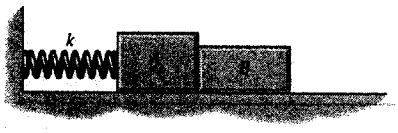
$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0 \sin\theta \, dt$

$v_{B/AC} = a_0 \sin\theta \, t$ **Ans**

$s_{B/AC} = s = \int_0^t a_0 \sin\theta \, t \, dt$

$s = \frac{1}{2} a_0 \sin\theta \, t^2$ **Ans**

13-49. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



Block A:

$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \quad -k(x-d) - N = m_A a_A$

Block B:

$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \quad N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x-d) - m_B a = m_A a$

$a = \frac{k(d-x)}{(m_A + m_B)} \qquad N = \frac{k m_B (d-x)}{(m_A + m_B)}$

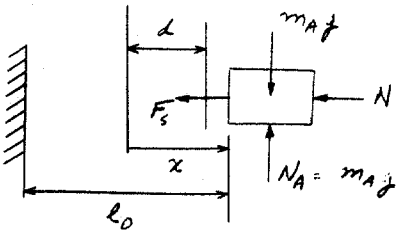
$N = 0$ when $d - x = 0$, or $x = d$ **Ans**

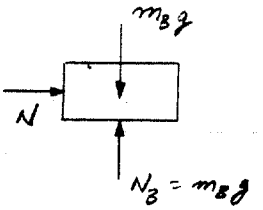
$v\,dv = a\,dx$

$\int_0^v v\,dv = \int_0^d \frac{k(d-x)}{(m_A + m_B)} dx$

$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A + m_B)}$

$v = \sqrt{\frac{kd^2}{(m_A + m_B)}}$ **Ans**





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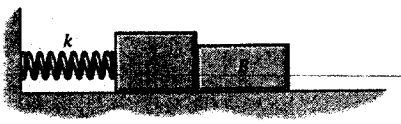
13-50. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Block A :

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -k(x-d) - N - \mu_k m_A g = m_A a_A$$

Block B :

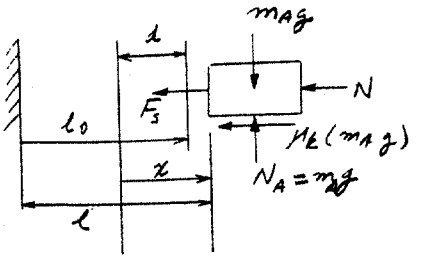
$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{km_B(d-x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans**



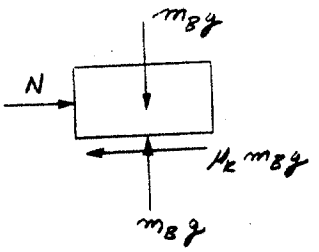
At the moment of separation :

$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)}\,dx - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$



Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0$$

Thus,

$$kd > 2\mu_k g(m_A + m_B)$$

$$d > \frac{2\mu_k g}{k}(m_A + m_B) \qquad \textbf{Q.E.D.}$$

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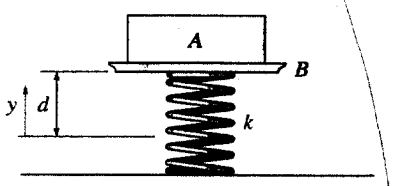
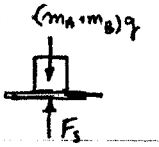
13-51. The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

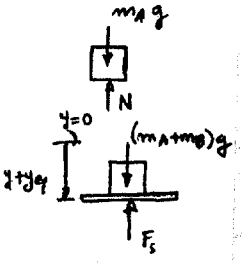




For Equilibrium

$$+\uparrow \Sigma F_y = ma_y; \quad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$



Block:

$$+\uparrow \Sigma F_y = ma_y; \quad -m_A g + N = m_A a$$

Block and pan

$$+\uparrow \Sigma F_y = ma_y; \quad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d$, $N = 0$

$$kd = -(m_A + m_B)g$$

Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans}$$

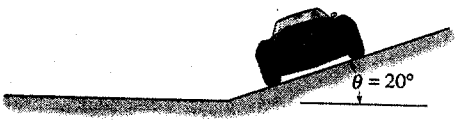
***13-52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is $149.6(10^6)$ km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

$$\Sigma F_n = ma_n; \quad G\frac{M_e M_s}{R^2} = M_e\frac{v^2}{R} \quad M_s = \frac{v^2 R}{G}$$

$$v = \frac{s}{t} = \frac{2\pi(149.6)(10^9)}{365(24)(3600)} = 29.81\,(10^3)\ \text{m/s}$$

$$M_s = \frac{[(29.81)(10^3)]^2(149.6)(10^9)}{66.73(10^{-12})} = 1.99\,(10^{30})\ \text{kg} \qquad \textbf{Ans}$$

13-53. The sports car, having a mass of 1700 kg, is traveling horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.

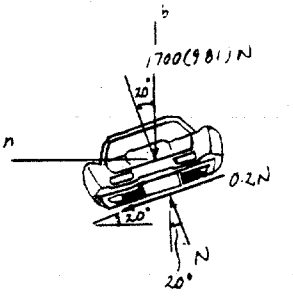


$$+\uparrow \Sigma F_b = 0; \quad N\cos 20° - 0.2N\sin 20° - 1700(9.81) = 0$$

$$N = 19\,140.6\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \quad 19\,140.6\sin 20° + 0.2(19\,140.6)\cos 20° = 1700\left(\frac{v_{max}^2}{100}\right)$$

$$v_{max} = 24.4\ \text{m/s} \qquad \textbf{Ans}$$



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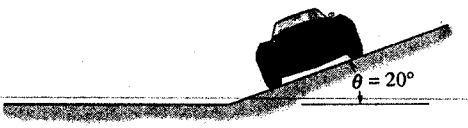
13-54. Using the data in Prob. 13-53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.

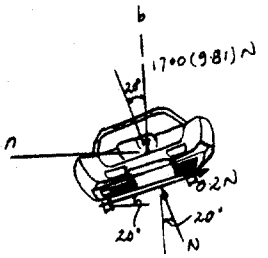


$+\uparrow \Sigma F_b = 0; \quad N\cos 20° + 0.2N\sin 20° - 1700(9.81) = 0$

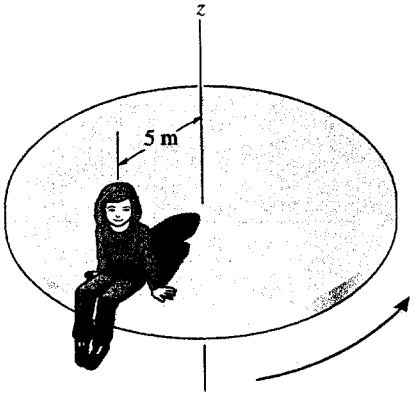
$N = 16543.1\ \text{N}$

$\stackrel{+}{\leftarrow} \Sigma F_n = ma_n; \quad 16543.1\sin 20° - 0.2(16543.1)\cos 20° = 1700\left(\frac{v_{min}^2}{100}\right)$

$v_{min} = 12.2\ \text{m/s}$ **Ans**



13-55. A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.

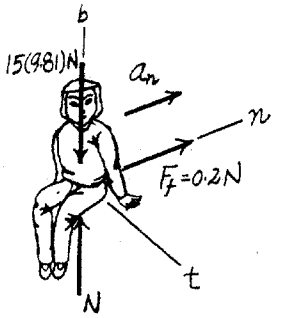


Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13 – 8, we have

$\Sigma F_b = 0; \quad N - 15(9.81) = 0 \quad N = 147.15\ \text{N}$

$\Sigma F_n = ma_n; \quad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$

$v = 3.13\ \text{m/s}$ **Ans**



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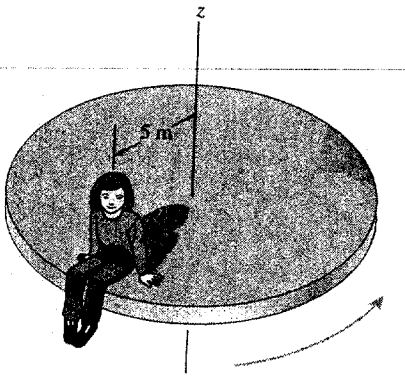
***13-56.** Solve Prob. 13–55 assuming that the platform starts rotating from rest so that the girl's speed is increased uniformly at $\dot{v} = 0.5\ \text{m/s}^2$.



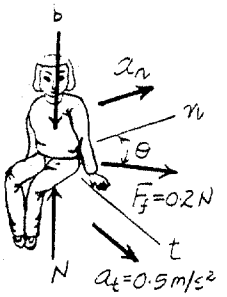
Equation of Motion : Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15\ \text{N}$$

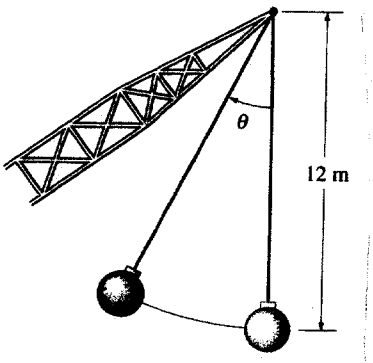
$$\Sigma F_t = ma_t; \qquad 0.2(147.15)\sin\theta = 15(0.5) \qquad \theta = 14.76°$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15)\cos 14.76° = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.08\ \text{m/s} \qquad \textbf{Ans}$$



13-57. The 600-kg wrecking ball is suspended from the crane by a cable having a negligible mass. If the ball has a speed $v = 8$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cable at this instant. Also, determine the angle θ to which the ball swings before it stops.



$$+\uparrow \Sigma F_n = m a_n; \qquad T - 600(9.81) = 600\left(\frac{8^2}{12}\right)$$

$$T = 9086\ \text{N} = 9.09\ \text{kN} \qquad \textbf{Ans}$$

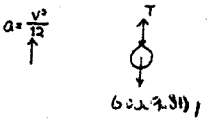
$$+\nwarrow \Sigma F_t = m a_t; \qquad -600(9.81)\sin\theta = 600\, a_t$$

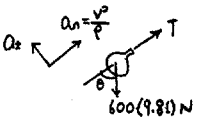
Set $a_t(12\, d\theta) = v\, dv$

$$-9.81(12)\int_0^\theta \sin\theta\, d\theta = \int_8^0 v\, dv$$

$$-9.81(12)(-\cos\theta + 1) = -\frac{1}{2}(8)^2$$

$$\theta = 43.3° \qquad \textbf{Ans}$$





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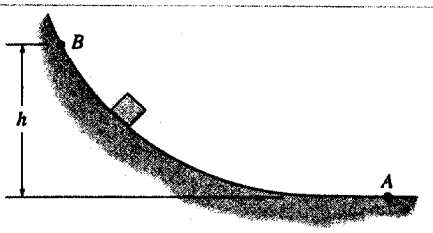
13-58. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



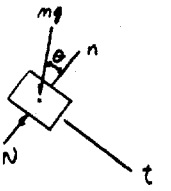
$+\nwarrow \Sigma F_t = ma_t$; $\quad mg\sin\theta = ma_t \quad a_t = g\sin\theta$

$v\,dv = a_t\,ds = g\sin\theta\,ds$ However $dy = ds\sin\theta$

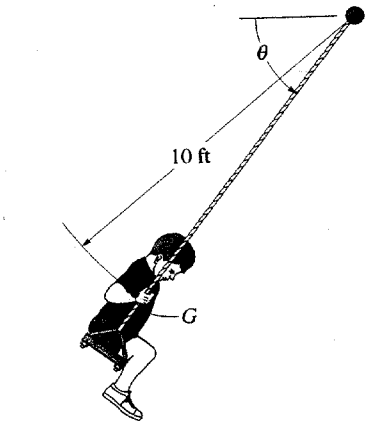
$\int_0^v v\,dv = \int_0^h g\,dy$

$\frac{v^2}{2} = gh$

$v = \sqrt{2gh}$ **Q.E.D.**



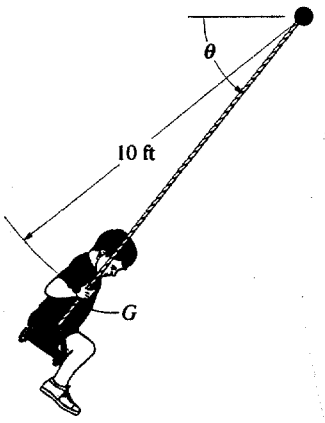
13-59. At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15$ ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



$+\nwarrow \Sigma F_t = ma_t$; $\quad 60\cos 60° = \frac{60}{32.2}a_t \quad a_t = 16.1\ \text{ft/s}^2$ **Ans**

$+\nearrow \Sigma F_n = ma_n$; $\quad 2T - 60\sin 60° = \frac{60}{32.2}\left(\frac{15^2}{10}\right) \quad T = 46.9$ lb **Ans**

***13-60.** At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



$+\nwarrow \Sigma F_t = ma_t$; $\quad 60\cos\theta = \frac{60}{32.2}a_t \quad a_t = 32.2\cos\theta$

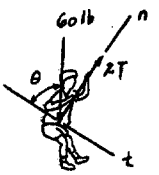
$+\nearrow \Sigma F_n = ma_n$; $\quad 2T - 60\sin\theta = \frac{60}{32.2}\left(\frac{v^2}{10}\right)$ [1]

$v\,dv = a\,ds$ however $ds = 10\,d\theta$

$\int_0^v v\,dv = \int_{60°}^{90°} 322\cos\theta\,d\theta$

$v = 9.289$ ft/s

From Eq.[1] $\quad 2T - 60\sin 90° = \frac{60}{32.2}\left(\frac{9.289^2}{10}\right) \quad T = 38.0$ lb **Ans**



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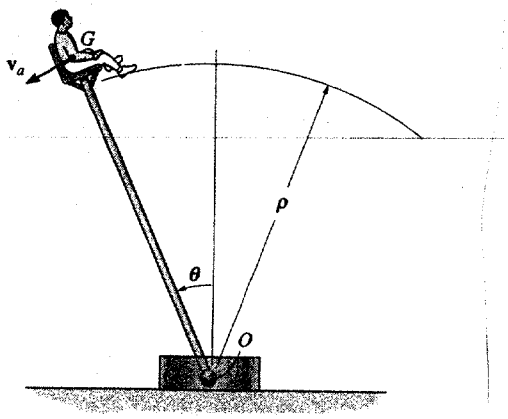
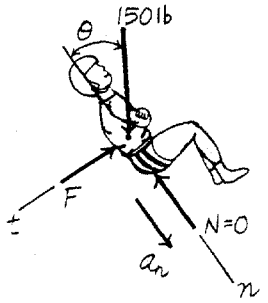
13-61. An acrobat has a weight of 150 lb and is sitting on a chair which is perched on top of a pole as shown. If by a mechanical drive the pole rotates downward at a constant rate from $\theta = 0°$, such that the acrobat's center of mass G maintains a *constant speed* of $v_a = 10$ ft/s, determine the angle θ at which he begins to "fly" out of the chair. Neglect friction and assume that the distance from the pivot O to G is $\rho = 15$ ft.



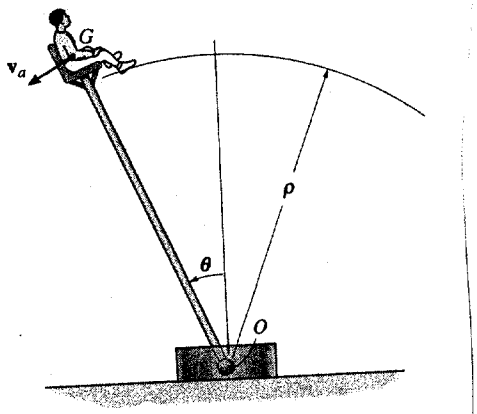
Equation of Motion : If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13 – 8, we have

$$\Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{10^2}{15}\right)$$

$$\theta = 78.1° \qquad \textbf{Ans}$$



13-62. Solve Prob. 13-61 if the speed of the acrobat's center of mass is increased from $(v_a)_0 = 10$ ft/s at $\theta = 0°$ by a constant rate of $\dot{v}_a = 0.5$ ft/s^2.



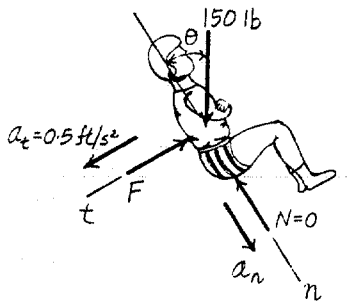
Kinematics : Applying equation $v^2 = v_0^2 + 2a_t(s - s_0)$, we have

$$v^2 = 10^2 + 2(0.5)(15\theta - 0) = 100 + 15\theta$$

Equation of Motion : If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13 – 8, we have

$$\Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{100 + 15\theta}{15}\right)$$

Solving for θ by trial and error, $\theta = 75.6°$ **Ans**



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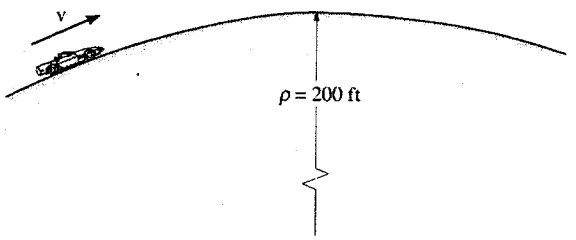
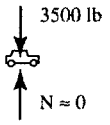
13-63. If the crest of the hill has a radius of curvature $\rho = 200$ ft, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has a weight of 3500 lb.

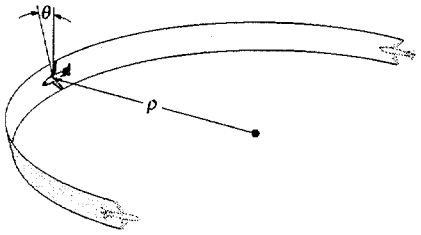


$$\downarrow \sum F_n = ma_n; \quad 3500 = \frac{3500}{32.2}\left(\frac{v^2}{200}\right)$$



$$v = 80.2 \text{ ft/s} \qquad \textbf{Ans}$$

***13-64.** The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg?

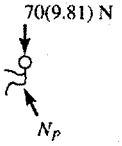


$$+\uparrow \sum F_b = ma_b; \quad N_P \sin 15° - 70(9.81) = 0$$

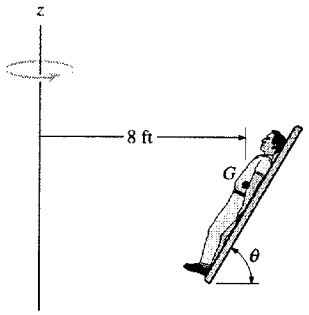
$$N_P = 2.65 \text{ kN} \qquad \textbf{Ans}$$

$$\overset{+}{\leftarrow} \sum F_n = ma_n; \quad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$$

$$\rho = 68.3 \text{ m} \qquad \textbf{Ans}$$



13-65. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



$$+\nwarrow \sum F_y = m(a_n)_y; \quad N - 150\cos 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin 60°$$

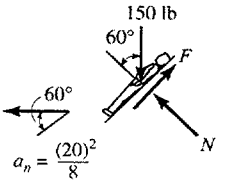
$$N = 277 \text{ lb} \qquad \textbf{Ans}$$

$$+\swarrow \sum F_x = m(a_n)_x; \quad -F + 150\sin 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60°$$

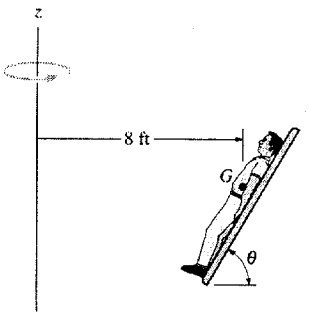
$$F = 13.4 \text{ lb} \qquad \textbf{Ans}$$

Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$



13-66. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.

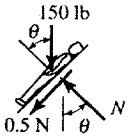


$$\overset{+}{\leftarrow} \sum F_n = ma_n; \quad 0.5N\cos\theta + N\sin\theta = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$+\uparrow \sum F_b = 0; \quad -150 + N\cos\theta - 0.5N\sin\theta = 0$$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$



$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

$$\theta = 47.5° \qquad \textbf{Ans}$$

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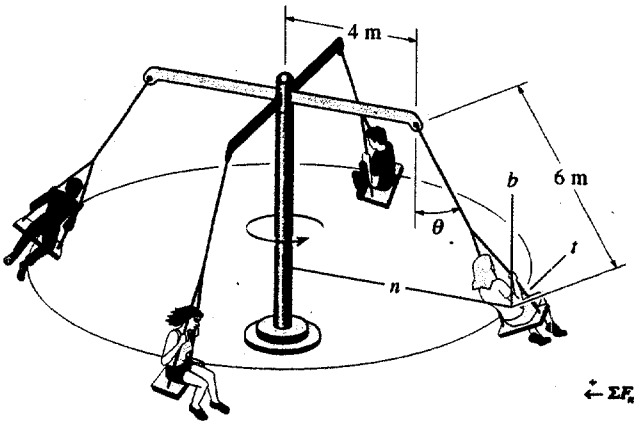
13-67. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



$\overset{+}{\leftarrow} \Sigma F_n = m a_n$; $T\sin 30° = 80\left(\frac{v^2}{4 + 6\sin 30°}\right)$

$+\uparrow \Sigma F_b = 0$; $T\cos 30° - 80(9.81) = 0$

$T = 906.2$ N

$v = 6.30$ m/s **Ans**

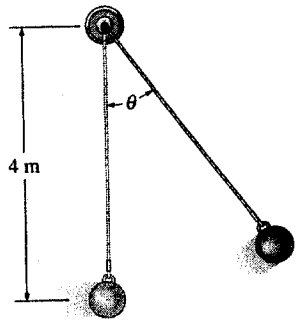
$\Sigma F_n = m a_n$; $F_n = 50\left(\frac{(6.30)^2}{7}\right) = 283$ N **Ans**

$\Sigma F_t = m a_t$; $F_t = 0$ **Ans**

$\Sigma F_b = m a_b$; $F_b - 490.5 = 0$

$F_b = 490$ N **Ans**

***13-68.** If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings at the instant it momentarily stops. Neglect the size of the ball.



$+\uparrow \Sigma F_n = ma_n$; $T - 30(9.81) = 30\left(\frac{(4)^2}{4}\right)$

$T = 414$ N **Ans**

$+\swarrow \Sigma F_t = ma_t$; $-30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$-9.81\int_0^\theta \sin\theta\,(4d\theta) = \int_4^0 v\, dv$

$9.81(4)\cos\theta\Big]_0^\theta = -\frac{1}{2}(4)^2$

$39.24(\cos\theta - 1) = -8$

$\theta = 37.2°$ **Ans**

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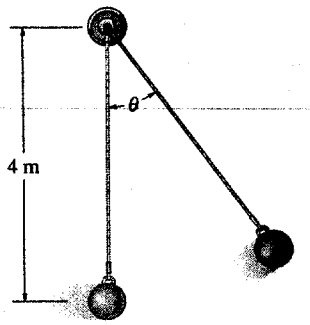
13-69. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n$; $\quad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\swarrow \Sigma F_t = ma_t$; $\quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$-9.81\int_0^\theta \sin\theta\,(4\, d\theta) = \int_4^v v\, dv$

$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$

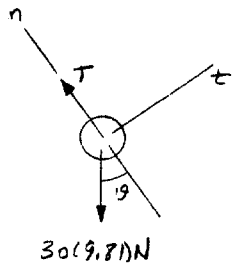
$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$

At $\theta = 20°$

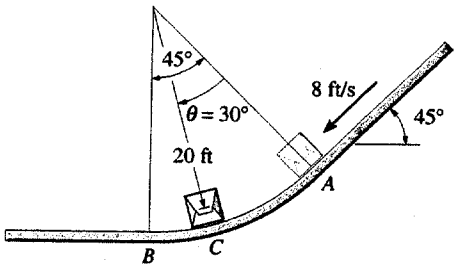
$v = 3.357$ m/s

$a_t = -3.36$ m/s^2 $= 3.36$ m/s^2 $\swarrow$ **Ans**

$T = 361$ N **Ans**



13-70. The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



$+\swarrow \Sigma F_t = ma_t$; $\quad 5\cos\phi = \frac{5}{32.2}a_t$

$a_t = 32.2\cos\phi$

$+\nwarrow \Sigma F_n = ma_n$; $\quad N - 5\sin\phi = \frac{5}{32.2}\left(\frac{v^2}{20}\right)$

$v\, dv = a_t\, ds$

$\int_8^v v\, dv = \int_{45°}^{\phi} 32.2\cos\phi\,(20\, d\phi)$

$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644(\sin\phi - \sin 45°)$

At $\phi = 45° + 30° = 75°$,

$v_C = 19.933$ ft/s $= 19.9$ ft/s **Ans**

$N_C = 7.91$ lb **Ans**

At $\phi = 45° + 45° = 90°$

$v_B = 21.0$ ft/s **Ans**

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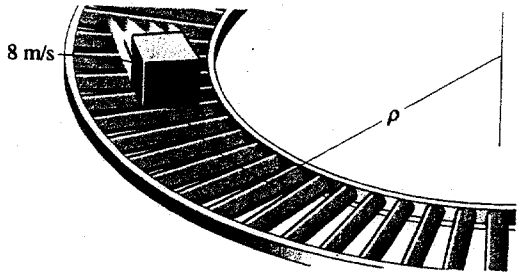
13-71. Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



$+\uparrow \Sigma F_b = m a_b;$ $\quad N - W = 0$

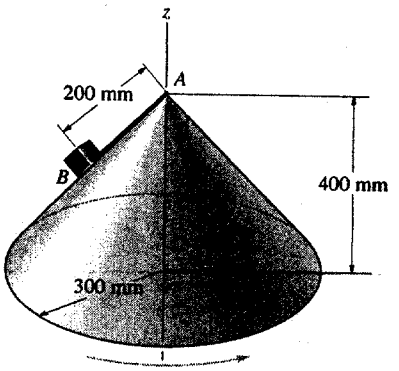
$N = W$

$F_s = 0.7W$

$\overset{+}{\leftarrow} \Sigma F_n = m a_n;$ $\quad 0.7W = \frac{W}{9.81}\left(\frac{8^2}{\rho}\right)$

$\rho = 9.32$ m **Ans**

***13-72.** The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the z axis such that the block attains a speed of 0.5 m/s. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



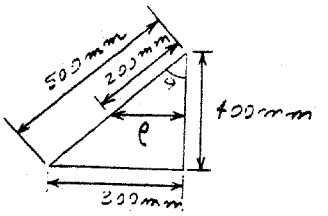
$\frac{\rho}{200} = \frac{300}{500};$ $\quad \rho = 120$ mm $= 0.120$ m

$+\nwarrow \Sigma F_y = ma_y;$ $\quad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$

$T = 1.82$ N **Ans**

$+\swarrow \Sigma F_x = ma_x;$ $\quad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$

$N_B = 0.844$ N **Ans**



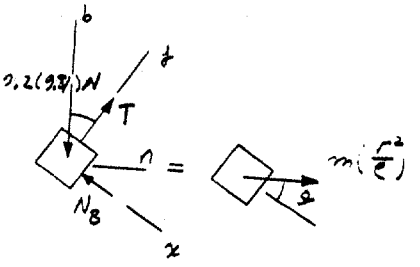
Also,

$\overset{+}{\rightarrow} \Sigma F_n = ma_n;$ $\quad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$

$+\uparrow \Sigma F_b = 0;$ $\quad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$

$T = 1.82$ N **Ans**

$N_B = 0.844$ N **Ans**



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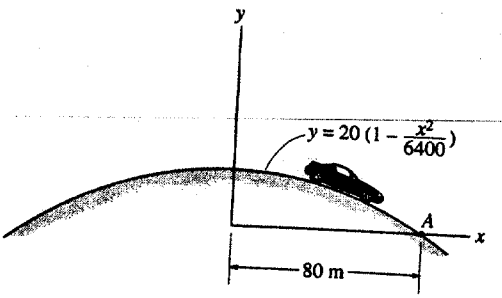
13-73. The 0.8-Mg car is traveling over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

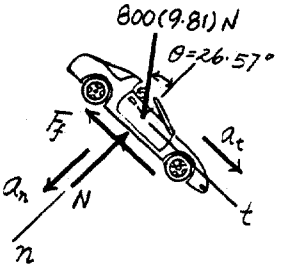


Geometry : Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\,\text{m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \left.\frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|}\right| = \left.\frac{[1+(-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=8\,\text{m}} = 223.61\ \text{m}$$

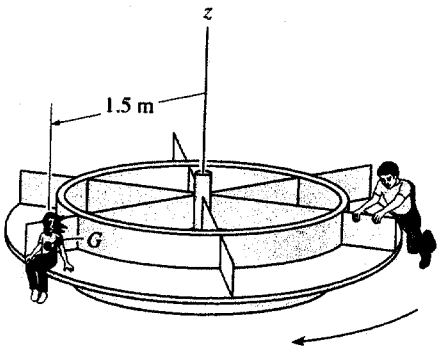


Equation of Motion : Here, $a_t = 0$. Applying Eq. 13 – 8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(0)$$
$$F_f = 3509.73\ \text{N} = 3.51\ \text{kN} \qquad \textbf{Ans}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$$
$$N = 6729.67\ \text{N} = 6.73\ \text{kN} \qquad \textbf{Ans}$$

13-74. A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



$$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$$

$$v = 2.10\ \text{m/s} \qquad \textbf{Ans}$$

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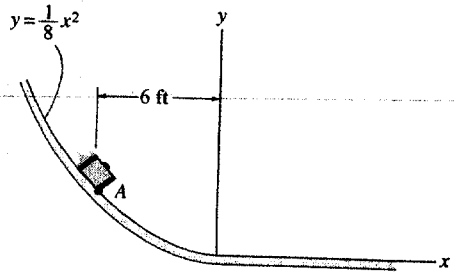
13-75. The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.

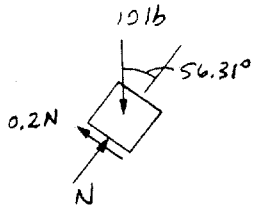


$y = \frac{1}{8}x^2$

$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x\Big|_{x=-6} = -1.5 \qquad \theta = -56.31°$

$\frac{d^2y}{dx^2} = \frac{1}{4}$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$



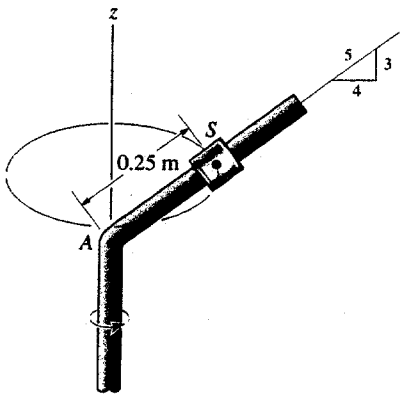
$+\nearrow \Sigma F_n = ma_n; \qquad N - 10\cos 56.31° = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$

$N = 5.8783 = 5.88 \text{ lb}$ **Ans**

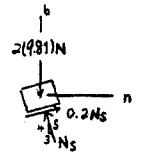
$+\searrow \Sigma F_t = ma_t; \qquad -0.2(5.8783) + 10\sin 56.31° = \left(\frac{10}{32.2}\right)a_t$

$a_t = 23.0 \text{ ft/s}^2$ **Ans**

***13-76.** The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



$\rho = 0.25(\frac{4}{5}) = 0.2 \text{ m}$



$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad N_s(\frac{3}{5}) - 0.2N_s(\frac{4}{5}) = 2(\frac{v^2}{0.2})$

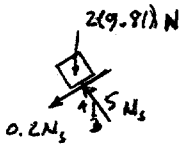
$+\uparrow \Sigma F_b = m a_b; \qquad N_s(\frac{4}{5}) + 0.2N_s(\frac{3}{5}) - 2(9.81) = 0$

$N_s = 21.3 \text{ N}$

$v = 0.969 \text{ m/s}$ **Ans**

13-77. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.

$\rho = 0.25(\frac{4}{5}) = 0.2 \text{ m}$



$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad N_s(\frac{3}{5}) + 0.2N_s(\frac{4}{5}) = 2(\frac{v^2}{0.2})$

$+\uparrow \Sigma F_b = m a_b; \qquad N_s(\frac{4}{5}) - 0.2N_s(\frac{3}{5}) - 2(9.81) = 0$

$N_s = 28.85 \text{ N}$

$v = 1.48 \text{ m/s}$ **Ans**

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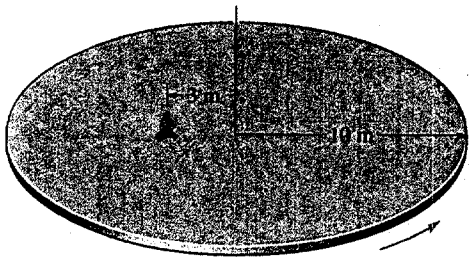
13-78. The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.



$\Sigma F_t = m a_t$; $\quad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m a_n$; $\quad F_n = (80)\frac{v^2}{3}$

$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\frac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\frac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

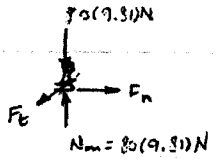
$a_t = \frac{dv}{dt} = 0.4$

$\int_0^v dv = \int_0^t 0.4\, dt$

$v = 0.4\, t$

$2.9575 = 0.4\, t$

$t = 7.39\ \text{s}$ **Ans**



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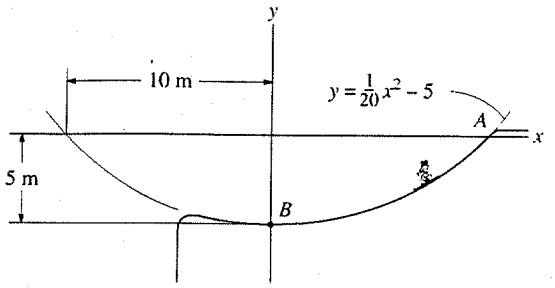
13-79. The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force she exerts on the ground at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–58.

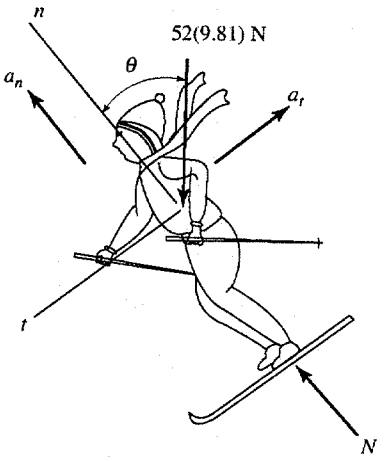


Geometry: Here, $\frac{dy}{dx} = \frac{1}{10}x$ and $\frac{d^2y}{dx^2} = \frac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \quad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{[1/10]}\right|_{x=0\text{ m}} = 10.0\text{ m}$$



Equation of Motion:

$$\sum F_t = ma_t; \quad 52(9.81)\sin\theta = -52a_t \quad a_t = -9.81\sin\theta$$

$$\sum F_n = ma_n; \quad N - 52(9.81)\cos\theta = m\left(\frac{v^2}{\rho}\right) \qquad [1]$$

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}\,dx = \sqrt{1 + \frac{1}{100}x^2}\,dx$. Here, $\tan\theta = \frac{1}{10}x$. Then, $\sin\theta = \frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+) \quad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^0 \left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \tfrac{1}{100}x^2}\,dx\right)$$

$$v^2 = 98.1\text{ m}^2/\text{s}^2$$

Also, one can obtain v^2 by using the result of Prob. 13–58.

$$v^2 = 2gh = 2(9.81)(5) = 98.1\text{ m}^2/\text{s}^2$$

Substitute $v^2 = 98.1\text{ m}^2/\text{s}^2, \theta = 0°$ and $\rho = 10.0\text{ m}$ into Eq. [1] yields

$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN} \qquad \textbf{Ans}$$

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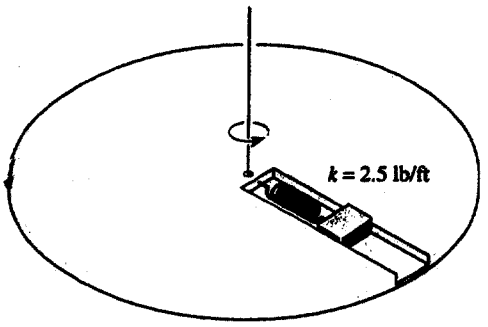
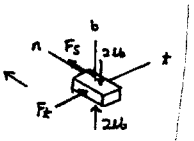
***13-80.** The block has a weight of 2 lb and it is free to move along the smooth slot in the rotating disk. The spring has a stiffness of 2.5 lb/ft and an unstretched length of 1.25 ft. Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with a constant speed of 12 ft/s.





$\Sigma F_n = ma_n$; $F_s = \frac{2}{32.2}(\frac{12^2}{\rho})$

$\Sigma F_t = ma_t$; $F_t = 0$ **Ans**

$F_s = ks$; $F_s = 2.5(\rho - 1.25)$

$2.5(32.2)(\rho^2 - 1.25\rho) = 288$

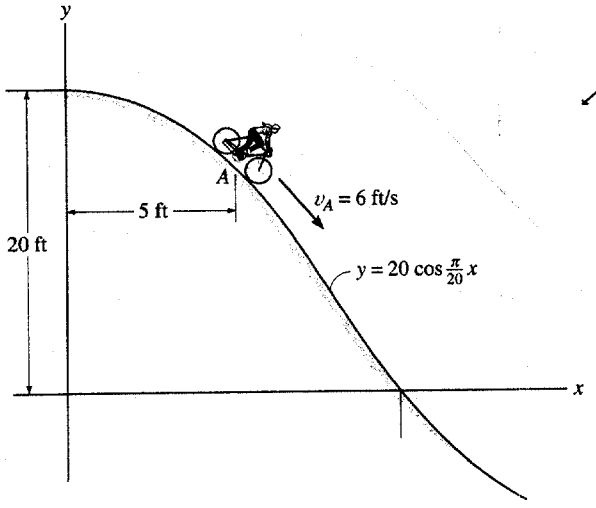
$\rho^2 - 1.25\rho - 3.58 = 0$

Choosing the positive root,

$\rho = 2.62$ ft

$F_s = 2.5(2.62 - 1.25) = 3.42$ lb **Ans**

13-81. If the bicycle and rider have a total weight of 180 lb, determine the resultant normal force acting on the bicycle when it is at point A while it is freely coasting at $v_A = 6$ ft/s. Also, compute the increase in the bicyclist's speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.



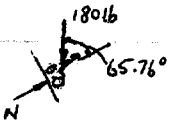
$y = 20\cos(\frac{\pi}{20}x)$

$\frac{dy}{dx} = -\pi\sin(\frac{\pi}{20}x)\Big|_{x=5\text{ ft}} = -2.221$

$\theta = \tan^{-1}(-2.221) = -65.76°$

$\frac{d^2y}{dx^2} = -\frac{\pi^2}{20}\cos(\frac{\pi}{20}x)\Big|_{x=5\text{ ft}} = -0.3489$

$\rho = \left|\frac{[1+(-2.221)^2]^{3/2}}{-0.3489}\right| = 41.43$ ft



$\searrow +\Sigma F_t = m a_t$; $180\sin 65.76° = \frac{180}{32.2}a_t$

$a_t = 29.4$ ft/s^2 **Ans**

$\swarrow +\Sigma F_n = m a_n$; $180\cos 65.76° - N = \frac{180}{32.2}(\frac{6^2}{41.43})$

$N = 69.0$ lb **Ans**

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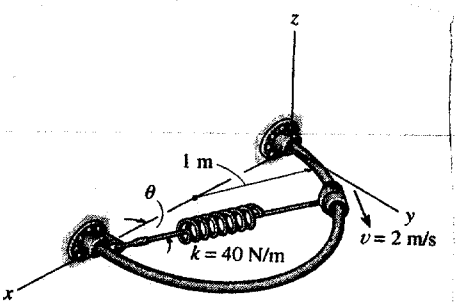
13-82. The collar has a mass of 5 kg and is confined to move along the smooth circular rod which lies in the horizontal plane. The attached spring has an unstretched length of 200 mm. If, at the instant $\theta = 30°$, the collar has a speed $v = 2\text{m/s}$, determine the magnitude of normal force of the rod on the collar and the collar's acceleration.

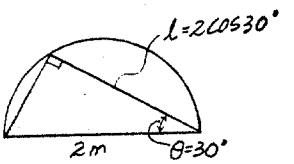


Equation of Motion : The spring force is given by $F_{sp} = k(l - l_0)$ $= 40(2\cos 30° - 0.2) = 61.28$ N. The normal component of acceleration is $a_n = \frac{v^2}{\rho} = \frac{2^2}{1} = 4\ \text{m/s}^2$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N_b - 5(9.81) = 0 \qquad N_b = 49.05\ \text{N}$$

$$\Sigma F_t = ma_t; \qquad 61.28\sin 30° = 5a_t \qquad a_t = 6.128\ \text{m/s}^2$$

$$\Sigma F_n = ma_n; \qquad 61.28\cos 30° - N_n = 5(4) \qquad N_n = 33.07\ \text{N}$$

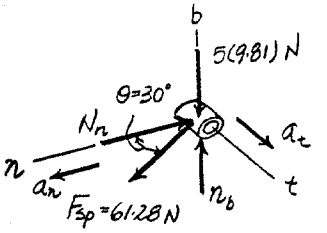


Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{6.128^2 + 4^2} = 7.32\ \text{m/s}^2 \qquad \textbf{Ans}$$

and the magnitude of normal force is

$$N = \sqrt{N_b^2 + N_n^2} = \sqrt{49.05^2 + 33.07^2} = 59.2\ \text{N} \qquad \textbf{Ans}$$



13-83. A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

$r = 4 + 3t|_{t=2\,s} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\,s} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\,s} = -12$ m/s^2

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160\ \text{m/s}^2$$

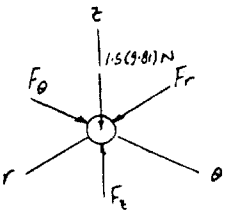
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44\ \text{m/s}^2$$

$$a_z = \ddot{z} = -12\ \text{m/s}^2$$

$\Sigma F_r = ma_r; \quad F_r = 1.5(-160) = -240$ N **Ans**

$\Sigma F_\theta = ma_\theta; \quad F_\theta = 1.5(44) = 66$ N **Ans**

$\Sigma F_z = ma_z; \quad F_z - 1.5(9.81) = 1.5(-12) \quad F_z = -3.28$ N **Ans**



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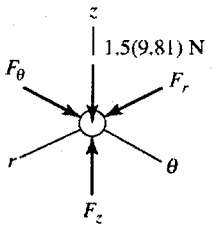
***13-84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.



$r = 2t + 1|_{t=2s} = 5$ ft $\quad \dot{r} = 2$ ft/s $\quad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2s} = 0$ rad $\quad \dot{\theta} = t - 1|_{t=2s} = 1$ rad/s $\quad \ddot{\theta} = 1$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5$ ft/s^2

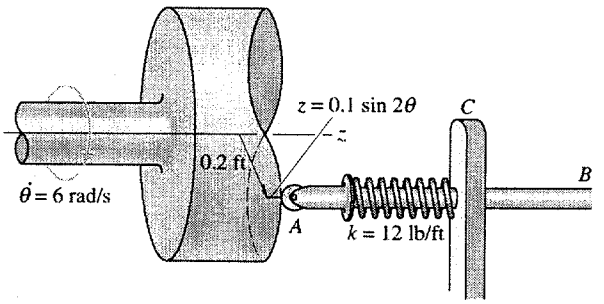
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9$ ft/s^2

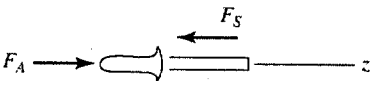
$\sum F_r = ma_r; \quad F_r = \frac{5}{32.2}(-5) = -0.7764$ lb

$\sum F_\theta = ma_\theta; \quad F_\theta = \frac{5}{32.2}(9) = 1.398$ lb

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60$ lb **Ans**

13-85. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2\theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 45°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.





$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z; \quad F_A - 12(z + 0.3) = m\ddot{z}$

$F_A - 12(0.1 \sin 2\theta + 0.3) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$

For $\theta = 45°$,

$F_A - 12(0.4) = \frac{0.75}{32.2}(-14.4)$

$F_A = 4.46$ lb **Ans**

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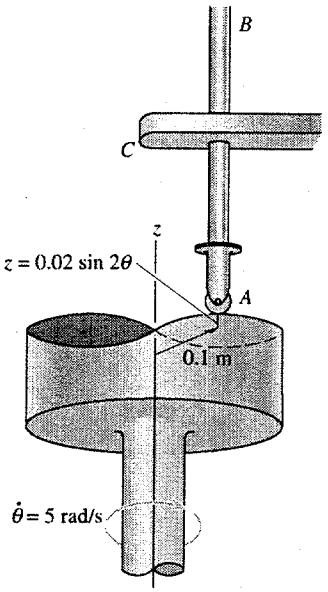
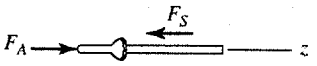
13-86. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2(\theta))$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the maximum and minimum force the follower exerts on the cam if the spring is compressed 0.2 ft when $\theta = 45$.





$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta \dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta \dot{\theta}^2 + 0.2 \cos 2\theta \ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z; \quad F_A - 12(z + 0.1) = m\ddot{z}$

$$F_A - 12(0.1 \sin 2\theta + 0.1) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

$F_A = 1.2 - 0.8646 \sin 2\theta$

$(F_A)_{max} = 2.06$ lb **Ans**

$(F_A)_{min} = 0.335$ lb **Ans**

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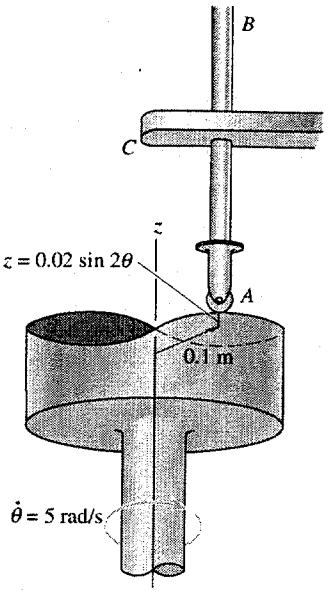
13-87. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin 2\theta)$ m. If the cam is rotating at a constant rate of 5 rad/s, determine the maximum and minimum force the cam exerts on the rod.



Kinematic: Taking the required time derivatives, we have

$\dot{\theta} = 5$ rad/s $\quad \ddot{\theta} = 0$

$z = 0.02 \sin 2\theta \quad \dot{z} = 0.04 \cos 2\theta\dot{\theta} \quad \ddot{z} = 0.04(\cos 2\theta\ddot{\theta} - 2 \sin 2\theta\dot{\theta}^2)$

Thus,

$a_z = \ddot{z} = 0.04[\cos 2\theta(0) - 2 \sin 2\theta(5^2)] = -2 \sin 2\theta$

At $\theta = 45°$, $\quad a_z = -2 \sin 90° = -2$ m/s^2

At $\theta = -45°$, $\quad a_z = -2 \sin(-90°) = 2$ m/s^2

Equation of Motion: At $\theta = 45°$, applying Eq. 13-9, we have

$\sum F_z = ma_z; \quad (F_z)_{\min} - 2(9.81) = 2(-2)$

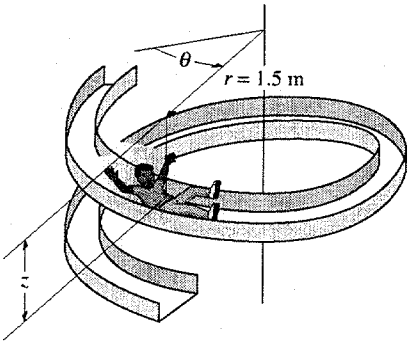
$(F_z)_{\min} = 15.6$ N **Ans**

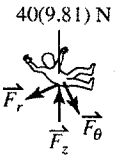
At $\theta = -45°$, we have

$\sum F_z = ma_z; \quad (F_z)_{\max} - 2(9.81) = 2(2)$

$(F_z)_{\max} = 23.6$ N **Ans**

***13-88.** The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.





$r = 1.5 \quad \theta = 0.7t \quad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \quad \dot{\theta} = 0.7 \quad \dot{z} = -0.5$

$\ddot{\theta} = 0 \quad \ddot{z} = 0$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

$\sum F_r = ma_r; \quad F_r = 40(-0.735) = -29.4$ N **Ans**

$\sum F_\theta = ma_\theta; \quad F_\theta = 0$ **Ans**

$\sum F_z = ma_z; \quad F_z - 40(9.81) = 0$

$F_z = 392$ N **Ans**

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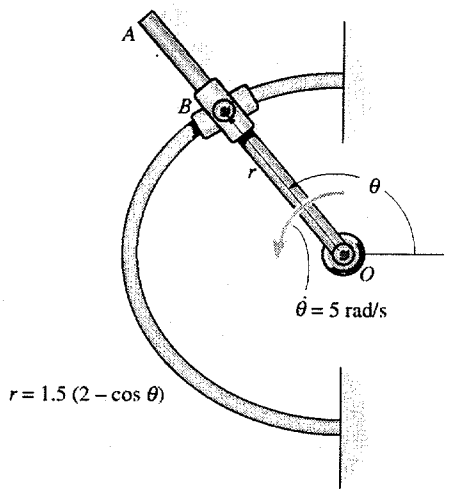
13-89. Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.



Kinematic: Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$$r = 1.5(2 - \cos\theta)|_{\theta=120°} = 3.75 \text{ ft}$$

$$\dot{r} = 1.5\sin\theta\dot{\theta}|_{\theta=120°} = 6.495 \text{ ft/s}$$

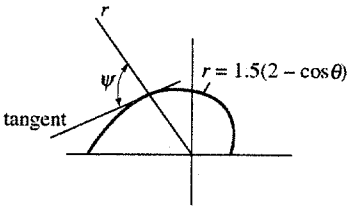
$$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)|_{\theta=120°} = -18.75 \text{ ft/s}^2$$

Applying Eqs. 12-29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5 \text{ ft/s}^2$$

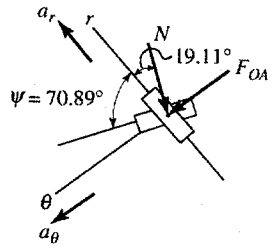
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952 \text{ ft/s}^2$$

Equation of Motion: The angle ψ must be obtained first.



$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{1.5(2 - \cos\theta)}{1.5\sin\theta}\right|_{\theta=120°} = 2.8867 \quad \psi = 70.89°$$

Applying Eq. 13-9, we have



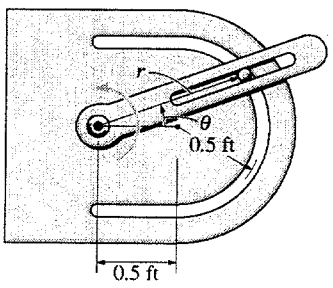
$$\sum F_r = ma_r; \quad -N\cos 19.11° = \frac{0.75}{32.2}(-112.5)$$

$$N = 2.773 \text{ lb} = 2.77 \text{ lb} \qquad \textbf{Ans}$$

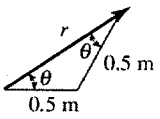
$$\sum F_\theta = ma_\theta; \quad F_{OA} + 2.773\sin 19.11° = \frac{0.75}{32.2}(64.952)$$

$$F_{OA} = 0.605 \text{ lb}$$

13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an "angular velocity $\ddot{\theta} = 4$ rad/s and an angular acceleration $\dot{\theta} = 8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



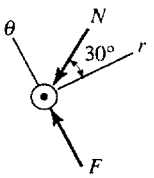
$$r = 2(0.5\cos\theta) = 1\cos\theta$$



$$\dot{r} = -\sin\theta\dot{\theta}$$

$$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$$

At $\theta = 30°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 8$ rad/s^2



$$r = 1\cos 30° = 0.8660 \text{ ft}$$

$$\dot{r} = -\sin 30°(4) = -2 \text{ ft/s}$$

$$\ddot{r} = -\cos 30°(4)^2 - \sin 30°(8) = -17.856 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072 \text{ ft/s}^2$$

$$\nearrow + \sum F_r = ma_r: \quad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686 \text{ lb}$$

$$+\nwarrow \sum F_\theta = ma_\theta; \quad F - 0.5686\sin 30° = \frac{0.5}{32.2}(-9.072)$$

$$F = 0.143 \text{ lb} \qquad \textbf{Ans}$$

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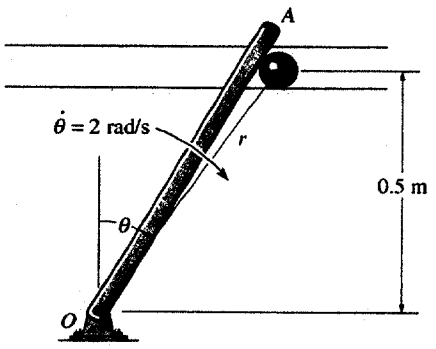
13-91. The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2$rad/s and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

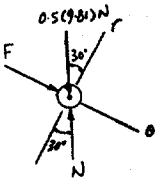
$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$



$+\nearrow \Sigma F_r = ma_r$; $N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N Ans

$+\nwarrow \Sigma F_\theta = ma_\theta$; $F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N Ans

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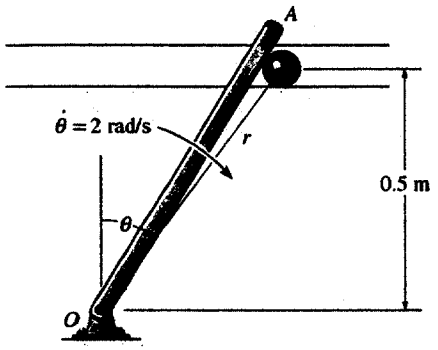
***13-92.** Solve Problem 13–91 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ and $\dot{\theta} = 2\ \text{rad/s}$ at this instant. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\ \text{m}$$

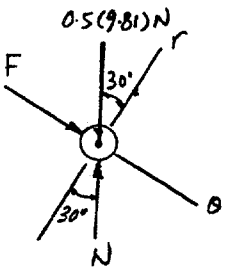
$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$



$\nearrow + \Sigma F_r = ma_r$; $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$$N = 6.3712 = 6.37\ \text{N} \qquad \textbf{Ans}$$

$+\searrow \Sigma F_\theta = ma_\theta$; $\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$$F = 2.93\ \text{N} \qquad \textbf{Ans}$$

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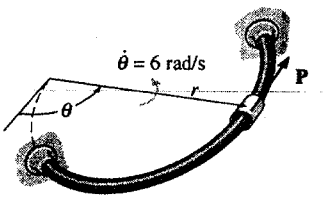
13-93. The spool, which has a mass of 2 kg, slides along the smooth *horizontal* spiral rod, $r = (0.4\theta)$ m, where θ is in radians. If its angular rate of rotation is constant and equals $\dot{\theta} = 6$ rad/s, determine the horizontal tangential force P needed to cause the motion and the horizontal normal force component that the spool exerts on the rod at the instant $\theta = 45°$.



$r = 0.4\theta \qquad \dot{r} = 0.4\dot{\theta} \qquad \ddot{r} = 0.4\ddot{\theta}$

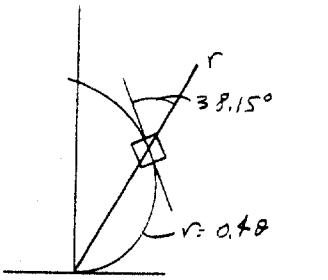
At $\theta = 45° = \frac{\pi}{4}$ rad, $\dot{\theta} = 6$ rad/s and $\ddot{\theta} = 0$

$r = 0.4\left(\frac{\pi}{4}\right) = (0.1)\ \pi$ m $\qquad \dot{r} = 0.4(6) = 2.4$ m/s $\qquad \ddot{r} = 0.4(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.1)\pi(6)^2 = -11.310$ m/s^2

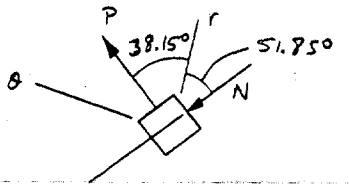
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.1)\pi(0) + 2(2.4)(6) = 28.8$ m/s^2

$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.4\theta}{0.4} = \theta = \frac{\pi}{4} \qquad \psi = 38.15°$



$+\nearrow \Sigma F_r = ma_r; \qquad P\cos 38.15° - N\cos 51.85° = 2(-11.310)$

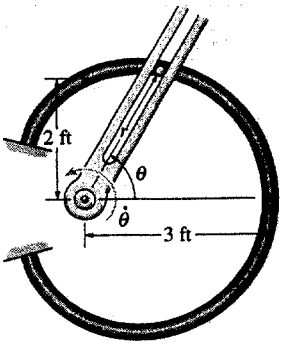
$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad P\sin 38.15° + N\sin 51.85° = 2(28.8)$



Solving

$N = 59.3$ N $\qquad P = 17.8$ N $\qquad$ **Ans**

13-94. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

$r = 2 + \cos 90° = 2$ ft

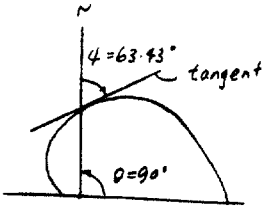
$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

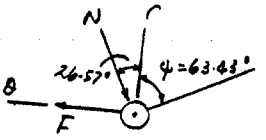
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5$ ft/s^2

$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta = 90°} = -2 \qquad \psi = -63.43°$



$+\uparrow \Sigma F_r = ma_r; \qquad -N\cos 26.57° = \frac{2}{32.2}(-0.5) \qquad N = 0.03472$ lb



$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad F - 0.03472\sin 26.57° = \frac{2}{32.2}(-0.5)$

$F = -0.0155$ lb $\qquad$ **Ans**

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13-95. Solve Prob. 13-94 at the instant $\theta = 60°$.

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

$r = 2 + \cos 60° = 2.5$ ft

$\dot{r} = -\sin 60°(0.5) = -0.4330$ ft/s

$\ddot{r} = -\cos 60°(0.5)^2 - \sin 60°(0) = -0.125$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75$ ft/s^2

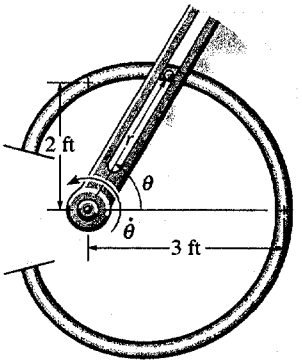
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330$ ft/s^2

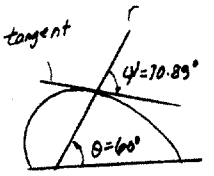
$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2+\cos\theta}{-\sin\theta}\Big|_{\theta=60°} = -2.887 \qquad \psi = -70.89°$

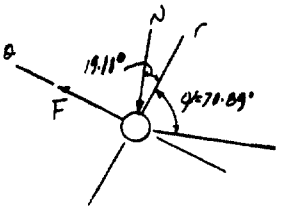
$+\nearrow \Sigma F_r = ma_r; \qquad -N\cos 19.11° = \dfrac{2}{32.2}(-0.75) \qquad N = 0.04930$ lb

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.04930\sin 19.11° = \dfrac{2}{32.2}(-0.4330)$

$F = -0.0108$ lb **Ans**







***13-96.** The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.

$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\dot{\theta} \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1$ rad/s^2

At $t = 1$ s, $\theta = 0.5$ rad, $\dot{\theta} = 1$ rad/s and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

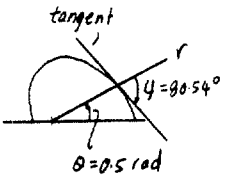
$\dot{r} = -\sin 0.5(1) = -0.4794$ ft/s

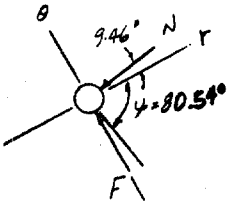
$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.357 - 2.8776(1)^2 = -4.2346$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187$ ft/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2+\cos\theta}{-\sin\theta}\Big|_{\theta=0.5\text{ rad}} = -6.002 \qquad \psi = -80.54°$

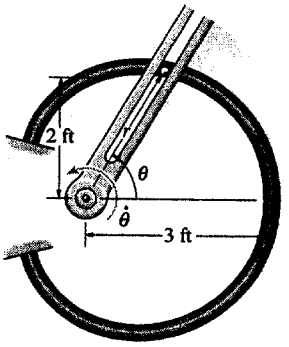




$+\nearrow \Sigma F_r = ma_r; \qquad -N\cos 9.46° = \dfrac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46° = \dfrac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans**



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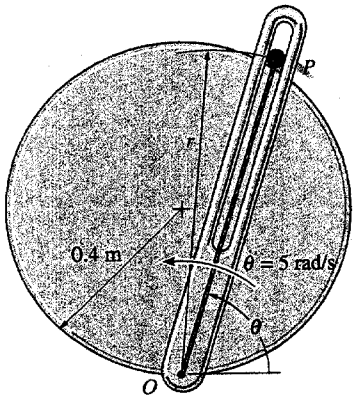
13-97. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin\theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\ \dot{\theta}$

$\ddot{r} = -0.8 \sin\theta\ (\dot{\theta})^2 + 0.8 \cos\theta\ \ddot{\theta}$

$\dot{\theta} = 5, \quad \ddot{\theta} = 0$

At $\theta = 60°$, $r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$

$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

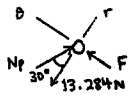
$F_s = ks; \quad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \quad -13.284 + N_P \cos 30° = 0.08(-34.641)$

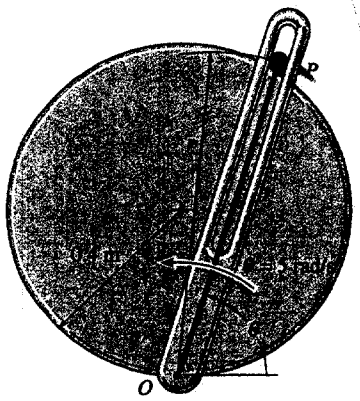
$\nwarrow + \Sigma F_\theta = m a_\theta; \quad F - N_P \sin 30° = 0.08(20)$

$F = 7.67$ N **Ans**

$N_P = 12.1$ N



13-98. Solve Prob. 13–97 if $\ddot{\theta} = 2$ rad/s^2 when $\dot{\theta} = 5$ rad/s and $\theta = 60°$.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\ \dot{\theta}$

$\ddot{r} = -0.8 \sin\theta\ (\dot{\theta})^2 + 0.8 \cos\theta\ \ddot{\theta}$

$\dot{\theta} = 5, \quad \ddot{\theta} = 2$

At $\theta = 60°$, $r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -16.521$

$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(2) + 2(2)(5) = 21.386$

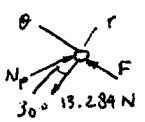
$F_s = ks; \quad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \quad -13.284 + N_P \cos 30° = 0.08(-33.841)$

$+\nwarrow \Sigma F_\theta = m a_\theta; \quad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82$ N **Ans**

$N_P = 12.2$ N



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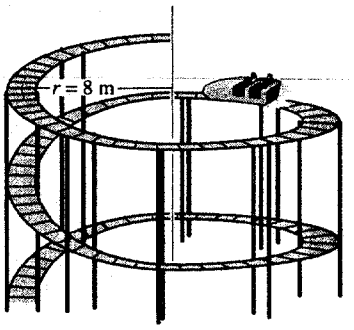
13-99. For a short time, the 250-kg roller coaster car is traveling along the spiral track such that its position measured from the top of the track has components $r = 8$ m, $\theta = (0.1t + 0.5)$ rad, and $z = (-0.2t)$ m, where t is in seconds. Determine the magnitudes of the components of force which the track exerts on the car in the r, θ, and z directions at the instant $t = 2$ s. Neglect the size of the car.



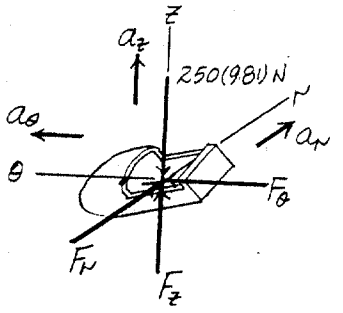
Kinematic : Here, $r = 8$ m, $\dot{r} = \ddot{r} = 0$. Taking the required time derivatives at $t = 2$ s, we have

$$\theta = 0.1t + 0.5\big|_{t=2s} = 0.700 \text{ rad} \qquad \dot{\theta} = 0.100 \text{ rad/s} \qquad \ddot{\theta} = 0$$

$$z = -0.2t\big|_{t=2s} = -0.400 \text{ m} \qquad \dot{z} = -0.200 \text{ m/s} \qquad \ddot{z} = 0$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 8\left(0.100^2\right) = -0.0800 \text{ m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8(0) + 2(0)(0.200) = 0$$
$$a_z = \ddot{z} = 0$$



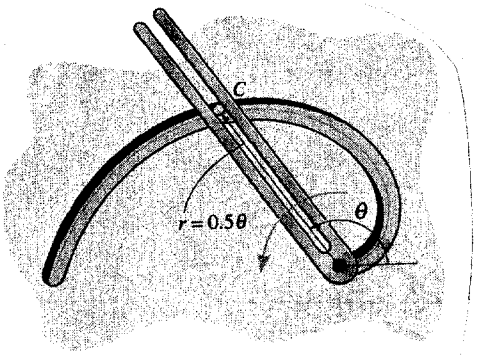
Equation of Motion :

$$\Sigma F_r = ma_r; \qquad F_r = 250(-0.0800) = -20.0 \text{ N} \qquad \textbf{Ans}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 250(0) = 0 \qquad \textbf{Ans}$$

$$\Sigma F_z = ma_z; \qquad F_z - 250(9.81) = 250(0)$$
$$F_z = 2452.5 \text{ N} = 2.45 \text{ kN} \qquad \textbf{Ans}$$

***13-100.** Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t = 2$ s. The cylinder is in contact with only *one edge* of the rod and slot at any instant.



$r = 0.5\theta \qquad \dot{r} = 0.5\dot{\theta} \qquad \ddot{r} = 0.5\ddot{\theta}$

$\theta = 0.5t^2 \qquad \dot{\theta} = t \qquad \ddot{\theta} = 1$

At $t = 2$ s,

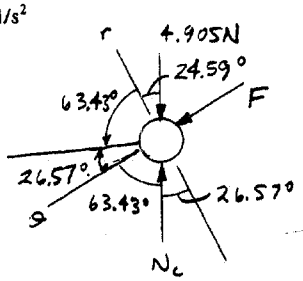
$\theta = 2 \text{ rad} = 114.59° \qquad \dot{\theta} = 2 \text{ rad/s} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

$r = 1 \text{ m} \qquad \dot{r} = 1 \text{ m/s} \qquad \ddot{r} = 0.5 \text{ m/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5(2)}{0.5} \qquad \psi = 63.43°$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 1(2)^2 = -3.5$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1(1) + 2(1)(2) = 5$$



$+\nearrow \Sigma F_r = ma_r; \qquad N_C\cos 26.57° - 4.905\cos 24.59° = 0.5(-3.5)$

$N_C = 3.030 = 3.03 \text{ N}$ **Ans**

$+\swarrow \Sigma F_\theta = ma_\theta; \qquad F - 3.030\sin 26.57° + 4.905\sin 24.59° = 0.5(5)$

$F = 1.81 \text{ N}$ **Ans**

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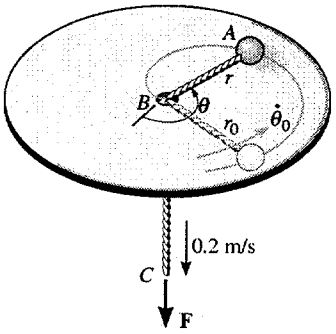
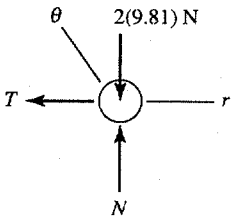
13-101. The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$$\sum F_\theta = ma_\theta; \quad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$$



Thus,

$$d(r^2\dot{\theta}) = 0$$

$$r^2\dot{\theta} = C$$

$$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$$

$$\dot{\theta} = 4.00 \text{ rad/s} \qquad \textbf{Ans}$$

Since $r = -0.2$ m/s, $\ddot{r} = 0$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2$$

$$\sum F_r = ma_r; \quad -T = 2(-4)$$

$$T = 8 \text{ N} \qquad \textbf{Ans}$$

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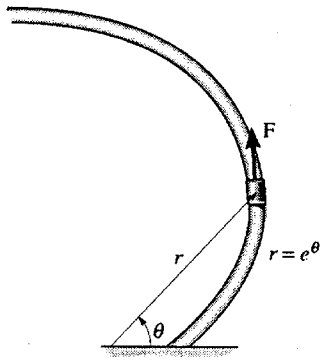
13-102. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\dot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

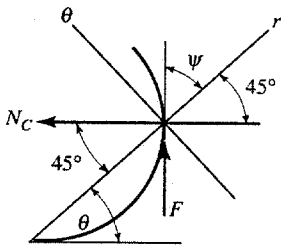
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$\tan \Psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\Psi = \theta = 45°$



$\nearrow + \sum F_r = ma_r; \quad -N_C \cos 45° + F \cos 45° = 2(0)$

$+\nwarrow \sum F_\theta = ma_\theta; \quad F \sin 45° + N_C \sin 45° = 2(17.5462)$

$N = 24.8$ N **Ans**

$F = 24.8$ N **Ans**

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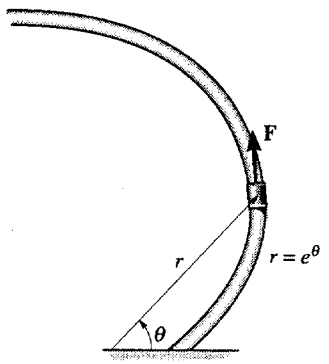
13-103. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 4.8105$

$\dot{r} = 9.6210$

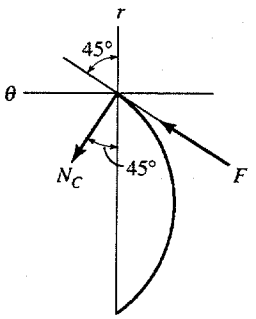
$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838$ m/s^2

$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = 45°$



$+\uparrow \sum F_r = ma_r; \quad -N_C\cos 45° + F\cos 45° = 2(0)$

$\overset{+}{\leftarrow} \sum F_\theta = ma_\theta; \quad F\sin 45° + N_C\sin 45° = 2(38.4838)$

$N_C = 54.4N$ **Ans**

$F = 54.4$ N **Ans**

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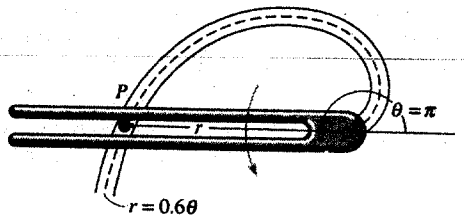
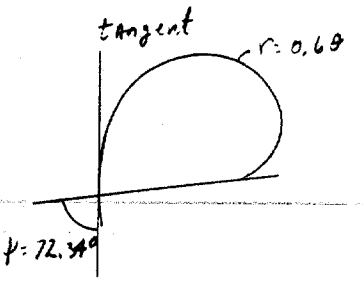
***13-104.** Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.





$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

$v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1+\theta^2}$

At $\theta = \pi$ rad, $\dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011$ rad/s

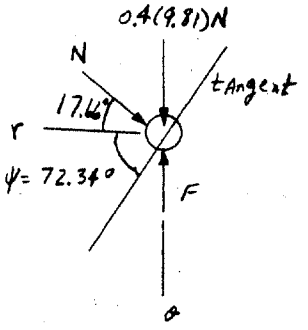
$\ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954$ rad/s^2

$r = 0.6(\pi) = 0.6\pi$ m $\qquad \dot{r} = 0.6(1.011) = 0.6066$ m/s

$\ddot{r} = 0.6(-0.2954) = -0.1772$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698$ m/s^2



$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883$ N **Ans**

$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

$F = 3.92$ N **Ans**

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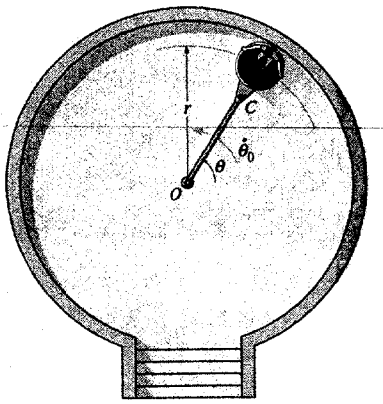
13-105. A ride in an amusement park consists of a cart which is supported by small wheels. Initially the cart is traveling in a circular path of radius $r_0 = 16$ ft such that the angular rate of rotation is $\dot{\theta}_0 = 0.2$ rad/s. If the attached cable OC is drawn inward at a constant speed of $\dot{r} = -0.5$ ft/s, determine the tension it exerts on the cart at the instant $r = 4$ ft. The cart and its passengers have a total weight of 400 lb. Neglect the effects of friction. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)d(r^2\dot{\theta})/dt = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$$+\nearrow \Sigma F_r = ma_r; \quad -T = \left(\frac{400}{32.2}\right)\left(\ddot{r} - r\dot{\theta}^2\right) \quad (1)$$

$$+\nwarrow \Sigma F_\theta = ma_\theta; \quad 0 = \left(\frac{400}{32.2}\right)\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \quad (2)$$

From Eq. (2), $\left(\frac{1}{r}\right)\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \quad r^2\dot{\theta} = c$

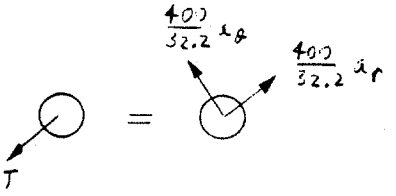
Since $\dot{\theta}_0 = 0.2$ rad/s when $r_0 = 16$ ft, $c = 51.2$.
Hence, when $r = 4$ ft,

$$\dot{\theta} = \left(\frac{51.2}{(4)^2}\right) = 3.2 \text{ rad/s}$$

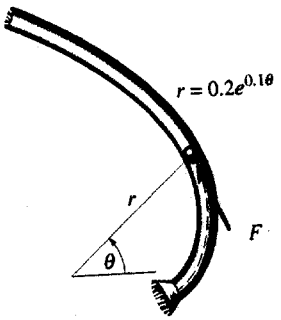
Since $\dot{r} = -0.5$ ft/s, $\ddot{r} = 0$, Eq. (1) becomes

$$-T = \left(\frac{400}{32.2}\right)\left(0 - (4)(3.2)^2\right)$$

$T = 509$ lb **Ans**



13-106. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. What direction does it act

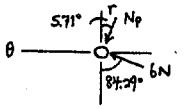


$$r = 0.2\,e^{0.1\theta}$$

$$\frac{dr}{d\theta} = 0.02\,e^{0.1\theta}$$

$$\tan\psi = \frac{r}{\frac{dr}{d\theta}} = \frac{0.2\,e^{0.1\theta}}{0.02\,e^{0.1\theta}} = 10$$

$$\psi = 84.29°$$



Rate of increase in speed is equivalent to the tangential component of acceleration.

$$\Sigma F_t = m a_t; \qquad 6 = 0.5\,a_t$$

$$a_t = 12 \text{ m/s}^2 \;\underset{5.71°}{\measuredangle} \quad \textbf{Ans}$$

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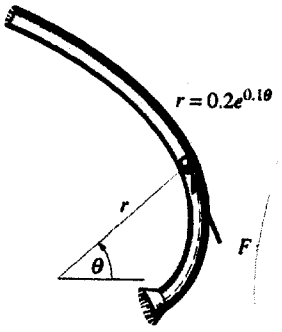
13-107. Solve Prob. 13–106 if the tube lies in a *vertical plane.*



$r = 0.2\,e^{0.1\theta}$

$\frac{dr}{d\theta} = 0.02\,e^{0.1\theta}$

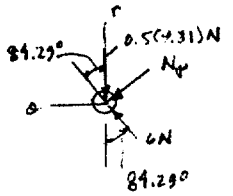
$\tan\psi = \frac{r}{\frac{dr}{d\theta}} = \frac{0.2\,e^{0.1\theta}}{0.02\,e^{0.1\theta}} = 10$

$\psi = 84.29°$

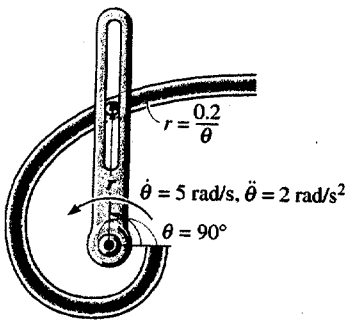
Rate of increase in speed is equivalent to the tangential component of acceleration.

$\Sigma F_t = 0.5\,a_t$; $\quad 6 - 0.5(9.81)\cos 84.29° = 0.5\,a_t$

$a_t = 11.0\ \text{m/s}^2$ ⦜ 5.71° Ans



***13-108.** The arm is rotating at a rate of $\dot\theta = 5$ rad/s when $\ddot\theta = 2\ \text{rad/s}^2$ and $\theta = 90°$. Determine the normal force it must exert on the 0.5-kg particle if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = 0.2$ m.



$\theta = \frac{\pi}{2} = 90°$

$\dot\theta = 5\ \text{rad/s}$

$\ddot\theta = 2\ \text{rad/s}^2$

$r = 0.2/\theta = 0.12732\ \text{m}$

$\dot r = -0.2\,\theta^{-2}\dot\theta = -0.40528\ \text{m/s}$

$\ddot r = -0.2[-2\theta^{-3}(\dot\theta)^2 + \theta^{-2}\ddot\theta] = 2.41801$

$a_r = \ddot r - r(\dot\theta)^2 = 2.41801 - 0.12732(5)^2 = -0.7651\ \text{m/s}^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0.12732(2) + 2(-0.40528)(5) = -3.7982\ \text{m/s}^2$

$\tan\psi = \frac{r}{(\frac{dr}{d\theta})} = \frac{0.2/\theta}{-0.2\theta^{-2}}$

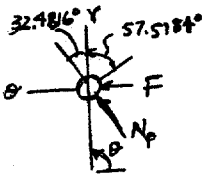
$\psi = \tan^{-1}(-\frac{\pi}{2}) = -57.5184°$

$+\uparrow \Sigma F_r = m\,a_r$; $\quad N_p\cos 32.4816° = 0.5(-0.7651)$

$\overset{+}{\leftarrow} \Sigma F_\theta = m\,a_\theta$; $\quad F + N_p\sin 32.4816° = 0.5(-3.7982)$

$N_p = -0.453$ N

$F = -1.66$ N Ans



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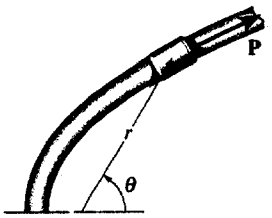
13-109. The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



$$r = \frac{4}{1-\cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\,\dot{\theta}}{(1-\cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\,\ddot{\theta}}{(1-\cos\theta)^2} + \frac{-4\cos\theta\,(\dot{\theta})^2}{(1-\cos\theta)^2} + \frac{8\sin^2\theta\,\dot{\theta}^2}{(1-\cos\theta)^3}$$

At $\theta = 90°$, $\dot{\theta} = 4$, $\ddot{\theta} = 0$

$$r = 4$$
$$\dot{r} = -16$$
$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1-\cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1-\cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{\frac{4}{1-\cos\theta}}{\frac{-4\sin\theta}{(1-\cos\theta)^2}}\bigg|_{\theta=90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$$+\uparrow \Sigma F_r = m a_r; \quad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$$

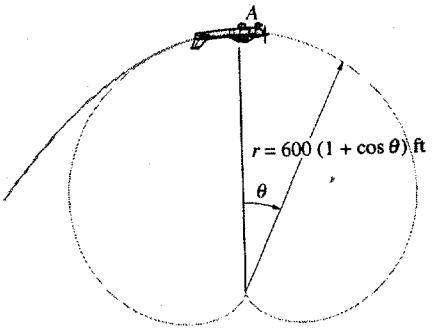
$$\overset{+}{\leftarrow} \Sigma F_\theta = m a_\theta; \quad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$$

Solving,

$P = 12.6$ lb **Ans**

$N = 4.22$ lb **Ans**

13-110. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft, where θ is in radians. If his speed at A ($\theta = 0°$) is a constant $v_P = 80$ ft/s, determine the vertical force the belt of his seat must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb. See hint related to Prob. 13–108.



$r = 600(1+\cos\theta)|_{\theta=0°} = 1200$ ft

$\dot{r} = -600\sin\theta\dot{\theta}|_{\theta=0°} = 0$

$\ddot{r} = -600\sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2|_{\theta=0°} = -600\dot{\theta}^2$

$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$

$2v_p\dot{v}_p = 2\dot{r}\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\dot{\theta} + r\ddot{\theta}\right)$

$0 = 0 + 0 + 2r^2\dot{\theta}\ddot{\theta} \qquad \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

$+\uparrow \Sigma F_r = ma_r; \quad N - 150 = \left(\frac{150}{32.2}\right)(-8) \quad N = 113$ lb **Ans**

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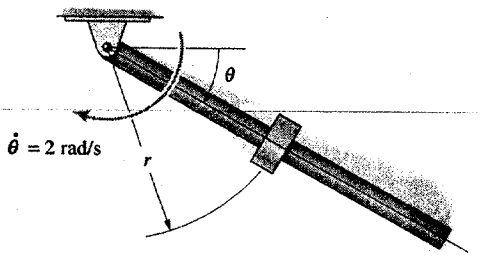
13-111. A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 determine r at the instant $\theta = \pi/4$ rad.



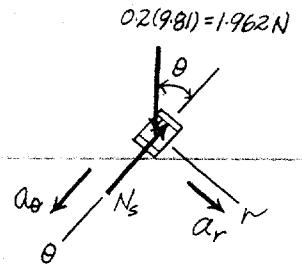
Kinematic : Here, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$. Applying Eqs. 12 – 29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$

Equation of Motion : Applying Eq. 13 – 9, we have

$$\Sigma F_r = ma_r; \quad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$$
$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad (\textbf{Q.E.D.}) \qquad [1]$$

$$\Sigma F_\theta = ma_\theta; \quad 1.962\cos\theta - N_s = 0.2(4\dot{r})$$
$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad (\textbf{Q.E.D.}) \qquad [2]$$



Since $\dot{\theta} = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^t 2\,dt$, $\theta = 2t$. The solution of the differential equation (Eq. [1]) is given by

$$r = C_1 e^{-2t} + C_2 e^{2t} - \frac{9.81}{8}\sin 2t \qquad [3]$$

Thus,

$$\dot{r} = -2C_1 e^{-2t} + 2C_2 e^{2t} - \frac{9.81}{4}\cos 2t \qquad [4]$$

At $t = 0$, $r = 0$. From Eq. [3] $\quad 0 = C_1(1) + C_2(1) - 0 \qquad [5]$

At $t = 0$, $\dot{r} = 0$. From Eq. [4] $\quad 0 = -2C_1(1) + 2C_2(1) - \frac{9.81}{4} \qquad [6]$

Solving Eqs. [5] and [6] yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$
$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$
$$= \frac{9.81}{8}(\sinh 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\quad r = \frac{9.81}{8}\left(\sinh\frac{\pi}{4} - \sin\frac{\pi}{4}\right) = 0.198$ m **Ans**

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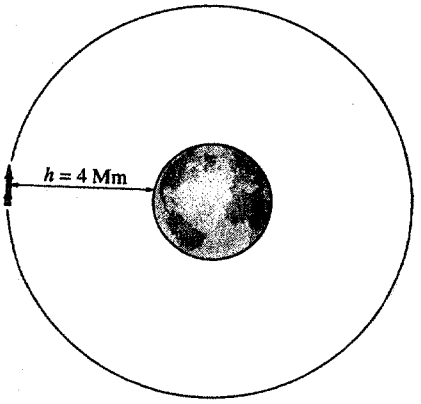
***13-112.** The rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Circular orbit :

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3)+6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic orbit :

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3)+6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$\Delta v = 2.57$ km/s **Ans**

13-113. Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

From Eq. 13 – 19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C,

From Eqs. 13 – 28 and 13 – 29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13 – 31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$ **Q.E.D.**

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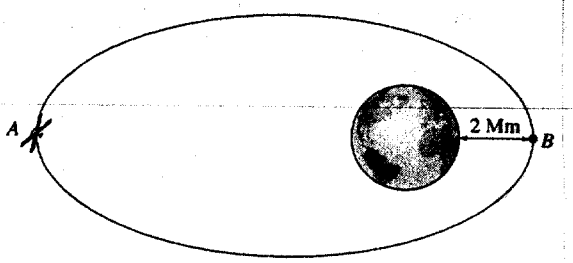
13-114. The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



$$e = \frac{Ch^2}{GM_e} \quad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

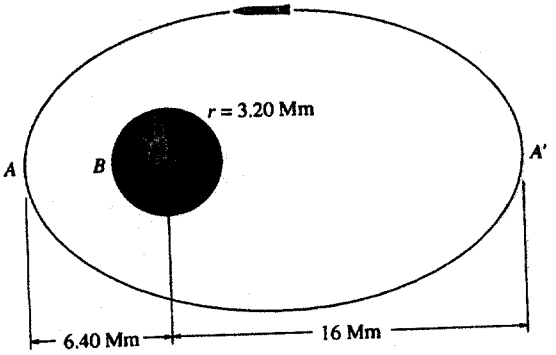
Where $r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)\,\text{m}$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713\ \text{m/s} = 7.71\ \text{km/s} \qquad \textbf{Ans}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96(10^6)$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628\ \text{m/s} = 4.63\ \text{km/s} \qquad \textbf{Ans}$$

13-115. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has a mass 0.60 times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



Central-Force Motion: Substitute Eq. 13–27, $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$,

with $r_a = 16(10^6)$ m, $r_0 = r_p = 6.40(10^6)$ m and $M = 0.60M_e$, we have

$$16(10^6) = \frac{6.40(10^6)}{\left(\frac{2(66.73)(10^{-12})(0.6)[5.976(10^{24})]}{6.40(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 7308.07\,\text{m/s} = 7.31\ \text{km/s} \qquad \textbf{Ans}$$

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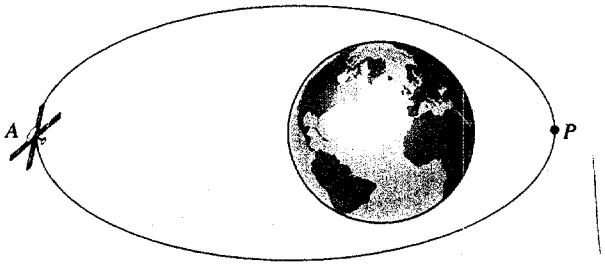
***13-116.** An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



$e = 0.130$

$v_P = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{[4.167(10^3)]^2}$$

$$= 25.96 \text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.6 \text{ Mm/h} \qquad \text{Ans}$$

$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3 \text{ Mm} \qquad \text{Ans}$$

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13-117. A satellite is launched with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})$ (lb·ft^2)/slug^2, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi = 5280 ft.

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $$e = \frac{C^2 h}{GM_e} = 0 \quad \text{or } C = 0$$

$$1 = \frac{GM_e}{r_0 v_0^2}$$

$$GM_e = 34.4(10^{-9})(409)(10^{21})$$

$$= 14.07(10^{15})$$

$$r_0 = \frac{GM_e}{v_0^2} = \frac{14.07(10^{15})}{[3.67(10^3)]^2} = 1.046(10^9) \text{ ft}$$

$$r = \frac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi} \qquad \textbf{Ans}$$

(b) $$e = \frac{C^2 h}{GM_e} = 1$$

$$\frac{1}{GM_e}(r_0^2 v_0^2)\left(\frac{1}{r_0}\right)\left(1 - \frac{GM_e}{r_0 v_0^2}\right) = 1$$

$$r_0 = \frac{2GM_e}{v_0^2} = \frac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$$

$$r = 396(10^3) - 3960 = 392(10^3) \text{ mi} \qquad \textbf{Ans}$$

(c) $e < 1$

$$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi} \qquad \textbf{Ans}$$

(d) $e > 1$

$$r > 392(10^3) \text{ mi} \qquad \textbf{Ans}$$

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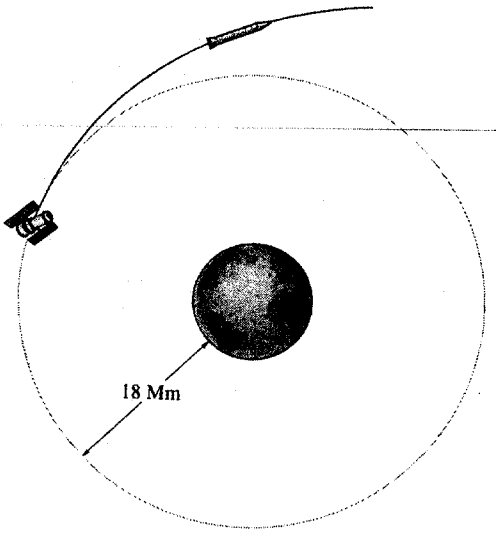
13-118. The rocket is docked next to a satellite located 18 Mm above the earth's surface. If the satellite is traveling in a circular orbit, determine the speed tangent to the earth's surface which must suddenly be given to the rocket, relative to the satellite, such that it travels in free flight away from the satellite along a parabolic trajectory as shown.



Central - Force Motion : In order for the satellite to have a circular free - flight trajectory, from Eq. 13 – 23, $e = 0$. Substitute $e = 0$ into Eq. 13 – 17, we have

$$e = \frac{Ch^2}{GM_e} = 0$$

Since $h = r_0 v_C \neq 0$, then $C = 0$. Using Eq. 13 – 21, we have

$$C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_c^2}\right) = 0 \qquad v_c = \sqrt{\frac{GM_e}{r_0}}$$

Here, $r_0 = (18 + 6.378)\left(10^6\right) = 24.378\left(10^6\right)$ m and $v_c = v_s$.

$$v_s = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{24.378(10^6)}} = 4044.52 \text{ m/s}$$

In order for the rocket to have a parabolic free - flight trajectory, from Eq. 13 – 23, $e = 1$. Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 – 21] and $h = r_0 v_0$ [Eq. 13 – 20]. Substitute these values into Eq. 13 – 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Substitute $e = 1$ and $v_0 = v_r$ into Eq. [1], we have

$$\frac{r_0 v_r^2}{GM_e} - 1 = 1$$

$$v_r = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{24.378(10^6)}} = 5719.81 \text{ m/s}$$

Thus, the required speed of the rocket relative to the satellite is given by

$$v_{rs} = v_r - v_s = 5719.81 - 4044.52 = 1675.29 \text{ m/s} = 1.68 \text{ km/s} \qquad \textbf{Ans}$$

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13-119. Show the speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

For a circle

$$e = \frac{ch^2}{GM_e} = 0 \text{ or } ch^2 = 0$$

but $h = r_0 v_0 \neq 0$,

so that $c = 0$

Hence, $$c = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) = 0$$

$$r_0 v_0^2 = GM_e$$

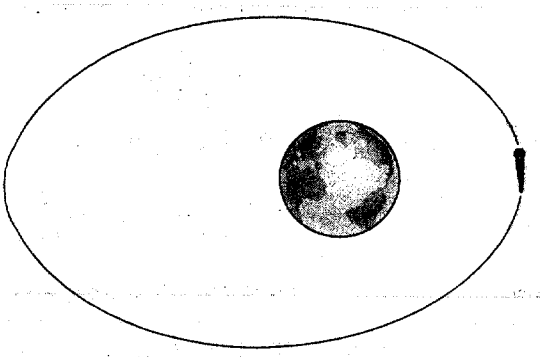
$$v_0 = \sqrt{\frac{GM_e}{r_0}} \qquad \text{QED}$$

For a 800 - km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800+6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \text{Ans}$$

***13-120.** The rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



To escape the earth's gravitation field, the rocket has to make a parabolic trajectory.

Parabolic trajectory :

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical orbit :

$$e = \frac{Ch^2}{GM_e} \quad \text{where} \quad C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \text{Ans}$$

The change in speed should occur at **perigee**. **Ans**

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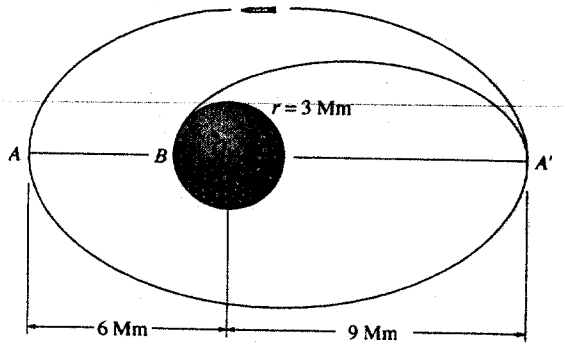
13-121. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth's. If the rocket has an apogee and perigee as shown in the figure, determine the speed of the rocket when it is at point A.



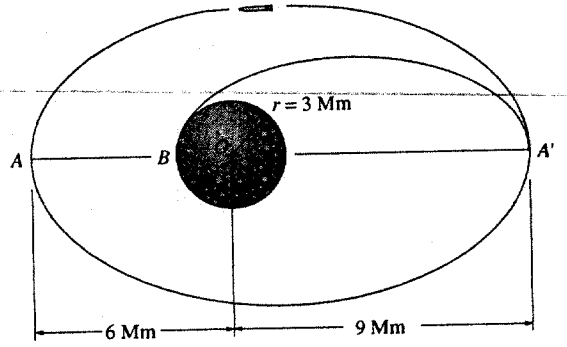
Central - Force Motion : Use $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$,

with $r_a = 9(10^6)$ m, $r_0 = r_p = 6(10^6)$ m and $M = 0.70M_e$, we have

$$9(10^6) = \frac{6(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{6(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans}$$

13-122. If the rocket in Prob. 13–121 is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path?



Central - Force Motion : Use $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$,

with $r_a = 9(10^6)$ m, $r_0 = r_p = 3(10^6)$ m and $M = 0.70M_e$, we have

$$9(10^6) = \frac{3(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{3(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 11814.08 \text{ m/s}$$

Applying Eq. 13 – 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{3(10^6)}{9(10^6)}\right](11814.08) = 3938.03 \text{ m/s} = 3.94 \text{ km/s} \qquad \textbf{Ans}$$

Eq. 13 – 20 gives $h = r_p v_p = 3(10^6)(11814.08) = 35.442(10^9)$ m^2/s. Thus, applying Eq. 13 – 31, we have

$$\begin{aligned} T &= \frac{\pi}{h}\left(r_p + r_a\right)\sqrt{r_p r_a} \\ &= \frac{\pi}{35.442(10^9)}\left[(9+3)(10^6)\right]\sqrt{3(10^6)9(10^6)} \\ &= 5527.03 \text{ s} \end{aligned}$$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} = 2763.51 \text{ s} = 46.1 \text{ min} \qquad \textbf{Ans}$$

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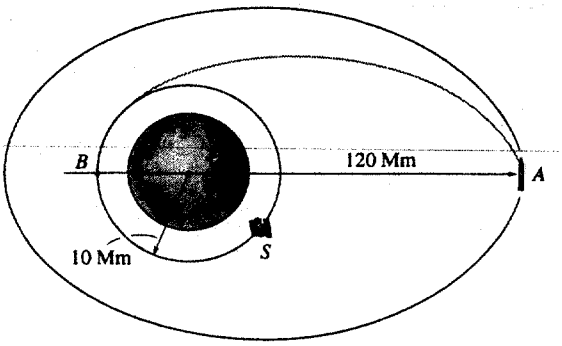
13-123. A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which $e = 0.58$. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the dashed elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.



Central - Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 – 21] and $h = r_0 v_0$ [Eq. 13 – 20]. Substitute these values into Eq. 13 – 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2], we have

$$v_0 = \sqrt{\frac{(1+e)GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 – 27, $r_a = \frac{r_0}{\left(2GM_e / r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \quad \text{or} \quad r_0 = \left(\frac{1-e}{1+e}\right) r_a \qquad [4]$$

For the first elliptical orbit $e = 0.58$, from Eq. [4]

$$(r_p)_1 = r_0 = \left(\frac{1-0.58}{1+0.58}\right)\left[120(10^6)\right] = 31.899(10^6) \text{ m}$$

Substitute $r_0 = (r_p)_1 = 31.899(10^6)$ m into Eq. [3] yields

$$(v_p)_1 = \sqrt{\frac{(1+0.58)(66.73)(10^{-12})(5.976)(10^{24})}{31.899(10^6)}} = 4444.34 \text{ m/s}$$

Applying Eq. 13 – 20, we have

$$(v_a)_1 = \left(\frac{r_p}{r_a}\right)(v_p)_1 = \left[\frac{31.899(10^6)}{120(10^6)}\right](4444.34) = 1181.41 \text{ m/s}$$

When the rocket travels along the second elliptical orbit, from Eq. [4], we have

$$10(10^6) = \left(\frac{1-e}{1+e}\right)\left[120(10^6)\right] \qquad e = 0.8462$$

Substitute $r_0 = (r_p)_2 = 10(10^6)$ m into Eq. [3] yields

$$(v_p)_2 = \sqrt{\frac{(1+0.8462)(66.73)(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 8580.25 \text{ m/s}$$

Applying Eq. 13 – 20, we have

$$(v_a)_2 = \left[\frac{(r_p)_2}{(r_a)_2}\right](v_p)_2 = \left[\frac{10(10^6)}{120(10^6)}\right](8580.25) = 715.02 \text{ m/s}$$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = (v_a)_1 - (v_a)_2 = 1184.41 - 715.02 = 466 \text{ m/s} \qquad \textbf{Ans}$$

If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 – 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 6314.89 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = (v_p)_2 - v_c = 8580.25 - 6314.89 = 2265.36 \text{ m/s} = 2.27 \text{ km/s} \qquad \textbf{Ans}$$

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***13-124.** An asteroid is in an elliptical orbit about the sun such that its periapsis is $9.30(10^9)$ km. If the eccentricity of the orbit is $e = 0.073$, determine the apoapsis of the orbit.

$r_p = r_0 = 9.30(10^9)$ km

$$e = \frac{Ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_s}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_s} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_s} = e + 1$$

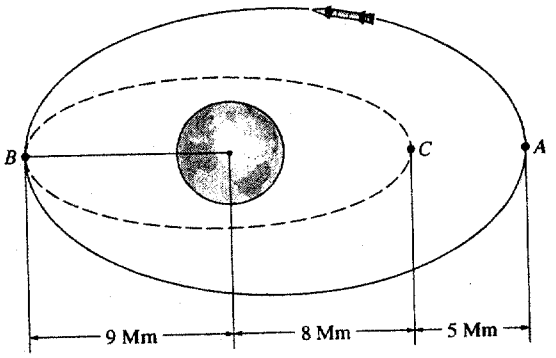
$$\frac{GM_s}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_a = \frac{r_0}{\frac{2GM_s}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_a = \frac{r_0(e+1)}{1-e}$$

$$= \frac{9.30(10^9)(1.073)}{0.927} = 10.8(10^9) \text{ km}$$ **Ans**

13-125. The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ as shown. Determine its speed when it is at point A. Also determine the sudden change in speed the rocket must experience at B in order to travel in free flight along the orbit indicated by the dashed path.



$$e = \frac{Ch^2}{GM_e} \quad \text{where} \quad C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \quad \text{or} \quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad (1)$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} \qquad (2)$$

Substituting Eq. (1) into (2) yields:

$$r_a = \frac{r_0}{2\left(\frac{1}{e+1}\right) - 1} = \frac{r_0(e+1)}{1-e} \qquad (3)$$

From Eq. (1) $\quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \quad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.76+1)}{9(10^6)}} = 8831 \text{ m/s}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{9(10^6)}{13(10^6)}(8831) = 6113 \text{ m/s} = 6.11 \text{ km/s}$$ **Ans**

From Eq. (3) $\quad r_a = \frac{r_0(e+1)}{1-e}$

$$9(10)^6 = \frac{8(10^6)(e+1)}{1-e} \qquad e = 0.05882$$

From Eq. (1) $\quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \quad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$

$$v_C = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.05882+1)}{8(10^6)}} = 7265 \text{ m/s}$$

$$v_B = \frac{r_p}{r_a} v_C = \frac{8(10^6)}{9(10^6)}(7265) = 6458 \text{ m/s}$$

$$\Delta v_B = 6458 - 8831 = -2374 \text{ m/s} = -2.37 \text{ km/s}$$ **Ans**

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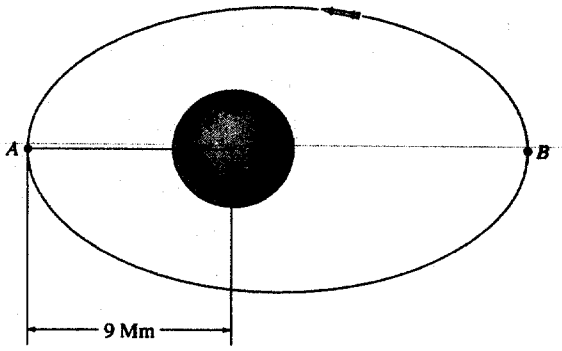
13-126. The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ and its perigee is 9 Mm as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



Central - Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 – 21] and $h = r_0 v_0$ [Eq. 13 – 20]. Substitute these values into Eq. 13 – 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2] , we have

$$v_0 = \sqrt{\frac{(1+e)GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 – 27, $r_a = \frac{r_0}{\left(2GM_e / r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \qquad [4]$$

Rearrange Eq. [4] , we have

$$r_a = \left(\frac{1+e}{1-e}\right) r_0 = \left(\frac{1+0.76}{1-0.76}\right)\left[9\left(10^6\right)\right] = 66.0\left(10^6\right) \text{ m}$$

Substitute $r_0 = r_p = 9\left(10^6\right)$ m into Eq. [3] yields

$$v_p = \sqrt{\frac{(1+0.76)(66.73)(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 8830.82 \text{ m/s}$$

Applying Eq. 13 – 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{9(10^6)}{66.0(10^6)}\right](8830.82) = 1204.2 \text{ m/s} = 1.20 \text{ km/s} \qquad \textbf{Ans}$$

If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 – 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 6656.48 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c = 8830.82 - 6656.48 = 2174.34 \text{ m/s} = 2.17 \text{ km/s} \qquad \textbf{Ans}$$

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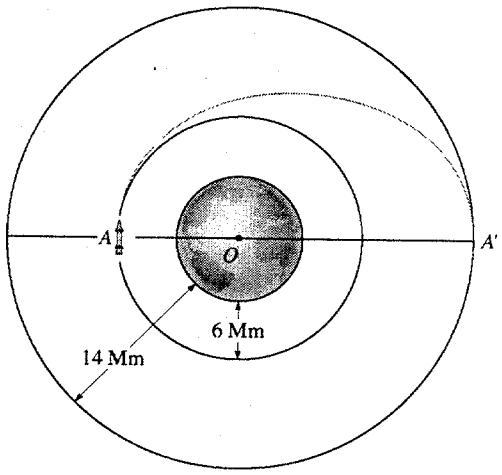
13-127. The rocket shown is originally in a circular orbit 6 Mm above the surface of the earth. It is required that it travel in another circular orbit having an altitude of 14 Mm. To do this, the rocket is given a short pulse of power at A so that it travels in free flight along the dashed elliptical path from the first orbit to the second orbit. Determine the necessary speed it must have at A just after the power pulse, and at the time required to get to the outer orbit along the path AA'. What adjustment in speed must be made at A' to maintain the second circular orbit?



Central-Force Motion: Substitute Eq. 13-27, $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$, with $r_a = (14 + 6.378)(10^6) = 20.378(10^6)$ m and $r_0 = r_p = (6 + 6.378)(10^6) = 12.378(10^6)$ m, we have

$$20.378(10^6) = \frac{12.378(10^6)}{\left(\dfrac{2(66.73)(10^{-12})[5.976(10^{24})]}{12.378(10^6)v_p^2}\right) - 1}$$

$$v_p = 6331.27 \text{ m/s}$$

Applying Eq. 13-20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{12.378(10^6)}{20.378(10^6)}\right](6331.27) = 3845.74 \text{ m/s}$$

Eq. 13-20 gives $h = r_p v_p = 12.378(10^6)(6331.27) = 78.368(10^9)$ m^2/s. Thus, applying Eq. 13-31, we have

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{78.368(10^9)}[(12.378 + 20.378)(10^6)]\sqrt{12.378(20.378)}(10^6)$$

$$= 20854.54 \text{ s}$$

The time required for the rocket to go from A to A' (half the orbit) is given by

$$t = \frac{T}{2} = 10427.38 \text{ s} = 2.90 \text{ hr} \quad \textbf{Ans}$$

In order for the satellite to stay in the second circular orbit, it must achieve a speed of (Eq. 13-25)

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{20.378(10^6)}} = 4423.69 \text{ m/s} = 4.42 \text{ km/s} \quad \textbf{Ans}$$

The speed for which the rocket must be increased in order to enter the second circular orbit at A' is

$$\Delta v = v_c - v_a = 4423.69 - 3845.74 = 578 \text{ m/s} \quad \textbf{Ans}$$

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14-1. A woman having a mass of 70 kg stands in an elevator which has a downward acceleration of 4 m/s^2 starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends 6 m. Explain why the work of these forces is different.

$+\downarrow \Sigma F_y = ma_y$: $\quad 70(9.81) - N_p = 70(4)$

$$N_p = 406.7 \text{ N}$$

$U_W = 6(686.7) = 4.12$ kJ **Ans**

$U_{N_p} = -6(406.7) = -2.44$ kJ **Ans**

The difference accounts for a change in kinetic energy. **Ans**

Note: $v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(4)(6 - 0)$

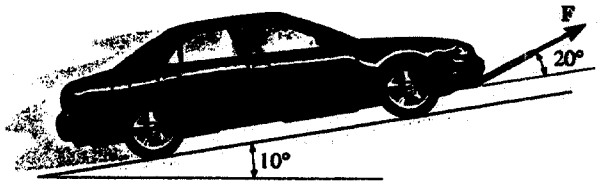
$v = 6.928$ m/s

$\Delta T = \frac{1}{2}(70)(6.928)^2 = 1.68$ kJ

Also, $T_1 + \Sigma U_{1-2} = T_2$

$\Delta T = \Sigma U_{1-2} = 4.12 - 2.44 = 1.68$ kJ

14-2. The car having a mass of 2 Mg is originally traveling at 2 m/s. Determine the distance it must be towed by a force $F = 4$ kN in order to attain a speed of 5 m/s. Neglect friction and the mass of the wheels.



$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(2000)(2)^2 + [4000\cos 20°(s) - 19\,620\sin 10°(s)] = \frac{1}{2}(2000)(5)^2$$

$s = 59.7$ m **Ans**

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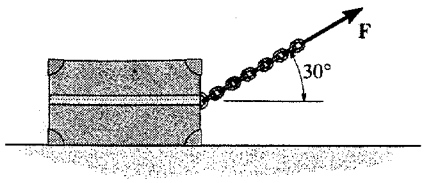
14-3. The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N, where s is measured in meters. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.

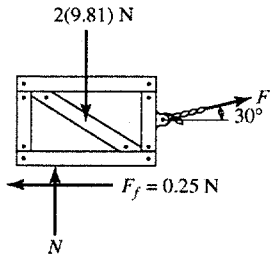


Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25$ N. Applying Eq. 13-7, we have

$$+\uparrow \sum F_y = ma_y; \quad N + \left(\frac{300}{1+s}\right)\sin 30° - 2(9.81) = 2(0)$$

$$N = \left(-\frac{150}{1+s} + 19.62\right) \text{N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25\left(-\frac{150}{1+s} + 19.62\right) = \left(-\frac{37.5}{1+s} + 4.905\right)$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq. 14-7, we have

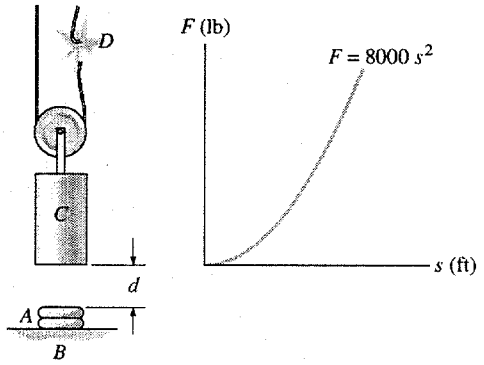


$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2)(8^2) + \int_{15\text{ m}}^{25\text{ m}} \left(\frac{300}{1+s}\right)\cos 30°\, ds - \int_{15\text{ m}}^{25\text{ m}} \left(-\frac{37.5}{1+s} + 4.905\right) ds = \frac{1}{2}(2)v^2$$

$$v = 12.6 \text{ m/s} \qquad \textbf{Ans}$$

***14-4.** The "air spring" A is used to protect the support structure B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the spring as a function of its deflection is shown by the graph. If the weight is 50 lb and is suspended a height $d = 1.5$ ft above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



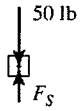
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \left[50(1.5 + s) - \int_0^s 8000\, s^2 ds\right] = 0$$

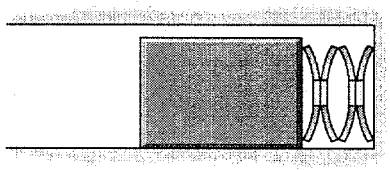
$$50(1.5 + s) - \frac{8000\, s^2}{3} = 0$$

$$8000\, s^2 - 150\, s - 225 = 0$$

$z = 0.3246$ ft $= 3.90$ in. **Ans**



14-5. The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (3s^{1/3})$ lb, where s is given in feet, determine the speed of the plug after it moves away from the spring. Neglect friction.

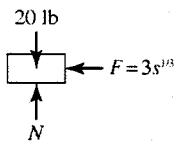


$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \int_0^{0.05} 3s^{\frac{1}{3}} ds = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$3\left(\frac{3}{4}\right)(0.05)^{\frac{1}{3}} = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$v = 0.365$ ft/s **Ans**



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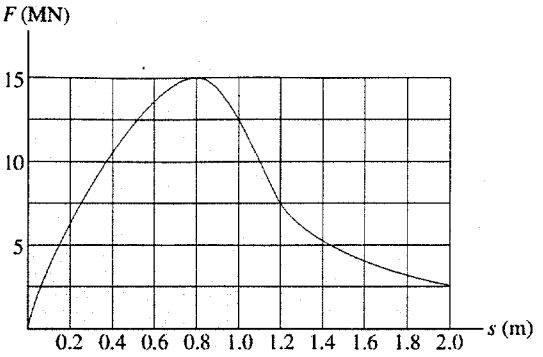
14-6. When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.

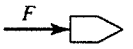


The work done is measured as the area under the force—displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5(10^6)(0.2)$

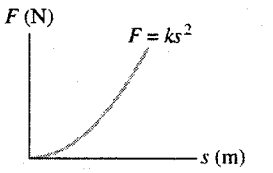
$T_1 + \sum U_{1-2} = T_2$

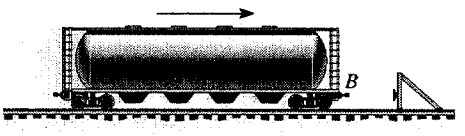
$0 + [(31.5)(2.5)(10^6)(0.2)] = \frac{1}{2}(7)(v_2)^2$

$v_2 = 2121 \text{ m/s} = 2.12 \text{ km/s}$ (approx.) **Ans**



14-7. Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.

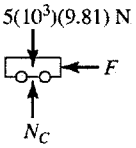




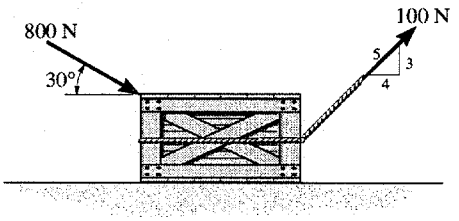
$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$

$40\,000 - k\frac{(0.2)^3}{3} = 0$

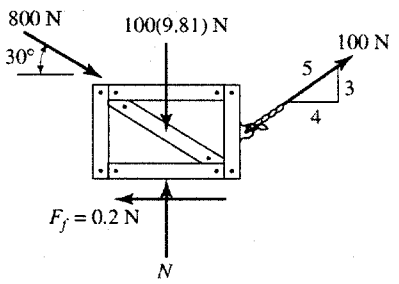
$k = 15.0 \text{ MN/m}^2$ **Ans**



***14-8.** The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2$ N. Applying Eq. 13-7, we have



$+\uparrow \sum F_y = ma_y; \quad N + 100\left(\frac{3}{5}\right) - 800\sin 30° - 100(9.81) = 100(0)$

$N = 1321$ N

Principle of Work and Energy: The horizontal components of forces 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(1321) = 264.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical components of the 800 N and 1000 N forces and the weight of the crate do not displace hence do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14-7, we have

$T_1 + \sum U_{1-2} = T_2$

$0 + 800\cos 30°(s) + 100\left(\frac{4}{5}\right)s - 264.2s = \frac{1}{2}(100)(6^2)$

$s = 3.54$ m **Ans**

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14-9. When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?

$40\text{ km/h} = \frac{40(10^3)}{3600} = 11.11\text{ m/s}$ $\quad 80\text{ km/h} = 22.22\text{ m/s}$

$T_1 + \Sigma U_{1-2} = T_2$

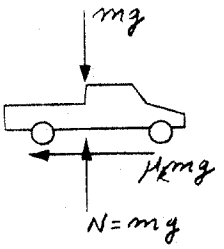
$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$

$\mu_k g = 20.576$

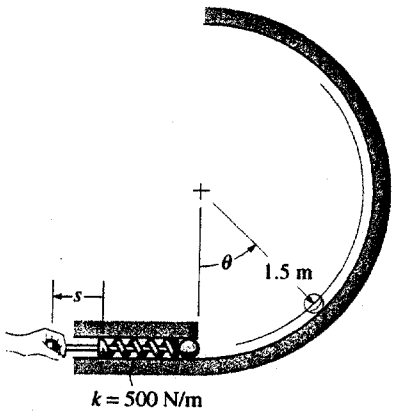
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$

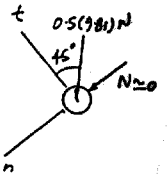
$d = 12\text{ m}$ **Ans**



14-10. The 0.5-kg ball of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



$\Sigma F_n = ma_n$; $\quad 0.5(9.81)\sin 45° = 0.5\left(\frac{v^2}{1.5}\right)$ $\quad v^2 = 10.41\text{ m}^2/\text{s}^2$



$T_1 + \Sigma U_{1-2} = T_2$

$0 + \left\{\left(\frac{1}{2}(500)(s+0.08)^2 - \frac{1}{2}(500)(0.08)^2\right) - 0.5(9.81)(1.5+1.5\sin 45°)\right\} = \frac{1}{2}(0.5)(10.41)$

$s = 0.1789\text{ m} = 179\text{ mm}$ **Ans**

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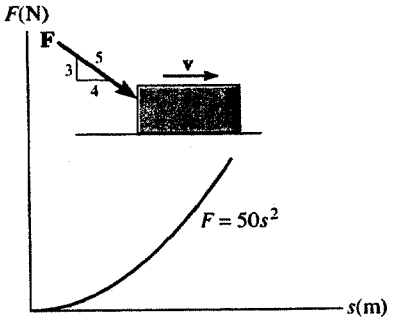
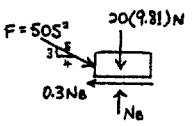
■14-11. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$$

$$N_B = 196.2 + 30\,s^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^s 50\,s^2\,ds - 0.3(196.2)(s) - 0.3\int_0^s 30\,s^2\,ds = \frac{1}{2}(20)(5)^2$$

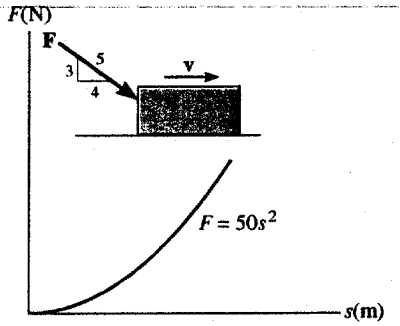
$$40 + 13.33\,s^3 - 58.86\,s - 3\,s^3 = 250$$

$$s^3 - 5.6961\,s - 20.323 = 0$$

Solving for the real root yields

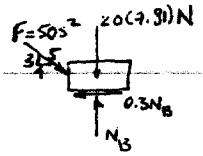
$$s = 3.41 \text{ m} \qquad \text{Ans}$$

***14-12.** The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position s of the block. Determine the speed of the block after it slides 3 m. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$$

$$N_B = 196.2 + 30\,s^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^3 50\,s^2\,ds - 0.3(196.2)(3) - 0.3\int_0^3 30\,s^2\,ds = \frac{1}{2}(20)(v)^2$$

$$40 + 360 - 176.58 - 81 = 10\,v^2$$

$$v = 3.77 \text{ m/s} \qquad \text{Ans}$$

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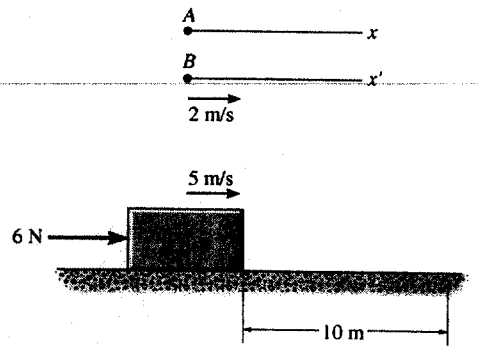
14-13. As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



Observer A :

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$

$v_2 = 6.08$ m/s Ans

Observer B :

$F = ma$

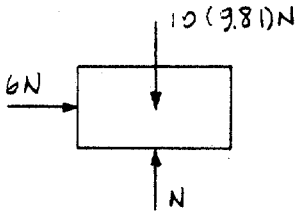
$6 = 10a \quad a = 0.6$ m/s^2

$\left(\stackrel{+}{\rightarrow}\right) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$

$t^2 + 16.67t - 33.33 = 0$

$t = 1.805$ s



At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$

$v_2 = 4.08$ m/s Ans

Note that this result is 2m/s less than that observed by A.

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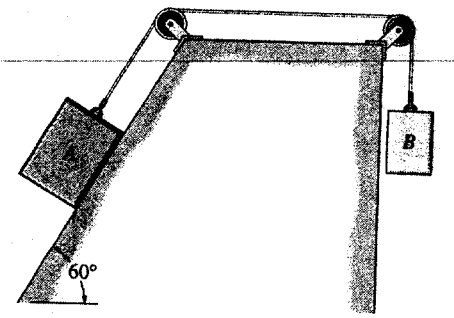
14-14. Determine the velocity of the 20-kg block A after it is released from rest and moves 2 m down the plane. Block B has a mass of 10 kg and the coefficient of kinetic friction between the plane and block A is $\mu_k = 0.2$. Also, what is the tension in the cord?



Block A :

$+\nwarrow \Sigma F_y = 0; \quad N_A - 20(9.81)\cos 60° = 0$

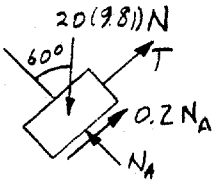
$N_A = 98.1$ N

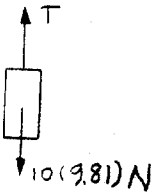
System :

$T_1 + \Sigma U_{1-2} = T_2$

$(0+0) + 20(9.81)(\sin 60°)2 - 0.2(98.1)(2) - 10(9.81)(2) = \frac{1}{2}(20)v^2 + \frac{1}{2}(10)v^2$

$v = 2.638 = 2.64$ m/s **Ans**





Block B :

$T_1 + \Sigma U_{1-2} = T_2$

$0 + T(2) - 10(9.81)(2) = \frac{1}{2}(10)(2.638)^2$

$T = 115$ N **Ans**

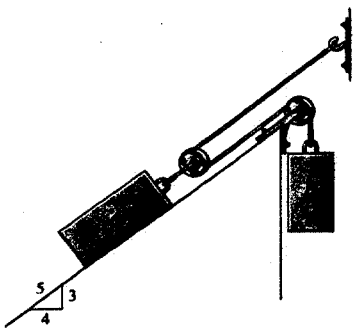
Also, block A :

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(9.81)(\sin 60°)(2) - T(2) - 0.2(98.1)(2) = \frac{1}{2}(20)(2.638)^2$

$T = 115$ N **Ans**

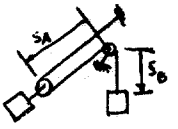
14-15. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.

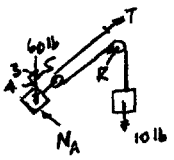


$2s_A + s_B = l$

$2\Delta s_A + \Delta s_B = 0$

$2v_A + v_B = 0$





$T_1 + \Sigma U_{1-2} = T_2$

$0 + 60\left(\frac{3}{5}\right)(5) - 10(10) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2$

$v_A = 7.18$ ft/s **Ans**

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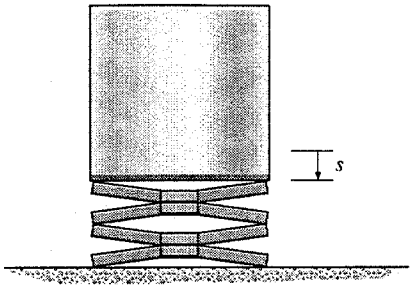
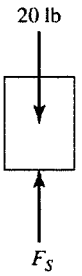
***14-16.** The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (100s^{1/3})$ lb, where s is given in feet, determine the speed of the plug just after it moves away from the spring, i.e., at $s = 0$.



Principle of Work and Energy: The spring force which acts in the direction of displacement does *positive* work, whereas the weight of the block does *negative* work since it acts in the opposite direction to that of displacement. Since he block is initially at rest, $T_1 = 0$. Applying Eq. 14-7, we have

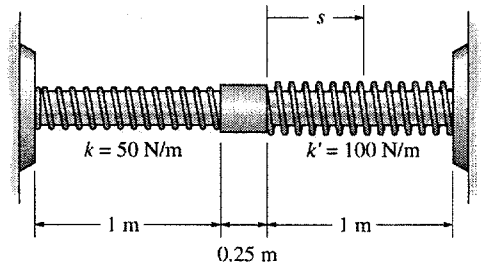


$$T_1 + \sum U_{1-2} = T_2$$

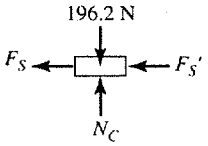
$$0 + \int_0^{0.05\text{ ft}} 100s^{1/3}\,ds - 20(0.05) = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$V = 1.11 \text{ ft/s} \qquad \textbf{Ans}$$

14-17. The collar has a mass of 20 kg and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length of 1 m. If the collar is displaced $s = 0.5$ m and released from rest, determine its velocity at the instant it returns to the point $s = 0$.



$$T_1 + \sum U_{1-2} = T_2$$



$$0 + \frac{1}{2}(50)(0.5)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(20)v_C^2$$

$$v_C = 1.37 \text{ m/s} \qquad \textbf{Ans}$$

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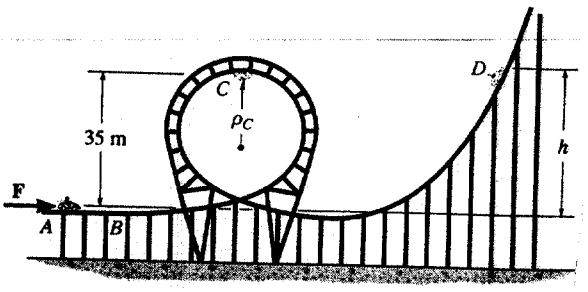
14-18. Determine the height h to the top of the incline D to which the 200-kg roller coaster car will reach, if it is launched at B with a speed just sufficient for it to round the top of the loop at C without leaving the track. The radius of curvature at C is $\rho_C = 25$ m.



Equation of Motion : Here, it is required that $N = 0$. Applying Eq. 13 – 8 to FBD(a), we have

$$\Sigma F_n = ma_n; \qquad 200(9.81) = 200\left(\frac{v_C^2}{25}\right) \qquad v_C^2 = 245.25 \text{ m}^2/\text{s}^2$$

Principle of Work and Energy : The weight of the roller coaster car and passengers do *negative* work since they act in the opposite direction to that of displacement. When the roller coaster car travels from B to C, applying Eq. 14 – 7, we have

$$T_B + \sum U_{B-C} = T_C$$

$$\frac{1}{2}(200)\,v_B^2 - 200(9.81)(35) = \frac{1}{2}(200)(245.25)$$

$$v_B = 30.53 \text{ m/s}$$

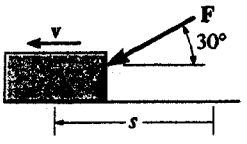
When the roller coaster car travels from B to D, it is required that the car stops at D, hence $T_D = 0$.

$$T_B + \sum U_{B-D} = T_D$$

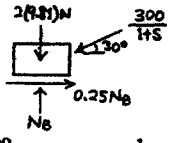
$$\frac{1}{2}(200)\left(30.53^2\right) - 200(9.81)(h) = 0$$

$$h = 47.5 \text{ m} \qquad \textbf{Ans}$$

14-19. The 2-kg block is subjected to a force having a constant direction and a magnitude $F = (300/(1 + s))$ N, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s. Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



$$+\uparrow \Sigma F_y = 0; \qquad N_B = 2(9.81) + \frac{150}{1+s}$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2)(8)^2 - 0.25[2(9.81)(12-4)] - 0.25\int_4^{12} \frac{150}{1+s}\,ds + \int_4^{12} \frac{300}{1+s}\,ds\cos 30° = \frac{1}{2}(2)(v_2^2)$$

$$v_2^2 = 24.76 - 37.5 \ln\left(\frac{1+12}{1+4}\right) + 259.81 \ln\left(\frac{1+12}{1+4}\right)$$

$$v_2 = 15.4 \text{ m/s} \qquad \textbf{Ans}$$

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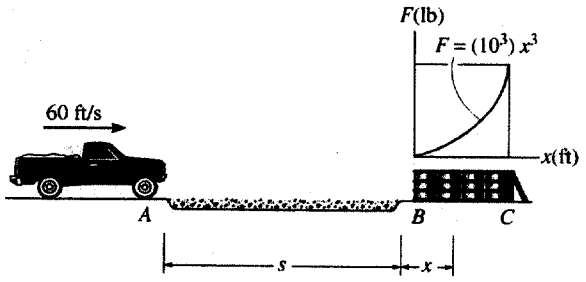
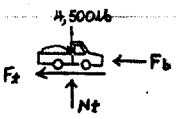
***14-20.** The motion of a truck is arrested using a bed of loose stones AB and a set of crash barrels BC. If experiments show that the stones provide a rolling resistance of 160 lb per wheel and the crash barrels provide a resistance as shown in the graph, determine the distance x the 4500-lb truck penetrates the barrels if the truck is coasting at 60 ft/s when it approaches A. Take $s = 50$ ft and neglect the size of the truck.



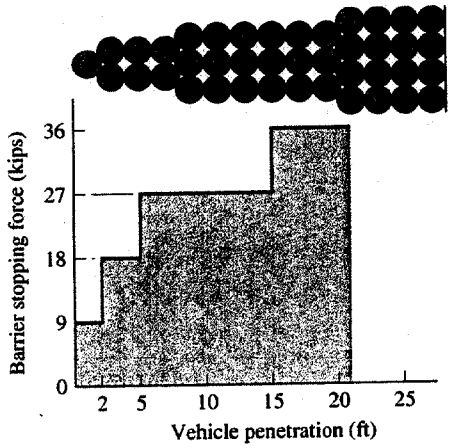
$$\frac{1}{2}\left(\frac{4500}{32.2}\right)(60)^2 - 4(160)(50) - \int_0^x (10)^3 x^3\,dx = 0$$

$$219\,552.80 - \frac{10^3}{4}x^4 = 0$$

$$x = 5.44 \text{ ft} \qquad \text{Ans}$$



14-21. The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

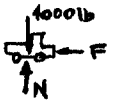
$$Area = 187.89 \text{ kip-ft}$$

$$2(9) + (5-2)(18) + x(27) = 187.89$$

$$x = 4.29 \text{ ft} < (15-5) \text{ ft} \qquad \text{(O.K!)}$$

Thus

$$s = 5 \text{ ft} + 4.29 \text{ ft} = 9.29 \text{ ft} \qquad \text{Ans}$$



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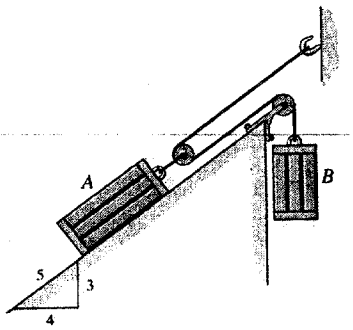
14-22. The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



Kinematics : The speed of the block A and B can be related by using position coordinate equation.

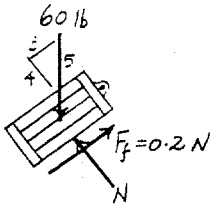
$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

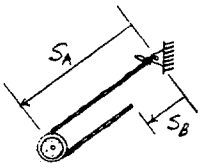
$$2v_A - v_B = 0 \qquad [1]$$

Equation of Motion : Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy : By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement. Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \qquad [2]$$

qs. [1] and [2] yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans}$$
$$v_B = 7.033 \text{ ft/s}$$

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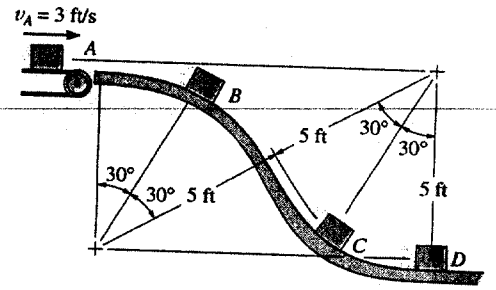
14-23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B, C, and D. Also calculate the normal force of the chute on the packages at B and C. Neglect friction and the size of the packages.

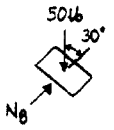


$T_A + \Sigma U_{A-B} = T_B$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5)(1 - \cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2$$

$v_B = 7.221 = 7.22$ ft/s **Ans**

$+\nearrow \Sigma F_n = ma_n$; $\quad -N_B + 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(7.221)^2}{5}\right]$



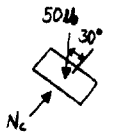
$N_B = 27.1$ lb **Ans**

$T_A + \Sigma U_{A-C} = T_C$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5\cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2$$

$v_C = 16.97 = 17.0$ ft/s **Ans**

$+\nearrow \Sigma F_n = ma_n$; $\quad N_C - 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(16.97)^2}{5}\right]$



$N_C = 133$ lb **Ans**

$T_A + \Sigma U_{A-D} = T_D$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2$$

$v_D = 18.2$ ft/s **Ans**

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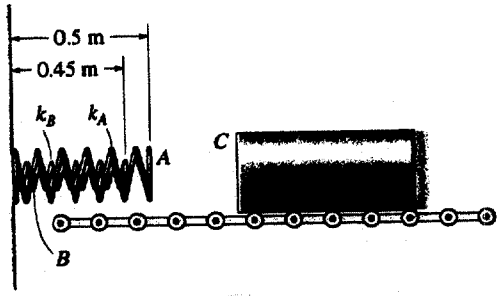
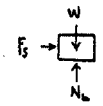
***14-24.** The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. Determine the maximum deflection in each spring needed to stop the motion of the ingot. Take $k_A = 5$ kN/m, $k_B = 3$ kN/m.



Assume both springs compress

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)s^2 - \frac{1}{2}(3000)(s-0.05)^2 = 0$



$225 - 2500\,s^2 - 1500(s^2 - 0.1\,s + 0.0025) = 0$

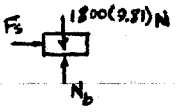
$s^2 - 0.0375\,s - 0.05531 = 0$

$s = 0.2547$ m > 0.05 m (O.K!)

$s_A = 0.255$ m Ans

$s_B = 0.205$ m Ans

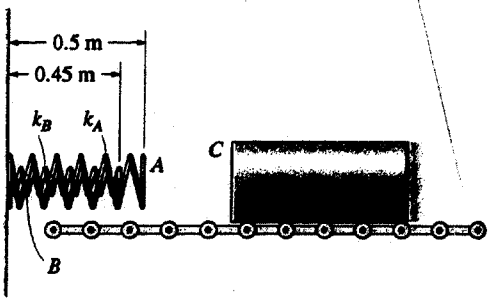
14-25. The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45-0.3)^2 = 0$

$k_B = 11.1$ kN/m Ans



14-26. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the distance A must descend from rest before it obtains a speed of 8 ft/s. Also, what is the tension in the cord supporting block A? Neglect the mass of the cord and pulleys.

$2\,s_A + s_B = l$

$2\,\Delta s_A = -\Delta s_B$

$2\,v_A = -v_B$

For $v_A = 8$ ft/s, $v_B = -16$ ft/s

For the system:

$T_1 + \Sigma U_{1-2} = T_2$

$[0+0] + [60(s_A) - 10(2s_A)] = \frac{1}{2}(\frac{60}{32.2})(8)^2 + \frac{1}{2}(\frac{10}{32.2})(-16)^2$

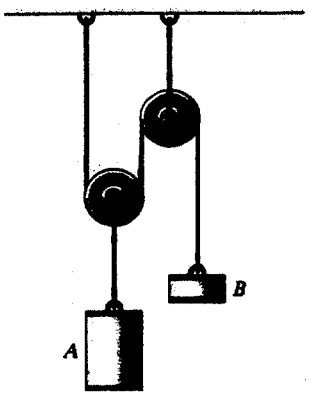
$s = 2.484 = 2.48$ ft Ans

For block A:

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 60(2.484) - T_A(2.484) = \frac{1}{2}(\frac{60}{32.2})(8)^2$

$T_A = 36.0$ lb Ans



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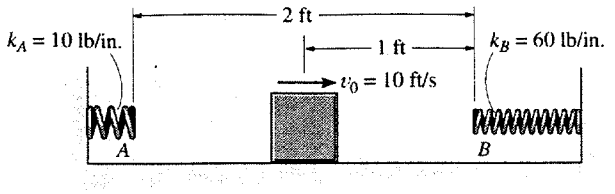
14-27. The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14-7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10^2) - 10(1 + s_1) - \frac{1}{2}(60)s_3^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14-7, we have

$$T_2 + \sum U_{2-3} = T_3$$

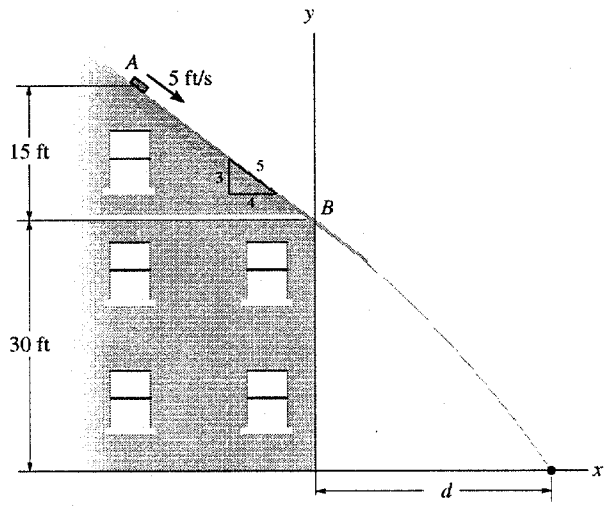
$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

$$s_2 = 1.227 \text{ ft}$$

Since $s_2 = 1.227$ ft < 2 ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \quad \textbf{Ans}$$

***14-28.** The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the block just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



$$T_A + \sum U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s} \quad \textbf{Ans}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

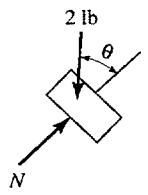
$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root.

$$t = 0.89916 \text{ s}$$

$$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft} \quad \textbf{Ans}$$



$$T_A + \sum U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$$v_C = 54.1 \text{ ft/s} \quad \textbf{Ans}$$

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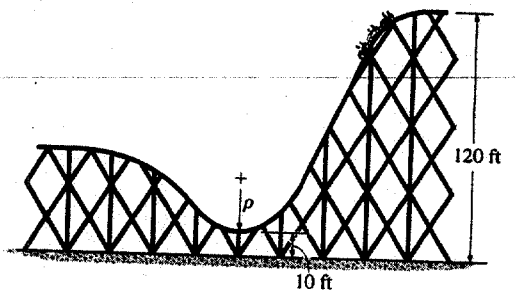
14-29. Roller coasters are designed so that riders will not experience more than 3.5 times their weight as a normal force against the seat of the car. Determine the smallest radius of curvature ρ of the track at its lowest point if the car has a speed of 5 ft/s at the crest of the drop. Neglect friction.



Principle of Work and Energy : Here, the rider is being displaced vertically (downward) by $s = 120 - 10 = 110$ ft and does *positive* work. Applying Eq. 14 – 7 we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{W}{32.2}\right)(5^2) + W(110) = \frac{1}{2}\left(\frac{W}{32.2}\right)v^2$$

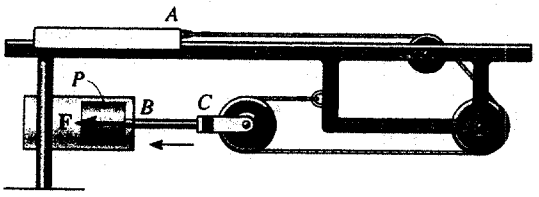
$$v^2 = 7109 \text{ ft}^2/\text{s}^2$$

Equation of Motion : It is required that $N = 3.5W$. Applying Eq. 13 – 7, we have

$$\Sigma F_n = ma_n; \quad 3.5W - W = \left(\frac{W}{32.2}\right)\left(\frac{7109}{\rho}\right)$$

$$\rho = 88.3 \text{ ft} \qquad \textbf{Ans}$$

14-30. The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.

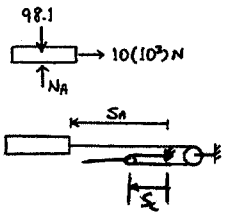


$$2s_C + s_A = l$$

$$2\Delta s_C + \Delta s_A = 0$$

$$2(0.2) = -\Delta s_A$$

$$-0.4 = \Delta s_A$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$$v_A = 28.3 \text{ m/s} \qquad \textbf{Ans}$$

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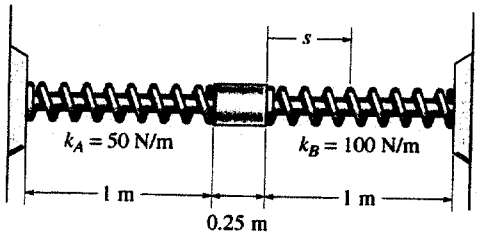
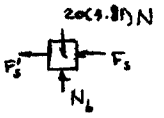
14-31. The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



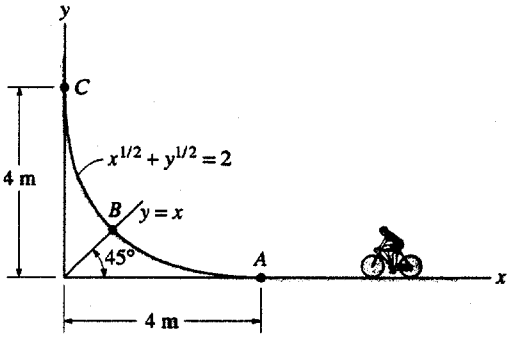
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$

$s = 0.730$ m Ans



***14-32.** The cyclist travels to point A, pedaling until he reaches a speed $v_A = 8$ m/s. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$

For $y = x$,

$2x^{\frac{1}{2}} = 2$

$x = 1, y = 1$ (Point B)

Thus,

$\tan\theta = \frac{dy}{dx} = -1$

$\theta = -45°$

$\frac{dy}{dx} = (-x^{-\frac{1}{2}})(y^{\frac{1}{2}})$

$\frac{d^2y}{dx^2} = y^{\frac{1}{2}}(\frac{1}{2}x^{-\frac{3}{2}}) - x^{-\frac{1}{2}}(\frac{1}{2})(y^{-\frac{1}{2}})(\frac{dy}{dx})$

$\frac{d^2y}{dx^2} = \frac{1}{2}y^{\frac{1}{2}}x^{-\frac{3}{2}} + \frac{1}{2}(\frac{1}{x})$

For $x = y = 1$

$\frac{dy}{dx} = -1, \quad \frac{d^2y}{dx^2} = 1$

$\rho = \frac{[1+(-1)^2]^{3/2}}{1} = 2.828$ m

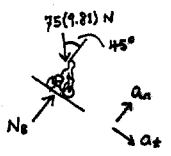
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(75)(8^2) - 75(9.81)(1) = \frac{1}{2}(75)(v_B^2)$

$v_B^2 = 44.38$

$+\nwarrow \Sigma F_n = m a_n; \quad N_B - 9.81(75)\cos 45° = 75(\frac{44.38}{2.828})$

$N_B = 1.70$ kN Ans



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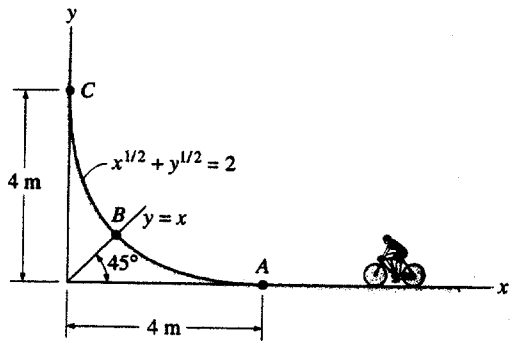
14-33. The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4$ m/s. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$$y = 0.81549 \text{ m} = 0.815 \text{ m} \qquad \text{Ans}$$

$$x^{1/2} + (0.81549)^{1/2} = 2$$

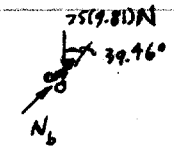
$$x = 1.2033 \text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46°$$

$$+\nearrow \Sigma F_n = m a_n; \quad N_b - 9.81(75)\cos 39.46° = 0$$

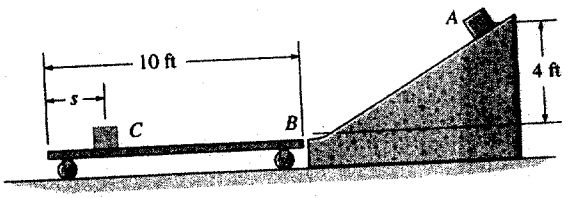
$$N_b = 568 \text{ N} \qquad \text{Ans}$$



$$+\nwarrow \Sigma F_t = m a_t; \quad 75(9.81)\sin 39.46° = 75 a_t$$

$$a = a_t = 6.23 \text{ m/s}^2 \qquad \text{Ans}$$

14-34. The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *fixed from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



Principle of Work and Energy : W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement. Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-C} = T_C$$

$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

Thus,

$$s = 10 - s' = 3.33 \text{ ft} \qquad \text{Ans}$$

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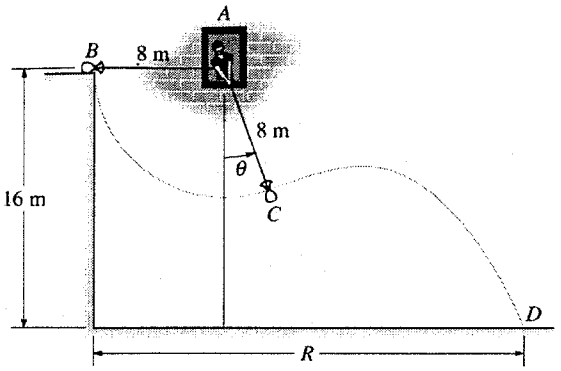
14-35. The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



$$T_B + \sum U_{B-C} = T_C$$

$$0 + 30(9.81)8\cos 30° = \frac{1}{2}(30)v_C^2$$

$$v_C = 11.659 \text{ m/s}$$

$$T_B + \sum U_{B-D} = T_D$$

$$0 + 30(9.81)(16) = \frac{1}{2}(30)v_D^2$$

$$v_D = 17.7 \text{ m/s} \quad \textbf{Ans}$$

During free flight:

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8\cos 30° - 11.659\sin 30° t + \frac{1}{2}(9.81)t^2$$

$$t^2 - 1.18848t - 1.8495 = 0$$

Solving for the positive root:

$$t = 2.0784 \text{ s}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$s = 8\sin 30° + 11.659\cos 30°(2.0784)$$

$$s = 24.985 \text{ m}$$

Thus,

$$R = 8 + 24.985 = 33.0 \text{ m} \quad \textbf{Ans}$$

Also,

$$(v_D)_x = 11.659\cos 30° = 10.097 \text{ m/s}$$

$$(+\downarrow) \quad (v_D)_y = -11.659\sin 30° + 9.81(2.0784) = 14.559 \text{ m/s}$$

$$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 17.7 \text{ m/s} \quad \textbf{Ans}$$

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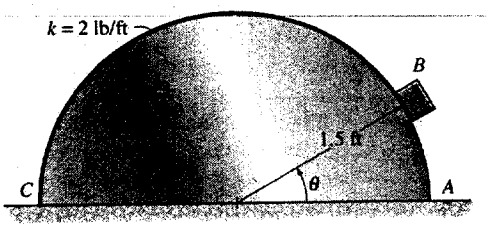
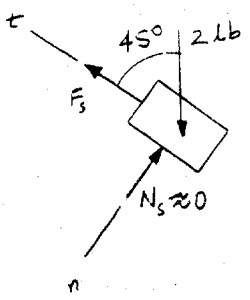
***14-36.** A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



$+\nearrow \Sigma F_n = ma_n$. $\quad 2\sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$v = 5.844$ ft/s

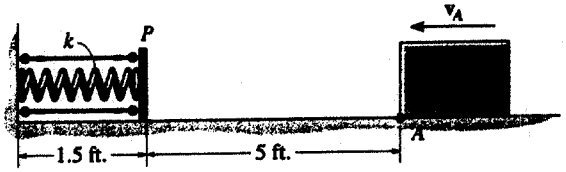


$T_1 + \Sigma U_{1-2} = T_2$

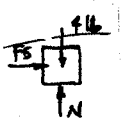
$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5\sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans**

14-37. The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



$T_1 + \Sigma U_{1-2} = T_2$



$$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s - 1.3)^2 - \frac{1}{2}(50)(s - 1.5)^2\right] = 0$$

$0.20124 = s^2 - 2.60s + 1.69 - (s^2 - 3.0s + 2.25)$

$0.20124 = 0.4s - 0.560$

$s = 1.90$ ft **Ans**

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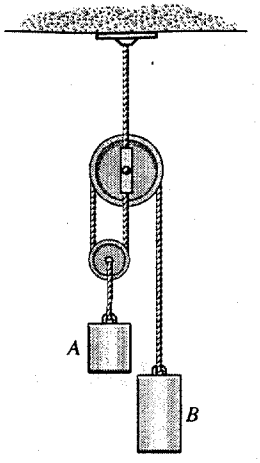
14-38. Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it moves upwards 2 m starting from rest. Neglect the mass of the cord and pulleys.



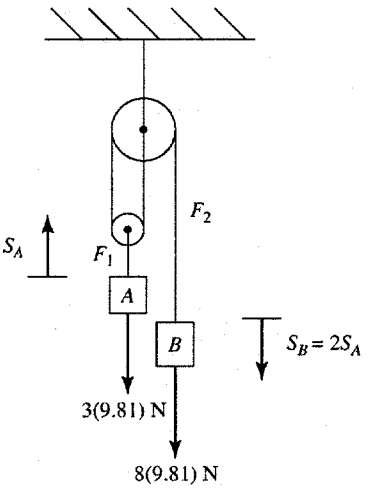
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

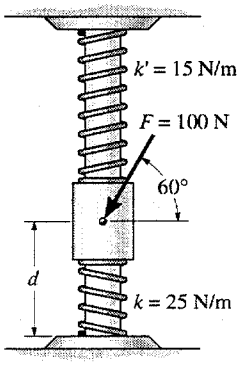
Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

$$255.06 = 17.5v_A^2$$

$$v_A = 3.82 \text{ m/s} \qquad \textbf{Ans}$$



14-39. The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

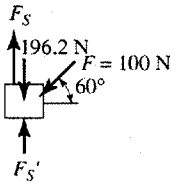


$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 100\sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

$$- \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

$$v_C = 2.36 \text{ m/s} \qquad \textbf{Ans}$$



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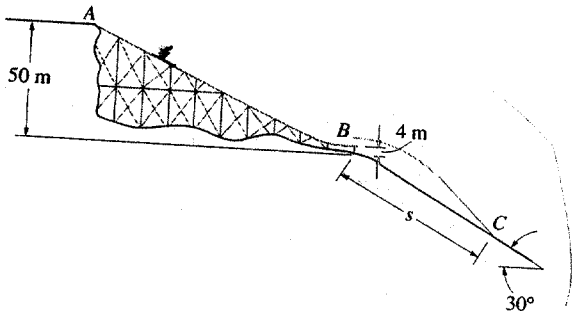
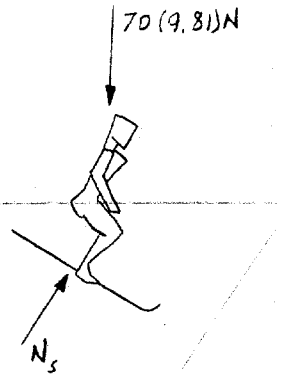
14-40. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.





$T_A + \Sigma U_{A-B} = T_B$

$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$

$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$ **Ans**

$\left(\overset{+}{\rightarrow}\right)$ $s = s_0 + v_0 t$

$s\cos 30° = 0 + 30.04t$

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s\sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$

Eliminating t,

$s^2 - 122.67s - 981.33 = 0$

Solving for the positive root

$s = 130 \text{ m}$ **Ans**

14-41. The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.

$T_1 + \Sigma U_{1-2} = T_2$

$0 + U_{1-2} = \frac{1}{2}(400)(10^3)(10)^2$

$U_{1-2} = 20(10^6) \text{ J}$

$P_{avg} = \frac{U_{1-2}}{t} = \frac{20(10^6)}{100} = 200 \text{ kW}$ **Ans**

Also,

$v = v_0 + a_c t$

$10 = 0 + a_c(100)$

$a_c = 0.1 \text{ m/s}^2$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x$: $F = 400(10^3)(0.1) = 40(10^3) \text{ N}$

$P_{ave} = \mathbf{F} \cdot \mathbf{v}_{avg} = 40(10^3)\left(\frac{10}{2}\right) = 200 \text{ kW}$ **Ans**

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14-42. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

Power: The power output can be obtained using Eq. 14-10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

Using Eq. 14-11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \quad \textbf{Ans}$$

14-43. An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how much time it takes to reach a speed of 40 ft/s.

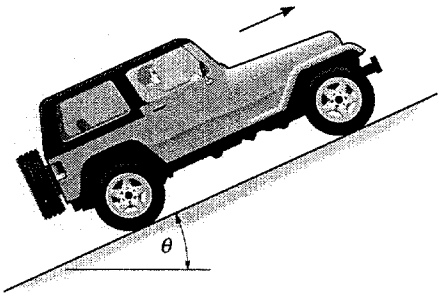
$$P = \frac{\Delta U}{\Delta t}$$

$$T_F = \Delta U = P \Delta t$$

$$\frac{1}{2}\left(\frac{15\,000}{32.2}\right)(40)^2 = 100(550)\Delta t$$

$$\Delta t = 6.78 \text{ s} \quad \textbf{Ans}$$

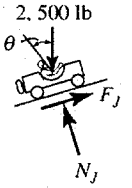
***14-44.** The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to all the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.



$$P = F_J v$$

$$100(550) = 2500 \sin\theta (30)$$

$$\theta = 47.2^\circ \quad \textbf{Ans}$$



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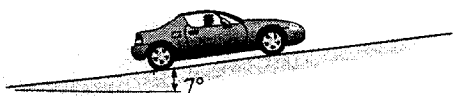
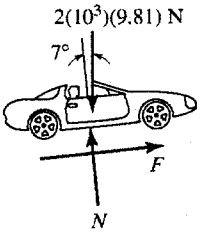
14-45. An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\sum F_{x'} = ma_{x'};\quad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\frac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. The power output can be obtained using Eq. 14-10.

$$P = \mathbf{F} \cdot \mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14-11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW}\quad \textbf{Ans}$$

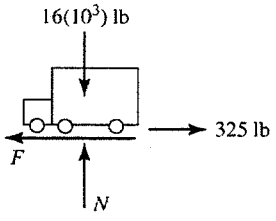
14-46. A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

$$a = \frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75\text{ ft/s}^2$$

$$\overset{+}{\leftarrow}\sum F_x = ma_x;\quad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

$$F = 2188.35\text{ lb}$$

$$P_{max} = \mathbf{F} \cdot \mathbf{v}_{max} = \frac{2188.35(30)}{550} = 119\text{ hp}\quad \textbf{Ans}$$



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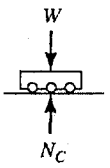
14-47. An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.



$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = \left(\frac{W}{g}\frac{v\,dv}{ds}\right)v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2\,dv$$

P = constant

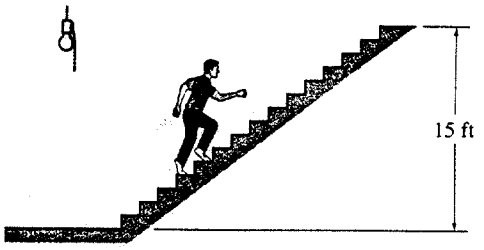
$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3$$

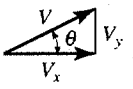
$$s = \frac{W}{3gP}v^3$$

$$s = \frac{15\,000(40)^3}{3(32.2)(100)(550)}$$

$s = 181$ ft **Ans**

***14-48.** The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.





Step height: 0.125 m

The number of steps: $\frac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \frac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v\sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans**

Also.

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans**

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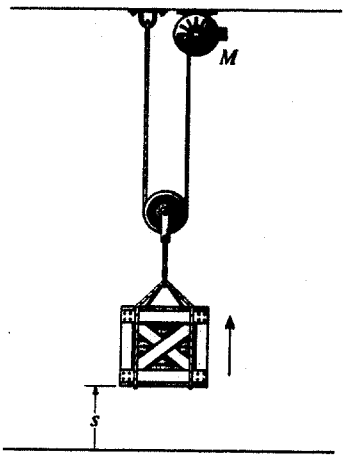
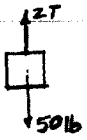
14-49. The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\epsilon = 0.76$. Neglect the mass of the pulley and cable.



$+\uparrow \Sigma F_y = ma_y;$ $\quad 2T - 50 = \frac{50}{32.2}a$



$(+\uparrow)\ v = v_0 + a_c t$

$10 = 0 + a(4)$

$a = 2.5\ \text{ft/s}^2$

$T = 26.94\ \text{lb}$

In $t = 2$ s,

$(+\uparrow)\ v = v_0 + a_c t$

$v = 0 + 2.5(2) = 5\ \text{ft/s}$

$s_C + (s_C - s_P) = l$

$2v_C = v_P$

$2(5) = v_P = 10\ \text{ft/s}$

$P_O = 26.94(10) = 269.2$

$P_{in} = \frac{269.2}{0.76} = 354.49\ \text{ft·lb/s}$

$P_{in} = 0.644\ \text{hp}$ **Ans**

14-50. A car has a mass m and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

Power : Since the power ouput is constant, then the traction force F varies with v. Applying Eq. 14 – 10, we have

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$P = Fv \qquad F = \frac{P}{v}$$

Equation of Motion :

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad \frac{P}{v} = ma \qquad a = \frac{P}{mv}$$

Kinematics : Applying equation $ds = \frac{v\,dv}{a}$, we have

$$\int_0^s ds = \int_0^v \frac{mv^2}{P}\,dv \qquad s = \frac{mv^3}{3P}$$ **Ans**

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14-51. To dramatize the loss of energy in an automobile, consider a car having a weight of 5 000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

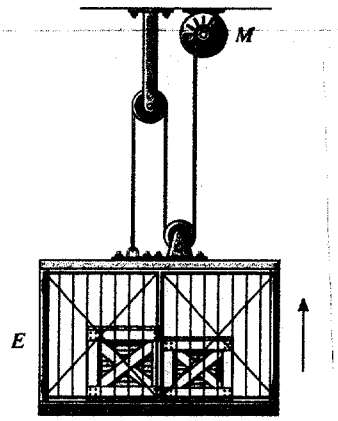
Energy : Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right)\times\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\times\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ = 51.33 ft/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right)\text{ ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W}\times\left(\frac{1\text{ hp}}{746\text{ W}}\right)\times\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right)$ = 73.73 ft · lb/s. Thus,

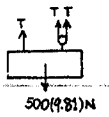
$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min}$$ **Ans**

***14-52.** The motor M is used to hoist the 500-kg elevator upward with a constant velocity $v_E = 8$ m/s. If the motor draws 60 kW of electrical power, determine the motor's efficiency. Neglect the mass of the pulleys and cable.



$+\uparrow\Sigma F_y = 0;\quad 3T - 500(9.81) = 0$

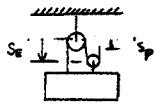
$T = 1635$ N



$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

$v_P = 24$ m/s

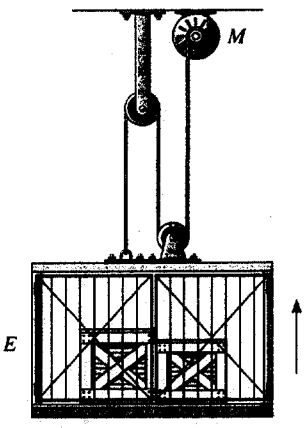


$P_O = 1635(24) = 39.24$ kW

$P_i = 60$ kW

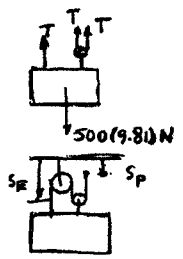
$\varepsilon = \frac{P_O}{P_i} = \frac{39.24}{60} = 0.654$ **Ans**

14-53. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\text{ m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



$+\uparrow\Sigma F_y = m a_y;\quad 3T - 500(9.81) = 500(2)$

$T = 1968.33$ N



$3s_E - s_P = l$

$3v_E = v_P$

When $t = 3$ s,

$(+\uparrow)\ v = v_0 + a_c t$

$v_E = 0 + 2(3) = 6$ m/s

$v_P = 3(6) = 18$ m/s

$P_O = 1968.33(18)$

$P_O = 35.4$ kW **Ans**

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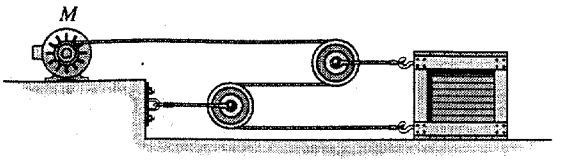
14-54. The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



Time to start motor, $\mu_s = 0.3$

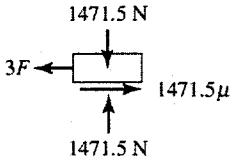
$\stackrel{+}{\leftarrow} \sum F_x = 0;\quad 3F - 1471.5(0.3) = 0$

$F = 147.15$ N

$F = 8t^2 + 20$

$147.15 = 8t^2 + 20$

$t = 3.987$ s



Motion: $\mu_k = 0.2$

$\stackrel{+}{\leftarrow} \sum F_x = ma_x;\quad 3F - 1471.5(0.2) = 150a$

$3(8t^2 + 20) - 294.3 = 150a$

$a = 0.160t^2 - 1.5620$

$$\int_0^v dv = \int_{3.987}^{5} (0.160t^2 - 1.5620)dt$$

When $t = 5$ s,

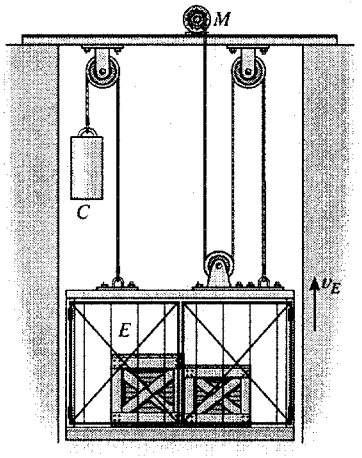
$$v = 0.160\left(\frac{t^3}{3}\right)\Big|_{3.987}^{5} - 1.5620t\Big|_{3.987}^{5}$$

$v = 1.7045$ ft/s

$P_O = 3[8(5)^2 + 20](1.7045) = 1124.97$ N · m/s

$P_O = 1.12$ kW **Ans**

14-55. The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



Elevator:

Since $a = 0$,

$+\uparrow \sum F_y = 0;\quad 60(9.81) + 3T - 400(9.81) = 0$

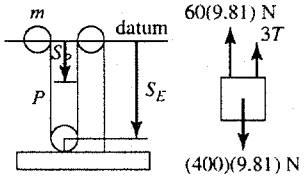
$T = 1111.8$ N

$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

Since $v_E = -4$ m/s, $\quad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans**



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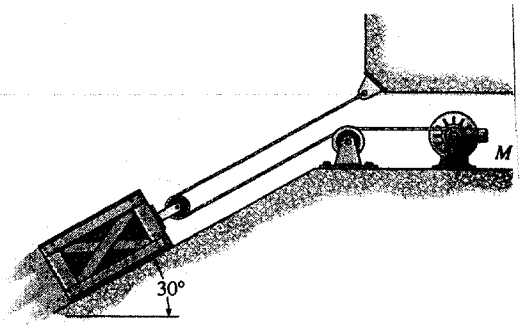
***14-56.** The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.

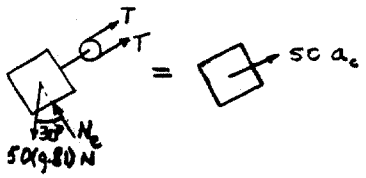


$v^2 = v_0^2 + 2a_c(s - s_0)$

$(4)^2 = 0 + 2a_c(8 - 0)$

$a_c = 1\ \text{m/s}^2$

$+\nearrow \Sigma F_x = ma_x;\quad 2T - 50(9.81)\sin 30° = (50)(1) \quad T = 147.6\ \text{N}$

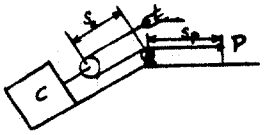


$2s_C + s_P = l$

$2v_C = -v_P$

$(2)(-4) = -v_P$

$v_P = 8\ \text{m/s}$



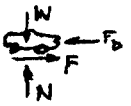
$P_o = \mathbf{T} \cdot v_P = 147.6(8) = 1181\ \text{W}$

$P_i = \frac{P_o}{e} = \frac{1181}{0.74} = 1595.9\ \text{W} = 1.60\ \text{kW}$ **Ans**

14-57. The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.

$\overset{+}{\rightarrow} \Sigma F_x = ma_x;\quad F - 0.3v^2 = 2.3(10^3)(5)$

$F = 0.3v^2 + 11.5(10^3)$



At $v = 28$ m/s

$F = 11\,735.2\ \text{N}$

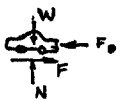
$P_o = (11\,735.2)(28) = 328.59\ \text{kW}$

$P_i = \frac{P_o}{e} = \frac{328.59}{0.68} = 483\ \text{kW}$ **Ans**

14-58. The sports car has a mass of 2.3 Mg and accelerates at 6 m/s^2, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.

$\overset{+}{\rightarrow} \Sigma F_x = ma_x;\quad F - 10v = 2.3(10^3)(6)$

$F = 13.8(10^3) + 10v$



$(\overset{+}{\rightarrow})\ v = v_0 + a_c t$

$v = 0 + 6(5) = 30\ \text{m/s}$

$P_o = \mathbf{F} \cdot v = [13.8(10^3) + 10(30)](30) = 423.0\ \text{kW}$

$P_i = \frac{P_o}{e} = \frac{423.0}{0.68} = 622\ \text{kW}$ **Ans**

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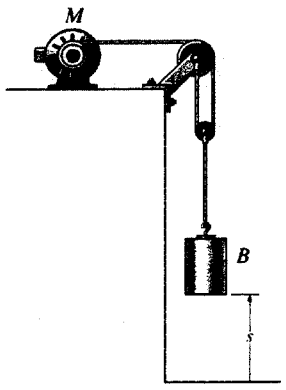
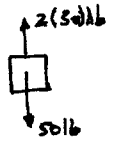
14-59. The 50-lb load is hoisted by the pulley system and motor M. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\epsilon = 0.76$.



$+\uparrow \Sigma F_y = m a_y; \quad 2(30) - 50 = \frac{50}{32.2} a_B$



$a_B = 6.44 \text{ m/s}^2$

$(+\uparrow) v^2 = v_0^2 + 2a_c(s - s_0)$

$v_B^2 = 0 + 2(6.44)(10 - 0)$

$v_B = 11.349 \text{ ft/s}$

$2s_B + s_M = l$

$2v_B = -v_M$

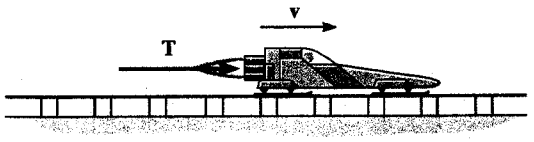
$v_M = -2(11.349) = -22.698 \text{ ft/s}$

$P_O = \mathbf{F} \cdot \mathbf{v} = 30(22.698) = 680.94 \text{ ft·lb/s}$

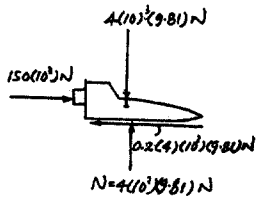
$P_i = \frac{680.94}{0.76} = 895.97 \text{ ft·lb/s}$

$P_i = 1.63 \text{ hp}$ **Ans**

***14-60.** The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Negiect the loss of fuel mass and air resistance.



$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \quad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3 a$



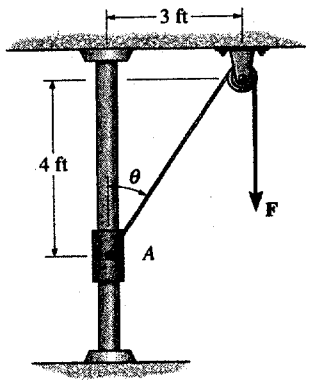
$a = 35.54 \text{ m/s}^2$

$(\stackrel{+}{\rightarrow}) v = v_0 + a_c t$

$= 0 + 35.54t = 35.54t$

$P = \mathbf{T} \cdot \mathbf{v} = 150(10)^3(35.54t) = 5.33t \text{ MW}$ **Ans**

14-61. The 10-lb collar starts from rest at A and is lifted by applying a constant force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.

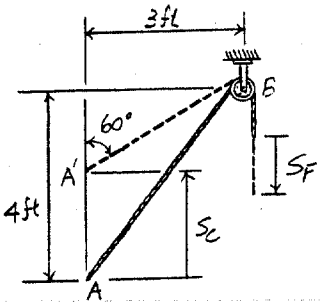


Principle of Work and Energy : The 25 - lb force which acts in the direction of displacement does *positive* work, whereas the weight of the collar does *negativ* work sinceit acts in the opposite direction to that of displacement. The 25 - lb force and the collar are being displaced vertically by $s_F = AB - A'B = \sqrt{3^2 + 4^2} - 3/\sin 60° = 1.5359$ ft and $s_C = 4 - 3/\tan 60° = 2.2679$ ft. Since the block is at rest initially, $T_1 = 0$. Applying Eq. 14 – 7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 25(1.5359) - 10(2.2679) = \frac{1}{2}\left(\frac{10}{32.2}\right)v^2$$

$$v = 10.06 \text{ ft/s}$$



Power : The power output at the instant when $s = 10$ ft can be obtained using Eq. 14 – 10.

$P = \mathbf{F} \cdot \mathbf{v} = 25\cos 60°(10.06) = 125.76 \text{ ft·lb/s} = 0.229 \text{ hp}$ **Ans**

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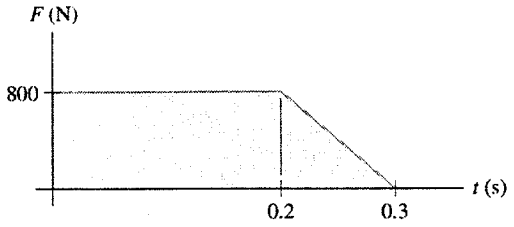
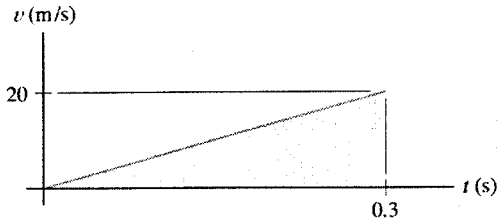
14-62. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





For $0 \le t \le 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67t$$

$P = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans**

For $0.2 \le t \le 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans**

$$U = \int_0^{0.3} P\,dt$$

$$U = \int_0^{0.2} 53.3t\,dt + \int_{0.2}^{0.3} (160t - 533t^2)dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans**

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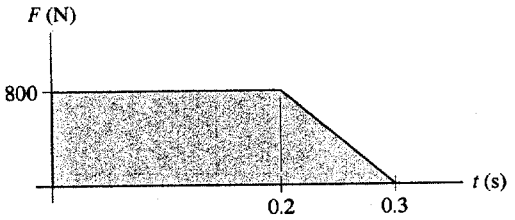
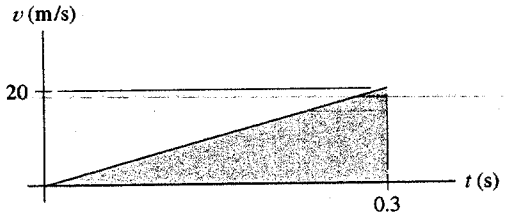
14-63. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





See solution to Prob. 14–62.

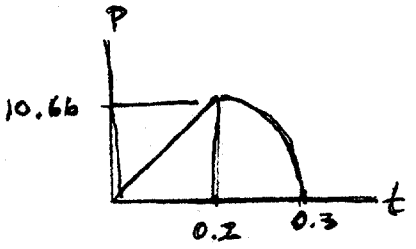
$P = 160\,t - 533\,t^2$

$\frac{dP}{dt} = 160 - 1066.6\,t = 0$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7 \text{ kW}$ **Ans**



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***14-64.** Solve Prob. 14-18 using the conservation of energy equation.

At C, with normal force from the track $N \approx 0$,

$$+\downarrow \sum F_y = ma_y; \quad mg = m\frac{v_C^2}{\rho_C}$$

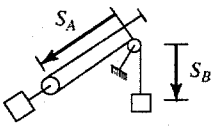
$$v_C^2 = (9.81)(25)\ (\text{m/s})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m(9.81)(25) + m(9.81)(35) = 0 + m(9.81)h$$

$$h = 47.5\ \text{m} \qquad \textbf{Ans}$$

14-65. Solve Prob. 14-15 using the conservation of energy equation.



$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$[0+0] + [0+0] = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2 + 10(10) - 60\left(\frac{3}{5}\right)(5)$$

$$v_A = 7.18\ \text{ft/s} \qquad \textbf{Ans}$$

14-66. Solve Prob. 14-17 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(100)(0.5)^2 + \frac{1}{2}(50)(0.5)^2 = \frac{1}{2}(20)v^2 + 0$$

$$v = 1.37\ \text{m/s} \qquad \textbf{Ans}$$

14-67. Solve Prob. 14-31 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(2)^2 + 0 = 0 + \frac{1}{2}(50)s^2 + \frac{1}{2}(100)s^2$$

$$s = 0.730\ \text{m} \qquad \textbf{Ans}$$

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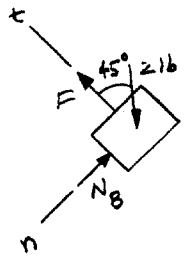
***14-68.** Solve Prob. 14-36 using the conservation of energy equation.

$+\nearrow \Sigma F_n = ma_n$; $\quad 2\sin 45° = \left(\frac{2}{32.2}\right)\left(\frac{v_B^2}{1.5}\right)$

$$v_B = 5.844 \text{ ft/s}$$



Datum at A:

$T_A + V_A = T_B + V_B$

$$0 + \frac{1}{2}(2)(\pi(1.5) - l_0)^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2 + \frac{1}{2}(2)\left[\pi(1.5) - \frac{\pi}{4}(1.5) - l_0\right]^2 + 2(1.5\sin 45°)$$

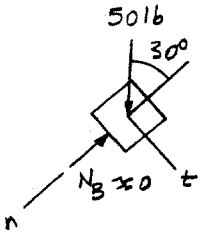
$l_0 = 2.77$ ft **Ans**

14-69. Solve Prob. 14-23 using the conservation of energy equation.

Datum at A:

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2 - 50(5)(1 - \cos 30°)$$



$v_B = 7.221$ ft/s $= 7.22$ ft/s **Ans**

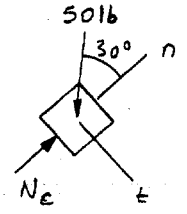
$+\nearrow \Sigma F_n = ma_n$; $\quad -N_B + 50\cos 30° = \left(\frac{50}{32.2}\right)\left(\frac{(7.221)^2}{5}\right)$

$$N_B = 27.1 \text{ lb}$$ **Ans**

$T_A + V_A = T_C + V_C$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2 - 50(5\cos 30°)$$

$v_C = 16.97$ ft/s $= 17.0$ ft/s **Ans**



$+\nearrow \Sigma F_n = ma_n$; $\quad N_C - 50\cos 30° = \left(\frac{50}{32.2}\right)\left(\frac{(16.97)^2}{5}\right)$

$$N_C = 133 \text{ lb}$$ **Ans**

$T_A + V_A = T_D + V_D$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2 - 50(5)$$

$v_D = 18.2$ ft/s **Ans**

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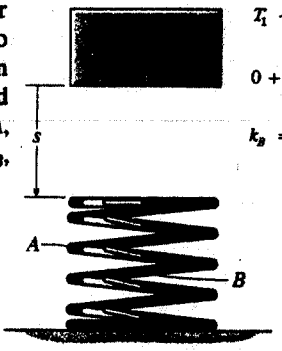
14-70. Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.

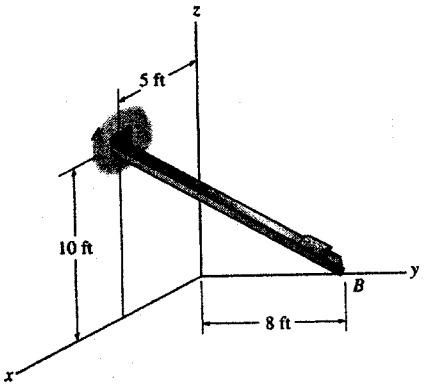


$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 - 2(9.81)(0.5+0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$

$k_B = 287$ N/m **Ans**

14-71. The block has a weight of 1.5 lb and slides along the smooth chute AB. It is released from rest at A, which has coordinates of A(5 ft, 0, 10 ft). Determine the speed at which it slides off at B, which has coordinates of B(0, 8 ft, 0).



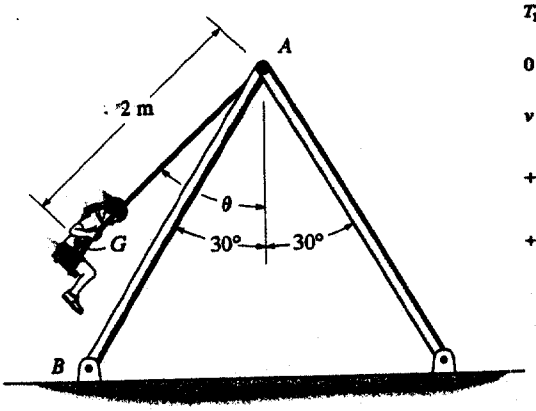
Datum at B :

$T_A + V_A = T_B + V_B$

$0 + 1.5(10) = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_B)^2 + 0$

$v_B = 25.4$ ft/s **Ans**

***14-72.** The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.

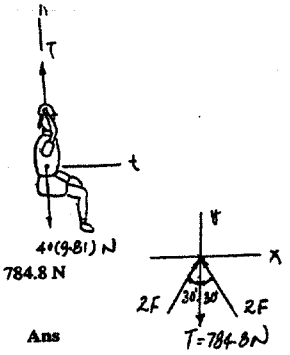


The maximum tension in the cable occurs when $\theta = 0°$.

$T_1 + V_1 = T_2 + V_2$

$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$

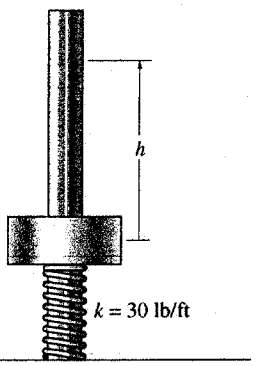
$v = 4.429$ m/s



$+\uparrow \Sigma F_n = ma_n$; $T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right)$ $T = 784.8$ N

$+\uparrow \Sigma F_y = 0$; $2(2F)\cos 30° - 784.8 = 0$ $F = 227$ N **Ans**

14-73. The collar has a weight of 8 lb. If it is pushed down so as to compress the spring 2 ft and then released from rest ($h = 0$), determine its speed when it is displaced $h = 4.5$ ft. The spring is not attached to the collar. Neglect friction.



$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(30)(2)^2 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 + 8(4.5)$

$v_2 = 13.9$ ft/s **Ans**

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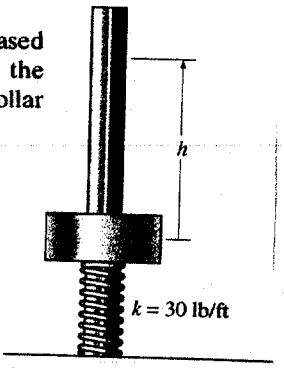
14-74. The collar has a weight of 8 lb. If it is released from rest at a height of $h = 2$ ft from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.3 ft.

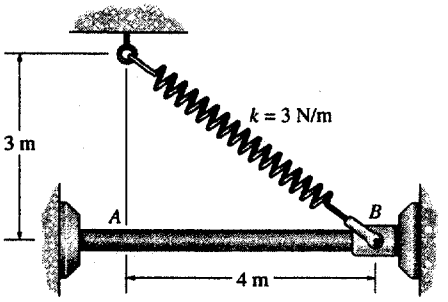


$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(\frac{8}{32.2})v_2^2 - 8(2.3) + \frac{1}{2}(30)(0.3)^2$

$v_2 = 11.7$ ft/s **Ans**

14-75. The 2-kg collar is attached to a spring that has an unstretched length of 3 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



Potential Energy : The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{3^2+4^2}-3\right)^2 = 6.00$ J and $\frac{1}{2}(3)(3-3)^2 = 0$, respectively. The gravitational potential energy remains the same since the elevation of collar does not change when it moves from B to A.

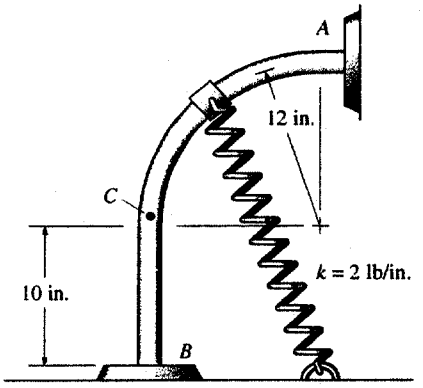
Conservation of Energy :

$T_B + V_B = T_A + V_A$

$0 + 6.00 = \frac{1}{2}(2)v_A^2 + 0$

$v_A = 2.45$ m/s **Ans**

***14-76.** The 5-lb collar is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length of 12 in.



$T_1 + V_1 = T_2 + V_2$

$0 + 5(\frac{22}{12}) + \frac{1}{2}(24)(\frac{10}{12})^2 = \frac{1}{2}(\frac{5}{32.2})v_B^2 + 0 + 0$

$v_B = 15.0$ ft/s **Ans**

14-77. The 5-lb collar is released from rest at A and travels along the smooth guide. Determine its speed when its center reaches point C and the normal force it exerts on the rod at this point. The spring has an unstretched length of 12 in., and point C is located just before the end of the curved portion of the rod.

$T_A + V_A = T_C + V_C$

$0 + 0 + \frac{1}{2}[2(12)](\frac{10}{12})^2 + 5(\frac{12}{12}) = \frac{1}{2}(\frac{5}{32.2})v^2 + \frac{1}{2}[2(12)](\sqrt{(\frac{12}{12})^2 + (\frac{10}{12})^2} - \frac{12}{12}]^2$

$v = 12.556$ ft/s $= 12.6$ ft/s **Ans**

$\stackrel{+}{\rightarrow}\Sigma F_n = m a_n;$ $\quad N_C + F_s \sin 50.1944° = \frac{5}{32.2}(\frac{(12.556)^2}{1})$

$F_s = ks;$ $\quad F_s = 2(12)[\sqrt{(\frac{12}{12})^2 + (\frac{10}{12})^2} - \frac{12}{12}] = 7.2410$ lb

Thus,

$N_C = 18.9$ lb **Ans**

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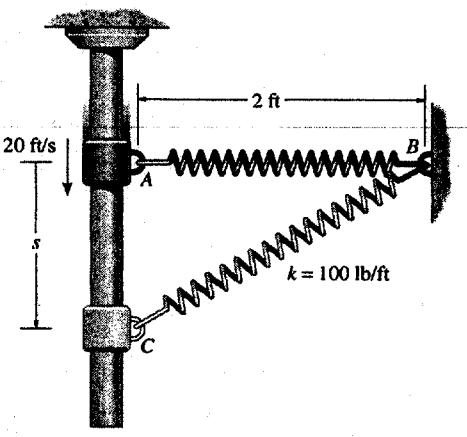
14-78. The 2-lb block is given an initial velocity of 20 ft/s when it is at A. If the spring has an unstretched length of 2 ft and a stiffness of $k = 100$ lb/ft, determine the velocity of the block when $s = 1$ ft.



Potential Energy : Datum is set along AB. The collar is 1 ft *below* the datum when it is at C. Thus, its gravitational potential energy at this point is $-2(1) = -2.00$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(100)(2-2)^2 = 0$ and $\frac{1}{2}(100)\left(\sqrt{2^2+1^2}-2\right)^2 = 2.786$ ft · lb, respectively.

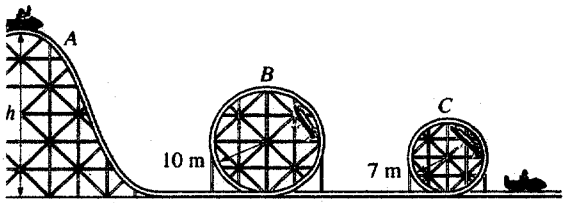
Conservation of Energy :

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(20^2) + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2.786 + (-2.00)$$

$$v_C = 19.4 \text{ ft/s} \qquad \textbf{Ans}$$

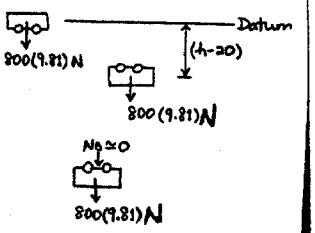
14-79. The roller-coaster car has a mass of 800 kg, including its passenger, and starts from the top of the hill A with a speed $v_A = 3$ m/s. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?



Since friction is neglected, the car will travel around the 7 m-loop provided it first travels around the 10 m-loop.

$T_A + V_A = T_B + V_B$

$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h-20)$



$+\downarrow \Sigma F_n = m a_n$; $\quad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$

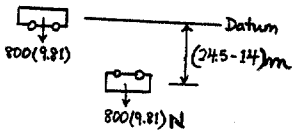
Thus, $\quad v_B = 9.90$ m/s

$h = 24.5$ m $\qquad$ **Ans**

At B: $\quad N_B = 0$ $\qquad$ **Ans** (For h to be minimum.)

$T_A + V_A = T_C + V_C$

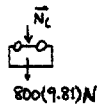
$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(24.5-14)$



$v_C = 14.69$ m/s

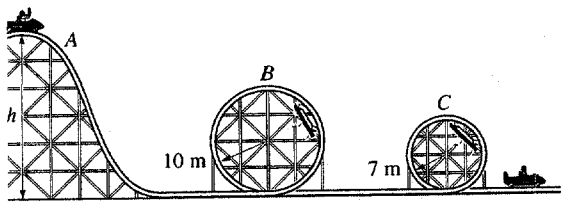
$a_n = \frac{14.69^2}{7}$

$+\downarrow \Sigma F_n = m a_n$; $\quad N_C + 800(9.81) = 800\left(\frac{14.69^2}{7}\right)$



$N_C = 16.8$ kN $\qquad$ **Ans**

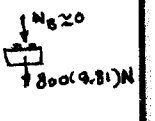
***14-80.** The roller-coaster car has a mass of 800 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?



Since friction is neglected, the car will travel around the 7 m-loop provided it first travels around the 10 m-loop.

$T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h-20)$



$+\downarrow \Sigma F_n = m a_n$; $\quad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$

Thus, $\quad v_B = 9.90$ m/s

$h = 25.0$ m $\qquad$ **Ans**

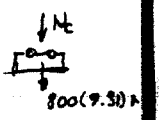
At B: $\quad N_B = 0$ $\qquad$ **Ans** (For h to be minimum.)

$T_A + V_A = T_C + V_C$

$0 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(25-14)$

$v_C = 14.69$ m/s

$+\downarrow \Sigma F_n = m a_n$; $\quad N_C + 800(9.81) = 800\left(\frac{(14.69)^2}{7}\right)$



$N_C = 16.8$ kN $\qquad$ **Ans**

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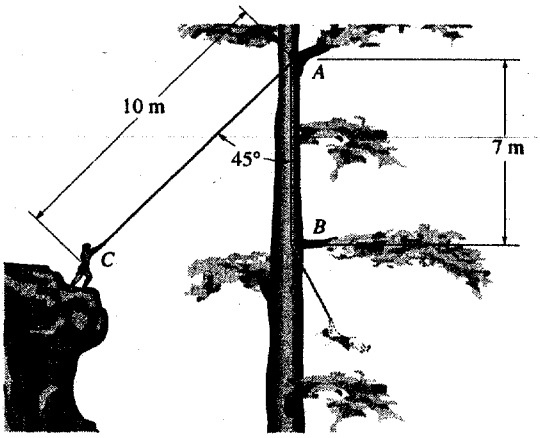
14-81. Tarzan has a mass of 100 kg and from rest swings from the cliff by rigidly holding on to the tree vine, which is 10 m measured from the supporting limb A to his center of mass. Determine his speed just after the vine strikes the lower limb at B. Also, with what force must he hold on to the vine just before and just after the vine contacts the limb at B?



Datum at C:

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(100)(v_C)^2 - 100(9.81)(10)(1 - \cos 45°)$

$v_C = 7.581 = 7.58$ m/s **Ans**

Just before striking B, $\rho = 10$ m

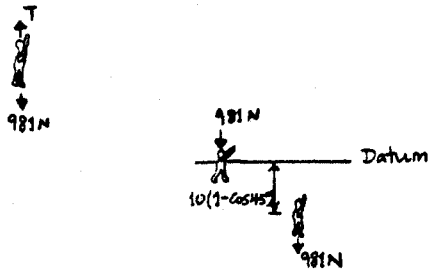
$+\uparrow \Sigma F_n = ma_n;\quad T - 981 = 100\left(\frac{(7.581)^2}{10}\right)$

$T = 1.56$ kN **Ans**

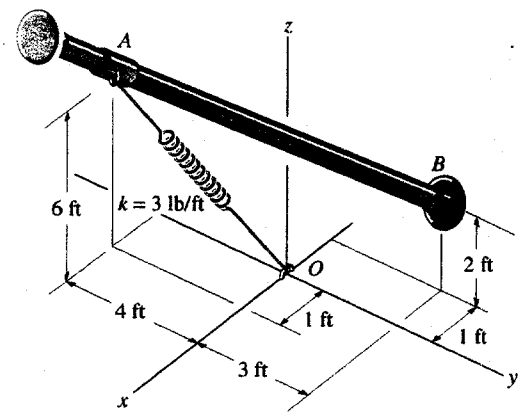
Just after striking B, $\rho = 3$ m

$+\uparrow \Sigma F_n = ma_n;\quad T - 981 = 100\left(\frac{(7.581)^2}{3}\right)$

$T = 2.90$ kN **Ans**



14-82. The spring has a stiffness $k = 3$ lb/ft and an unstretched length of 2 ft. If it is attached to the 5-lb smooth collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.



Datum at B.

$|r_{OA}| = \sqrt{(1)^2 + (4)^2 + (6)^2} = 7.28$ ft

$|r_{OB}| = \sqrt{(1)^2 + (3)^2 + (2)^2} = 3.74$ ft

$T_1 + V_1 = T_2 + V_2$

$0 + (5)(6-2) + \frac{1}{2}(3)(7.28-2)^2 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + \frac{1}{2}(3)(3.74-2)^2$

$v_B = 27.2$ ft/s **Ans**

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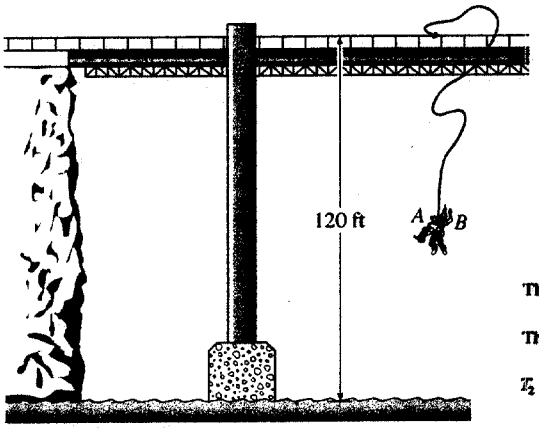
14-83. Just for fun, two 150-lb engineering students A and B intend to jump off the bridge from rest using an elastic cord (bungee cord) having a stiffness $k = 80$ lb/ft. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.



$T_1 + V_1 = T_2 + V_2$

$0 + 2(150)(120) = 0 + \frac{1}{2}(80)(x)^2$

$x = 30$ ft

Unstretched length of cord.

$120 = l + 30$

$l = 90$ ft Ans

When A lets go of B.

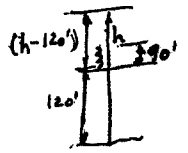
$T_2 + V_2 = T_3 + V_3$

$0 + \frac{1}{2}(80)(30)^2 = 0 + (150)h$

$h = 240$ ft

This is not possible since 90 ft cord would have to stretch again, i.e., $h_{max} = 120 + 90 = 210$ ft

Thus $h > 120 + 90 = 210$ ft



$T_2 + V_2 = T_3 + V_3$

$0 + \frac{1}{2}(80)(30)^2 = 0 + 150h + \frac{1}{2}(80)[(h-120) - 90]^2$

$36\,000 = 150h + 40(h^2 - 420h + 44\,100)$

$h^2 - 416.25h + 43\,200 = 0$

Choosing the root > 210 ft

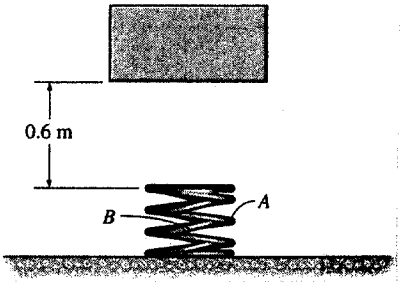
$h = 219$ ft Ans

$+\uparrow \Sigma F_y = ma_y$; $80(30) - 150 = \frac{150}{32.2}a$

$a = 483$ ft/s^2 Ans

It would not be a good idea to perform the stunt since $a = 15$ g which is excessive and A rises $219' - 120' = 99$ ft above the bridge!

***14-84.** Two equal-length springs having a stiffness $k_A = 300$ N/m and $k_B = 200$ N/m are "nested" together in order to form a shock absorber. If a 2-kg block is dropped from an at-rest position 0.6 m above the top of the springs, determine their deformation when the block momentarily stops.



Datum at initial position :

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 - 2(9.81)(0.6 + x) + \frac{1}{2}(300 + 200)(x)^2$

$250x^2 - 19.62x - 11.772 = 0$

Solving for the positive root,

$x = 0.260$ m Ans

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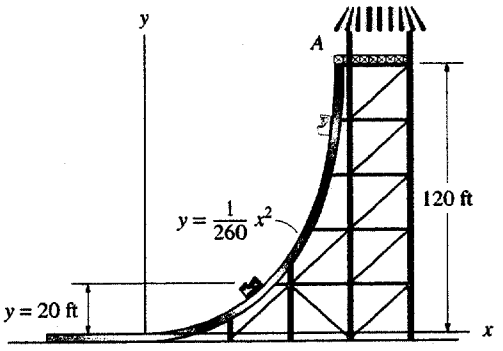
14-85. The ride at an amusement park consists of a gondola which is lifted to a height of 120 ft at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = 20$ ft. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight of 500 lb. Neglect the effects of friction and the mass of the wheels.



$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{130}$$

At $y = 120 - 100 = 20$ ft

$x = 72.11$ ft

$$\tan\theta = \frac{dy}{dx} = 0.555, \quad \theta = 29.02°$$

$$\rho = \frac{[1+(0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$

$$+\nwarrow \Sigma F_n = m a_n; \quad N_G - 500\cos 29.02° = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad (1)$$

$$T_1 + V_1 = T_2 + V_2$$

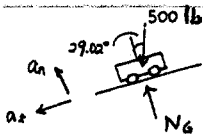
$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

$$v^2 = 6440$$

$v = 80.2$ ft/s **Ans**

Substituting into Eq. (1) yields

$N_G = 952$ lb **Ans**



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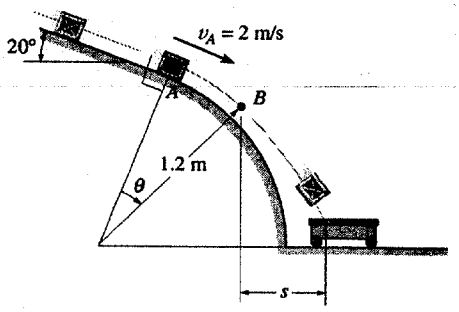
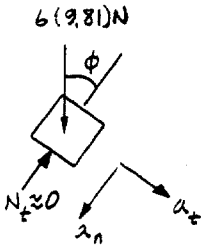
14-86. When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.

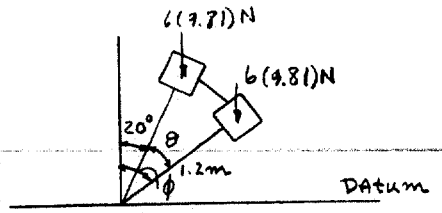




At point B :

$+\swarrow \Sigma F_n = ma_n$: $\quad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right)$ (1)

Datum at bottom of curve :



$T_A + V_A = T_B + V_B$

$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$

$13.062 = 0.5v_B^2 + 11.772\cos\phi$ (2)

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$v_B = 2.951$ m/s

Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

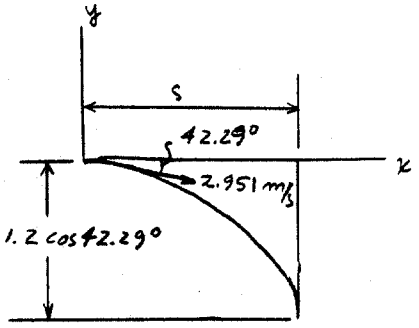
$\theta = \phi - 20° = 22.3°$ **Ans**

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 + 1.9857t - 0.8877 = 0$

Solving for the positive root : $\quad t = 0.2687$ s



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$s = 0 + (2.951\cos 42.29°)(0.2687)$

$s = 0.587$ m **Ans**

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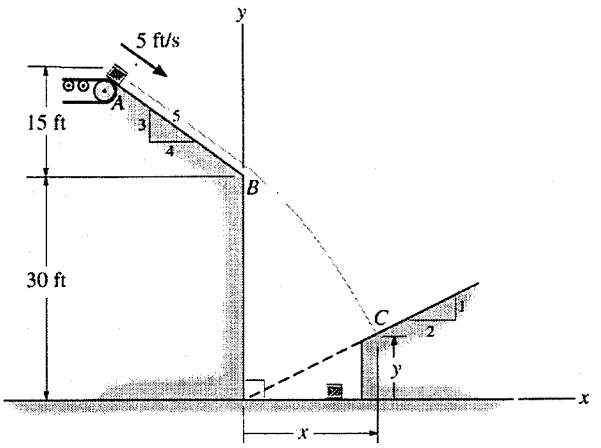
14-87. The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine the point $C(x, y)$ where it strikes the lower incline.

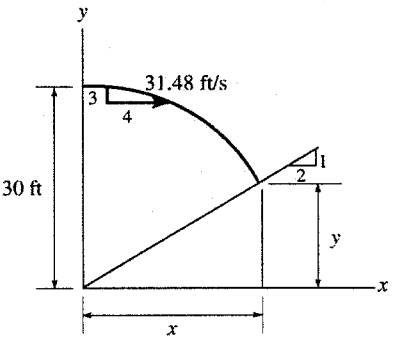


Datum at A:

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$v_B = 31.48$ ft/s



$$\left(\overset{+}{\rightarrow}\right) \qquad s = s_0 + v_0 t$$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad (1)$$

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad (2)$$

Thus,

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

Solving for the positive root,

$t = 0.7014$ s

From Eqs. (1) and (2):

$$x = 31.48\left(\frac{4}{5}\right)(0.7014) = 17.66 = 17.7 \text{ ft} \quad \textbf{Ans}$$

$$y = \frac{1}{2}(17.664) = 8.832 = 8.83 \text{ ft} \quad \textbf{Ans}$$

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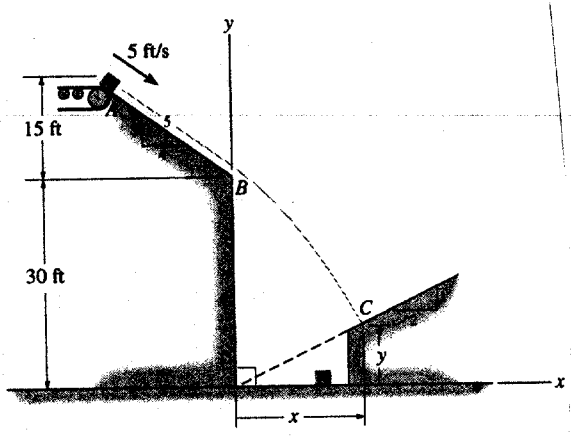
***14-88.** The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine its speed just before hitting the surface at C and the time to travel from A to C. The coordinates of point C are $x = 17.66$ ft, and $y = 8.832$ ft.



Datum at A :

$T_A + V_A = T_C + V_C$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2 - 2[15 + (30 - 8.832)]$$

$v_C = 48.5$ ft/s **Ans**

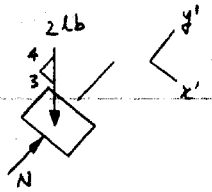
$+\nwarrow \Sigma F_{x'} = ma_{x'}$; $\quad 2\left(\frac{3}{5}\right) = \left(\frac{2}{32.2}\right)a_{x'}$

$a_{x'} = 19.32$ ft/s^2

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$v_B = 31.48$ ft/s



$(+\nwarrow)$ $\quad v_B = v_A + a_c t$

$31.48 = 5 + 19.32 t_{AB}$

$t_{AB} = 1.371$ s

$\left(\overset{+}{\rightarrow}\right)$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad (1)$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface :

$$\frac{y}{x} = \frac{1}{2}; \quad y = \frac{1}{2}x \qquad (2)$$

Thus

$30 - 18.888t - 16.1t^2 = 12.592t$

$-16.1t^2 - 31.480t + 30 = 0$

Solving for the positive root :

$t = 0.7014$ s

Total time is

$t = 1.371 + 0.7014 = 2.07$ s **Ans**

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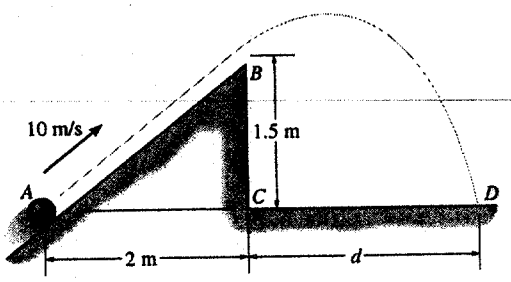
14-89. The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?



Datum at A :

$T_A + V_A = T_B + V_B$

$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$

$v_B = 8.401$ m/s

$\left(\overset{+}{\rightarrow}\right)$ $s = s_0 + v_0 t$

$d = 0 + 8.401\left(\frac{4}{5}\right)t$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

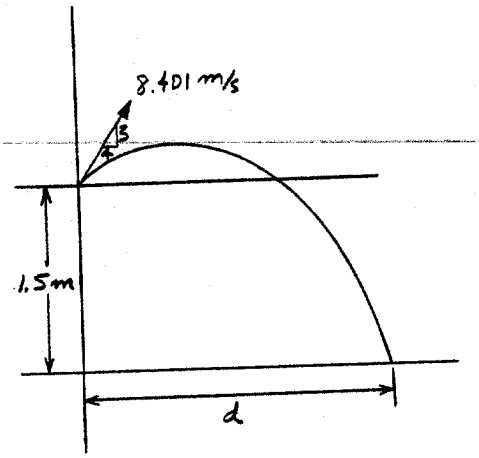
$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$

$-4.905t^2 + 5.040t + 1.5 = 0$

Solving for the positive root,

$t = 1.269$ s

$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53$ m **Ans**



Datum at A :

$T_A + V_A = T_D + V_D$

$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$

$v_D = 10$ m/s **Ans**

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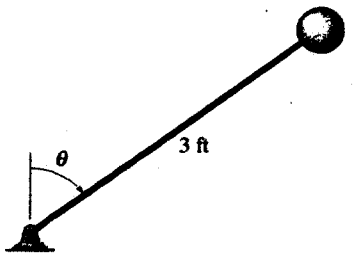
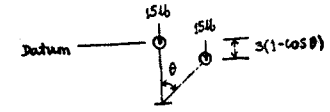
14-90. The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2 - 15(3)(1-\cos\theta)$



$v^2 = 193.2(1-\cos\theta)$

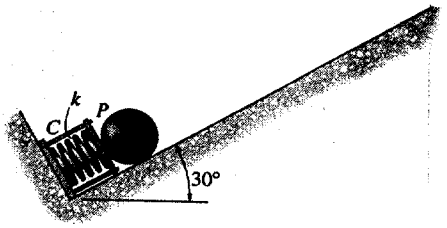
$+\nearrow \Sigma F_n = m a_n;\quad 15\cos\theta = \frac{15}{32.2}\left[\frac{193.2(1-\cos\theta)}{3}\right]$

$\cos\theta = 2 - 2\cos\theta$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 48.2°$ **Ans**

14-91. The 0.5-lb ball is shot from the spring device shown. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position, so that $s = 30$ in. If it is then released from rest, determine the speed of the ball when it leaves the surface of the smooth inclined plane.

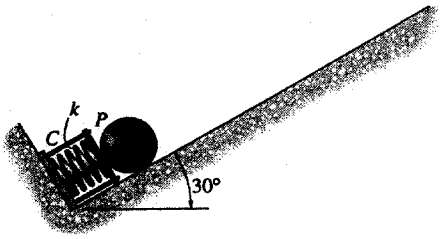


Potential Energy : The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42$ ft · lb and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667$ ft · lb, respectively.

Conservation of Energy :

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$
$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$
$$v = 32.3 \text{ ft/s}$$ **Ans**

***14-92.** The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance $s = 30$ in. up the plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



Potential Energy : The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

Conservation of Energy :

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$
$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$
$$k = 8.57 \text{ lb/ft}$$ **Ans**

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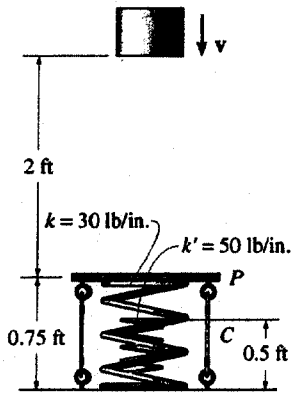
14-93. Four inelastic cables C are attached to a plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also an undeformed spring nested within this compressed spring. If the block, having a weight of 10 lb, is moving downward at $v = 4$ ft/s, when it is 2 ft above the plate, determine the maximum compression in each spring after it strikes the plate. Neglect the mass of the plate and spring and any energy lost in the collision.



$k = 30(12) = 360$ lb/ft

$k' = 50(12) = 600$ lb/ft

Assume both springs compress;

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + 0 + \frac{1}{2}(360)(0.25)^2 = 0 + \frac{1}{2}(360)(s+0.25)^2 + \frac{1}{2}(600)(s-0.25)^2 - 10(s+2)$$

$$13.73 = 180(s+0.25)^2 + 300(s-0.25)^2 - 10s - 20 \qquad (1)$$

$$33.73 = 180(s+0.25)^2 + 300(s-0.25)^2 - 10s$$

$$480s^2 - 70s - 3.73 = 0$$

Choose the positive root;

$s = 0.1873$ ft < 0.25 ft NG!

The nested spring does not deform.

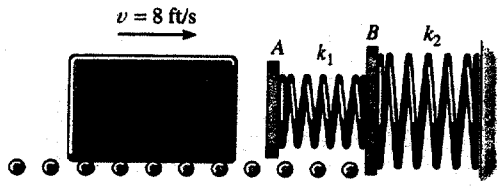
Thus Eq. (1) becomes

$$13.73 = 180(s+0.25)^2 - 10s - 20$$

$$180s^2 + 80s - 22.48 = 0$$

$s = 0.195$ ft **Ans**

14-94. The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum deflection of the plate A caused by the billet if it strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 4500$ lb/ft.



$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \tfrac{1}{2}(3000)s_1^2 + \tfrac{1}{2}(4500)s_2^2 \qquad [1]$$

$F_s = 3000s_1 = 4500s_2$; [2]

$s_1 = 1.5s_2$

Solving Eqs. [1] and [2] yields:

$s_2 = 0.5148$ ft $s_1 = 0.7722$ ft

$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29$ ft **Ans**

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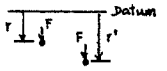
14-95. If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_em/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_em/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.

The work is computed by moving F from position r to a greater position r'.

$$V = -U = -\int F\,dr$$

$$= -GM_e m \int_r^{r'} \frac{dr}{r^2}$$



$$= -GM_e m\left(\frac{1}{r} - \frac{1}{r'}\right)$$

As $r' \to \infty$,

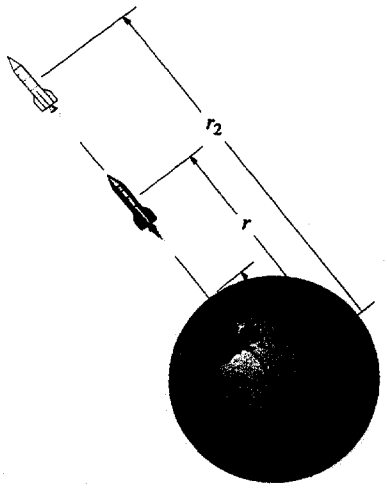
$$V = \frac{-GM_e m}{r}$$

To be conservative, require

$$F = -\nabla V = -\frac{\partial}{\partial r}\left(-\frac{GM_e m}{r}\right)$$

$$= \frac{-GM_e m}{r^2} \qquad \text{Q.E.D.}$$

***14-96.** A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming that no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_em/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



$$F = G\frac{M_e m}{r^2}$$

$$U_{1-2} = \int F\,dr = -GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$U_{1-2} = GM_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{Ans}$$

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15-1. A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

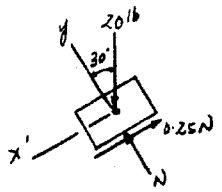
$\left(+\nwarrow\right)$ $m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$

$0 + N(3) - 20\cos 30°(3) = 0 \qquad N = 17.32 \text{ lb}$

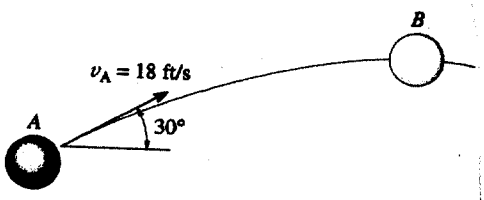
$\left(+\swarrow\right)$ $m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$

$\frac{20}{32.2}(2) + 20\sin 30°(3) - 0.25(17.32)(3) = \frac{20}{32.2}v$

$v = 29.4 \text{ ft/s}$ **Ans**



15-2. A 2-lb ball is thrown in the direction shown with an initial speed $v_A = 18$ ft/s. Determine the time needed for it to rereach its highest point B and the speed at which it is traveling at B. Use the principle of impulse and momentum for the solution.



$(+\uparrow)$ $m(v_y)_1 + \Sigma \int F\,dt = m(v_y)_2$

$\frac{2}{32.2}(18\sin 30°) - 2(t) = 0$

$t = 0.2795 = 0.280 \text{ s}$ **Ans**

$(\overset{+}{\rightarrow})$ $m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$

$\frac{2}{32.2}(18\cos 30°) + 0 = \frac{2}{32.2}(v_B)$

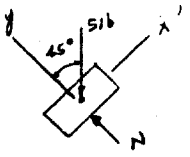
$v_B = 15.588 = 15.6 \text{ ft/s}$ **Ans**

15-3. A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time it will take to travel up the slope before it stops.

$\left(\nearrow+\right)$ $m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$

$\frac{5}{32.2}(10) + (-5\sin 45°)t = 0$

$t = 0.439 \text{ s}$ **Ans**



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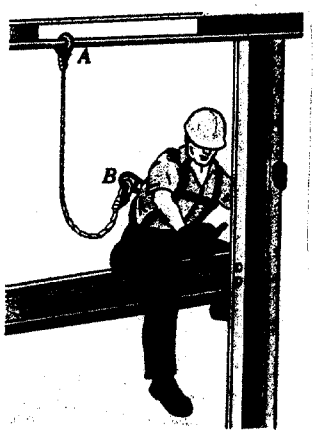
***15-4.** The 180-lb iron worker is secured by a fall-arrest system consisting of a harness and lanyard AB, which is fixed to the beam. If the lanyard has a slack of 4 ft, determine the average impulsive force developed in the lanyard if he happens to fall 4 feet. Neglect his size in the calculation and assume the impulse takes place in 0.6 seconds.



$T_1 + \Sigma U_{1-2} = T_2$

$0 + 180(4) = \frac{1}{2}\left(\frac{180}{32.2}\right)v^2$

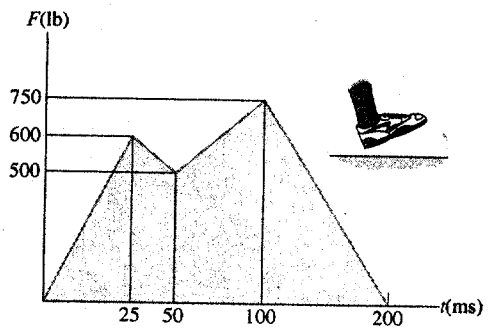
$v = 16.05$ ft/s

$(+\downarrow)$ $mv_1 + \int F\,dt = mv_2$

$\frac{180}{32.2}(16.05) + 180(0.6) - F(0.6) = 0$

$F = 329.5$ lb $= 330$ lb **Ans**

15-5. The graph shows the vertical reaction force of the shoe-ground interaction as a function of time. The first peak acts on the heel, and the second peak acts on the forefoot. Determine the total impulse acting on the shoe during the interaction.

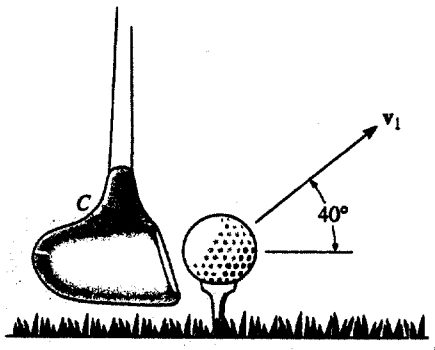


Impulse : The total impluse acting on the shoe can be obtained by evaluating the area under the $F-t$ graph.

$$I = \frac{1}{2}(600)\left[25\left(10^{-3}\right)\right] + \frac{1}{2}(500+600)(50-25)\left(10^{-3}\right) + \frac{1}{2}(500+750)(100-50)\left(10^{-3}\right) + \frac{1}{2}(750)\left[(200-100)\left(10^{-3}\right)\right]$$

$= 90.0$ lb · s **Ans**

15-6. A man hits the 50-g golf ball such that it leaves the tee at an angle of 40° with the horizontal and strikes the ground at the same elevation a distance of 20 m away. Determine the impulse of the club C on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.



$(\stackrel{+}{\rightarrow})$ $s_x = (s_0)_x + (v_0)_x t$

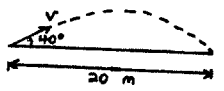
$20 = 0 + v\cos 40°(t)$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0 = 0 + v\sin 40°(t) - \frac{1}{2}(9.81)t^2$

$t = 1.85$ s

$v = 14.115$ m/s



$(+\nearrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$

$0 + \int F\,dt = (0.05)(14.115)$

$\int F\,dt = 0.706$ N · s $\angle 40°$ **Ans**

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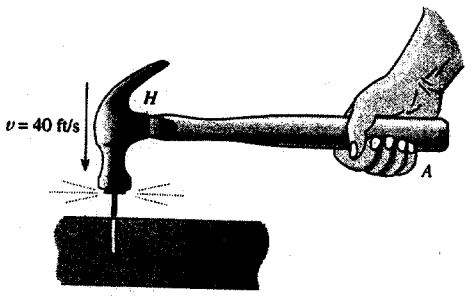
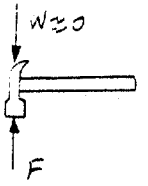
15-7. A hammer head H having a weight of 0.25 lb is moving vertically downward at 40 ft/s when it strikes the head of a nail of negligible mass and drives it into a block of wood. Find the impulse on the nail if it is assumed that the grip at A is loose, the handle has a negligible mass, and the hammer stays in contact with the nail while it comes to rest. Neglect the impulse caused by the weight of the hammer head during contact with the nail.



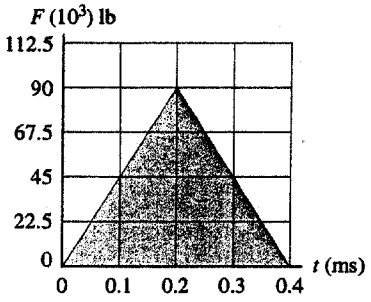
$$(+\downarrow) \quad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$$

$$\left(\frac{0.25}{32.2}\right)(40) - \int F\, dt = 0$$

$$\int F\, dt = 0.311 \text{ lb} \cdot \text{s} \quad \textbf{Ans}$$



***15-8.** During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the 2-lb spike S is fired from rest into the surface at 200 ft/s. Determine the speed of the spike just after rebounding.



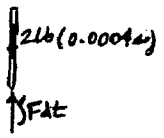
$$(+\downarrow) \quad mv_1 + \int F\, dt = mv_2$$

$$\frac{2}{32.2}(200) + 2(0.0004) - Area = \frac{-2}{32.2}(v)$$

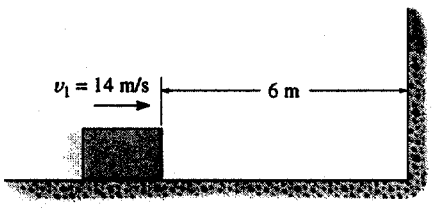
$$Area = \frac{1}{2}(90)(10^3)(0.4)(10^{-3}) = 18 \text{ lb}\cdot\text{s}$$

Thus,

$$v = 89.8 \text{ ft/s} \quad \textbf{Ans}$$



15-9. When the 5-kg block is 6 m from the wall, it is sliding at $v_1 = 14$ m/s. If the coefficient of kinetic friction between the block and the horizontal plane is $\mu_k = 0.3$, determine the impulse of the wall on the block necessary to stop the block. Neglect the friction impulse acting on the block during the collision.



Equation of Motion : The acceleration of the block must be obtained first before one can determine the velocity of the block before it strikes the wall.

$$+\uparrow \Sigma F_y = ma_y; \quad N - 5(9.81) = 5(0) \quad N = 49.05 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad -0.3(49.05) = -5a \quad a = 2.943 \text{ m/s}^2$$

Kinematics : Applying the equation $v^2 = v_0^2 + 2a_c(s - s_0)$ yields

$$\left(\overset{+}{\rightarrow}\right) \quad v^2 = 14^2 + 2(-2.943)(6 - 0) \quad v = 12.68 \text{ m/s}$$

Principle of Linear Impulse and Momentum : Applying Eq. 15–4, we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$\left(\overset{+}{\rightarrow}\right) \quad 5(12.68) - I = 5(0)$$

$$I = 63.4 \text{ N}\cdot\text{s} \quad \textbf{Ans}$$

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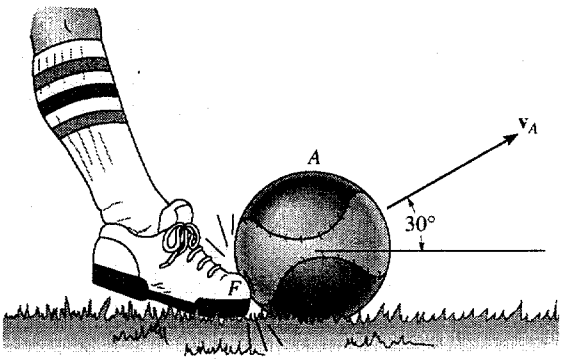
15-10. A man kicks the 200-g ball such that it leaves the ground at an angle of 30° with the horizontal and strikes the ground at the same elevation a distance of 15 m away. Determine the impulse of his foot F on the ball. Neglect the impulse caused by the ball's weight while its being kicked.



$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t + \frac{1}{2}a_c t^2$$

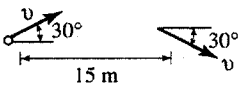
$$15 = 0 + v\cos 30°\, t + 0$$

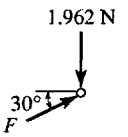
$$(+\uparrow) \quad v_y = (v_0)_y + a_c t$$

$$-v\sin 30° = v\sin 30° - 9.81t$$

$$t = 1.329 \text{ s}$$

$$v = 13.04 \text{ m/s}$$



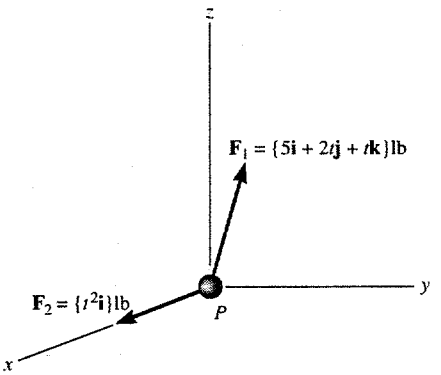


$$(+\nearrow) \quad mv_1 + \sum \int F\,dt = mv_2$$

$$0 + \int F\,dt = 0.2(13.04)$$

$$I = \int F\,dt = 2.608 = 2.61 \text{ N}\cdot\text{s} \quad \text{Ans}$$

15-11. The particle P is acted upon by its weight of 3 lb and forces F_1 and F_2, where t is in seconds. If the particle originally has a velocity of $v_1 = \{3i + 1j + 6k\}$ ft/s, determine its speed after 2 s.



$$mv_1 + \sum \int_0^2 F\,dt = mv_2$$

Resolving into scalar components,

$$\frac{3}{32.2}(3) + \int_0^2 (5 + t^2)dt = \frac{3}{32.2}(v_x)$$

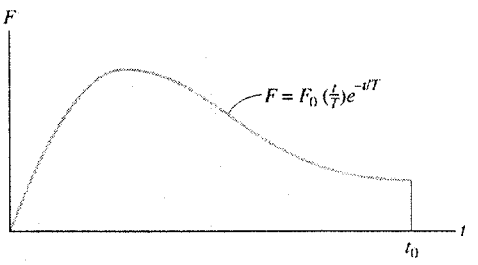
$$\frac{3}{32.2}(1) + \int_0^2 2t\,dt = \frac{3}{32.2}(v_y)$$

$$\frac{3}{32.2}(6) + \int_0^2 (t - 3)dt = \frac{3}{32.2}(v_z)$$

$$v_x = 138.96 \text{ ft/s} \quad v_y = 43.933 \text{ ft/s} \quad v_z = -36.933 \text{ ft/s}$$

$$v = \sqrt{(138.96)^2 + (43.933)^2 + (-36.933)^2} = 150 \text{ ft/s} \quad \text{Ans}$$

***15-12.** The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time t_0, determine the impulse developed by the muscle.



$$I = \int F\,dt = \int_0^{t_0} F_0\left(\frac{t}{T}\right)e^{-t/T}dt$$

$$I = \frac{F_0}{T}\int_0^{t_0} te^{-(t/T)}dt$$

$$I = -F_0\left[Te^{-t/T}\left(\frac{t}{T} + 1\right)\right]_0^{t_0}$$

$$I = -F_0\left[Te^{-t_0/T}\left(\frac{t_0}{T} + 1\right) - T\right]$$

$$I = TF_0\left[1 - e^{-t_0/T}\left(1 + \frac{t_0}{T}\right)\right] \quad \text{Ans}$$

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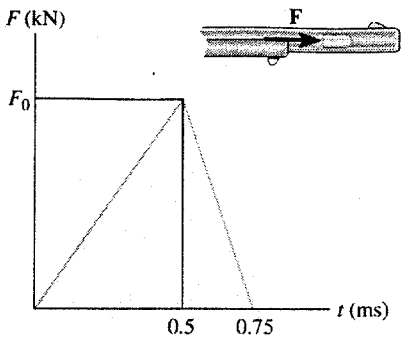
15-13. Assuming that the force acting on a 2-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force F_0, applied to the bullet when it is fired. The muzzle velocity is 500 m/s when $t = 0.75$ ms. Neglect friction between the bullet and the rifle barrel.

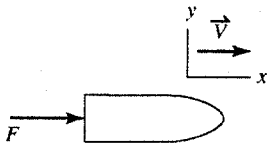


Principle of Linear Impulse and Momentum: The total impluse acting on the bullet can be obtained by evaluating the area under the $F - t$ graph. Thus, $I = \sum \int_{t_1}^{t_2} F_x dt = \frac{1}{2}(F_0)[0.5(10^{-3})] + \frac{1}{2}(F_0)[(0.75 - 0.5)(10^{-3})] = 0.375(10^{-3})F_0$. Applying Eq. 15-4, we have

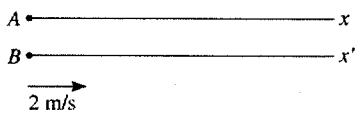
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_k)_2$$

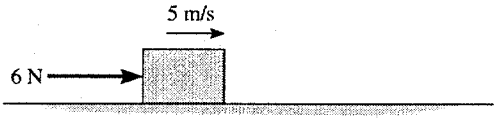
$$(\overset{+}{\rightarrow}) \quad 0 + 0.375(10^{-3})F_0 = 2(10^{-3})(500)$$

$$F_0 = 2666.67 \text{ N} = 2.67 \text{ kN} \qquad \textbf{Ans}$$



15-14. As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a fixed frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



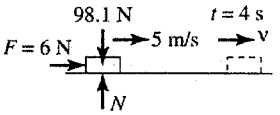


Observer A:

$$(\overset{+}{\rightarrow}) \quad mv_1 + \sum \int F dt = mv_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s} \qquad \textbf{Ans}$$



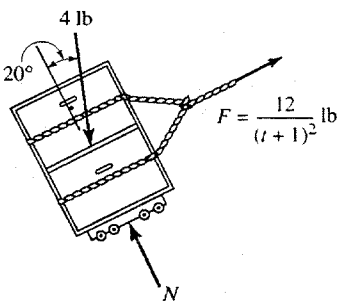
Observer B:

$$(\overset{+}{\rightarrow}) \quad mv_1 + \sum \int F dt = mv_2$$

$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s} \qquad \textbf{Ans}$$

15-15. The 4-lb cabinet is subjected to the force $F = 12(t + 1)^2$ lb where t is in seconds. If the cabinet is initially moving up the plane with a velocity of 10 ft/s, determine how long it will take before the cabinet comes to a stop. F always acts parallel to the plane. Neglect the size of the rollers.



Principle of Linear Impulse and Momentum: Applying Eq. 15-4, we have

$$m(v_{x'})_1 + \sum \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$$

$$\left(\frac{4}{32.2}\right)(10) + \int_0^t \frac{12}{(t+1)^2} dt - 4 \sin 20° t = \left(\frac{4}{32.2}\right)(0)$$

$t = 8.78$ s **Ans**

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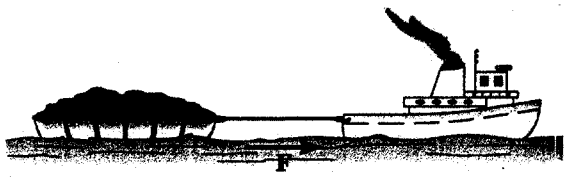
***15-16.** If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



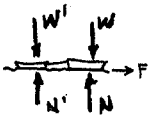
$25\left(\frac{1000}{3600}\right) = 6.944$ m/s

System:

$(\overset{+}{\rightarrow})$ $mv_1 + \Sigma\int F\,dt = mv_2$

$[0+0] + F(35) = (50+75)(10^3)(6.944)$

$F = 24.8$ kN **Ans**



Barge:

$(\overset{+}{\rightarrow})$ $mv_1 + \Sigma\int F\,dt = mv_2$

$0 + T(35) = (75)(10^3)(6.944)$

$T = 14.881 = 14.9$ kN **Ans**

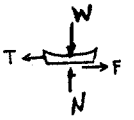
Also, using this result for T,

Tugboat:

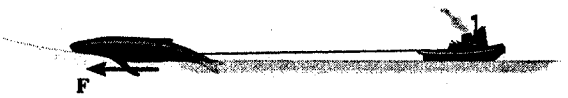
$(\overset{+}{\rightarrow})$ $mv_1 + \Sigma\int F\,dt = mv_2$

$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$

$F = 24.8$ kN **Ans**



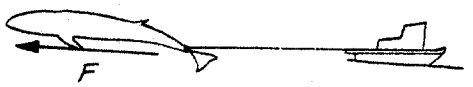
15-17. The 5.5-Mg humpback whale is stuck on the shore due to changes in the tide. In an effort to rescue the whale, a 12-Mg tugboat is used to pull it free using an inextensible rope tied to its tail. To overcome the frictional force of the sand on the whale, the tug backs up so that the rope becomes slack and then the tug proceeds forward at 3 m/s. If the tug then turns the engines off, determine the average frictional force **F** on the whale if sliding occurs for 1.5 s before the tug stops after the rope becomes taut. Also, what is the average force on the rope during the tow?



$\left(\overset{+}{\rightarrow}\right)$ $m_1(v_x)_1 + \Sigma\int F_x\,dt = m_2(v_x)_2$

$0 + 12(10^3)(3) - F(1.5) = 0 + 0$

$F = 24$ kN **Ans**



Tug:

$\left(\overset{+}{\rightarrow}\right)$ $m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$12(10^3)(3) - T(1.5) = 0$

$T = 24$ kN **Ans**

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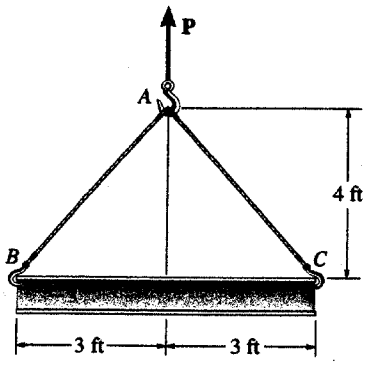
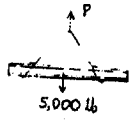
15-18. The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.



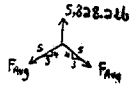
$(+\uparrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$



$$0 + P_{avg}(1.5) - 5000(1.5) = \frac{5000}{32.2}(8)$$

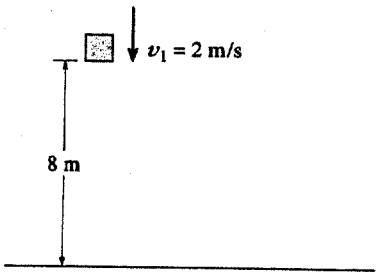
$$P_{avg} = 5828.157 = 5828.2 \text{ lb}$$

$+\uparrow \Sigma F_y = 0;$ $\quad 5828.157 - 2\left(\frac{4}{5}\right)F_{avg} = 0$



$$F_{avg} = 3642.598 \text{ lb} = 3.64 \text{ kip}$$ **Ans**

15-19. The 5-kg block is moving downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

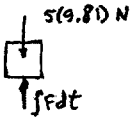


Just before impact

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.687 \text{ m/s}$$

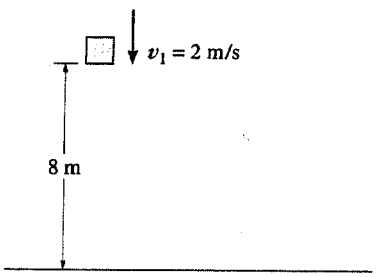


$(+\downarrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$

$$5(12.687) - \int F\,dt = 0$$

$$I = \int F\,dt = 63.4 \text{ N}\cdot\text{s}$$ **Ans**

***15-20.** The 5-kg block is falling downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in 0.9 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

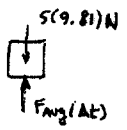


Just before impact

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.69 \text{ m/s}$$



$(+\downarrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$

$$5(12.69) - F_{avg}(0.9) = 0$$

$$F_{avg} = 70.5 \text{ N}$$ **Ans**

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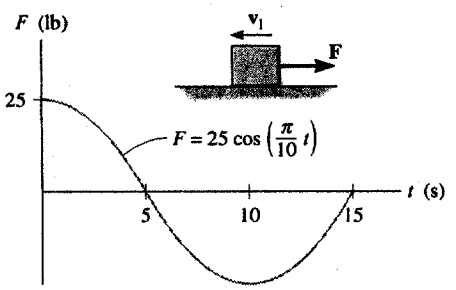
15-21. A 30-lb block is initially moving along a smooth horizontal surface with a speed of $v_1 = 6$ ft/s to the left. If it is acted upon by a force **F**, which varies in the manner shown, determine the velocity of the block in 15 s.



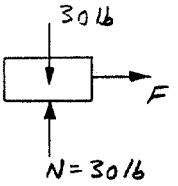
$$\left(\overset{+}{\rightarrow}\right) \quad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$-\left(\frac{30}{32.2}\right)(6) + \int_0^{15} 25\cos\left(\frac{\pi}{10}t\right) dt = \left(\frac{30}{32.2}\right)(v_x)_2$$

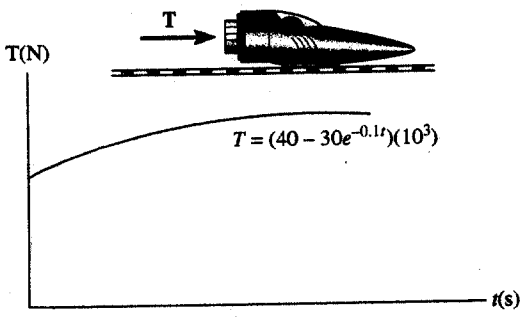
$$-5.59 + (25)\left[\sin\left(\frac{\pi}{10}t\right)\right]_0^{15}\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$-5.59 + (25)[-1]\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$(v_x)_2 = -91.4 = 91.4 \text{ ft/s} \leftarrow \quad \textbf{Ans}$$



***15-22.** The rocket sled has a mass of 3 Mg and starts from rest when $t = 0$. If the engines provide a horizontal thrust T which varies as shown in the graph, determine the sled's velocity in $t = 4$ s. Neglect air resistance, friction, and the loss of fuel during the motion.



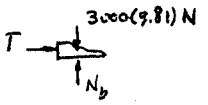
$$\left(\overset{+}{\rightarrow}\right) \quad mv_1 + \Sigma \int F \, dt = mv_2$$

$$0 + (10^3)\int_0^4 (40 - 30e^{-0.1t}) \, dt = 3(10^3)v$$

$$(10^3)[40t + 300e^{0.1t}]_0^4 = 3(10^3)v$$

$$(10^3)[160 + 300(\tfrac{1}{e^{0.4}}) - 300] = 3(10^3)v$$

$$v = 20.4 \text{ m/s} \quad \textbf{Ans}$$



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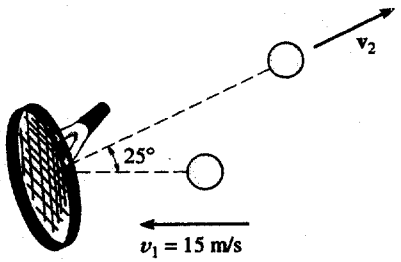
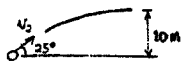
15-23. The tennis ball has a horizontal speed of 15 m/s when it is struck by the racket. If it then travels away at an angle of 25° from the horizontal and reaches a maximum altitude of 10 m, measured from the height of the racket, determine the magnitude of the net impulse of the racket on the ball. The ball has a mass of 180 g. Neglect the weight of the ball during the time the racket strikes the ball.



$(+\uparrow)$ $v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$

$(v_2 \sin 25°)^2 = 0 + 2(9.81)(10-0)$

$v_2 = 33.14$ m/s



$(\overset{+}{\rightarrow})$ $m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$-0.180(15) + \int F_x\,dt = 0.180(33.14\cos 25°)$

$\int F_x\,dt = 8.107$ N·s

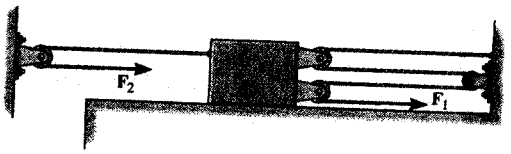
$(+\uparrow)$ $m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

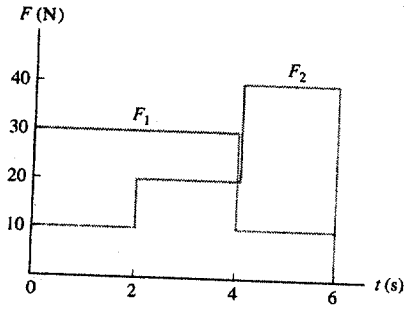
$0 + \int F_y\,dt = 0.180(33.14\sin 25°)$

$\int F_y\,dt = 2.521$ N·s

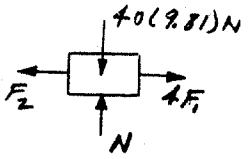
$I = \int F\,dt = \sqrt{(8.107)^2 + (2.521)^2} = 8.49$ N·s $\underset{17.3°}{\angle}$ **Ans**

***15-24.** The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





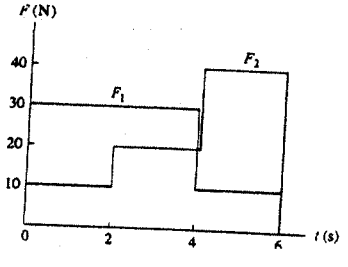
The impulses acting on the block are equal to the areas under the graph.



$(\overset{+}{\rightarrow})$ $m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$40(1.5) + 4[(30)4 + 10(6-4)] - [10(2) + 20(4-2) + 40(6-4)] = 40v_2$

$v_2 = 12.0$ m/s $(\rightarrow)$ **Ans**



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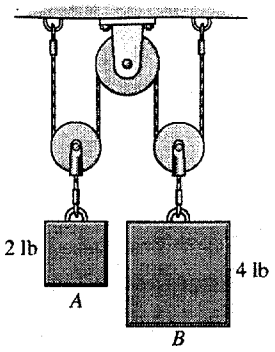
15-25. Determine the velocities of blocks A and B 2 s after they are released from rest. Neglect the mass of the pulleys and cables.



$2s_A + 2s_B = l$

$v_A = -v_B$

Block A:

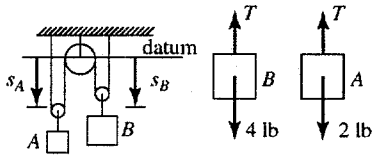
$(+\downarrow)$ $\quad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$$0 - T(2) + 2(2) = \left(\frac{2}{32.2}\right) v_A$$

Block B:

$(+\downarrow)$ $\quad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$$0 + 4(2) - T(2) = \left(\frac{4}{32.2}\right) v_B$$



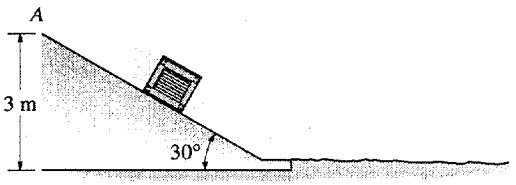
Solving,

$T = 2.67$ lb

$v_B = 21.5$ ft/s $\downarrow$ **Ans**

$v_A = -21.5$ ft/s $= 21.5$ ft/s $\uparrow$ **Ans**

15-26. The 5-kg package is released from rest at A. It slides down the smooth plane onto the rough surface having a coefficient of kinetic friction of $\mu_k = 0.2$. Determine the total time of travel before the package stops sliding. Neglect the size of the package.



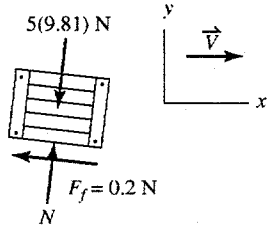
Potential Energy: The datum is set at point A. When the package reaches the toe of the inclined plane, its position is 3 m *below* the datum. Its gravitational potential energy is $5(9.81)(-3) = -147.15$ N · m.

Conservation of Energy: Applying Eq. 14-21, we have

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}(5)v^2 + (-147.15)$$

$v = 7.672$ m/s

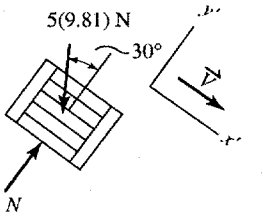


Principle of Linear Impulse and Momentum: The time taken for the package to reach the toe of the inclined plane can be obtained by applying Eq. 15-4.

$$m(v_{x'})_1 + \sum \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$$

$(\searrow +)$ $\quad 5(0) + 5(9.81)\sin 30° t_1 = 5(7.672)$

$t_1 = 1.564$ s



The time taken for the package to stop when it moves along the horizontal rough surface can be obtained by applying Eq. 15-4.

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$(+\uparrow)$ $\quad 5(0) + N(t_2) - 5(9.81)t_2 = 5(0)$

$N = 49.05$ N

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$(\overset{+}{\rightarrow})$ $\quad 5(7.672) + [-0.2(49.05)t_2] = 5(0)$

$t = 3.910$ s

Thus, the total package's traveling time is

$t = t_1 + t_2 = 1.564 + 3.910 = 5.47$ s **Ans**

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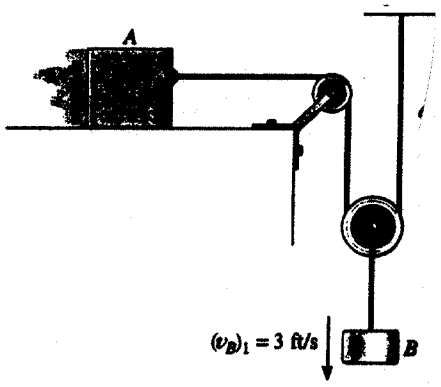
15-27. Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s.



$$s_A + 2s_B = l$$

$$v_A = -2v_B$$

$$(\overset{+}{\leftarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2$$

$$(+\downarrow) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

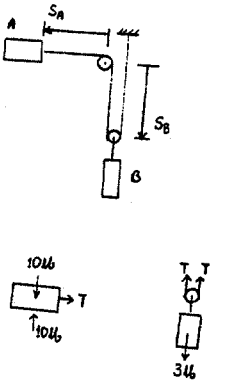
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

$$-32.2T - 10(v_A)_2 = 60$$

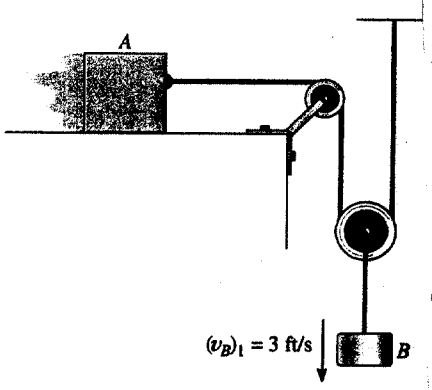
$$-64.4T + 1.5(v_A)_2 = -105.6$$

$$T = 1.40 \text{ lb}$$

$$(v_A)_2 = -10.5 \text{ ft/s} = 10.5 \text{ ft/s} \rightarrow \quad \textbf{Ans}$$



***15-28.** Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(\mu_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. The coefficient of kinetic friction between the horizontal plane and block A is $\mu_A = 0.15$.



$$s_A + 2s_B = l$$

$$v_A = -2v_B$$

$$(\overset{+}{\leftarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) + 0.15(10) = \frac{10}{32.2}(v_A)_2$$

$$(+\downarrow) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

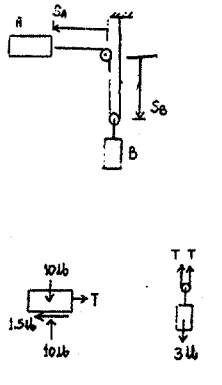
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

$$-32.2T - 10(v_A)_2 = 11.70$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

$$T = 1.50 \text{ lb}$$

$$(v_A)_2 = -6.00 \text{ ft/s} = 6.00 \text{ ft/s} \rightarrow \quad \textbf{Ans}$$



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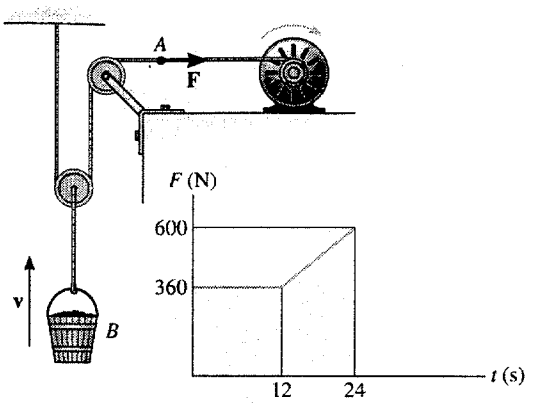
15-29. The winch delivers a horizontal towing force F to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket B when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.

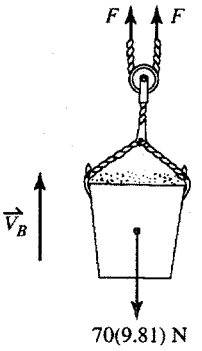


Principle of Linear Impulse and Momentum: For the time period $12\text{ s} \le t < 18\text{ s}$, $\frac{F - 360}{t - 12} = \frac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15-4 to bucket B, we have

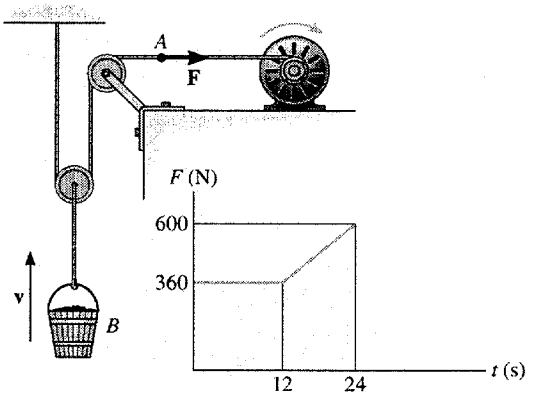
$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$(+\uparrow) \quad 70(3) + 2\left[360(12) + \int_{12s}^{18s} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$$v_2 = 21.8 \text{ m/s} \qquad \textbf{Ans}$$



15-30. The winch delivers a horizontal towing force F to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket B when $t = 24$ s. Originally the bucket is moving downward at 20 m/s.



Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the $F - t$ graph. Thus,

$$I = \sum \int_{t_1}^{t_2} F_y dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right]$$

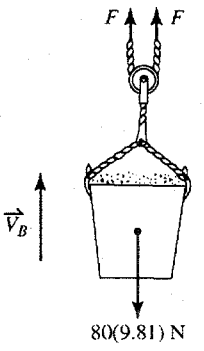
$$= 20\,160 \text{ N} \cdot \text{s}.$$

Applying Eq. 15-4 to the bucket B, we have

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$(+\uparrow) \quad 80(-20) + 20\,160 - 80(9.81)(24) = 80v_2$$

$$v_2 = -3.44 \text{ m/s} = 3.44 \text{ m/s} \downarrow \qquad \textbf{Ans}$$



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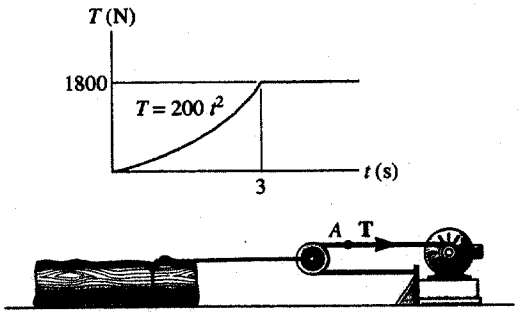
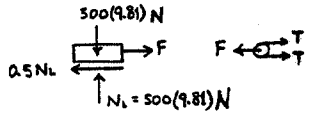
15-31. The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.



$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad F - 0.5(500)(9.81) = 0$



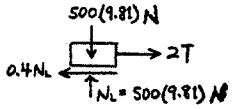
$F = 2452.5$ N

Thus,

$2T = F$

$2(200t^2) = 2452.5$

$t = 2.476$ s to start log moving



$(\overset{+}{\rightarrow})\quad m v_1 + \Sigma \int F\,dt = m v_2$

$0 + 2\int_{2.476}^{3} 200t^2\,dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$

$400\left(\frac{t^3}{3}\right)\Big|_{2.476}^{3} + 2247.91 = 500v_2$

$v_2 = 7.65$ m/s **Ans**

***15-32.** A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

$(\overset{+}{\rightarrow})\quad \Sigma m v_1 = \Sigma m v_2$

$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$

$v_2 = 0.5$ m/s **Ans**

$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25$ kJ

$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375$ kJ

$\Delta T = T_2 - T_1$

$= 3.375 - 20.25 = -16.9$ kJ **Ans**

This energy is dissipated as noise, shock, and heat during the coupling.

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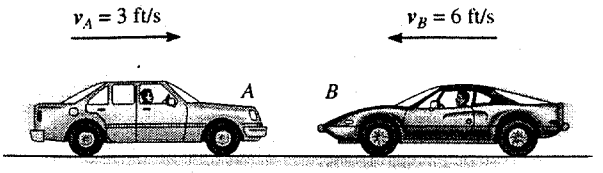
15-33. The car A has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car B is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

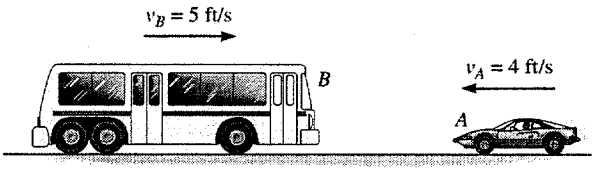


$(\overset{+}{\rightarrow})$ $\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans}$$

15-34. The bus B has a weight of 15 000 lb and is traveling to the right at 5 ft/s. Meanwhile a 3000-lb car A is traveling at 4 ft/s to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

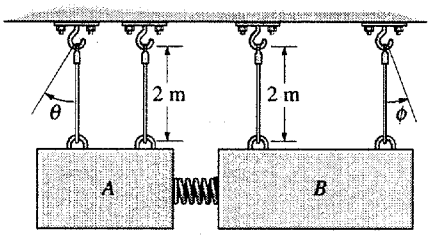


$(\overset{+}{\rightarrow})$ $\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v$

$$\frac{15\,000}{32.2}(5) - \frac{3000}{32.2}(4) = \frac{18\,000}{32.2}v$$

$$v = 3.5 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

15-35. The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



$(\overset{+}{\rightarrow})$ $\quad \sum mv_1 = \sum mv_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348 \text{ m/s}$$

For A or B:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$$\theta = \phi = 9.52^\circ \qquad \textbf{Ans}$$

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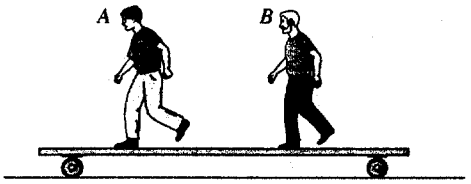
***15-36.** Two men A and B, each having a weight of 160 lb, stand on the 200-lb, cart. Each runs with a speed of 3 ft/s measured relative to the cart. Determine the final speed of the cart if (a) A runs and jumps off, then B runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.



(a) A jumps first.

$(\overset{+}{\leftarrow}) \; 0+0 = m_A v_A - (m_C + m_B) v'_C$ However, $v_A = -v'_C + 3$

$0 = \frac{160}{32.2}(-v'_C + 3) - \frac{360}{32.2} v'_C$

$v'_C = 0.9231 \text{ ft/s} \rightarrow$

And then B jumps

$0 + (m_C + m_B) v'_C = m_B v_B - m_C v_C$ However, $v_B = -v_C + 3$

$\frac{360}{32.2}(-0.9231) = \frac{160}{32.2}(-v_C + 3) - \frac{200}{32.2} v_C$

$v_C = 2.26 \text{ ft/s} \rightarrow$ Ans

(b) Both men jump at the same time

$(\overset{+}{\leftarrow}) \; 0+0 = (m_A + m_B) v - m_C v_C$ However, $v = -v_C + 3$

$0 = \left(\frac{160}{32.2} + \frac{160}{32.2}\right)(-v_C + 3) - \frac{200}{32.2} v_C$

$v_C = 1.85 \text{ ft/s} \rightarrow$ Ans

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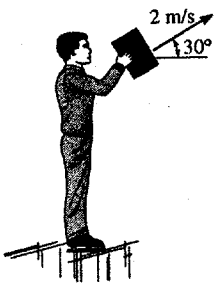
15-37. A man wearing ice skates throws an 8-kg block with an initial velocity of 2 m/s, measured relative to himself, in the direction shown. If he is originally at rest and completes the throw in 1.5 s while keeping his legs rigid, determine the horizontal velocity of the man just after releasing the block. What is the vertical reaction of both his skates on the ice during the throw? The man has a mass of 70 kg. Neglect friction and the motion of his arms.



$\left(\overset{+}{\rightarrow}\right)$ $0 = -m_M v_M + m_B (v_B)_x$ (1)

However, $\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$

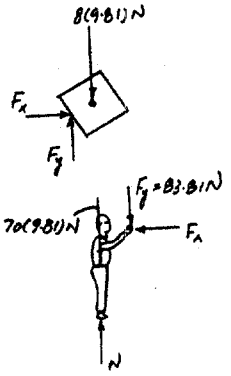
$\left(\overset{+}{\rightarrow}\right)$ $(v_B)_x = -v_M + 2\cos 30°$ (2)

$(+\uparrow)$ $(v_B)_y = 0 + 2\sin 30° = 1$ m/s

Substituting Eq. (2) into (1) yields:

$0 = -m_M v_M + m_B (-v_M + 2\cos 30°)$

$$v_M = \frac{2m_B \cos 30°}{m_B + m_M} = \frac{2(8)\cos 30°}{8+70} = 0.178 \text{ m/s}$$ **Ans**



For the block

$(+\uparrow)$ $m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$

$0 + F_y(1.5) - 8(9.81)(1.5) = 8(2\sin 30°)$ $F_y = 83.81$ N

For the man

$(+\uparrow)$ $m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$

$0 + N(1.5) - 70(9.81)(1.5) - 83.81(1.5) = 0$

$N = 771$ N **Ans**

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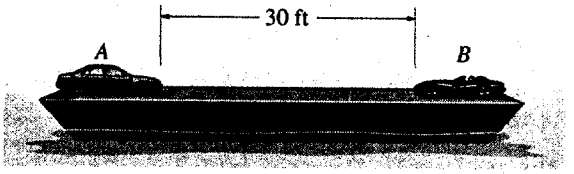
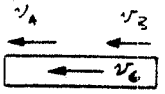
15-38. The barge weighs 45 000 lb and supports two automobiles A and B, which weigh 4000 lb and 3000 lb, respectively. If the automobiles start from rest and drive towards each other, accelerating at $a_A = 4\ \text{ft/s}^2$ and $a_B = 8\ \text{ft/s}^2$ until they reach a constant speed of 6 ft/s relative to the barge, determine the speed of the barge just before the automobiles collide. How much time does this take? Originally the barge is at rest. Neglect water resistance.



$\left(\overset{+}{\leftarrow}\right)$ $v_A = v_C + v_{A/C} = v_C - 6$

$\left(\overset{+}{\leftarrow}\right)$ $v_B = v_C + v_{B/C} = v_C + 6$



$\left(\overset{+}{\leftarrow}\right)$ $\Sigma m v_1 = \Sigma m v_2$

$0 = m_A(v_C - 6) + m_B(v_C + 6) + m_C v_C$

$0 = \left(\frac{4000}{32.2}\right)(v_C - 6) + \left(\frac{3000}{32.2}\right)(v_C + 6) + \left(\frac{45\,000}{32.2}\right)v_C$

$v_C = 0.1154\ \text{ft/s} = 0.115\ \text{ft/s}$ **Ans**

For A:

$\left(\overset{+}{\rightarrow}\right)$ $v = v_0 + a_c t$

$6 = 0 + 4t_A$

$t_A = 1.5\ \text{s}$

$\left(\overset{+}{\rightarrow}\right)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 0 + \frac{1}{2}(4)(1.5)^2 = 4.5\ \text{ft}$

For B:

$\left(\overset{+}{\leftarrow}\right)$ $v = v_0 + a_c t$

$6 = 0 + 8t_B$

$t_B = 0.75\ \text{s}$

$\left(\overset{+}{\leftarrow}\right)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 0 + \frac{1}{2}(8)(0.75)^2 = 2.25\ \text{ft}$

For the remaining $(1.5 - 0.75)\text{s} = 0.75\ \text{s}$

$s = vt = 6(0.75) = 4.5\ \text{ft}$

Thus,

$s = 30 - 4.5 - 4.5 - 2.25 = 18.75\ \text{ft}$

$t' = \frac{s/2}{v} = \frac{18.75/2}{6} = 1.5625$

$t = 1.5 + 1.5625 = 3.06\ \text{s}$ **Ans**

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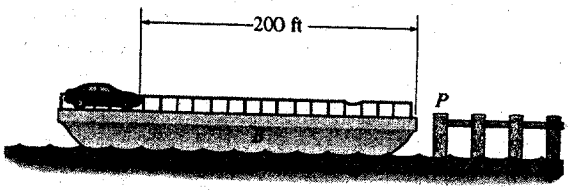
15-39. The barge B weighs 30 000 lb and supports an automobile weighing 3000 lb. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.



Relative Velocity : The relative velocity of the car with respect to the barge is $v_{c/b}$. Thus, the velocity of the car is

$$(\overset{+}{\rightarrow}) \qquad v_c = -v_b + v_{c/b} \qquad [1]$$

Conservation of Linear Momentum : If we consider the car and the barge as a system, then the impulsive force caused by the traction of the tires is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$0 = m_c v_c + m_b v_b$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 0 = \left(\frac{3000}{32.2}\right) v_c + \left(\frac{30\,000}{32.2}\right) v_b \qquad [2]$$

Substituting Eq. [1] into [2] yields

$$11 v_b - v_{c/b} = 0 \qquad [3]$$

Integrating Eq. [3] becomes

$$(\overset{+}{\rightarrow}) \qquad 11 s_b - s_{c/b} = 0 \qquad [4]$$

Here, $s_{c/b} = 200$ ft. Then, from Eq. [4]

$$11 s_b - 200 = 0 \qquad s_b = 18.2 \text{ ft} \qquad \textbf{Ans}$$

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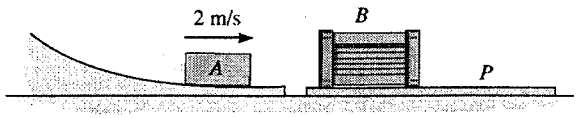
***15-40.** The block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.5$.



Equations of Equilibrium: From FBD(a),

$+\uparrow \sum F_y = 0; \quad N_B - (3+2)(9.81) = 0 \quad N_B = 49.05 \text{ N}$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b), and assuming the plate does not move.

$+\uparrow \sum F_y = 0; \quad N_P - 49.05 - 3(9.81) = 0 \quad N_P = 78.48 \text{ N}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 9.81 - (F_f)_P = 0 \quad (F_f)_P = 9.81 \text{ N}$

Since $(F_f)_P < [(F_f)_P]_{\max} = \mu'_s N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$s_P = 0$ **Ans**

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.

$$m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$(\overset{+}{\rightarrow}) \quad 2(2) + 0 = (2+3)v_2$

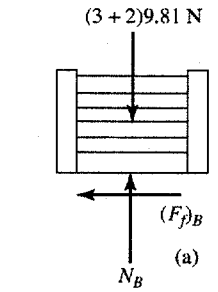
$$v_2 = 0.800 \text{ m/s} \rightarrow$$

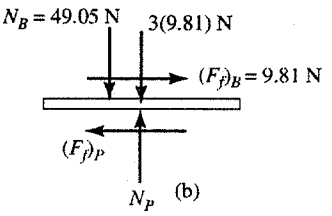
Principle of Linear Impulse and Momentum: Applying Eq. 15-4, we have

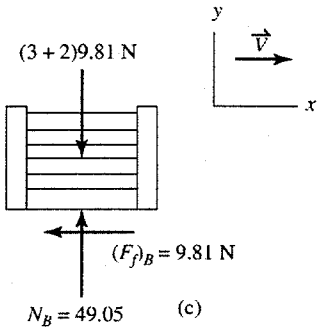
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$(\overset{+}{\rightarrow}) \quad 5(0.8) + [-9.81(t)] = 5(0)$

$t = 0.408$ s **Ans**







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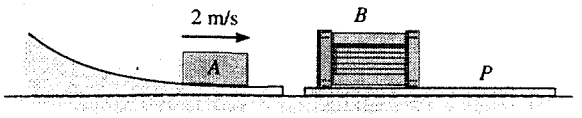
15-41. The block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.12$.



Equations of Equilibrium: From FBD(a),

$+\uparrow \sum F_y = 0; \quad N_B - (3+2)(9.81) = 0 \quad N_B = 49.05 \text{ N}$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b), and assuming the plate does not move,

$+\uparrow \sum F_y = 0; \quad N_P - 49.05 - 3(9.81) = 0 \quad N_P = 78.48 \text{ N}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 9.81 - (F_f)_P = 0 \quad (F_f)_P = 9.81 \text{ N}$

Since $(F_f)_P > [(F_f)_P]_{\max} = \mu'_s N_P = 0.12(78.48) = 9.418$ N, plate P slides. Thus, $(F_f)_P = \mu'_k N_P = 0.1(78.48) = 7.848$ N.

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$(\overset{+}{\rightarrow}) \quad 2(2) + 0 = (2+3)v_2$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15-4 to box B [FBD(a)], we have

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$(\overset{+}{\rightarrow}) \quad 5(0.8) + [-9.81(t_1)] = 5v_3 \qquad [1]$

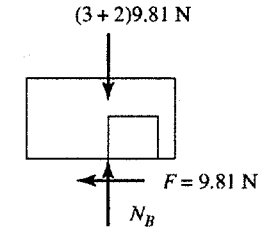
Applying Eq. 15-4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

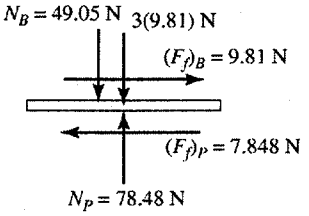
$(\overset{+}{\rightarrow}) \quad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \quad [2]$

Solving Eqs. [1] and [2] yields

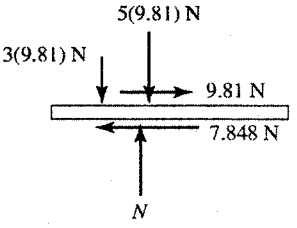
$t_1 = 0.3058$ s $\quad v_3 = 0.200$ m/s



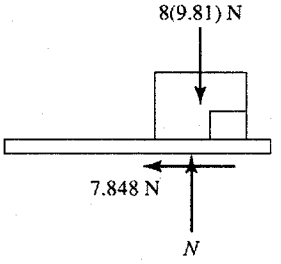
(a)



(b)



(c)



(d)

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15-41. (Cont.)

Equation of Motion: From FBD(c), the acceleration of plate P when box B still slides on top of it is given by

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 9.81 - 7.848 = 3(a_P)_1 \quad (a_P)_1 = 0.654 \text{ m/s}^2$$

When box B stops sliding on top of plate, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad -7.848 = 8(a_P)_2 \quad (a_P)_2 = -0.981 \text{ m/s}^2$$

Kinematics: Plate P travels a distance d_1 before box B stop sliding.

$$(\overset{+}{\rightarrow}) \quad d_1 = (v_0)_P t_1 + \frac{1}{2}(a_P)_1 t_1^2$$

$$= 0 + \frac{1}{2}(0.654)(0.3058^2) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop sliding is given by

$$(\overset{+}{\rightarrow}) \quad v_4 = v_3 + (a_P)_2 t_2$$

$$0 = 0.200 + (-0.981)t_2 \quad t_2 = 0.2039 \text{ s}$$

The distance d_2 traveled by plate P after box B stop sliding is given by

$$(\overset{+}{\rightarrow}) \quad v_4^2 = v_3^2 + 2(a_P)_2 d_2$$

$$0 = 0.200^2 + 2(-0.981)d_2 \quad d_2 = 0.02039 \text{ m}$$

The total distance traveled by plate P is

$$d_P = d_1 + d_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 0.0510 \text{ m} \quad \textbf{Ans}$$

The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s} \quad \textbf{Ans}$$

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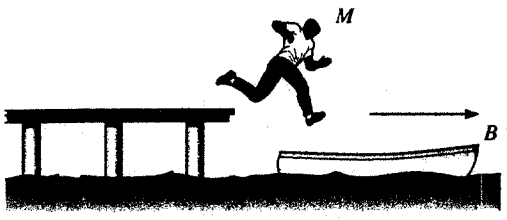
***15-42.** The man M weighs 150 lb and jumps onto the boat B which has a weight of 200 lb. If he has a horizontal component of velocity *relative to the boat* of 3 ft/s, just before he enters the boat, and the boat is traveling $v_B = 2$ ft/s away from the pier when he makes the jump, determine the resulting velocity of the man and boat.



$(\overset{+}{\rightarrow})$ $v_M = v_B + v_{M/B}$

$v_M = 2 + 3$

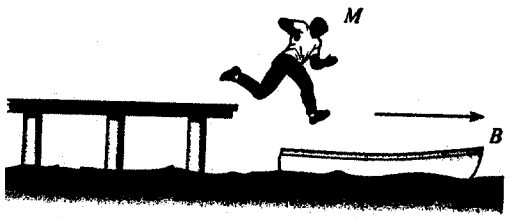
$v_M = 5$ ft/s

$(\overset{+}{\rightarrow})$ $\Sigma m\, v_1 = \Sigma m\, v_2$

$$\frac{150}{32.2}(5) + \frac{200}{32.2}(2) = \frac{350}{32.2}(v_B)_2$$

$(v_B)_2 = 3.29$ ft/s Ans

***15-43.** The man M weighs 150 lb and jumps onto the boat B which is originally at rest. If he has a horizontal component of velocity of 3 ft/s just before he enters the boat, determine the weight of the boat if it has a velocity of 2 ft/s once the man enters it.



$(\overset{+}{\rightarrow})$ $v_M = v_B + v_{M/B}$

$v_M = 0 + 3$

$v_M = 3$ ft/s

$(\overset{+}{\rightarrow})$ $\Sigma m(v_1) = \Sigma m(v_2)$

$$\frac{150}{32.2}(3) + \frac{W_B}{32.2}(0) = \frac{(W_B + 150)}{32.2}(2)$$

$W_B = 75$ lb Ans

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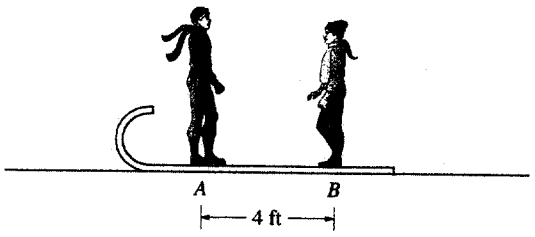
***15-44.** A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If A walks to B and stops, and both walk back together to the original position of A, determine the final position of the toboggan just after the motion stops. Neglect friction.



A goes to B,

$$(\overset{+}{\rightarrow}) \quad \Sigma m v_1 = \Sigma m v_2$$

$$0 = m_A v_A - (m_t + m_B) v_B$$

$$0 = m_A s_A - (m_t + m_B) s_B$$

Assume B moves x to the left, then A moves $(4-x)$ to the right

$$0 = m_A(4-x) - (m_t + m_B)x$$

$$x = \frac{4m_A}{m_A + m_B + m_t}$$

$$= \frac{4(80)}{80+65+20} = 1.939 \text{ ft} \leftarrow$$

A and B go to other end.

$$(\overset{+}{\rightarrow}) \quad \Sigma m v_1 = \Sigma m v_2$$

$$0 = -m_B v - m_A v + m_t v_t$$

$$0 = -m_B s - m_A s + m_t s_t$$

Assume the toboggan moves x' to the right, then A and B move $(4-x')$ to the left

$$0 = -m_B(4-x') - m_A(4-x') + m_t x'$$

$$x' = \frac{4(m_B + m_A)}{m_A + m_B + m_t}$$

$$= \frac{4(65+80)}{80+65+20} = 3.515 \text{ ft} \rightarrow$$

Net movement of sled is

$$(\overset{+}{\rightarrow}) \quad x = 3.515 - 1.939 = 1.58 \text{ ft} \rightarrow$$ Ans

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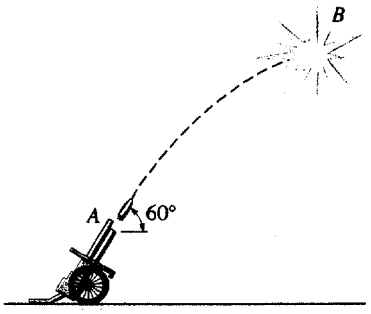
15-45. The 10-lb projectile is fired from ground level with an initial velocity of $v_A = 80$ ft/s in the direction shown. When it reaches its highest point B it explodes into two 5-lb fragments. If one fragment travels vertically upward at 12 ft/s, determine the distance between the fragments after they strike the ground. Neglect the size of the gun.



$(\overset{+}{\rightarrow})$ $v_x = (v_0)_x$

$v_B = 80\cos 60° = 40$ ft/s

$(+\uparrow)$ $v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$

$0 = (80\sin 60°)^2 + 2(-32.2)(h - 0)$

$h = 74.53$ ft

$(\overset{+}{\rightarrow})$ $\Sigma mv_1 = \Sigma mv_2$

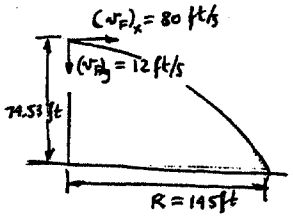
$\frac{10}{32.2}(80\cos 60°) = \frac{5}{32.2}(v_F)_x$

$(v_F)_x = 80$ ft/s $\rightarrow$

$(+\uparrow)$ $\Sigma mv_1 = \Sigma mv_2$

$0 = \frac{5}{32.2}(v_F)_y + \frac{5}{32.2}(12)$

$(v_F)_y = -12$ ft/s $= 12$ ft/s $\downarrow$



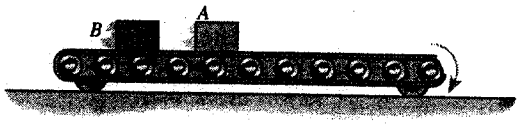
$(+\downarrow)$ $s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_c t^2$

$74.53 = 0 + 12t + \frac{1}{2}(32.2)t^2$

$t = 1.81$ s

$(\overset{+}{\rightarrow})$ $R = 80(1.81) = 145$ ft **Ans**

15-46. Two boxes A and B, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



a) Let v_b be the velocity of A and B.

$(\overset{+}{\rightarrow})$ $\Sigma mv_1 = \Sigma mv_2$

$$0 = \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)$$

$(\overset{+}{\rightarrow})$ $v_b = v_c + v_{b/c}$

$v_b = -v_c + 3$

Thus, $v_b = 1.83$ ft/s $\rightarrow$ $v_c = 1.17$ ft/s $\leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

a) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

b) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

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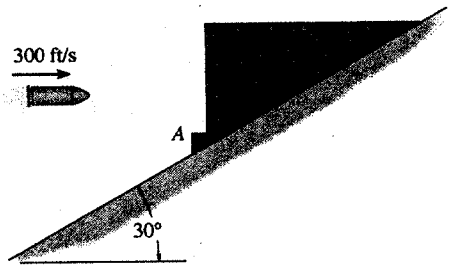
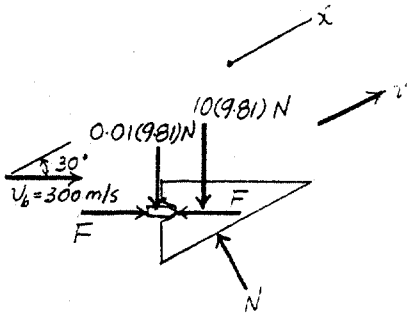
15-47. The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



Conservation of Linear Momentum : If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.

$$(+) \qquad m_b(v_b)_{x'} = (m_b + m_B)v_x$$
$$0.01(300\cos 30°) = (0.01 + 10)v$$
$$v = 0.2595 \text{ m/s}$$

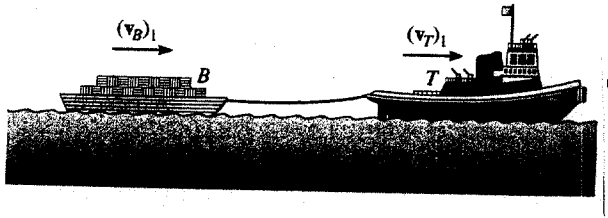


Conservation of Energy : The datum is set at the block's initial position. When the block and the embedded bullet is at their highest point, they are h *above* the datum. Their gravitational potential energy is $(10+0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + \frac{1}{2}(10+0.01)\left(0.2595^2\right) = 0 + 98.1981h$$
$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm} \qquad \textbf{Ans}$$

***15-48.** A tugboat T having a mass of 19 Mg is tied to a barge B having a mass of 75 Mg. If the rope is "elastic" such that it has a stiffness $k = 600$ kN/m, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds $(v_T)_1 = 15$ km/h and $(v_B)_1 = 10$ km/h, respectively. Neglect the resistance of the water.

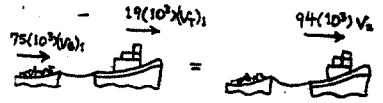


$(v_T)_1 = 15 \text{ km/h} = 4.167 \text{ m/s}$

$(v_B)_1 = 10 \text{ km/h} = 2.778 \text{ m/s}$

When the rope is stretched to its maximum, both the tug and barge have a common velocity. Hence,

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$
$$19\,000(4.167) + 75\,000(2.778) = (19\,000 + 75\,000)v_2$$
$$v_2 = 3.059 \text{ m/s}$$



$$T_1 + V_1 = T_2 + V_2$$
$$T_1 = \frac{1}{2}(19\,000)(4.167)^2 + \frac{1}{2}(75\,000)(2.778)^2 = 454.282 \text{ kJ}$$
$$T_2 = \frac{1}{2}(19\,000 + 75\,000)(3.059)^2 = 439.661 \text{ kJ}$$

Hence,

$$454.282(10^3) + 0 = 439.661(10^3) + \frac{1}{2}(600)(10^3)x^2$$
$$x = 0.221 \text{ m} \qquad \textbf{Ans}$$

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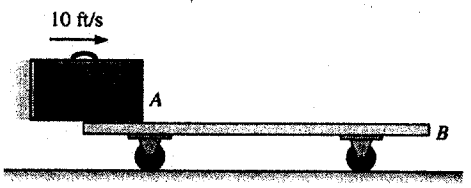
15-49. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally on it at 10 ft/s, determine the length of time that A slides relative to B, and the final velocity of A and B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.

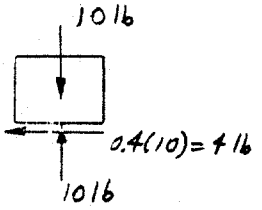


System :

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10+20}{32.2}\right)v$$

$v = 3.33$ ft/s **Ans**



For A :

$m v_1 + \Sigma \int F\, dt = m v_2$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$t = 0.5176 = 0.518$ s **Ans**

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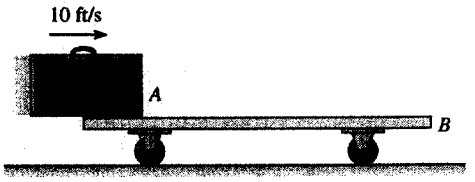
15-50. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally on it at 10 ft/s, determine the time t and the distance B moves before A stops relative to B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.



System :

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10+20}{32.2}\right)v$$

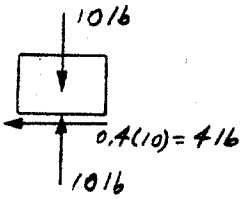
$$v = 3.33 \text{ ft/s}$$

For A :

$$mv_1 + \Sigma \int F\,dt = mv_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$t = 0.5176 = 0.518$ s **Ans**



For B :

$\left(\overset{+}{\rightarrow}\right)$ $v = v_0 + a_c t$

$$3.33 = 0 + a_c\,(0.5176)$$

$$a_c = 6.440 \text{ ft/s}^2$$

$\left(\overset{+}{\rightarrow}\right)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(6.440)(0.5176)^2 = 0.863 \text{ ft}$$ **Ans**

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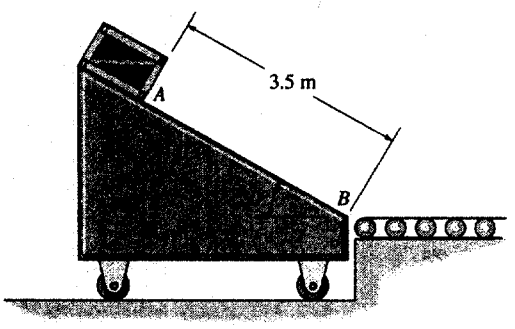
15-51. The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



Conservation of Energy : The datum is set at lowest point B. When the crate is at point A, it is $3.5\sin 30° = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675\ \text{N}\cdot\text{m}$. Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)\,v_C^2 + \frac{1}{2}(40)\,v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad [1]$$

Relative Velocity : The velocity of the crate is given by

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_R + \mathbf{v}_{C/R}\\ &= -v_R\mathbf{i} + (v_{C/R}\cos 30°\mathbf{i} - v_{C/R}\sin 30°\mathbf{j})\\ &= (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j}\end{aligned} \qquad [2]$$

The magnitude of $\mathbf{v}_C$ is

$$\begin{aligned}v_C &= \sqrt{(0.8660v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2}\\ &= \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_R\,v_{C/R}}\end{aligned} \qquad [3]$$

Conservation of Linear Momentum : If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other . As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660v_{C/B} - 50v_B \qquad [4]$$

Solving Eqs. [1], [3] and [4] yields

$$v_R = 1.101\ \text{m/s} = 1.10\ \text{m/s} \qquad v_C = 5.43\ \text{m/s} \qquad \textbf{Ans}$$

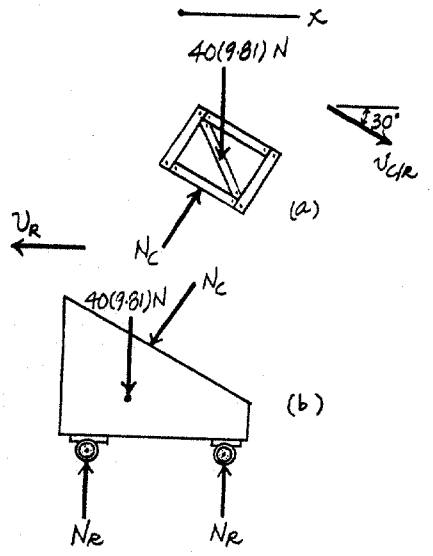
$$v_{C/R} = 6.356\ \text{m/s}$$

From Eq. [1]

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\}\ \text{m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8° \qquad \textbf{Ans}$$



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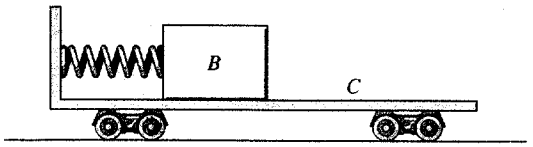
***15-52.** The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$T_1 + V_1 = T_2 + V_2$

$[0+0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$

$12 = 50\,v_b^2 + 75\,v_c^2$

$(\stackrel{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

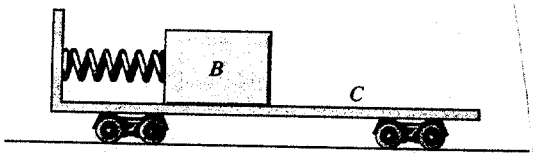
$0 + 0 = 50\,v_b - 75\,v_c$

$v_b = 1.5 v_c$

$v_c = 0.253$ m/s $\leftarrow$

$v_b = 0.379$ m/s $\rightarrow$ **Ans**

15-53. The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the cart after the spring becomes undeformed. Neglect the mass of the wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$T_1 + V_1 = T_2 + V_2$

$[0+0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$

$12 = 50\,v_b^2 + 75\,v_c^2$

$(\stackrel{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$0 + 0 = 50\,v_b - 75\,v_c$

$v_b = 1.5 v_c$

$v_c = 0.253$ m/s $\leftarrow$

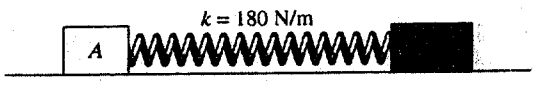
$v_b = 0.379$ m/s $\rightarrow$

$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$

$(\stackrel{+}{\rightarrow})$ $0.379 = -0.253 + v_{b/c}$

$v_{b/c} = 0.632$ m/s $\rightarrow$ **Ans**

15-54. Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



$(\stackrel{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$0 + 0 = 40\,v_A - 60\,v_B$

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$

$v_A = 3.29$ m/s **Ans**

$v_B = 2.19$ m/s **Ans**

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15-55. An ivory ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

Before impact

$T_1 + V_1 = T_2 + V_2$

$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)v_1^2 + 0$

$v_1 = 2.801$ m/s

After the impact

$\frac{1}{2}(0.2)v_2^2 = 0 + 0.2(9.81)(0.325)$

$v_2 = 2.525$ m/s

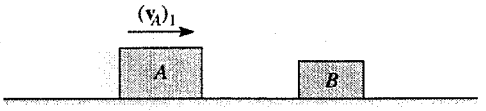
Coefficient of restitution:

$(+\downarrow) \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$= \frac{0 - (-2.525)}{2.801 - 0}$

$= 0.901$ **Ans**

***15-56.** Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(v_A)_1 = 2$ m/s when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic ($e = 1$), determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.



$(\overset{+}{\rightarrow}) \quad \sum mv_1 = \sum mv_2$

$3(2) + 0 = 3(v_A)_2 + 2(v_B)_2$

$(\overset{+}{\rightarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}$

Solving

$(v_A)_2 = 0.400$ m/s **Ans**

$(v_B)_2 = 2.40$ m/s **Ans**

Block A:

$T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}(3)(0.400)^2 - 3(9.81)(0.3)d_A = 0$

$d_A = 0.0272$ m

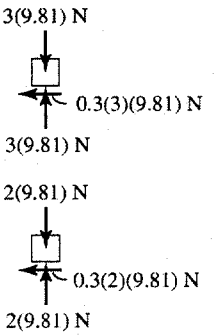
Block B:

$T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}(2)(2.40)^2 - 2(9.81)(0.3)d_B = 0$

$d_B = 0.9786$ m

$d = d_B - d_A = 0.951$ m **Ans**



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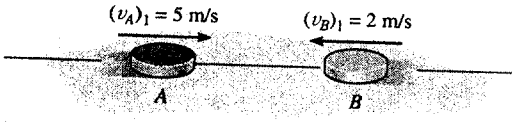
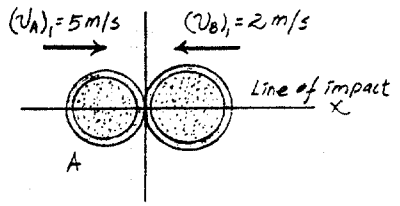
15-57. Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.



Conservation of Momentum :

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad [1]$$



Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$(v_A)_2 = -1.53$ m/s $= 1.53$ m/s $\leftarrow$ $\qquad (v_B)_2 = 1.27$ m/s $\rightarrow$ **Ans**

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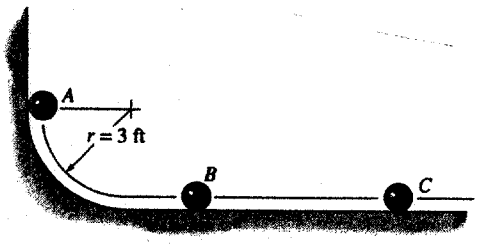
15-58. The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. Neglect the size of each ball.



Ball A :
Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$$0 + (0.5)(3) = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v_A)_1^2 + 0$$

$(v_A)_1 = 13.90$ ft/s

Balls A and B :

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m v_1 = \Sigma m v_2$

$$\left(\frac{0.5}{32.2}\right)(13.90) + 0 = \left(\frac{0.5}{32.2}\right)(v_A)_2 + \left(\frac{0.5}{32.2}\right)(v_B)_2$$

$\left(\overset{+}{\rightarrow}\right)$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}$$

Solving :

$(v_A)_2 = 1.04$ ft/s **Ans**

$(v_B)_2 = 12.86$ ft/s

Balls B and C :

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m v_2 = \Sigma m v_3$

$$\left(\frac{0.5}{32.2}\right)(12.86) + 0 = \left(\frac{0.5}{32.2}\right)(v_B)_3 + \left(\frac{0.5}{32.2}\right)(v_C)_3$$

$\left(\overset{+}{\rightarrow}\right)$ $e = \dfrac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}$

$$0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}$$

Solving :

$(v_B)_3 = 0.964$ ft/s **Ans**

$(v_C)_3 = 11.9$ ft/s **Ans**

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15-59. If two disks A and B have the same mass and are subjected to direct central impact such that the collision is perfectly elastic ($e = 1$), prove that the kinetic energy before collision equals the kinetic energy after collision. The surface upon which they slide is smooth.

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m v_1 = \Sigma m v_2$

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$m_A\left[(v_A)_1 - (v_A)_2\right] = m_B\left[(v_B)_2 - (v_B)_1\right] \quad (1)$$

$\left(\overset{+}{\rightarrow}\right)$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = 1$$

$$(v_B)_2 - (v_A)_2 = (v_A)_1 - (v_B)_1 \quad (2)$$

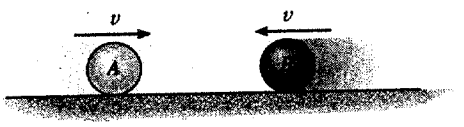
Combining Eqs. (1) and (2):

$$m_A\left[(v_A)_1 - (v_A)_2\right]\left[(v_A)_1 + (v_A)_2\right] = m_B\left[(v_B)_2 - (v_B)_1\right]\left[(v_B)_2 + (v_B)_1\right]$$

Expand and multiply by $\frac{1}{2}$:

$$\frac{1}{2}m_A (v_A)_1^2 + \frac{1}{2}m_B (v_B)_1^2 = \frac{1}{2}m_A (v_A)_2^2 + \frac{1}{2}m_B (v_B)_2^2 \quad \textbf{Q.E.D.}$$

***15-60.** Each ball has a mass m and the coefficient of restitution between the balls is e. If they are moving towards one another with a velocity v, determine their speeds after collision. Also, determine their common velocity when they reach the state of maximum deformation. Neglect the size of each ball.



$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m v_1 = \Sigma m v_2$

$$mv - mv = mv_A + mv_B$$

$$v_A = -v_B$$

$\left(\overset{+}{\rightarrow}\right)$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{v_B - v_A}{v - (-v)}$$

$$2ve = 2v_B$$

$v_B = ve \rightarrow$ **Ans**

$v_A = -ve = ve \leftarrow$ **Ans**

At maximum deformation $v_A = v_B = v'$.

$\left(\overset{+}{\rightarrow}\right)$ $\Sigma m v_1 = \Sigma m v_2$

$$mv - mv = (2m)v'$$

$v' = 0$ **Ans**

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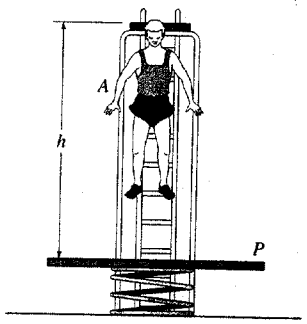
15-61. The man A has a weight of 175 lb and jumps from rest $h = 8$ ft onto a platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion.



$$T_0 + V_0 = T_1 + V_1$$

$$0 + 175(8) = \frac{1}{2}\left(\frac{175}{32.2}\right)(v_{m1})^2 + 0$$

$$v_{m1} = 22.698 \text{ ft/s}$$

$$(+\downarrow) \quad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{175}{32.2}(22.698) + 0 = \frac{175}{32.2}(v_{m2}) + \frac{60}{32.2}(v_{p2})$$

$$(+\downarrow) \quad e = \frac{v_{p2} - v_{m2}}{v_{m1} - v_{p1}}$$

$$0.6 = \frac{v_{p2} - v_{m2}}{22.698 - 0}$$

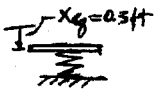
Solving.

$$v_{p2} = 27.04 \text{ ft/s} = 27.0 \text{ ft/s} \qquad \textbf{Ans}$$

$$v_{m2} = 13.4 \text{ ft/s} \qquad \textbf{Ans}$$

$$60 = 200(x_{eq})$$

$$x_{eq} = 0.3 \text{ ft}$$



$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)(27.04)^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(x + 0.3)^2 - 60(x)$$

$$100x^2 - 681.206 = 0$$

$$x = 2.61 \text{ ft} \qquad \textbf{Ans}$$

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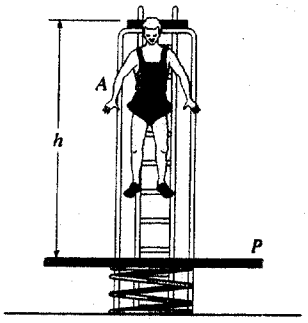
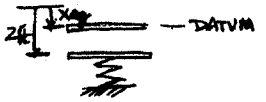
15-62. The man A has a weight of 100 lb and jumps from rest onto the platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. If the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring becomes 2 ft.



For the platform after collision:



$60 = 200(x_{eq})$

$x_{eq} = 0.3$ ft

$T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)(v_{p2})^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(2)^2 - 60(2-0.3)$$

$v_{p2} = 17.612$ ft/s

$$(+\downarrow) \quad e = \frac{v_{p2} - v_{m2}}{v_{m1} - v_{p1}}$$

$$0.6 = \frac{17.612 - v_{m2}}{v_{m1} - 0}$$

$$(+\downarrow) \quad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{100}{32.2} v_{m1} + 0 = \frac{100}{32.2}(v_{m2}) + \frac{60}{32.2}(17.612)$$

Solving,

$v_{m1} = 17.612$ m/s

$v_{m2} = 7.045$ m/s

For the man just before striking the platform

$T_0 + V_0 = T_1 + V_1$

$$0 + 100\,h = \frac{1}{2}\left(\frac{100}{32.2}\right)(17.612)^2 + 0$$

$h = 4.82$ ft **Ans**

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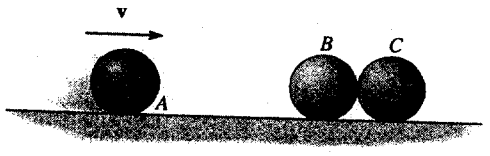
15-63. The three balls each have the same mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each ball is e. Neglect the size of each ball.



Conservation of Momentum : When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad [1]$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = \frac{v(1-e)}{2} \qquad (v_B)_2 = \frac{v(1+e)}{2}$$

Conservation of Momentum : When ball B strikes ball C, we have

$$m_B(v_B)_2 + m_C(v_C)_1 = m_B(v_B)_3 + m_C(v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1+e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad [3]$$

Coefficient of Restitution :

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1+e)}{2} - 0} \qquad [4]$$

Solving Eqs. [3] and [4] yields

$$(v_C)_2 = \frac{v(1+e)^2}{4} \qquad \textbf{Ans}$$

$$(v_B)_3 = \frac{v(1-e^2)}{4}$$

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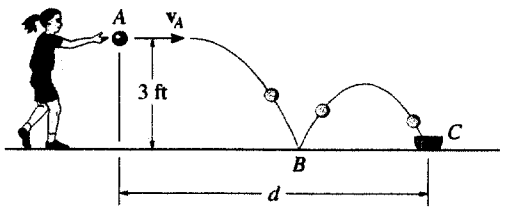
***15-64.** If the girl throws the ball with a horizontal velocity of 8 ft/s, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C. Take $e = 0.8$.



$$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(v_1)_y^2 = 0 + 2(32.2)(3)$$

$$(v_1)_y = 13.90 \downarrow$$

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$3 = 0 + 0 + \frac{1}{2}(32.2)(t_{AB})^2$$

$$t_{AB} = 0.43167 \text{ s}$$

$$(+\downarrow) \quad e = \frac{(v_2)_y}{(v_1)_y}$$

$$0.8 = \frac{(v_2)_y}{13.90}$$

$$(v_2)_y = 11.1197 \uparrow$$

$$(+\downarrow) \quad v = v_0 + a_c t$$

$$11.1197 = -11.1197 + 32.2(t_{BC})$$

$$t_{BC} = 0.6907 \text{ s}$$

Total time is $t_{AC} = 1.1224$ s

Since the x component of momentum is conserved

$$d = v_A(t_{AC})$$

$$d = 8(1.1224)$$

$$d = 8.98 \text{ ft} \qquad \text{Ans}$$

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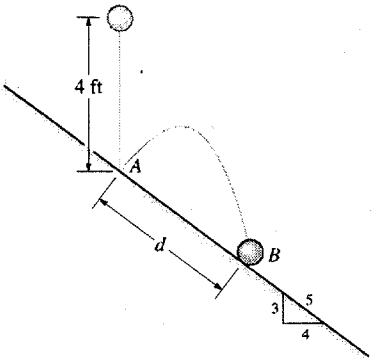
15-65. The ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If $e = 0.8$, determine the distance d to where it again strikes the plane at B.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$

$\searrow +$ $\quad (v_A)_{2x'} = \frac{3}{5}(16.05) = 9.63$ ft/s

$\nearrow +$ $\quad (v_A)_{2y'} = 0.8\left(\frac{4}{5}\right)(16.05) = 10.27$ ft/s

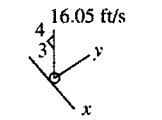
$$(v_A)_2 = \sqrt{(9.63)^2 + (10.27)^2} = 14.08 \text{ ft/s}$$

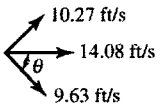
$$\theta = \tan^{-1}\left(\frac{10.27}{9.63}\right) = 46.85°$$

$$\phi = 46.85° - \tan^{-1}\left(\frac{3}{4}\right) = 9.977°$$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$d\left(\frac{4}{5}\right) = 0 + 14.08\cos 9.977°(t)$$





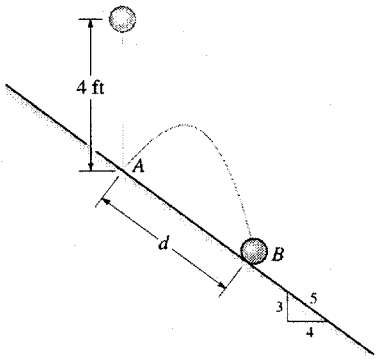
$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\left(\frac{3}{5}\right) = 0 - 14.08\sin 9.977°(t) + \frac{1}{2}(32.2)t^2$$

$$t = 0.798 \text{ s}$$

$$d = 13.8 \text{ ft} \qquad \textbf{Ans}$$

15-66. The ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If it rebounds and in $t = 0.5$ s again strikes the plane at B, determine the coefficient of restitution e between the ball and the plane. Also, what is the distance d?



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$

$+\searrow$ $\quad (v_A)_{2x'} = \frac{3}{5}(16.05) = 9.63$ ft/s

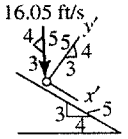
$\nearrow +$ $\quad (v_A)_{2y'} = e\left(\frac{4}{5}\right)(16.05) = 12.84e$ ft/s

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$\frac{4}{5}(d) = 0 + v_{A2x}(0.5)$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$\frac{3}{5}(d) = 0 - v_{A2y}(0.5) + \frac{1}{2}(32.2)(0.5)^2$$



$(\overset{+}{\rightarrow})$ $\quad 0.5\left[9.63\left(\frac{4}{5}\right) + 12.84e\left(\frac{3}{5}\right)\right] = \frac{4}{5}d$

$(+\uparrow)$ $\quad 0.5\left[-9.63\left(\frac{3}{5}\right) + 12.84e\left(\frac{4}{5}\right)\right] = 4.025 - \frac{3}{5}d$

Solving,

$$e = 0.502 \qquad \textbf{Ans}$$

$$d = 7.23 \text{ ft} \qquad \textbf{Ans}$$

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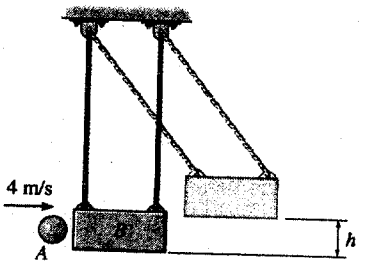
15-67. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



System :

$\left(\stackrel{+}{\rightarrow}\right)$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$

$(v_A)_2 + 10(v_B)_2 = 4$

$\left(\stackrel{+}{\rightarrow}\right)$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$0.8 = \dfrac{(v_B)_2 - (v_A)_2}{4 - 0}$

$(v_B)_2 - (v_A)_2 = 3.2$

Solving :

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

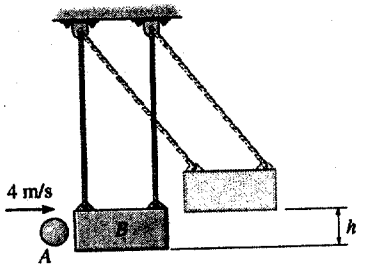
Block :
Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$\dfrac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$

$h = 0.0218$ m $= 21.8$ mm **Ans**

***15-68.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



System :

$\left(\stackrel{+}{\rightarrow}\right)$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$

$(v_A)_2 + 10(v_B)_2 = 4$

$\left(\stackrel{+}{\rightarrow}\right)$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$0.8 = \dfrac{(v_B)_2 - (v_A)_2}{4 - 0}$

$(v_B)_2 - (v_A)_2 = 3.2$

Solving :

$(v_A)_2 = -2.545$ m/s

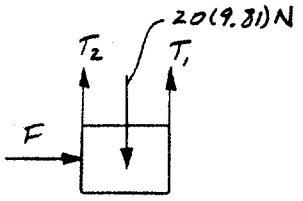
$(v_B)_2 = 0.6545$ m/s

Block :

$\left(\stackrel{+}{\rightarrow}\right)$ $m v_1 + \Sigma \int F\, dt = m v_2$

$0 + F(0.005) = 20(0.6545)$

$F = 2618$ N $= 2.62$ kN **Ans**



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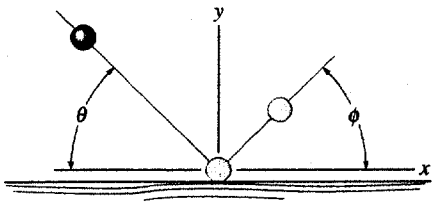
15-69. A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



$$(+\downarrow) \qquad e = \frac{0-[-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$$

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad [2]$$

$$(+\downarrow) \qquad m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

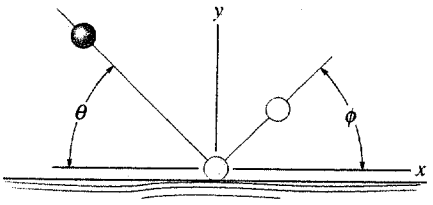
$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

Substituting Eq.[4] into [1] yields :

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right) \qquad \text{Ans}$$

15-70. A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.

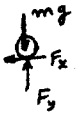


$$(+\downarrow) \qquad e = \frac{0-[-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$$

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad [2]$$



$$(+\downarrow) \qquad m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

Substituting Eq.[4] into [1] yields :

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right)$$

$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25 \qquad \text{Ans}$$

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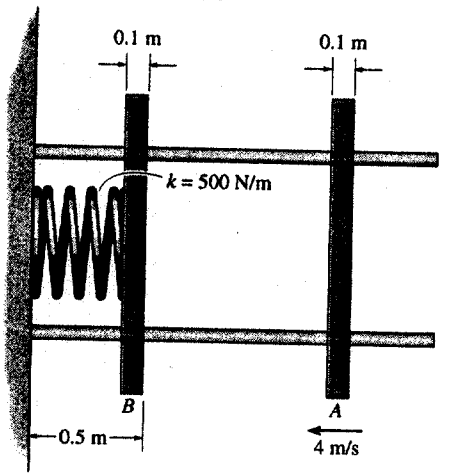
15-71. Plates A and B each have a mass of 4 kg and are restricted to move along the smooth guides. If the coefficient of restitution between the plates is $e = 0.7$, determine (a) the speed of both plates just after collision and (b) the maximum compression of the spring. Plate A has a velocity of 4 m/s just before striking B. Plate B is originally at rest and the spring is unstretched.



Conservation of Momentum :

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\leftarrow}) \qquad 4(4) + 0 = 4(v_A)_2 + 4(v_B)_2 \qquad [1]$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.7 = \frac{(v_B)_2 - (v_A)_2}{4 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 0.600 \text{ m/s} \leftarrow \qquad (v_B)_2 = 3.40 \text{ m/s} \leftarrow \qquad \textbf{Ans}$$

Conservation of Energy : When plate B stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(500)s^2 = 250s^2$. Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(4)\left(3.40^2\right) + 0 = 0 + 250s^2$$

$$s = 0.3041 \text{ m} = 304 \text{ mm} \qquad \textbf{Ans}$$

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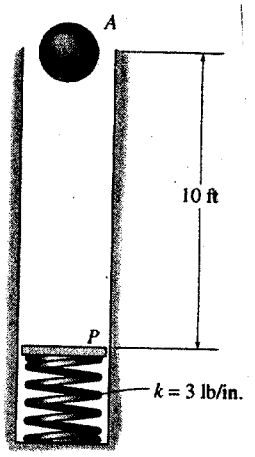
***15-72.** The 8-lb ball is released from rest 10 ft from the surface of a flat plate P which weighs 6 lb. Determine the maximum compression in the spring if the impact is perfectly elastic.



Datum at lowest point :

$T_1 + V_1 = T_2 + V_2$

$$0 + (8)(10) = \frac{1}{2}\left(\frac{8}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 25.377 \text{ ft/s}$$

$$(+\downarrow) \quad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{8}{32.2}(25.377) + 0 = \frac{8}{32.2}(v_A)_2 + \frac{6}{32.2}(v_P)_2$$

$$e = \frac{(v_P)_2 - (v_A)_2}{(v_A)_1 - (v_P)_1}$$

$$1 = \frac{(v_P)_2 - (v_A)_2}{25.377 - 0}$$

Solving,

$$(v_A)_2 = 3.625 \text{ ft/s}$$

$$(v_P)_2 = 29.00 \text{ ft/s}$$

Initially the plate is compressed

$$F_s = kx; \quad 6 = (3)x, \quad x = 2 \text{ in.} = \frac{1}{6} \text{ ft}$$

Datum at final plate position :

$T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}\left(\frac{6}{32.2}\right)(29.00)^2 + \frac{1}{2}(3)(12)\left(\frac{1}{6}\right)^2 + 6x = 0 + \frac{1}{2}(3)(12)\left(x + \frac{1}{6}\right)^2$$

$$18x^2 - 78.367 = 0$$

$$x = 2.087 \text{ ft}$$

Maximum spring compression is

$$x_{max} = 2.087 + \frac{1}{6} = 2.25 \text{ ft} \qquad \text{Ans}$$

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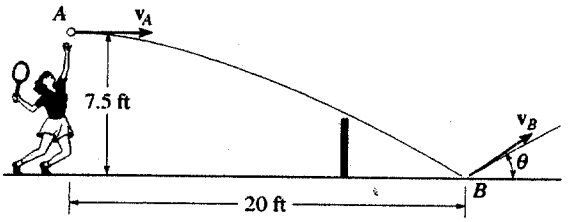
15-73. It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and θ) of the ball just after it strikes the court at B. Take $e = 0.7$.



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$20 = 0 + v_A t$

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$7.5 = 0 + 0 + \frac{1}{2}(32.2) t^2$

$t = 0.682524$

$v_A = 29.303 = 29.3$ ft/s **Ans**

$v_{Bx1} = 29.303$ ft/s

$(+\downarrow)$ $v = v_0 + a_c t$

$v_{By1} = 0 + 32.2(0.68252) = 21.977$ ft/s

$(\overset{+}{\rightarrow})$ $mv_1 = mv_2$

$v_{B2x} = v_{B1x} = 29.303$ ft/s $\rightarrow$

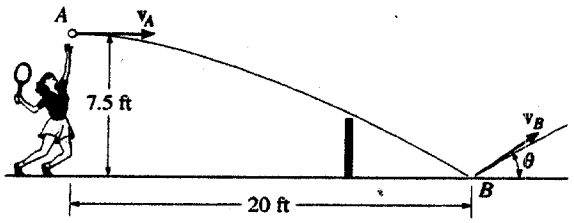
$e = \frac{v_{By2}}{v_{By1}}$

$0.7 = \frac{v_{By2}}{21.977}$, $v_{By2} = 15.384$ ft/s $\uparrow$

$v_{B2} = \sqrt{(29.303)^2 + (15.384)^2} = 33.1$ ft/s **Ans**

$\theta = \tan^{-1}\frac{15.384}{29.303} = 27.7°$ $\measuredangle$ **Ans**

15-74. The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at B, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.



$(+\downarrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_{By})_1^2 = 0 + 2(32.2)(7.5 - 0)$

$v_{By1} = 21.9773$ m/s

$(+\downarrow)$ $v = v_0 + a_c t$

$21.9773 = 0 + 32.2t$

$t = 0.68252$ s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$20 = 0 + v_A(0.68252)$

$v_A = 29.303 = 29.3$ ft/s **Ans**

$(\overset{+}{\rightarrow})$ $mv_1 = mv_2$

$v_{Bx2} = v_{Bx1} = v_A = 29.303$

$v_{B_2} = 29.303/\cos 30° = 33.8$ ft/s **Ans**

$v_{By2} = 29.303 \tan 30° = 16.918$ ft/s

$e = \frac{v_{By2}}{v_{By1}} = \frac{16.918}{21.9773} = 0.770$ **Ans**

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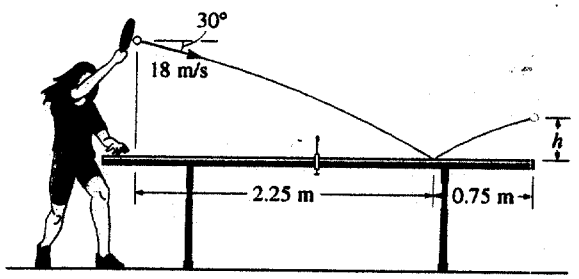
15-75. The ping-pong ball has a mass of 2 g. If it is struck with the velocity shown, determine how high h it rises above the end of the smooth table after the rebound. Take $e = 0.8$.



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$2.25 = 0 + 18\cos 30° t$

$t = 0.14434$ s

$(v_x)_1 = (v_x)_2 = 18\cos 30° = 15.5885$ m/s

$(+\downarrow)$ $v = v_0 + a_c t$

$(v_y)_1 = 18\sin 30° + 9.81(0.14434)$

$(v_y)_1 = 10.4160$ m/s

$(+\uparrow)$ $e = 0.8 = \dfrac{(v_y)_2}{10.4160}$

$(v_y)_2 = 8.3328$ m/s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$0.75 = 0 + 15.5885t$

$t = 0.048112$ s

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$h = 0 + 8.3328(0.048112) - \frac{1}{2}(9.81)(0.048112)^2$

$h = 0.390$ m **Ans**

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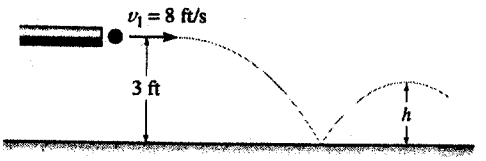
***15-76.** The ball is ejected from the tube with a horizontal velocity of $v_1 = 8$ ft/s as shown. If the coefficient of restitution between the ball and the ground is $e = 0.8$, determine (a) the velocity of the ball just after it rebounds from the ground and (b) the maximum height to which the ball rises after the first bounce.



$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_y)_1^2 = 0 + 2(-32.2)(-3 - 0)$

$(v_y)_1 = 13.90$ ft/s $\downarrow$

In y direction :

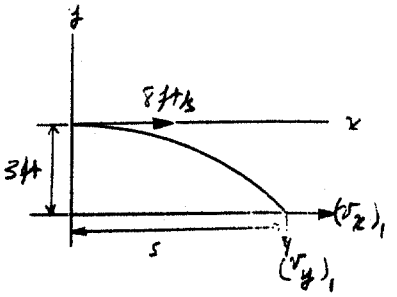
$(+\uparrow)$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$; $0.8 = \dfrac{(v_y)_2 - 0}{0 - (-13.90)}$

$(v_y)_2 = 11.12$ ft/s $\uparrow$

$(v_x)_2 = 8$ ft/s $\rightarrow$

$v_2 = \sqrt{(8)^2 + (11.12)^2} = 13.7$ ft/s **Ans**

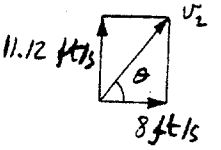
$\theta = \tan^{-1}\left(\dfrac{11.12}{8}\right) = 54.3°$ **Ans**



$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (11.12)^2 + 2(-32.2)(h - 0)$

$h = 1.92$ ft **Ans**



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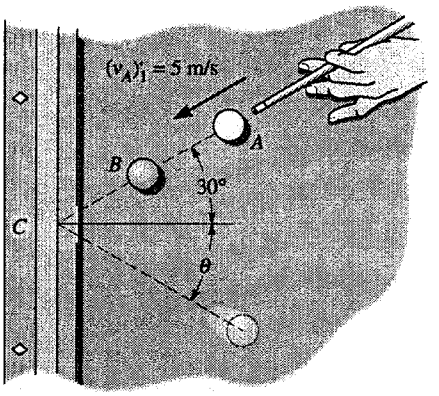
15-77. The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball $B(e = 0.8)$, determine the velocity of B and the angle θ just after it rebounds from the cushion at $C(e' = 0.6)$. Each ball has a mass of 0.4 kg. Neglect he size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(+) \quad 0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad [1]$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \quad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 0.500 \text{ m/s} \quad (v_B)_2 = 4.50 \text{ m/s}$$

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_x})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \quad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \quad \textbf{Ans} \qquad [3]$$

Coefficient of Restitution (x):

$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

$$(\overset{+}{\leftarrow}) \quad 0.6 = \frac{0 - [-(v_B)_3 \cos\theta]}{4.50 \cos 30° - 0} \qquad [4]$$

Solving Eqs. [1] and [2] yields

$$(v_B)_3 = 3.24 \text{ m/s} \quad \theta = 43.9° \quad \textbf{Ans}$$

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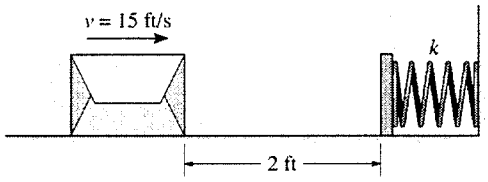
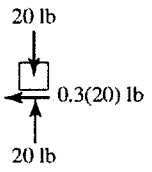
15-78. The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate.



$T_1 + \sum U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(15)^2 - (0.3)(20)(2) = \frac{1}{2}\left(\frac{20}{32.2}\right)(v_2)^2$$



$v_2 = 13.65$ ft/s

$(\overset{+}{\rightarrow})$ $\sum m v_1 = \sum m v_2$

$$\left(\frac{20}{32.2}\right)(13.65) = \left(\frac{20}{32.2}\right)v_A + \frac{10}{32.2}v_B$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{v_P - v_A}{13.65}$$

Solving,

$v_P = 16.38$ ft/s, $v_A = 5.46$ ft/s

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(16.38)^2 + 0 = 0 + \frac{1}{2}(400)(s)^2$$

$s = 0.456$ ft **Ans**

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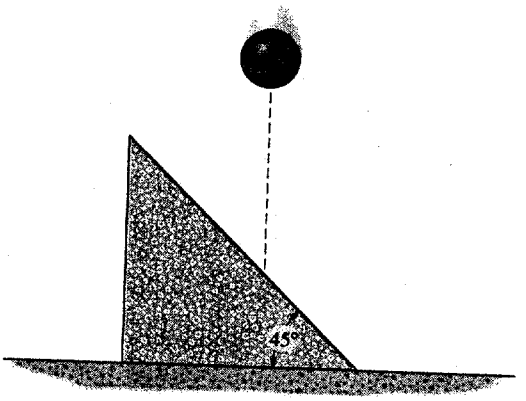
15-79. The sphere of mass m falls and strikes the triangular block with a vertical velocity v. If the block rests on a smooth surface and has a mass $3m$, determine its velocity just after the collision. The coefficient of restitution is e.



Conservation of "x'" Momentum :

$$m(v)_1 = m(v)_2$$

$$(+) \qquad m(v \sin 45°) = m(v_{sx'})_2$$

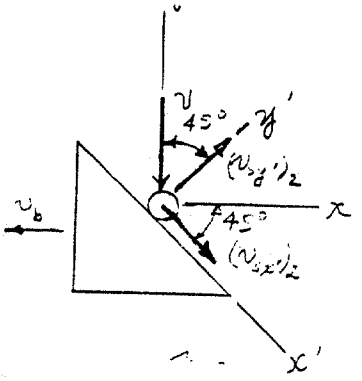
$$(v_{sx'})_2 = \frac{\sqrt{2}}{2} v$$

Coefficient of Restitution (y') :

$$e = \frac{(v_b)_2 - (v_{b_{y'}})_2}{(v_{s_{y'}})_1 - (v_b)_1}$$

$$(+) \qquad e = \frac{v_b \cos 45° - [-(v_{sy'})_2]}{v \cos 45° - 0}$$

$$(v_{sy'})_2 = \frac{\sqrt{2}}{2}(ev - v_b)$$



Conservation of "x" Momentum :

$$0 = m_s(v_s)_x + m_b v_b$$

$$(\overset{+}{\leftarrow}) \qquad 0 + 0 = 3m v_b - m(v_{sy'})_2 \cos 45° - m(v_{sx'})_2 \cos 45°$$

$$3v_b - \frac{\sqrt{2}}{2}(ev - v_b)\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} v \frac{\sqrt{2}}{2} = 0$$

$$v_b = \left(\frac{1+e}{7}\right) v \qquad \textbf{Ans}$$

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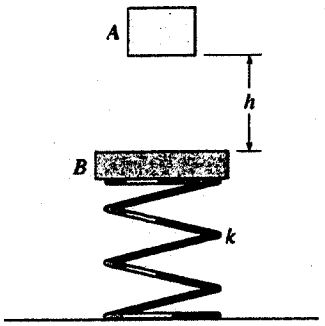
***15-80.** Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad (1)$$

$$(+\downarrow) \quad \Sigma mv_1 = \Sigma mv_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad (2)$$

Solving Eqs.(1) and (2) for $(v_B)_2$ yields;

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1+e) \qquad \text{Ans}$$

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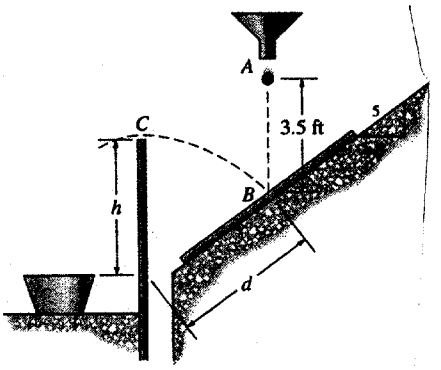
15-81. Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e \geq 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the plate at B and bounces over the barrier at C.



Conservation of Energy : The datum is set at point B. When the cranberry falls from a height of 3.5 ft *above* the datum, its initial gravitational potential energy is $W(3.5) = 3.5W$. Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3.5W = \frac{1}{2}\left(\frac{W}{32.2}\right)(v_c)_1^2 + 0$$

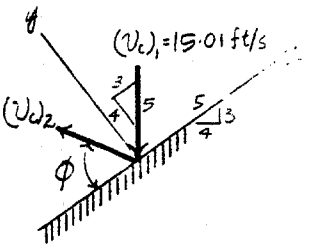
$$(v_c)_1 = 15.01 \text{ ft/s}$$

Conservation of "x'" Momentum : When the cranberry strikes the plate with a speed of $(v_c)_1 = 15.01$ ft/s, it rebounds with a speed of $(v_c)_2$.

$$m_c\left(v_{c_{x'}}\right)_1 = m_c\left(v_{c_{x'}}\right)_2$$

$$(+\ ) \qquad m_c(15.01)\left(\frac{3}{5}\right) = m_c\left[(v_c)_2 \cos\phi\right]$$

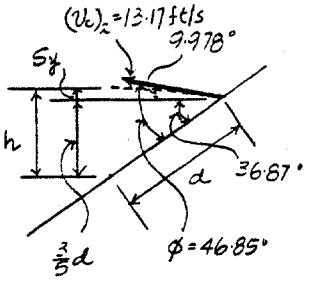
$$(v_c)_2 \cos\phi = 9.008 \qquad [1]$$



Coefficient of Restitution (y') :

$$e = \frac{(v_P)_2 - \left(v_{c_{y'}}\right)_2}{\left(v_{c_{y'}}\right)_1 - (v_P)_1}$$

$$(+) \qquad 0.8 = \frac{0 - (v_c)_2 \sin\phi}{-15.01\left(\frac{4}{5}\right) - 0} \qquad [2]$$



Solving Eqs. [1] and [2] yields

$$\phi = 46.85° \qquad (v_c)_2 = 13.17 \text{ ft/s}$$

Kinematics : By considering the vertical motion of the cranberry after the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + a_c t$$

$$0 = 13.17\sin 9.978° + (-32.2)\,t \qquad t = 0.07087 \text{ s}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$= 0 + 13.17\sin 9.978°(0.07087) + \frac{1}{2}(-32.2)\left(0.07087^2\right)$$

$$= 0.080864 \text{ ft}$$

By considering the horizontal motion of the cranberry after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$\frac{4}{5}d = 0 + 13.17\cos 9.978°(0.07087)$$

$$d = 1.149 \text{ ft} = 1.15 \text{ ft} \qquad \textbf{Ans}$$

Thus,

$$h = s_y + \frac{3}{5}d = 0.080864 + \frac{3}{5}(1.149) = 0.770 \text{ ft} \qquad \textbf{Ans}$$

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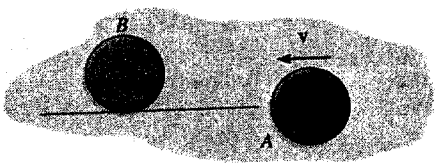
15-82. If disk A is sliding along the tangent to disk B and strikes B with a velocity $\mathbf{v}$, determine the velocity of B after the collision and compute the loss of kinetic energy during the collision. Neglect friction. Disk B is originally at rest. The coefficient of restitution is e, and each disk has the same size and mass m.



Impact : This problem involves *oblique impact* where the *line of impact* lies along x' axis (line jointing the mass center of the two impact bodies). From the geometry $\theta = \sin^{-1}\left(\frac{r}{2r}\right) = 30°$. The x' andy' components of velocity for disk A just before impact are

$$(v_{A_{x'}})_1 = -v\cos 30° = -0.8660v \qquad (v_{A_{y'}})_1 = -v\sin 30° = -0.5v$$

Conservation of "x'" Momentum :

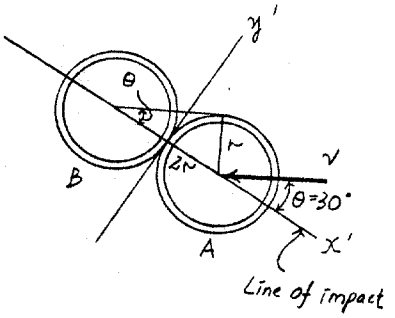
$$m_A(v_{A_{x'}})_1 + m_B(v_{B_{x'}})_1 = m_A(v_{A_{x'}})_2 + m_B(v_{B_{x'}})_2$$

$$(+) \qquad m(-0.8660v) + 0 = m(v_{A_{x'}})_2 + m(v_{B_{x'}})_2 \qquad [1]$$

Coefficient of Restitution (x') :

$$e = \frac{(v_{B_{x'}})_2 - (v_{A_{x'}})_2}{(v_{A_{x'}})_1 - (v_{B_{x'}})_1}$$

$$(+) \qquad e = \frac{(v_{B_{x'}})_2 - (v_{A_{x'}})_2}{-0.8660v - 0} \qquad [2]$$



Solving Eqs. [1] and [2] yields

$$(v_{B_{x'}})_2 = -\frac{\sqrt{3}}{4}(1+e)v \qquad (v_{A_{x'}})_2 = \frac{\sqrt{3}}{4}(e-1)v$$

Conservation of "y'" Momentum : The momentum is conserved along y' axis for both disks A and B.

$$(+\uparrow) \qquad m_B(v_{B_{y'}})_1 = m_B(v_{B_{y'}})_2; \qquad (v_{B_{y'}})_2 = 0$$

$$(+\uparrow) \qquad m_A(v_{A_{y'}})_1 = m_A(v_{A_{y'}})_2; \qquad (v_{A_{y'}})_2 = -0.5v$$

Thus, the magnitude the velocity for disk B just after the impact is

$$(v_B)_2 = \sqrt{(v_{B_{x'}})_2^2 + (v_{B_{y'}})_2^2} = \sqrt{\left(-\frac{\sqrt{3}}{4}(1+e)v\right)^2 + 0} = \frac{\sqrt{3}}{4}(1+e)v$$ **Ans**

nd directed toward **negative x' axis.** **Ans**

he magnitude of the velocity for disk A just after the impact is

$$(v_A)_2 = \sqrt{(v_{A_{x'}})_2^2 + (v_{A_{y'}})_2^2} = \sqrt{\left[\frac{\sqrt{3}}{4}(e-1)v\right]^2 + (-0.5v)^2} = \sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}$$

of Kinetic Energy : Kinetic energy of the system before the impact is

$$U_k = \frac{1}{2}mv^2$$

etic energy of the system after the impact is

$$U_k' = \frac{1}{2}m\left[\sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}\right]^2 + \frac{1}{2}m\left[\frac{\sqrt{3}}{4}(1+e)v\right]^2 = \frac{mv^2}{32}(6e^2 + 10)$$

Thus, the kinetic energy loss is

$$\Delta U_k = U_k - U_k' = \frac{1}{2}mv^2 - \frac{mv^2}{32}(6e^2 + 10) = \frac{3mv^2}{16}(1 - e^2)$$ **Ans**

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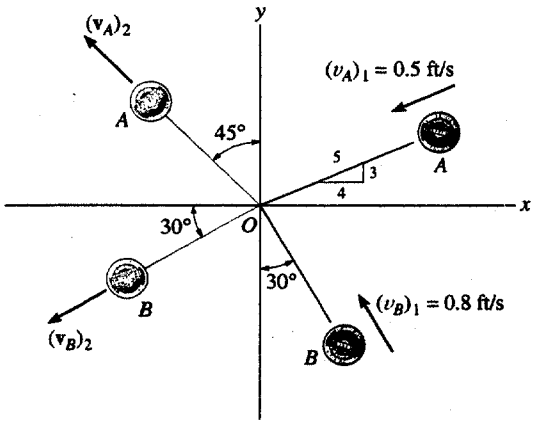
15-83. Two smooth coins A and B, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the dashed paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the x and y axes, respectively.



$$\Sigma m v_1 = \Sigma m v_2$$

$$(\overset{+}{\rightarrow}) \quad -m(0.8)\sin 30° - m(0.5)\left(\tfrac{4}{5}\right) = -m(v_A)_2 \sin 45° - m(v_B)_2 \cos 30°$$

$$0.8 = 0.707(v_A)_2 + 0.866(v_B)_2$$

$$(+\uparrow) \quad m(0.8)\cos 30° - m(0.5)\left(\tfrac{3}{5}\right) = m(v_A)_2 \cos 45° - m(v_B)_2 \sin 30°$$

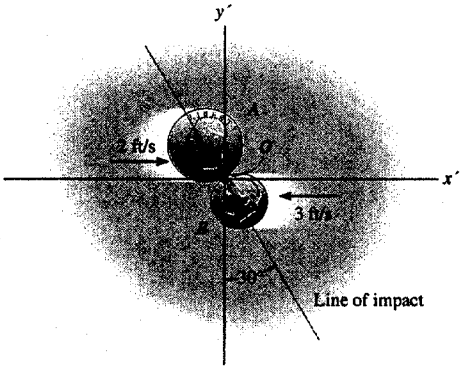
$$-0.3928 = -0.707(v_A)_2 + 0.5(v_B)_2$$

Solving,

$(v_B)_2 = 0.298$ ft/s **Ans**

$(v_A)_2 = 0.766$ ft/s **Ans**

***15-84.** Two coins A and B have the initial velocities shown just before they collide at point O. If they have weights of $W_A = 13.2(10^{-3})$ lb and $W_B = 6.60(10^{-3})$ lb and the surface upon which they slide is smooth, determine their speeds just after impact. The coefficient of restitution is $e = 0.65$.



$$(+\swarrow) \quad m_A (v_{A_x})_1 + m_B (v_{B_x})_1 = m_A (v_{A_x})_2 + m_B (v_{B_x})_2$$

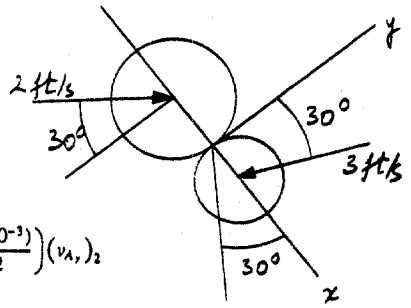
$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\sin 30° - \left(\frac{6.6(10^{-3})}{32.2}\right)3\sin 30° = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_x})_2 + \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_x})_2$$

$$(+\swarrow) \quad e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1}; \qquad 0.65 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{2\sin 30° - (-3\sin 30°)}$$

Solving:

$(v_{A_x})_2 = -0.3750$ ft/s

$(v_{B_x})_2 = 1.250$ ft/s



$$(+\nearrow) \quad m_A (v_{A_y})_2 = m_A (v_{A_y})_2$$

$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\cos 30° = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_y})_2$$

$$(v_{A_y})_2 = 1.732 \text{ ft/s}$$

$$(+\nearrow) \quad m_B (v_{B_y})_1 = m_B (v_{B_y})_2$$

$$\left(\frac{6.6(10^{-3})}{32.2}\right)3\cos 30° = \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_y})_2$$

$$(v_{B_y})_2 = 2.598 \text{ ft/s}$$

Thus,

$(v_B)_2 = \sqrt{(1.250)^2 + (2.598)^2} = 2.88$ ft/s **Ans**

$(v_A)_2 = \sqrt{(-0.3750)^2 + (1.732)^2} = 1.77$ ft/s **Ans**

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15-85. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.

$$(\overset{+}{\rightarrow}) \quad \Sigma m v_1 = \Sigma m v_2$$

$$0.5(4)\left(\tfrac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$$(+\uparrow) \quad m v_1 = m v_2$$

$$0.5\left(\tfrac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

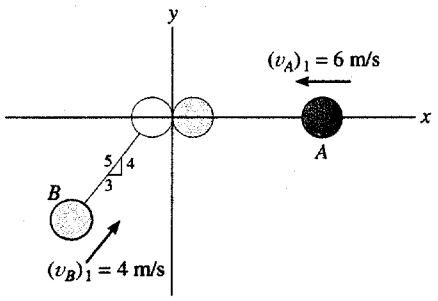
$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$v_A = 1.35 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

$$v_B = \sqrt{(4.95)^2 + (3.20)^2} = 5.89 \text{ m/s} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{3.20}{4.95} = 32.9° \text{ } \measuredangle \qquad \textbf{Ans}$$

15-86. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



$$\Sigma m v_1 = \Sigma m v_2$$

$$(\overset{+}{\rightarrow}) \quad 0.5(4)\left(\tfrac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

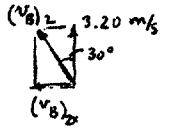
$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$$(+\uparrow) \quad 0.5(4)\left(\tfrac{4}{5}\right) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$(v_B)_{2x} = 3.20\tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$



$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752 - (-1.8475)}{4\left(\frac{3}{5}\right) - (-6)} = 0.0113 \qquad \textbf{Ans}$$

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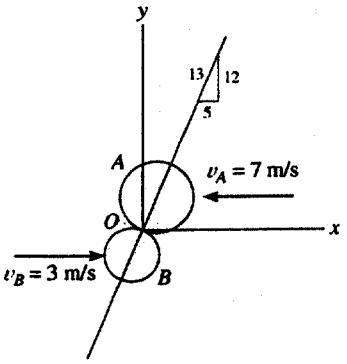
15-87. Two smooth disks A and B have the initial velocities shown just before they collide at O. If they have masses $m_A = 8\text{ kg}$ and $m_B = 6\text{ kg}$, determine their speeds just after impact. The coefficient of restitution is $e = 0.5$.

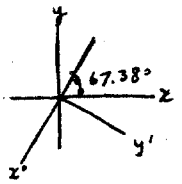


$$+\checkmark \Sigma mv_1 = \Sigma mv_2$$

$$-6(3\cos 67.38°) + 8(7\cos 67.38°) = 6(v_B)_{x'} + 8(v_A)_{x'}$$

$$(+\checkmark) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.5 = \frac{(v_B)_{x'} - (v_A)_{x'}}{7\cos 67.38° + 3\cos 67.38°}$$



Solving,

$(v_B)_{x'} = 2.14$ m/s

$(v_A)_{x'} = 0.220$ m/s

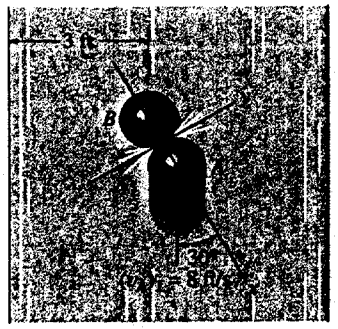
$(v_B)_{y'} = 3\sin 67.38° = 2.769$ m/s

$(v_A)_{y'} = -7\sin 67.38° = -6.462$ m/s

$v_B = \sqrt{(2.14)^2 + (2.769)^2} = 3.50$ m/s **Ans**

$v_A = \sqrt{(0.220)^2 + (6.462)^2} = 6.47$ m/s **Ans**

***15-88.** The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially A has a velocity of 8 ft/s and B is at rest. Neglect friction.

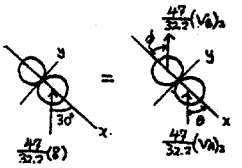


Line of impact (x-axis):

$\Sigma mv_1 = \Sigma mv_2$

$$(+\searrow) \quad 0 + \frac{47}{32.2}(8)\cos 30° = \frac{47}{32.2}(v_B)_{2x} + \frac{47}{32.2}(v_A)_{2x}$$

$$(+\nwarrow) \quad e = 0.8 = \frac{(v_B)_{2x} - (v_A)_{2x}}{8\cos 30° - 0}$$



Solving:

$(v_A)_{2x} = 0.6928$ ft/s

$(v_B)_{2x} = 6.235$ ft/s

Plane of impact (y-axis):

Stone A:

$mv_1 = mv_2$

$$(+\uparrow) \quad \frac{47}{32.2}(8)\sin 30° = \frac{47}{32.2}(v_A)_{2y}$$

$$(v_A)_{2y} = 4$$

Stone B:

$mv_1 = mv_2$

$$(+\uparrow) \quad 0 = \frac{47}{32.2}(v_B)_{2y}$$

$$(v_B)_{2y} = 0$$

$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06$ ft/s **Ans**

$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24$ ft/s **Ans**

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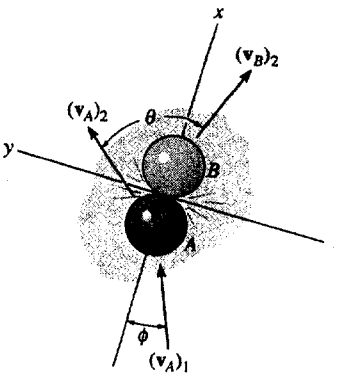
15-89. Ball A strikes ball B with an initial velocity of $(v_A)_1$ as shown. If both balls have the same mass and the collision is perfectly elastic, determine the angle θ after collision. Ball B is originally at rest. Neglect the size of each ball.



Velocity before impact :

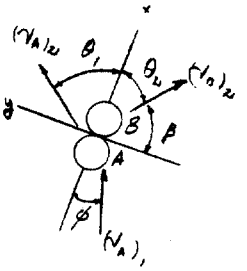
$$(v_{Ax})_1 = (v_A)_1 \cos\phi \qquad (v_{Ay})_1 = (v_A)_1 \sin\phi$$
$$(v_{Bx})_1 = 0 \qquad (v_{By})_1 = 0$$

Velocity after impact :

$$(v_{Ax})_2 = (v_A)_2 \cos\theta_1 \qquad (v_{Ay})_2 = (v_A)_2 \sin\theta_1$$
$$(v_{Bx})_2 = (v_B)_2 \cos\theta_2 \qquad (v_{By})_2 = -(v_B)_2 \sin\theta_2$$

Conservation of " y " momentum :

$$m_B (v_{By})_1 = m_B (v_{By})_2$$
$$0 = m\left[-(v_B)_2 \sin\theta_2\right] \qquad \theta_2 = 0°$$



Conservation of " x " momentum :

$$m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$
$$m(v_A)_1 \cos\phi + 0 = m(v_A)_2 \cos\theta_1 + m(v_B)_2 \cos 0°$$
$$(v_A)_1 \cos\phi = (v_A)_2 \cos\theta_1 + (v_B)_2 \qquad (1)$$

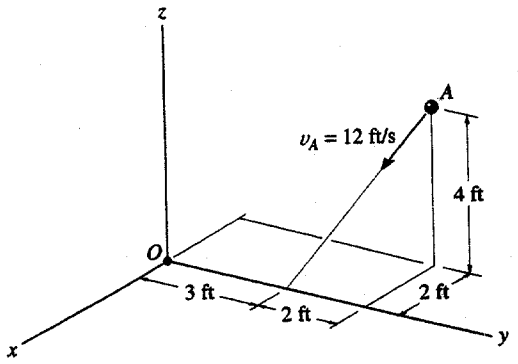
Coefficient of Restitution (x direction) :

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \qquad 1 = \frac{(v_B)_2 \cos 0° - (v_A)_2 \cos\theta_1}{(v_A)_1 \cos\phi - 0}$$
$$(v_A)_1 \cos\phi = -(v_A)_2 \cos\theta_1 + (v_B)_2 \qquad (2)$$

Subtracting Eq.(1) from Eq. (2) yields :

$$2(v_A)_2 \cos\theta_1 = 0 \qquad \text{Since } 2(v_A)_2 \neq 0$$
$$\cos\theta_1 = 0 \qquad \theta_1 = 90°$$
$$\theta = \theta_1 + \theta_2 = 90° + 0° = 90° \qquad \textbf{Ans}$$

15-90. Determine the angular momentum of the 2-lb particle A about point O. Use a Cartesian vector solution.



$$m\mathbf{v}_A = \frac{2}{32.2}(12)\left(\frac{2}{\sqrt{24}}\mathbf{i} - \frac{2}{\sqrt{24}}\mathbf{j} - \frac{4}{\sqrt{24}}\mathbf{k}\right)$$

$$= \{0.3043\mathbf{i} - 0.3043\mathbf{j} - 0.6086\mathbf{k}\} \text{ slug}\cdot\text{ft/s}$$

$$(\mathbf{H}_A)_O = \mathbf{r}_A \times m\mathbf{v}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & 4 \\ 0.3043 & -0.3043 & -0.6086 \end{vmatrix}$$

$$= \{-1.83\mathbf{i} - 0.913\mathbf{k}\} \text{ slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans}$$

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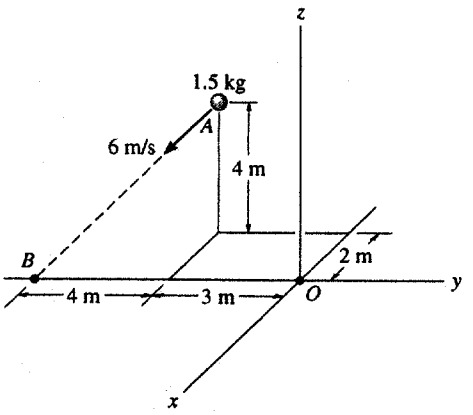
15-91. Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.

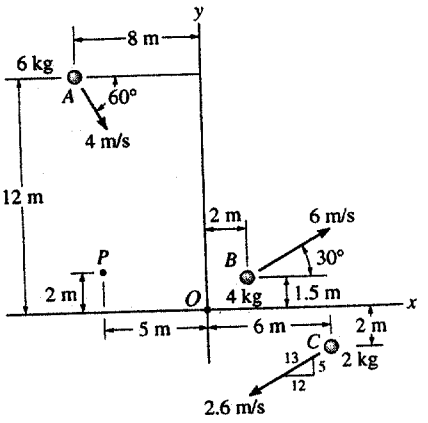


$$\mathbf{r}_{OB} = \{-7\mathbf{j}\}\ \text{m} \qquad \mathbf{v}_A = 6\left(\frac{2\mathbf{i}-4\mathbf{j}-4\mathbf{k}}{\sqrt{2^2+(-4)^2+(-4)^2}}\right) = \{2\mathbf{i}-4\mathbf{j}-4\mathbf{k}\}\ \text{m/s}$$

$$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -7 & 0 \\ 1.5(2) & 1.5(-4) & 1.5(-4) \end{vmatrix} = \{42\mathbf{i}+21\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

***15-92.** Determine the angular momentum $\mathbf{H}_O$ of each of the particles about point O.

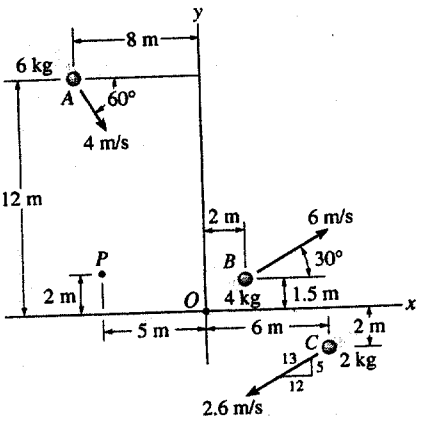


$$(H_A)_O = 8(6)(4\sin 60°) - 12(6)(4\cos 60°) = 22.3\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

$$(H_B)_O = -1.5(4)(6\cos 30°) + 2(4)(6\sin 30°) = -7.18\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

$$(H_C)_O = -2(2)\left(\frac{12}{13}\right)(2.6) - 6(2)\left(\frac{5}{13}\right)(2.6) = -21.6\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

15-93. Determine the angular momentum $\mathbf{H}_P$ of each of the particles about point P.



$$(H_A)_P = -6(4\cos 60°)(10) + 6(4\sin 60°)(3) = -57.6\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

$$(H_B)_P = 4(6\cos 30°)(0.5) + 4(6\sin 30°)(7) = 94.4\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

$$(H_C)_P = -2(2.6)\left(\frac{5}{13}\right)(11) - 2(2.6)\left(\frac{12}{13}\right)(4) = -41.2\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

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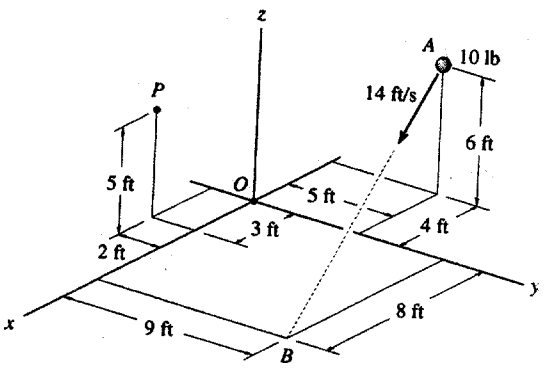
15-94. Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.



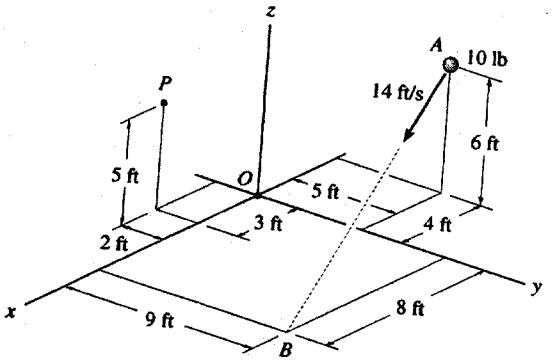
$$\mathbf{r}_{OB} = \{8\mathbf{i} + 9\mathbf{j}\}\ \text{ft} \qquad \mathbf{v}_A = 14\left(\frac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{12^2 + 4^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}\ \text{ft/s}$$

$$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 9 & 0 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-16.8\mathbf{i} + 14.9\mathbf{j} - 23.6\mathbf{k}\}\ \text{slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans}$$

15-95. Determine the angular momentum $\mathbf{H}_P$ of the particle about point P.



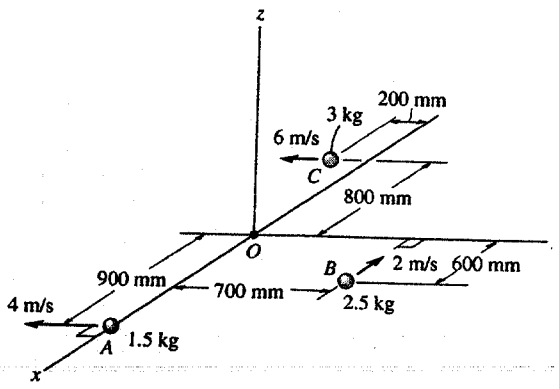
$$\mathbf{r}_{PB} = \{5\mathbf{i} + 11\mathbf{j} - 5\mathbf{k}\}\ \text{ft}$$

$$\mathbf{v}_A = 14\left(\frac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{(12)^2 + (4)^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}\ \text{ft/s}$$

$$\mathbf{H}_P = \mathbf{r}_{PB} \times m\mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 11 & -5 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-14.3\mathbf{i} - 9.32\mathbf{j} - 34.8\mathbf{k}\}\ \text{slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans}$$

***15-96.** Determine the total angular momentum $\mathbf{H}_O$ for the system of three particles about point O. All the particles are moving in the x–y plane.



$$\mathbf{H}_O = \Sigma \mathbf{r} \times m\mathbf{v}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0 \\ 0 & -1.5(4) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.7 & 0 \\ -2.5(2) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.8 & -0.2 & 0 \\ 0 & 3(-6) & 0 \end{vmatrix}$$

$$= \{12.5\mathbf{k}\}\ \text{kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans}$$

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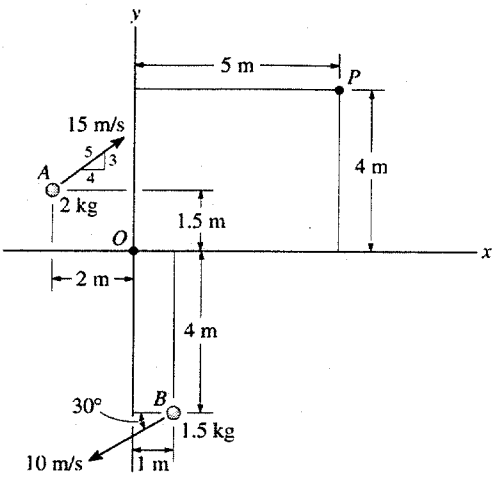
15-97. Determine the angular momentum $\mathbf{H}_O$ of each of the two particles about point O. Use a scalar solution.



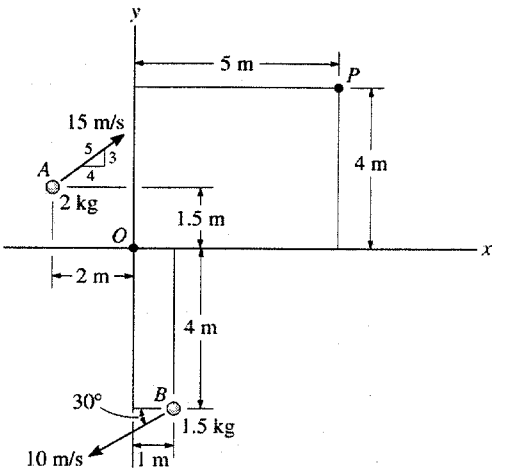
$$\curvearrowleft +(H_A)_O = -2(15)\left(\frac{4}{5}\right)(1.5) - 2(15)\left(\frac{3}{5}\right)(2)$$

$$= -72.0 \text{ kg} \cdot \text{m}^2/\text{s} = 72.0 \text{ kg} \cdot \text{m}^2/\text{s} \curvearrowright \quad \textbf{Ans}$$

$$\curvearrowleft +(H_B)_O = -1.5(10)(\cos 30°)(4) - 1.5(10)(\sin 30°)(1)$$

$$= -59.5 \text{ kg} \cdot \text{m}^2/\text{s} = 59.5 \text{ kg} \cdot \text{m}^2/\text{s} \curvearrowright \quad \textbf{Ans}$$

15-98. Determine the angular momentum $\mathbf{H}_P$ of each of the two particles about point P. Use a scalar solution.



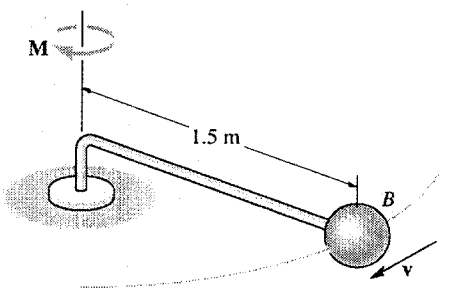
$$\curvearrowleft +(H_A)_P = 2(15)\left(\frac{4}{5}\right)(2.5) - 2(15)\left(\frac{3}{5}\right)(7)$$

$$= -66.0 \text{ kg} \cdot \text{m}^2/\text{s} = 66.0 \text{ kg} \cdot \text{m}^2/\text{s} \curvearrowright \quad \textbf{Ans}$$

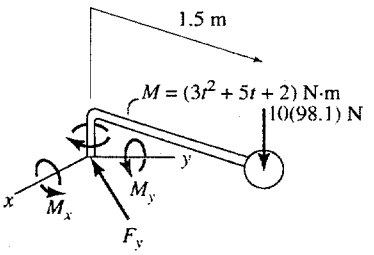
$$\curvearrowleft +(H_B)_P = -1.5(10)(\cos 30°)(8) + 1.5(10)(\sin 30°)(4)$$

$$= -73.9 \text{ kg} \cdot \text{m}^2/\text{s} = 73.9 \text{ kg} \cdot \text{m}^2/\text{s} \curvearrowright \quad \textbf{Ans}$$

15-99. The ball B has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)$ N · m, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.



Principle of Angular Impluse and Momentum: Applying Eq. 15-22, we have



$$(H_z)_1 + \sum \int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$1.5(10)(2) + \int_0^{2s} (3t^2 + 5t + 2)dt = 1.5(10)v$$

$$v = 3.47 \text{ m/s} \quad \textbf{Ans}$$

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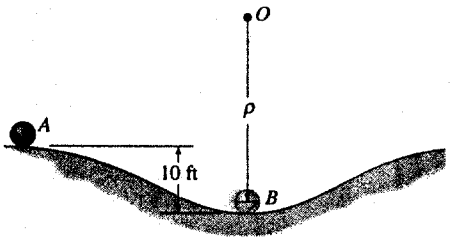
***15-100.** The 3-lb ball located at A is released from rest and travels down the curved path. If the ball exerts a normal force of 5 lb on the path when it reaches point B, determine the angular momentum of the ball about the center of curvature, point O. *Hint:* Neglect the size of the ball. The radius of curvature at point B must be determined.



Datum at B :

$T_1 + V_1 = T_2 + V_2$

$$0 + 3(10) = \frac{1}{2}\left(\frac{3}{32.2}\right)(v_B)^2 + 0$$

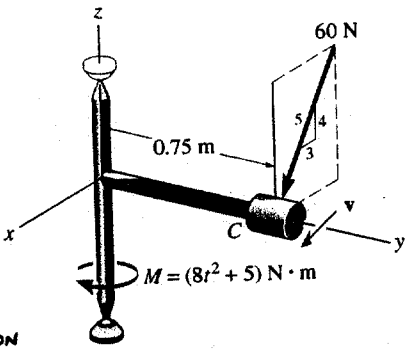
$v_B = 25.38$ ft/s

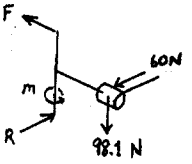
$$(+\uparrow)\Sigma F_n = ma_n; \quad 5 - 3 = \left(\frac{3}{32.2}\right)\left(\frac{(25.38)^2}{\rho}\right)$$

$\rho = 30$ ft

$$H_B = 30\left(\frac{3}{32.2}\right)(25.38) = 70.9 \text{ slug} \cdot \text{ft}^2/\text{s} \quad \textbf{Ans}$$

15-101. The small cylinder C has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)$ N · m, where t is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t = 2$ s. The cylinder has a speed $v_0 = 2$ m/s when $t = 0$.





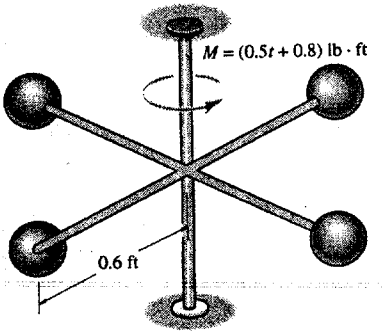
$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$(10)(2)(0.75) + 60(2)\left(\frac{3}{5}\right)(0.75) + \int_0^2 (8t^2 + 5)dt = 10v(0.75)$$

$$69 + \left[\frac{8}{3}t^3 + 5t\right]_0^2 = 7.5v$$

$v = 13.4$ m/s **Ans**

15-102. The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb·ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.



$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$0 + \int_0^4 (0.5t + 0.8)\, dt = 4\left[\left(\frac{5}{32.2}\right)(0.6v_2)\right]$$

$7.2 = 0.37267\, v_2$

$v_2 = 19.3$ ft/s **Ans**

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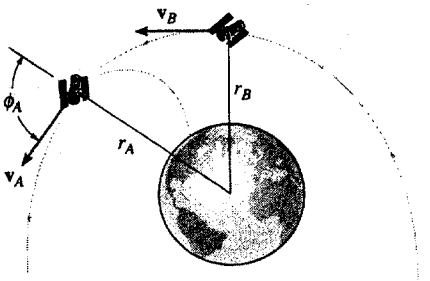
15-103. An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy (see Prob. 14-97).



$(H_O)_1 = (H_O)_2$

$m_s(v_A \sin\phi_A) r_A = m_s(v_B) r_B$

$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B)$ (1)

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}m_s(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B}$$ (2)

Solving,

$v_B = 10.2$ km/s Ans

$r_B = 13.8$ Mm Ans

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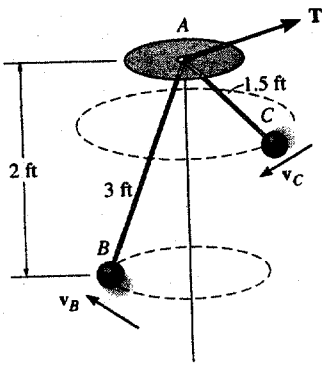
***15-104.** The ball B has a weight of 5 lb and is originally rotating in a circle. As shown, the cord AB has a length of 3 ft and passes through the hole A, which is 2 ft above the plane of motion. If 1.5 ft of cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at C.



Equation of Motion : When the ball is travelling around the first circular path, $\theta = \sin^{-1}\frac{2}{3} = 41.81°$ and $r_1 = 3\cos 41.81° = 2.236$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0; \qquad T_1\left(\frac{2}{3}\right) - 5 = 0 \qquad T_1 = 7.50 \text{ lb}$$

$$\Sigma F_n = ma_n; \qquad 7.50\cos 41.81° = \frac{5}{32.2}\left(\frac{v_1^2}{2.236}\right)$$

$$v_1 = 8.972 \text{ ft/s}$$

When the ball is traveling around *the* second circular path, $r_2 = 1.5\cos\phi$. Applying Eq. 13 – 8 , we have

$$\Sigma F_b = 0; \qquad T_2 \sin\phi - 5 = 0 \qquad [1]$$

$$\Sigma F_n = ma_n; \qquad T_2\cos\phi = \frac{5}{32.2}\left(\frac{v_2^2}{1.5\cos\phi}\right) \qquad [2]$$

Conservation of Angular Momentum : Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about z axis. Applying Eq. 15 – 23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$2.236\left(\frac{5}{32.2}\right)(8.972) = 1.5\cos\phi\left(\frac{5}{32.2}\right)v_2 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$\phi = 13.8768° \qquad T_2 = 20.85 \text{ lb}$$

$$v_2 = 13.8 \text{ ft/s} \qquad \textbf{Ans}$$

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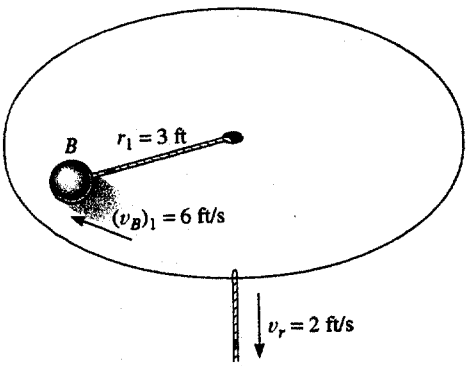
15-105. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine the ball's speed at the instant $r_2 = 2$ ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.



$H_1 = H_2$

$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}v_\theta(2)$

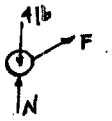
$v_\theta = 9$ ft/s

$v_2 = \sqrt{9^2 + 2^2} = 9.22$ ft/s **Ans**

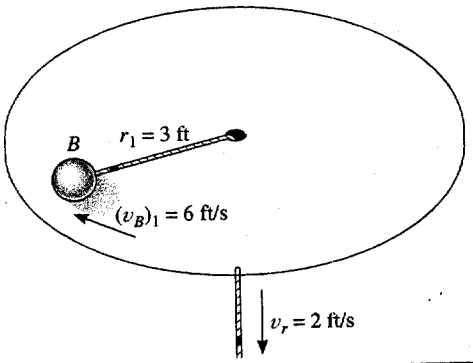
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(\frac{4}{32.2})(6)^2 + \Sigma U_{1-2} = \frac{1}{2}(\frac{4}{32.2})(9.22)^2$

$\Sigma U_{1-2} = 3.04$ ft·lb **Ans**



15-106. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



$v = \sqrt{(v_\theta)^2 + (2)^2}$

$12 = \sqrt{(v_\theta)^2 + (2)^2}$

$v_\theta = 11.832$ ft/s

$H_1 = H_2$

$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$

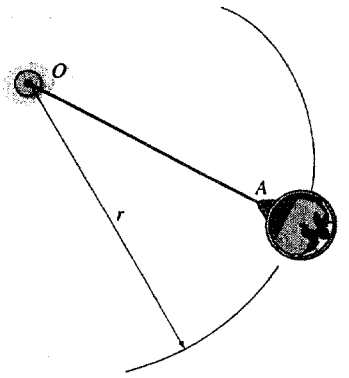
$r_2 = 1.5213 = 1.52$ ft **Ans**

$\Delta r = v_r t$

$(3 - 1.5213) = 2t$

$t = 0.739$ s **Ans**

15-107. An amusement park ride consists of a car which is attached to the cable OA. The car rotates in a horizontal circular path and is brought to a speed $v_1 = 4$ ft/s when $r = 12$ ft. The cable is then pulled in at the constant rate of 0.5 ft/s. Determine the speed of the car in 3 s.



Conservation of Angular Momentum : Cable OA is shorten by $\Delta r = 0.5(3) = 1.50$ ft in 3 s. Thus, at this instant $r_2 = 12 - 1.50 = 10.5$ ft. Since no force acts on the car along the tangent of the moving path, the angular momentum is conserved about point O. Applying Eq. 15 – 23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$
$$r_1 m v_1 = r_2 m v'$$
$$12(m)(4) = 10.5(m)\, v'$$
$$v' = 4.571 \text{ ft/s}$$

The speed of car after 3 s is

$$v_2 = \sqrt{0.5^2 + 4.571^2} = 4.60 \text{ ft/s} \quad \textbf{Ans}$$

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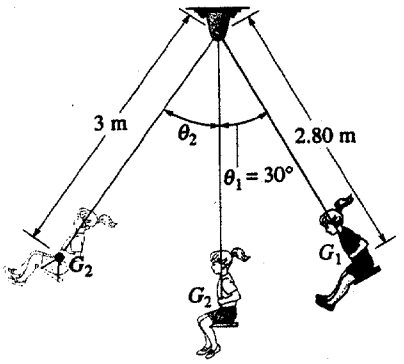
***15-108.** A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30°$. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.



First before $\theta = 0°$;

$T_1 + V_1 = T_2 + V_2$

$0 + 2.80(1-\cos 30°)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$

$v_1 = 2.713$ m/s

$H_1 = H_2$

$50(2.713)(2.80) = 50(v_2)(3)$

$v_2 = 2.532 = 2.53$ m/s Ans

Just after $\theta = 0°$;

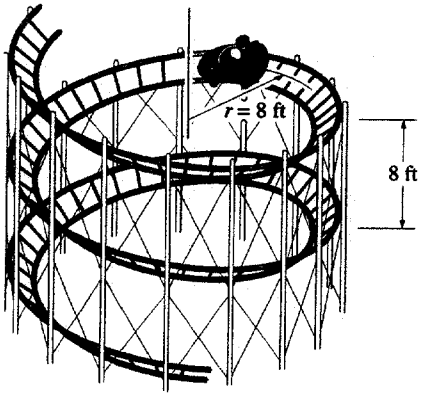
$T_2 + V_2 = T_3 + V_3$

$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1-\cos\theta_2)$

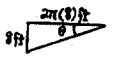
$0.1089 = 1-\cos\theta_2$

$\theta_2 = 27.0°$ Ans

15-109. The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car in $t = 4$ s. Also, how far has the car descended in this time? Neglect friction and the size of the car.

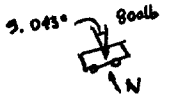


$\theta = \tan^{-1}(\frac{8}{2\pi(8)}) = 9.043°$



$\Sigma F_y = 0$; $N - 800\cos 9.043° = 0$

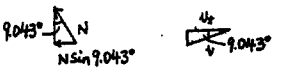
$N = 790.1$ lb



$H_1 + \int M\,dt = H_2$

$0 + \int_0^4 8(790.1\sin 9.043°)\,dt = \frac{800}{32.2}(8)v_t$

$v_t = 20.0$ ft/s



$v = \frac{20}{\cos 9.043°} = 20.2$ ft/s Ans

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 800h = \frac{1}{2}(\frac{800}{32.2})(20.2)^2$

$h = 6.36$ ft Ans

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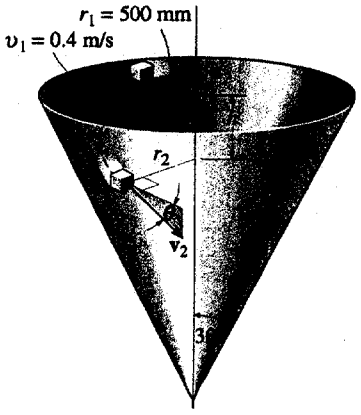
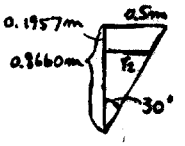
15-110. A small block having a mass of 0.1 kg is given a horizontal velocity of $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. Determine the distance h it must descend for it to reach a speed of $v_2 = 2$ m/s. Also, what is the angle of descent θ, that is, the angle measured from the horizontal to the tangent of the path?



$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(0.1)(0.4)^2 + 0.1(9.81)(h) = \frac{1}{2}(0.1)(2)^2$

$h = 0.1957$ m $= 196$ mm **Ans**



From similar triangles

$r_2 = \frac{(0.8660 - 0.1957)}{0.8660}(0.5) = 0.3870$ m

$(H_0)_1 = (H_0)_2$

$0.5(0.1)(0.4) = 0.3870(0.1)(v_2')$

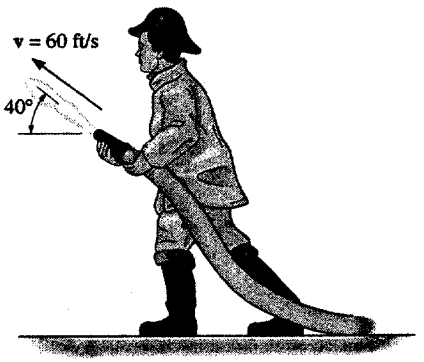
$v_2' = 0.5168$ m/s

$v_2 \cos\theta = v_2'$

$2\cos\theta = 0.5168$

$\theta = 75.0°$ **Ans**

15-111. The 150-lb fireman is holding a hose which has a nozzle diameter of 1 in. and hose diameter of 2 in. If the velocity of the water at discharge is 60 ft/s, determine the resultant normal and frictional force acting on the man's feet at the ground. Neglect the weight of the hose and the water within it. $\gamma_w = 62.4$ lb/ft^3.



Originally, the water flow is horizontal. The fireman alters the direction of flow to 40° from the horizontal.

$$\frac{dm}{dt} = \rho v_B A_B = \frac{62.4}{32.2}(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right) = 0.6342 \text{ slug/s}$$

Also, the velocity of the water through the hose is

$\rho v_A A_A = \rho v_B A_B$

$$\rho v_A\left(\frac{\pi(1)^2}{(12)^2}\right) = \rho(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right)$$

$v_A = 15$ ft/s

$\stackrel{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}((v_B)_x - (v_A)_x)$

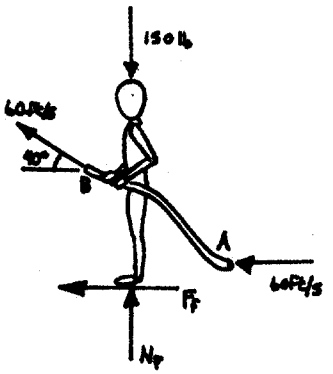
$F_f = 0.6342[60\cos 40° - 15]$

$F_f = 19.6$ lb **Ans**

$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$

$N_f - 150 = 0.6342[60\sin 40° - 0]$

$N_f = 174$ lb **Ans**



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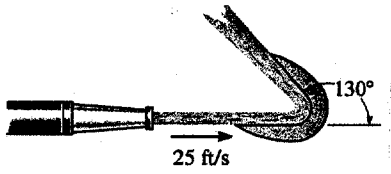
***15-112.** A jet of water having a cross-sectional area of 4 in^2 strikes the fixed blade with a speed of 25 ft/s. Determine the horizontal and vertical components of force which the blade exerts on the water. $\gamma_w = 62.4$ lb/ft^3.



$$Q = Av = \left(\frac{4}{144}\right)(25) = 0.6944 \text{ ft}^3/\text{s}$$

$$\frac{dm}{dt} = \rho Q = \left(\frac{62.4}{32.2}\right)(0.6944) = 1.3458 \text{ slug/s}$$

$$v_{Ax} = 25 \text{ ft/s} \qquad v_{Ay} = 0$$

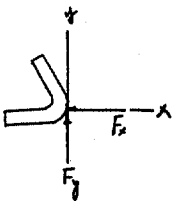
$$v_{Bx} = -25\cos 50° \text{ ft/s} \qquad v_{By} = 25\sin 50° \text{ ft/s}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax}); \quad -F_x = 1.3458[-25\cos 50° - 25]$$

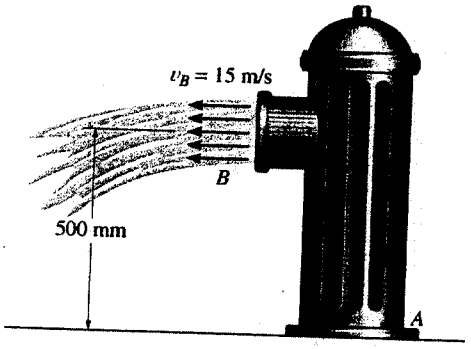
$$F_x = 55.3 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay}); \quad F_y = 1.3458(25\sin 50° - 0)$$

$$F_y = 25.8 \text{ lb} \qquad \textbf{Ans}$$



15-113. Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is 50 kPa. The diameter of the fire hydrant at A is 200 mm. $\rho_w = 1$ Mg/m^3.



$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07 \text{ kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$$v_A = 8.4375 \text{ m/s}$$

$$\stackrel{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$$A_x = 265.07(15 - 0) = 3.98 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

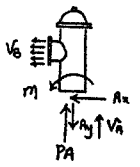
$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0 - 8.4375)$$

$$A_y = 3.81 \text{ kN} \qquad \textbf{Ans}$$

$$(+\Sigma M_A = \frac{dm}{dt}(d_{AB} v_B - d_{AA} v_A)$$

$$M = 265.07(0.5(15) - 0)$$

$$M = 1.99 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$



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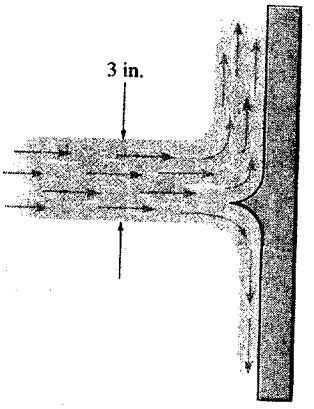
15-114. The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5$ ft^3/s, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4$ lb/ft^3.



Equations of Steady Flow : Here, the flow rate $Q = 0.5$ ft^3/s. Then, $v = \frac{Q}{A}$

$= \frac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19$ ft/s. Also, $\frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2}(0.5) = 0.9689$ slug/s.

Applying Eq. 15 – 26, we have

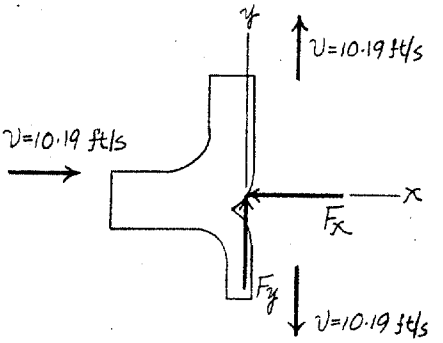
$$\Sigma F_x = \Sigma \frac{dm}{dt}\left(v_{out_x} - v_{in_x}\right);$$

$$-F_x = 0 - 0.9689(10.19) \qquad F_x = 9.87 \text{ lb} \qquad \textbf{Ans}$$

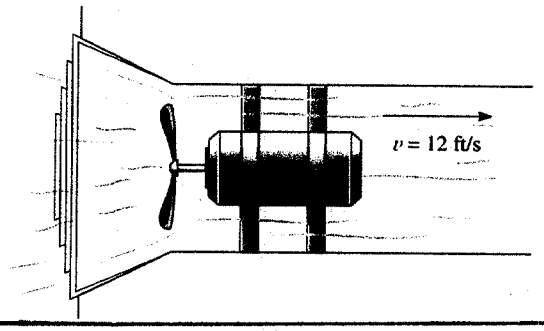
$$\Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{out_y} - v_{in_y}\right);$$

$$F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

$$F_y = 4.93 \text{ lb} \qquad \textbf{Ans}$$



15-115. The fan draws air through a vent with a speed of 12 ft/s. If the cross-sectional area of the vent is 2 ft^2, determine the horizontal thrust on the blade. The specific weight of the air is $\gamma_a = 0.076$ lb/ft^3.



$$\frac{dm}{dt} = \rho v A$$

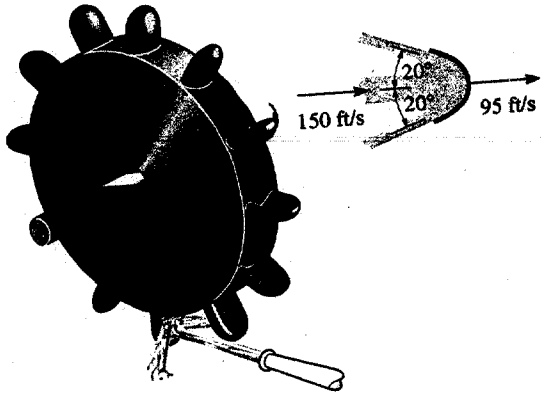
$$= \frac{0.076}{32.2}(12)(2)$$

$$= 0.05665 \text{ slug/s}$$

$$\Sigma F = \frac{dm}{dt}(v_B - v_A)$$

$$T = 0.05665(12 - 0) = 0.680 \text{ lb} \qquad \textbf{Ans}$$

***15-116.** The buckets on the *Pelton wheel* are subjected to a 2-in-diameter jet of water, which has a velocity of 150 ft/s. If each bucket is traveling at 95 ft/s when the water strikes it, determine the power developed by the wheel. $\gamma_w = 62.4$ lb/ft^3.



$v_A = 150 - 95 = 55$ ft/s $\rightarrow$

$(\overset{+}{\rightarrow})(v_B)_x = -55\cos 20° + 95 = 43.317$ ft/s $\rightarrow$

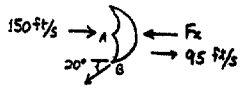
$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$

$F_x = \left(\frac{62.4}{32.2}\right)(\pi)\left(\frac{1}{12}\right)^2(55)(-43.317 - (-55))$

$F_x = 27.1667$ lb

$P = 27.1667(95) = 2580.835$ ft·lb/s

$P = 4.69$ hp **Ans**



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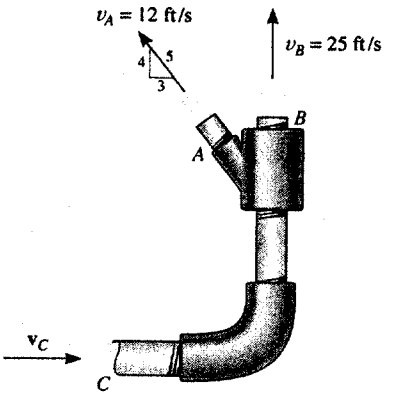
15-117. The static pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities $v_A = 12$ ft/s and $v_B = 25$ ft/s, determine the horizontal and vertical components of force exerted on the elbow at C necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4$ lb/ft^3.



$$\frac{dm_A}{dt} = \frac{62.4}{32.2}(12)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm_B}{dt}v_{B_x} + \frac{dm_A}{dt}v_{A_x} - \frac{dm_C}{dt}v_{C_x}$$

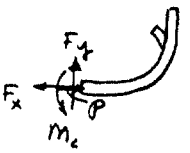
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

$$F_x = 19.5 \text{ lb} \qquad \textbf{Ans}$$

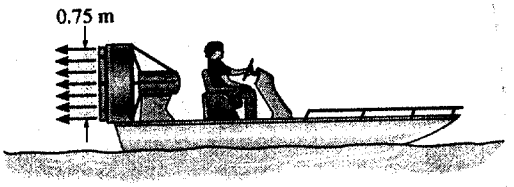
$$+\uparrow \Sigma F_y = \frac{dm_B}{dt}v_{B_y} + \frac{dm_A}{dt}v_{A_y} - \frac{dm_C}{dt}v_{C_y}$$

$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

$$F_y = 1.9559 = 1.96 \text{ lb} \qquad \textbf{Ans}$$

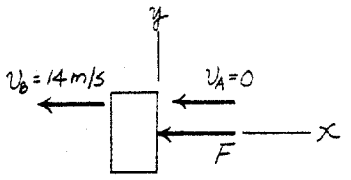


15-118. The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_a = 1.22$ kg/m^3 and that the entering air is essentially at rest. Neglect the drag resistance of the water.



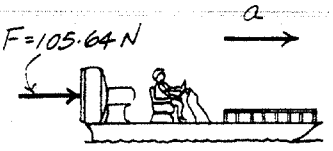
Equations of Steady Flow : Initially, the boat is at rest hence $v_B = v_{a/b}$ = 14 m/s. Then, $Q = v_B A = 14\left[\frac{\pi}{4}(0.75^2)\right] = 6.185$ m^3/s and $\frac{dm}{dt} = \rho_a Q$ = 1.22(6.185) = 7.546 kg/s. Applying Eq. 15 – 26, we have

$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}): \quad -F = 7.546(-14 - 0) \quad F = 105.64 \text{ N}$$



Equation of Motion :

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 105.64 = 200a \quad a = 0.528 \text{ m/s}^2 \qquad \textbf{Ans}$$



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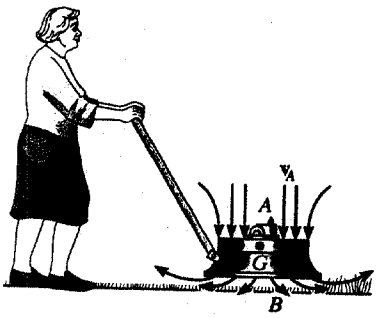
15-119. A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ m^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ m^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ kg/m^3$.

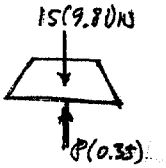


$$\frac{dm}{dt} = \rho A_A v_A = 1.22(0.25)(6) = 1.83\ kg/s$$

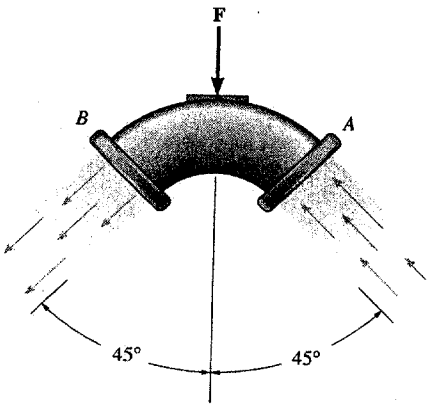
$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$p(0.35) - 15(9.81) = 1.83(0-(-6))$$

$$p = 452\ Pa \qquad \text{Ans}$$



***15-120.** The elbow for a 5-in-diameter buried pipe is subjected to a static pressure of 10 lb/in^2. The speed of the water passing through it is $v = 8$ ft/s. Assuming the pipe connection at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. $\gamma_w = 62.4\ lb/ft^3$.



Equations of Steady Flow : Here, $Q = vA = 8\left[\frac{\pi}{4}\left(\frac{5}{12}\right)^2\right] = 1.091\ ft^3/s$.

Then, the mass flow rate is $\frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2}(1.091) = 2.114\ slug/s$.

Also, the force induced by the water pressure at A is $F = pA = 10\left[\frac{\pi}{4}(5^2)\right]$ $= 62.5\pi$ lb. Applying Eq. 15 – 26, we have

$$\Sigma F_y = \frac{dm}{dt}(v_{B_y} - v_{A_y});$$

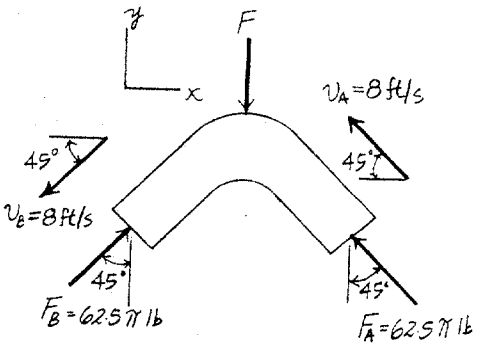
$$-F + 2(62.5\pi\cos 45°) = 2.114(-8\sin 45° - 8\sin 45°)$$

$$F = 302\ lb \qquad \text{Ans}$$

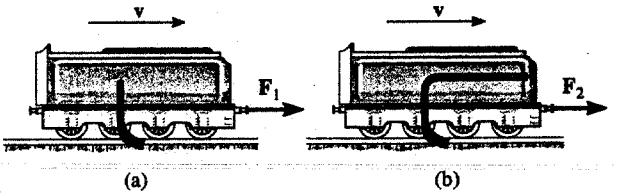
$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x});$$

$$62.5\pi\sin 45° - 62.5\pi\sin 45° = 2.114[-8\cos 45° - (-8\cos 45°)]$$

$$0 = 0 \qquad (Check!)$$



15-121. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



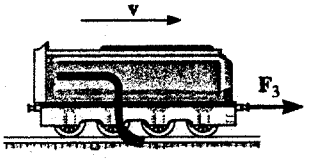
(a) (b)

The system consists of the car and the scoop. In all cases

$$\Sigma F_x = m\frac{dv}{dt} - v_{Dle}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)v$$

$$F = v^2 \rho A \qquad \text{Ans}$$



(c)

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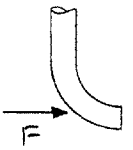
15-122. A plow located on the front of a locomotive scoops up snow at the rate of 10 ft^3/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6$ lb/ft^3.

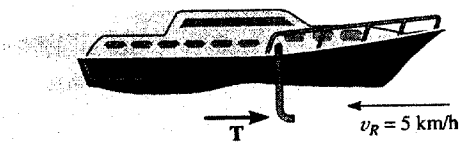
$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

$F = 22.4$ lb **Ans**



15-123. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance to the water collection. $\rho_w = 1$ Mg/m^3.



$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

$$v_{D/i} = (70)\left(\frac{1000}{3600}\right) = 19.444 \text{ m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$T = 0 + 19.444(0.5) = 9.72$ N **Ans**

***15-124.** The second stage of the two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

Initially,

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$0 = \frac{3000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$a = 133$ ft/s^2 **Ans**

Finally,

$$0 = \frac{2000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$a = 200$ ft/s^2 **Ans**

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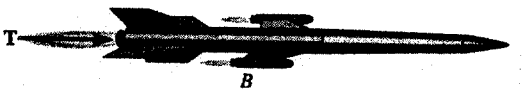
15-125. The missile weighs 40 000 lb. The constant thrust provided by the turbojet engine is $T = 15\,000$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.



$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} - v_{Dle}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \frac{dm_e}{dt}$.

$$T = (m_0 - ct)\frac{dv}{dt} - v_{Dle}c$$

$$\int_{v_0}^{v} dv = \int_0^t \left(\frac{T + cv_{Dle}}{m_0 - ct}\right)dt$$

$$v = \left(\frac{T + cv_{Dle}}{c}\right)\ln\left(\frac{m_0}{m_0 - ct}\right) + v_0 \qquad (1)$$

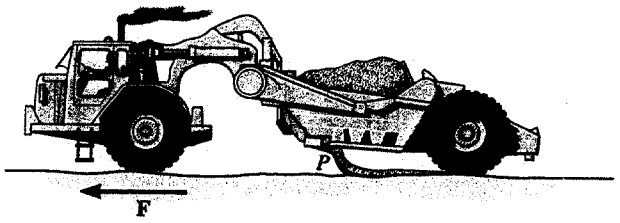
Here, $m_0 = \frac{40\,000}{32.2} = 1242.24$ slug, $c = 2\left(\frac{150}{32.2}\right) = 9.3168$ slug/s, $v_{Dle} = 3000$ ft/s, $t = 4$ s, $v_0 = \frac{300(5280)}{3600} = 440$ ft/s.

Substitute the numerical values into Eq. (1):

$$v_{max} = \left(\frac{15\,000 + 9.3168(3000)}{9.3168}\right)\ln\left(\frac{1242.24}{1242.24 - 9.3168(4)}\right) + 440$$

$v_{max} = 580$ ft/s **Ans**

15-126. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho v A$$

$$900 = 1520(v)(2.5)$$

$$v = 0.237 \text{ m/s}$$

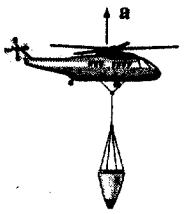
$$a = \frac{dv}{dt} = 0$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,000(0.1) - 900(0.237)$$

$F = 3.49$ kN **Ans**

15-127. The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{Dle}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$$0 = 10.5(10^3)a - (10)(50)$$

$a = 0.0476$ m/s^2 **Ans**

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***15-128.** The rocket has a mass of 65 Mg including the fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed of 200 ft/s in 10 s starting from rest. The fuel is expelled from the rocket at a relative speed of 3000 ft/s. Neglect the effects of air resistance and assume that g is constant.

A System That Loses Mass : Here, $W = \left(m_0 - \frac{dm_e}{dt}t\right)g$. Applying Eq. 15–29, we have

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

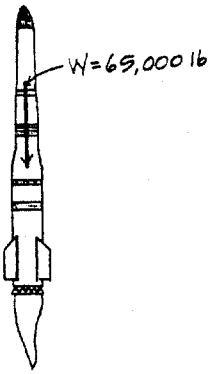
$$-\left(m_0 - \frac{dm_e}{dt}t\right)g = \left(m_0 - \frac{dm_e}{dt}t\right)\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$\frac{dv}{dt} = \frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g$$

$$\int_0^v dv = \int_0^t \left(\frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g\right)dt$$

$$v = \left[-v_{D/e}\ln\left(m_0 - \frac{dm_e}{dt}t\right) - gt\right]\Big|_0^t$$

$$v = v_{D/e}\ln\left(\frac{m_0}{m_0 - \frac{dm_e}{dt}t}\right) - gt \qquad [1]$$



Substitute Eq. [1] with $m_0 = \frac{65\,000}{32.2} = 2018.63$ slug, $v_{D/e} = 3000$ ft/s, $v = 200$ ft/s and $t = 10$ s, we have

$$200 = 3000\ln\left[\frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}\right] - 32.2(10)$$

$$e^{0.174} = \frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}$$

$$\frac{dm_e}{dt} = 32.2 \text{ slug/s} \qquad \textbf{Ans}$$

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15-129. The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed $v_{e/r}$ determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.

$$a_0 = \frac{dv}{dt}$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$-mg = ma_0 - v_{e/r}\frac{dm}{dt}$$

$$v_{e/r}\frac{dm}{m} = (a_0 + g)\,dt \qquad (1)$$

Since $v_{e/r}$ is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{e/r}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t$$

$$\frac{m}{m_0} = e^{[(a_0+g)/v_{e/r}]t}$$

The time rate of fuel consumption is determined from Eq. (1).

$$\frac{dm}{dt} = m\left(\frac{(a_0 + g)}{v_{e/r}}\right)$$

$$\frac{dm}{dt} = m_0\left(\frac{(a_0 + g)}{v_{e/r}}\right)e^{[(a_0+g)/v_{e/r}]t} \qquad \text{Ans}$$

Note: $v_{e/r}$ must be considered a negative quantity.

15-130. The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–29 and 15–30 must be combined to yield

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad (1)$$

$v = 950$ km/h $= 0.2639$ km/s, $\quad \frac{dv}{dt} = 0$

$v_{D/E} = 0.45$ km/s

$v_{D/i} = 0.2639$ km/s

$\frac{dm_i}{dt} = 50(1.22) = 61.0$ kg/s

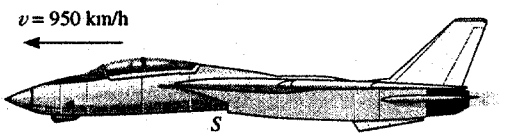
$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4$ kg/s

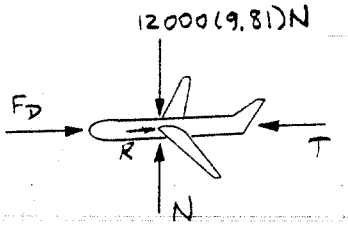
Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

$$(\overset{+}{\leftarrow})\ -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \text{Ans}$$

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$





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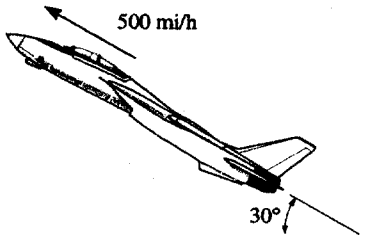
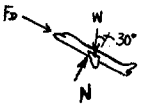
15-131. The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–130.





$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

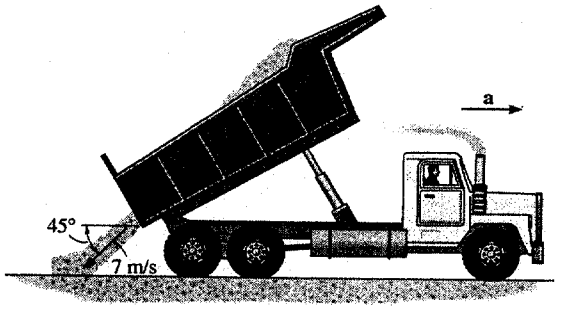
$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$

$$\nwarrow + \Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2 \qquad \textbf{Ans}$$

***15-132.** The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520 \text{ kg/m}^3$.



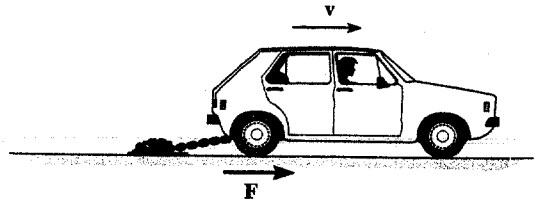
A System That Loses Mass : Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\frac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15 – 29, we have

$$\stackrel{+}{\rightarrow} \Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$0 = 57.6(10^3)\,a - (-7\cos 45°)(1216)$$

$$a = 0.104 \text{ m/s}^2 \qquad \textbf{Ans}$$

15-133. The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



$$\stackrel{+}{\rightarrow} \Sigma F_t = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \frac{dm_i}{dt} = \frac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v$, $\frac{dv}{dt} = 0$.

$$F = (m_0 + m'vt)(0) + v(m'v) = m'v^2 \qquad \textbf{Ans}$$

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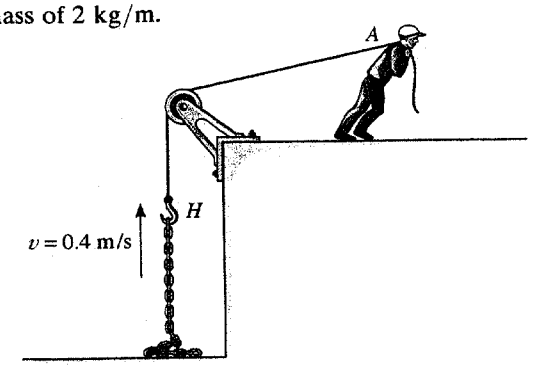
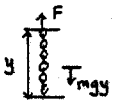
15-134. Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



$\frac{dv}{dt} = 0, \quad y = vt$

$m_i = my = mvt$

$\frac{dm_i}{dt} = mv$



$+\uparrow \Sigma F_y = m\frac{dv}{dt} + v_{Dli}(\frac{dm_i}{dt})$

$F - mgvt = 0 + v(mv)$

$F = m(gvt + v^2)$

$= 2[9.81(0.4)t + (0.4)^2]$

$F = (7.85t + 0.320)$ N **Ans**

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R1-1. A sports car can accelerate at 6 m/s^2 and decelerate at 8 m/s^2. If the maximum speed it can attain is 60 m/s, determine the shortest time it takes to travel 900 m starting from rest and then stopping when s = 900 m.

Time to accelerate to 60 m/s,

$(\overset{+}{\rightarrow})$ $v = v_0 + a_c t$

$60 = 0 + 6t$

$t = 10$ s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 0 + 0 + \frac{1}{2}(6)(10^2)$

$s = 300$ m

Time to decelerate to a stop,

$(\overset{+}{\rightarrow})$ $v = v_0 + a_c t$

$0 = 60 - 8t$

$t = 7.5$ s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 0 + 60(7.5) - \frac{1}{2}(8)(7.5^2)$

$s = 225$ m

Time to travel at 60 m/s,

$900 - 300 - 225 = 375$ m

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$375 = 0 + 60t$

$t = 6.25$ s

Total time $t = 10 + 7.5 + 6.25 = 23.8$ s **Ans**

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R1-2. A 2-kg particle rests on a smooth horizontal plane and is acted upon by forces $F_x = 0$ and $F_y = 3$ N. If $x = 0$, $y = 0$, $v_x = 6$ m/s, and $v_y = 2$ m/s when $t = 0$, determine the equation $y = f(x)$ which describes the path.

$$+\uparrow \Sigma F_y = ma_y; \quad 3 = 2a_y \quad a_y = 1.5\ \text{m/s}^2 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 0 = 2a_x \quad a_x = 0 \qquad [2]$$

$$a_y = \frac{dv_y}{dt} = 1.5$$

$$\int_2^{v_y} dv_y = 1.5\int_0^t dt$$

$$v_y = \frac{dy}{dt} = 1.5t + 2$$

$$\int_0^y dy = \int_0^t (1.5t + 2)\,dt$$

$$y = 0.75t^2 + 2t \qquad [3]$$

$$a_x = \frac{dv_x}{dt} = 0$$

$$\int_6^{v_x} dv_x = \int_0^t 0\,dt$$

$$v_x = \frac{dx}{dt} = 6$$

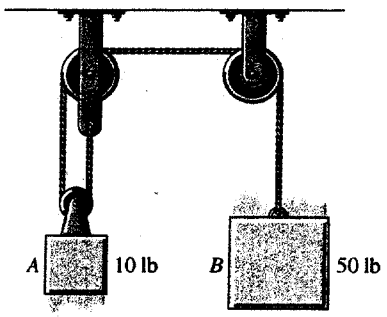
$$\int_0^x dx = \int_0^t 6\,dt$$

$$x = 6t \qquad [4]$$

Eliminating t from Eq.[3] and [4] yields :

$y = 0.0208x^2 + 0.333x$ **(Parabola)** **Ans**

R1-3. Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.



$$2s_A + s_B = l$$

$$2v_A + v_B = 0$$

$$v_A = \frac{v_B}{2}$$

$$(+\uparrow) \quad mv_1 + \Sigma\int F dt = mv_2$$

$$0 + 2T(2) - 10(2) = \frac{10}{32.2}\left(\frac{v_B}{2}\right)$$

$$(+\downarrow) \quad mv_1 + \Sigma\int F dt = mv_2$$

$$0 - T(2) + 50(2) = \frac{50}{32.2}v_B$$

solving,

$$T = 7.14\ \text{lb}$$

$$v_B = 55.2\ \text{ft/s} \qquad \textbf{Ans}$$

Thus, $v_A = -\frac{v_B}{2} = -27.6\ \text{ft/s} = 27.6\ \text{ft/s}$ **Ans**

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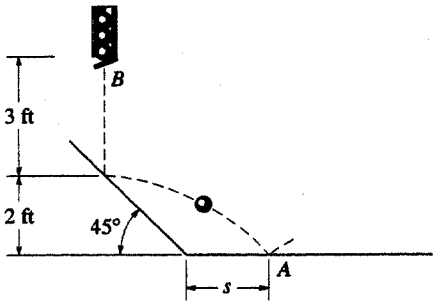
***R1-4.** To test the manufactured properties of 2-lb steel balls, each ball is released from rest as shown and strikes a 45° inclined surface. If the coefficient of restitution is to be $e = 0.8$, determine the distance s to where the ball must strike the horizontal plane at A. At what speed does the ball strike A?

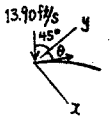


Just before impact,

$T_1 + V_1 = T_2 + V_2$

$0 + 2(3) = \frac{1}{2}(\frac{2}{32.2})(v_B)_1^2 + 0$

$(v_B)_1 = 13.90 \text{ ft/s} \downarrow$



Conservation of momentum in $x-$direction,

$\frac{2}{32.2}(13.9)\sin 45° = \frac{2}{32.2}(v_B)_2 \sin\theta$

$(v_B)_2 \sin\theta = 9.83 \qquad (1)$

Coefficient of restitution,

$e = 0.8 = \frac{-(v_B)_2 \cos\theta - 0}{0 - (13.9)\cos 45°}$

$(v_B)_2 \cos\theta = 7.86 \qquad (2)$

Solving Eqs.(1) and (2)

$(v_B)_2 = 12.59 \text{ ft/s}$

$\theta = 51.3°$, so that $\nabla\theta$ 6.39°

In $y-$direction:

$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_B)_{A_y}^2 = (12.59\sin 6.34°)^2 + 2(32.2)(2-0)$

$(v_B)_{A_y} = 11.43 \text{ ft/s} \downarrow$

$(+\downarrow) \quad v = v_0 + a_c t$

$11.43 = 12.59\sin 6.34° + 32.2t$

$t = 0.312 \text{ s}$

In $x-$direction:

$(\overset{+}{\rightarrow}) \quad (v_B)_{Ax} = 12.59\cos 6.34° = 12.51 \text{ ft/s}$

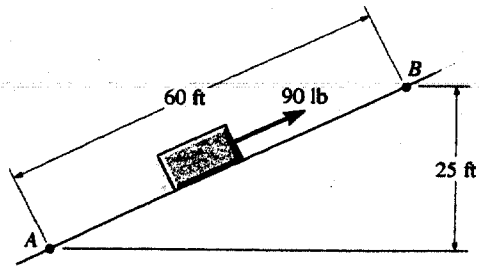
$s = v_B t$

$s + 2 = 12.51(0.312)$

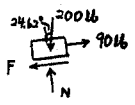
$s = 1.90 \text{ ft}$ **Ans**

$v_A = \sqrt{(12.51)^2 + (11.43)^2} = 16.9 \text{ ft/s}$ **Ans**

R1-5. The 90-lb force is required to drag the 200-lb block 60 ft up the *rough* inclined plane at constant velocity. If the force is removed when the block reaches point B, and the block is then released from rest, determine the block's velocity when it slides back down the plane and reaches point A.



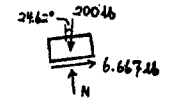
$\theta = \sin^{-1}(\frac{25}{60}) = 24.62°$



$+\nearrow \Sigma F_x = 0; \quad 90 - 200\sin 24.62° - F = 0$

$F = 6.667 \text{ lb}$

$T_1 + \Sigma U_{1-2} = T_2$



$0 + 200\sin 24.62°(60) - 6.667(60) = \frac{1}{2}(\frac{200}{32.2})v^2$

$v = 38.5 \text{ ft/s}$ **Ans**

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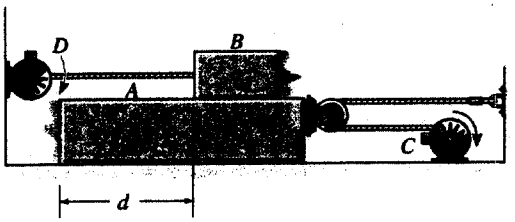
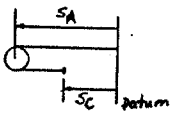
R1-6. The motor at C pulls in the cable with an acceleration $a_c = (3t^2)$ m/s^2, where t is in seconds. The motor at D draws in its cable at $a_D = 5$ m/s^2. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs.



$$s_A + (s_A - s_C) = l$$

$$2s_A - s_C = l$$

$$2a_A - a_C = 0 \qquad a_A = 0.5a_C = 1.5t^2$$



$$(\overset{+}{\rightarrow})\ a_{A/B} = a_A - a_B$$

$$= 1.5t^2 - (-5) = 1.5t^2 + 5$$

$$(\overset{+}{\rightarrow})\ v_{A/B} = \int_0^t (1.5t^2 + 5)\,dt = 0.5t^3 + 5t \qquad [1]$$

$$(\overset{+}{\rightarrow})\ s_{A/B} = \int_0^t (0.5t^3 + 5t)\,dt$$

$$3 = 0.125t^4 + 2.5t^2$$

$$t = 1.067 \text{ s} = 1.07 \text{ s} \qquad \textbf{Ans}$$

From Eq.[1] $\quad v_{A/B} = 0.5(1.067)^3 + 5(1.067) = 5.93 \text{ m/s} \rightarrow \qquad \textbf{Ans}$

R1-7. A spring having a stiffness of 5 kN/m is compressed 400 mm. The stored energy in the spring is used to drive a machine which requires 80 W of power. Determine how long the spring can supply energy at the required rate.

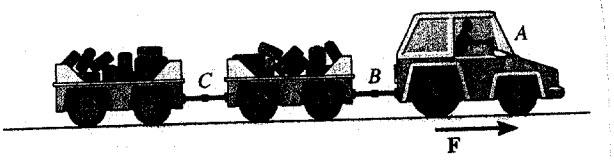
$$U = \frac{1}{2}kx^2 = \frac{1}{2}(5000)(0.4)^2 = 400 \text{ J}$$

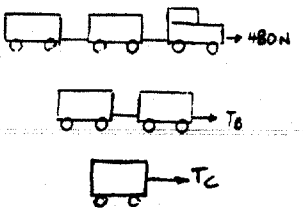
$$P = \frac{U}{t}$$

$$80 \text{ W} = \frac{400 \text{ J}}{t}$$

$$t = 5 \text{ s} \qquad \textbf{Ans}$$

***R1-8.** **The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. Determine the tension in the couplings at B and C if the tractive force F on the truck is $F = 480$ N. What is the speed of the truck when $t = 2$ s, starting from rest? The car wheels are free to roll. Neglect the mass of the wheels.**





$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad 480 = [800 + 2(300)]a$

$$a = 0.3429 \text{ m/s}^2$$

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$$v = 0 + 0.3429(2) = 0.686 \text{ m/s} \qquad \textbf{Ans}$$

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad T_B = 2(300)(0.3429)$

$$T_B = 205.71 = 206 \text{ N} \qquad \textbf{Ans}$$

$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad T_C = (300)(0.3429)$

$$T_C = 102.86 = 103 \text{ N}$$

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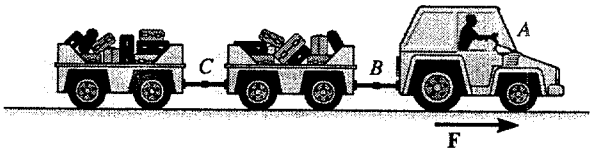
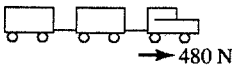
R1-9. The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. If the tractive force **F** on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.



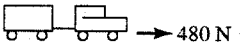
$\overset{+}{\rightarrow} \sum F_x = ma_x;$ $\quad 480 = [800 + 2(300)]a$

$a = 0.3429 = 0.343 \text{ m/s}^2$ **Ans**

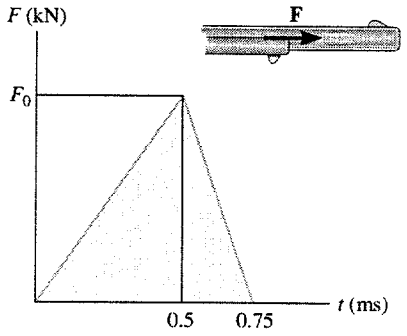


$\overset{+}{\rightarrow} \sum F_x = ma_x;$ $\quad 480 = (800 + 300)a$

$a = 0.436 \text{ m/s}^2$ **Ans**



R1-10. Assuming that the force acting on a 2.5-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force, F_0, applied to the bullet when it is fired. The muzzle velocity is 800 m/s when $t = 0.75$ m/s. Neglect friction between the bullet and rifle barrel.



The impulse is determined from the area under the curve:

$$mv_1 + \sum \int F dt = mv_2$$

$$0 + \frac{1}{2}(0.75)(10^{-3})F_0 = (0.0025)(800)$$

$F_0 = 5.33$ kN **Ans**

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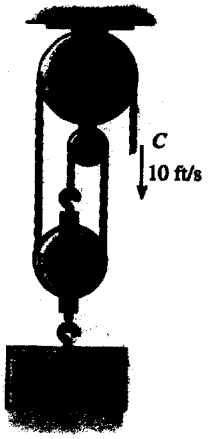
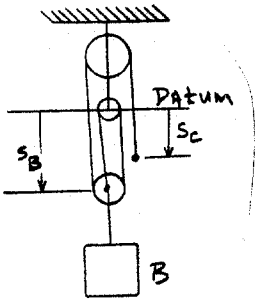
R1-11. Determine the speed of block B if the end of the cable at C is pulled downward with a speed of 10 ft/s. What is the relative velocity of the block with respect to C?





$3s_B + s_C = l$

$3v_B = -v_C$

$3v_B = -(10)$

$v_B = -3.33 \text{ ft/s} = 3.33 \text{ ft/s} \uparrow$ **Ans**

$(+\downarrow)$ $v_B = v_C + v_{B/C}$

$-3.33 = 10 + v_{B/C}$

$v_{B/C} = -13.3 \text{ ft/s} = 13.3 \text{ ft/s} \uparrow$ **Ans**

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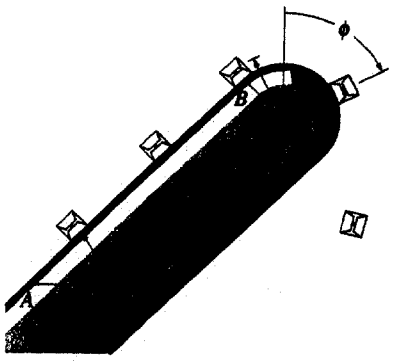
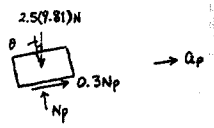
***■R1-12.** Packages having a mass of 2.5 kg ride on the surface of the conveyor belt. If the belt starts from rest and with constant acceleration increases to a speed of 0.75 m/s in 2 s, determine the maximum angle of tilt, θ, so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and each package is $\mu_s = 0.3$. At what angle ϕ do the packages first begin to slip off the surface of the belt if the belt is moving at a constant speed of 0.75 m/s?



$\nwarrow +\Sigma F_y = ma_y$; $N_p - 2.5(9.81)\cos\theta = 0$ (1)

$\nearrow +\Sigma F_x = ma_x$; $0.3N_p - 2.5(9.81)\sin\theta = 2.5a_p$ (2)



Kinematics:

$v = v_0 + a_c t$

$0.75 = 0 + a_p(2)$

$a_p = 0.375\ \text{m/s}^2$

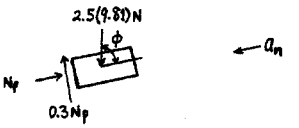
Combining Eqs.(1) and (2), using a_p,

$0.3\cos\theta - \sin\theta = 0.0382$

$\theta = 14.6°$ **Ans**

$\searrow +\Sigma F_n = ma_n$; $2.5(9.81)\cos\phi - N_p = 2.5[\frac{(0.75)^2}{0.350}]$

$\searrow +\Sigma F_t = ma_t$; $2.5(9.81)\sin\phi - 0.3N_p = 0$



Combining these equations,

$\cos\phi - 3.33\sin\phi = 0.164$

$\phi = 14.0°$

R1-13. A projectile, initially at the origin, moves along a straight-line path through a fluid medium such that its velocity is $v = 1800(1 - e^{-0.3t})$ mm/s, where t is in seconds. Determine the displacement of the projectile during the first 3 s.

$$v = \frac{ds}{dt} = 1800(1 - e^{-0.3t})$$

$$\int_0^s ds = \int_0^t 1800(1 - e^{-0.3t})\,dt$$

$$s = 1800(t + \frac{1}{0.3}e^{-0.3t}) - 6000$$

Thus, in $t = 3$ s

$$s = 1800(3 + \frac{1}{0.3}e^{-0.3(3)}) - 6000$$

$s = 1839.4$ mm $= 1.84$ m

R1-14. The speed of a train during the first minute of its motion has been recorded as follows:

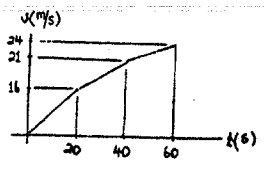
t(s)	0	20	40	60
v(m/s)	0	16	21	24

Plot the v–t graph, approximating the curve as straight line segments between the given points. Determine the total distance traveled.

s = area under curve

$$= \frac{1}{2}(20)(16) + (40-20)(\frac{21+16}{2}) + (60-40)(\frac{24+21}{2})$$

$= 980$ m **Ans**



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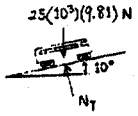
R1-15. A train car, having a mass of 25 Mg, travels up a 10° incline with a constant speed of 80 km/h. Determine the power required to overcome the force of gravity.

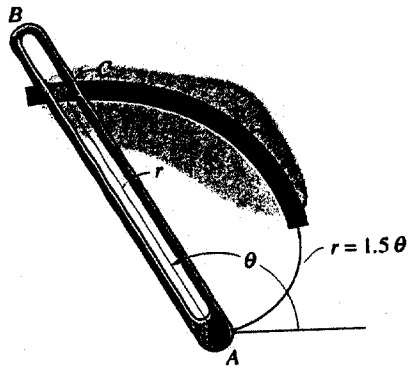
$v = 80 \text{ km/h} = 22.22 \text{ m/s}$

$P = \mathbf{F} \cdot \mathbf{v} = 25(10^3)(9.81)(22.22)(\sin 10°)$

$P = 946 \text{ kW}$ **Ans**



***R1-16.** The slotted arm AB drives the pin C through the spiral groove described by the equation $r = (1.5\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular rate of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin when $t = 1$ s.



$\dot{\theta} = 4t$

$\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$

$\theta - \frac{1}{3}\pi = 2t^2$

$\theta = (2t^2 + \frac{1}{3}\pi) \text{ rad}$

$\dot{\theta} = 4t$

$\ddot{\theta} = 4$

$r = 1.5\theta = 1.5(2t^2 + \frac{1}{3}\pi)$

$\dot{r} = 6t$

$\ddot{r} = 6$

When $t = 1$ s,

$\dot{\theta} = 4 \quad r = 4.5708$

$\ddot{\theta} = 4 \quad \dot{r} = 6$

$\ddot{r} = 6$

$v_r = \dot{r} = 6.00 \text{ ft/s}$ **Ans**

$v_\theta = r\dot{\theta} = 4.5708(4) = 18.3 \text{ ft/s}$ **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.5708(4)^2 = -67.1 \text{ ft/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5708(4) + 22(6)(4) = 66.3 \text{ ft/s}^2$ **Ans**

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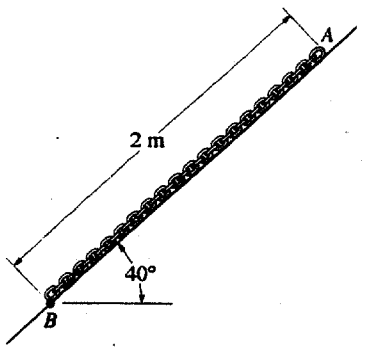
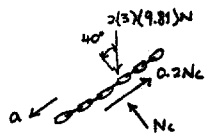
R1-17. The chain has a mass of 3 kg/m. If the coefficient of kinetic friction between the chain and the plane is $\mu_k = 0.2$, determine the velocity at which the end A will pass point B when the chain is released from rest.



$+\nwarrow \Sigma F_y = ma_y;$ $\quad -2(3)(9.81)\cos 40° + N_C = 0$

$N_C = 45.09$ N



$+\swarrow \Sigma F_x = ma_x;$ $\quad 2(3)(9.81)\sin 40° - 0.2(45.09) = 2(3)\,a$

$a = 4.80$ m/s^2

$+\swarrow \quad v_2^2 = v_1^2 + 2as$

$v_2^2 = 0 + 2(4.80)(2)$

$v_2 = 4.38$ m/s **Ans**

Also,

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 2(3)(9.81)(2\sin 40°) - 0.2(45.09)(2) = \frac{1}{2}(2)(3)(v^2)$

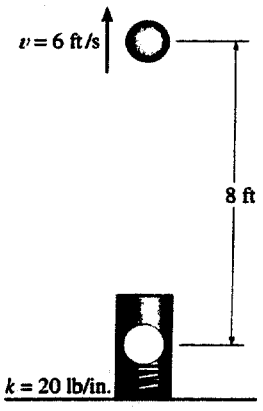
$v = 4.38$ m/s **Ans**

R1-18. The 6-lb ball is fired from a tube by a spring having a stiffness $k = 20$ lb/in. Determine how far the spring must be compressed to fire the ball from the compressed position to a height of 8 ft, at which point it has a velocity of 6 ft/s.

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(20)(12)(x^2) = \frac{1}{2}(\frac{6}{32.2})(6)^2 + 8(6)$

$x = 0.654$ ft $= 7.85$ in. **Ans**



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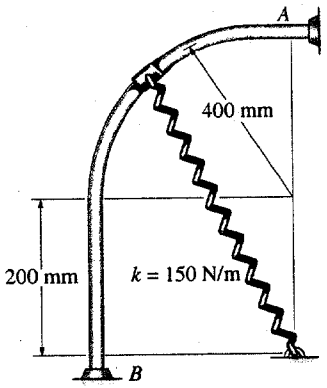
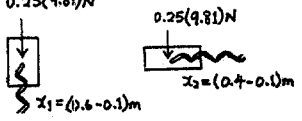
R1-19. The collar of negligible size has a mass of 0.25 kg and is attached to a spring having an unstretched length of 100 mm. If the collar is released from rest at A and travels along the smooth guide, determine its speed just before it strikes B.



$T_A + V_A = T_B + V_B$

$$0 + (0.25)(9.81)(0.6) + \frac{1}{2}(150)(0.6-0.1)^2 = \frac{1}{2}(0.25)(v_B)^2 + \frac{1}{2}(150)(0.4-0.1)^2$$

$v_B = 10.4$ m/s **Ans**



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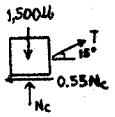
***R1-20.** A crate has a weight of 1500 lb. If it is pulled along the ground at a constant speed for a distance of 20 ft, and the towing cable makes an angle of 15° with the horizontal, determine the tension in the cable and the work done by the towing force. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.55$.

$+\uparrow \Sigma F_y = 0;\quad N_C - 1500 + T\sin 15° = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad T\cos 15° - 0.55N_C = 0$

$T = 744.4 \text{ lb} = 744 \text{ lb}$ **Ans**

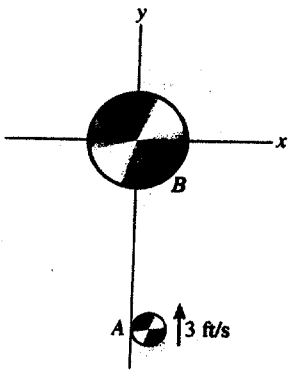
$N_C = 1307.3 \text{ lb}$



$U_{1-2} = (744.4\cos 15°)(20) = 14\,380.7 \text{ ft·lb}$

$U_{1-2} = 14.4 \text{ ft·kip}$ **Ans**

R1-21. Disk A weighs 2 lb and is sliding on a smooth horizontal plane with a velocity of 3 ft/s. Disk B weighs 11 lb and is initially at rest. If after the impact A has a velocity of 1 ft/s directed along the positive x axis, determine the velocity of B after impact. How much kinetic energy is lost in the collision?



Conservation of momentum for A and B in $x-$direction

$0 + 0 = \frac{2}{32.2}(1) + \frac{11}{32.2}(v_B)_{2x}$

$(v_B)_{2x} = -0.1818 \text{ ft/s}$

Conservation of momentum for A and B in $y-$direction

$\frac{2}{32.2}(3) + 0 = 0 + \frac{11}{32.2}(v_B)_{2y}$

$(v_B)_{2y} = 0.54545 \text{ ft/s}$

Thus,

$(v_B)_2 = \sqrt{(-0.1818)^2 + (0.54545)^2} = 0.575 \text{ ft/s}$ **Ans**

$\theta = \tan^{-1}\left(\frac{0.54545}{0.1818}\right) = 71.5°$ ∢ **Ans**

$T_1 = \frac{1}{2}\left(\frac{2}{32.2}\right)(3)^2 = 0.280 \text{ ft·lb}$

$T_2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(1)^2 + \frac{1}{2}\left(\frac{11}{32.2}\right)(0.575)^2 = 0.0875 \text{ ft·lb}$

$\Delta T = T_2 - T_1 = -0.192 \text{ ft·lb}$ **Ans**

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R1-22. A particle is moving along a circular path of 2-m radius such that its position as a function of time is given by $\theta = (5t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

$r = 2$ m $\quad \theta = 5t^2$

$\dot{r} = 0 \quad \dot{\theta} = 10t$

$\ddot{r} = 0 \quad \ddot{\theta} = 10$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$= [0 - 2(10t)^2]\mathbf{u}_r + [2(10) + 0]\mathbf{u}_\theta$$

$$= \{-200t^2\mathbf{u}_r + 20\mathbf{u}_\theta\} \text{ m/s}^2$$

When $\theta = 30° = 30(\frac{\pi}{180}) = 0.524$ rad

Then,

$0.524 = 5t^2$

$t = 0.324$ s

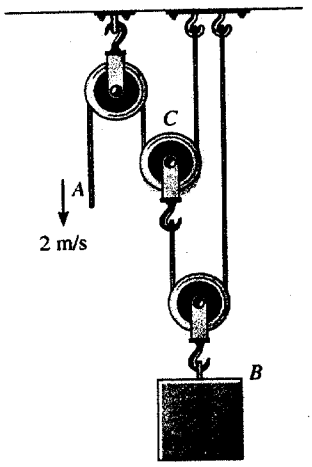
Hence,

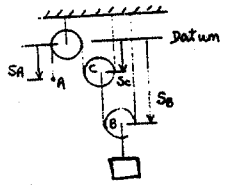
$$\mathbf{a} = [-200(0.324)^2]\mathbf{u}_r + 20\mathbf{u}_\theta$$

$$= \{-20.9\mathbf{u}_r + 20\mathbf{u}_\theta\} \text{ m/s}^2$$

$a = \sqrt{(-20.9)^2 + (20)^2} = 29.0$ m/s^2 Ans

R1-23. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.





Two cords :

$s_A + 2s_C = l$

$s_B + (s_B - s_C) = l'$

Thus, $v_A = -2v_C$

$2v_B = v_C$

$4v_B = -v_A$

$v_B = \frac{-2}{4} = -0.5$ m/s $= 0.5$ m/s $\uparrow$ Ans

***R1-24.** A rifle has a mass of 2.5 kg. If it is loosely gripped and a 1.5-g bullet is fired from it with a horizontal muzzle velocity of 1400 m/s, determine the recoil velocity of the rifle just after firing.

$\overset{+}{\rightarrow} \quad \Sigma mv_1 = \Sigma mv_2$

$0 + 0 = 0.0015(1400) - 2.5(v_R)_2$

$(v_R)_2 = 0.840$ m/s Ans

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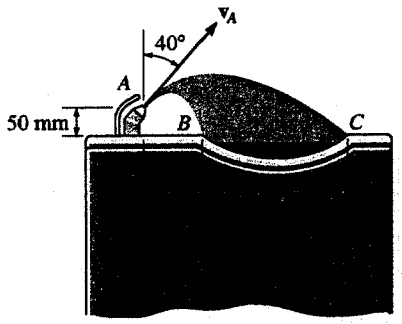
R1-25. The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



$$(\overset{+}{\rightarrow})\ s_x = v_x t$$

$$R = v_A \sin 40°\, t \qquad t = \frac{R}{v_A \sin 40°} \qquad [1]$$

$$(+\uparrow)\ s_y = (s_y)_0 + v_y t + \tfrac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40° t + \tfrac{1}{2}(-9.81) t^2 \qquad [2]$$

Substituting Eq.[1] into [2] yields :

$$-0.05 = v_A \cos 40° \left(\frac{R}{v_A \sin 40°}\right) + \tfrac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40°}\right)^2$$

$$v_A = \sqrt{\frac{4.905 \sin 40° R^2}{\sin^2 40° (R \cos 40° + 0.05 \sin 40°)}}$$

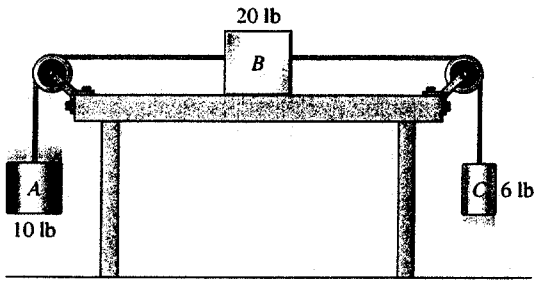
At point B, $R = 0.1$ m.

$$v_A = \sqrt{\frac{4.905 \sin 40° (0.1)^2}{\sin^2 40° (0.1 \cos 40° + 0.05 \sin 40°)}} = 0.838 \text{ m/s} \qquad \textbf{Ans}$$

At point C, $R = 0.35$ m.

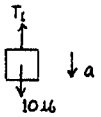
$$v_A = \sqrt{\frac{4.905 \sin 40° (0.35)^2}{\sin^2 40° (0.35 \cos 40° + 0.05 \sin 40°)}} = 1.76 \text{ m/s} \qquad \textbf{Ans}$$

R1-26. The 20-lb block B rests on the surface of a table for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the speed of the 10-lb block A after it has moved downward 2 ft from rest. Neglect the mass of the pulleys and cords.



Block A :

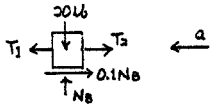
$$+\downarrow \Sigma F_y = ma_y; \qquad -T_1 + 10 = \frac{10}{32.2} a \qquad (1)$$



Block B :

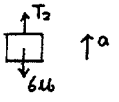
$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T_2 + T_1 - 0.1 N_B = \frac{20}{32.2} a \qquad (2)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B - 20 = 0 \qquad (3)$$



Block C :

$$+\uparrow \Sigma F_y = ma_y; \qquad T_2 - 6 = \frac{6}{32.2} a \qquad (4)$$



Solving Eqs.(1) – (4) for a,

$$a = 1.79 \text{ ft/s}^2$$

$$(+\downarrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(1.79)(2 - 0)$$

$$v = 2.68 \text{ ft/s} \qquad \textbf{Ans}$$

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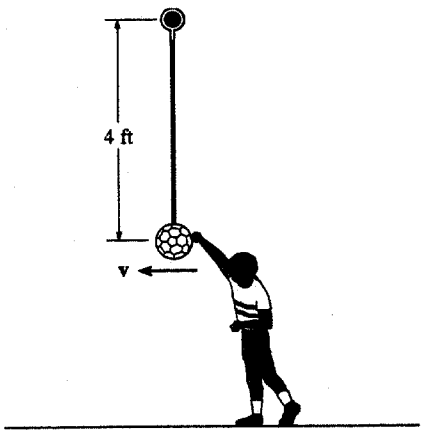
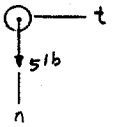
R1-27. The 5-lb ball, attached to the cord, is struck by the boy. Determine the smallest speed he must impart to the ball so that it will swing around in a vertical circle, without causing the cord to become slack.



$+\downarrow \Sigma F_n = ma_n;\quad 5 = \frac{5}{32.2}\left(\frac{v_2^2}{4}\right)\quad v_2^2 = 128.8\ \text{ft}^2/\text{s}^2$

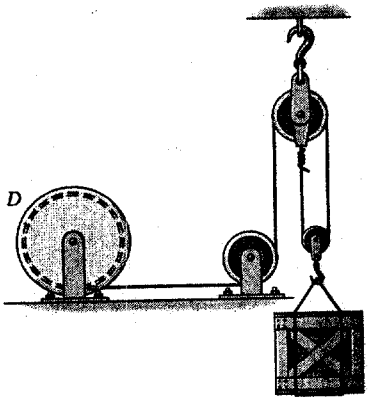


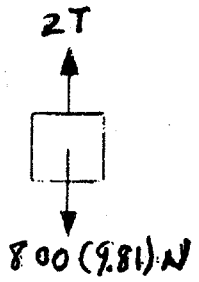
$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}\left(\frac{5}{32.2}\right)v^2 + 0 = \frac{1}{2}\left(\frac{5}{32.2}\right)(128.8) + 5(8)$

$v = 25.4\ \text{ft/s}$ **Ans**

***R1-26.** The winding drum D is drawing in the cable at an accelerated rate of 5 m/s^2. Determine the cable tension if the suspended crate has a mass of 800 kg.





$s_A + 2s_B = l$

$a_A = -2a_B$

$5 = -2a_B$

$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2\ \uparrow$

$+\uparrow \Sigma F_y = ma_y;\quad 2T - 800(9.81) = 800(2.5)$

$T = 4924\ \text{N} = 4.92\ \text{kN}$ **Ans**

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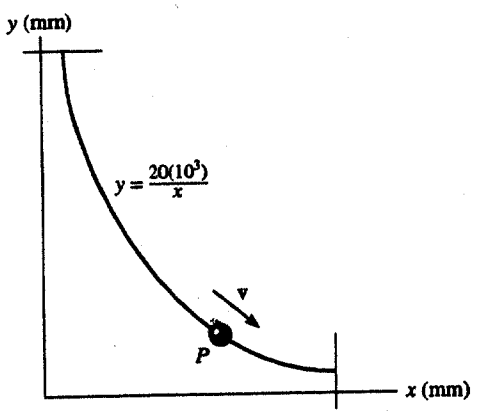
R1-29. The particle P travels with a constant speed of 300 mm/s along the curve. Determine its acceleration when it is located at point (200 mm, 100 mm).



$v = 300$ mm/s

$a_t = \frac{dv}{dt} = 0$

$y = \frac{2(10^4)}{x}$

$\left.\frac{dy}{dx}\right|_{x=200} = \frac{-2(10^4)}{x^2} = -0.5$

$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{4(10^4)}{x^3} = 5(10^{-3})$

$\rho = \left|\frac{[1+(-0.5)^2]^{3/2}}{5(10^{-3})}\right| = 279.5$ mm

$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322$ mm/s^2

$a = \sqrt{a_t^2 + a_n^2} = 322$ mm/s^2

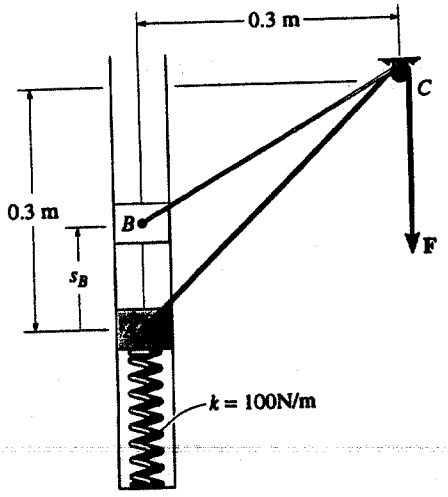
Since $\frac{dy}{dx} = -0.5$,

$\theta = \tan^{-1}(-0.5) = -26.6°$

Thus,

$a = 322$ m/s^2 $\angle 26.6°$ Ans

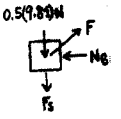
R1-30. The block has a mass of 0.5 kg and moves within the smooth vertical slot. If the block starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the mass of the cord and pulley.



The work done by F depends upon the difference in the cord length $AC - BC$.

$T_A + \Sigma U_{A-B} = T_B$

$0 + F[\sqrt{(0.3)^2 + (0.3)^2} - \sqrt{(0.3)^2 + (0.3-0.15)^2}\,] - 0.5(9.81)(0.15)$
$-\frac{1}{2}(100)(0.15)^2 = \frac{1}{2}(0.5)(2.5)^2$



$F(0.0889) = 3.423$

$F = 38.5$ N Ans

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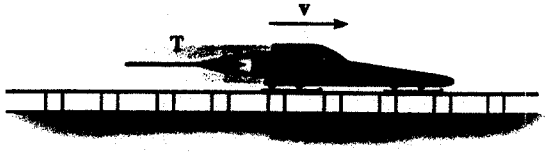
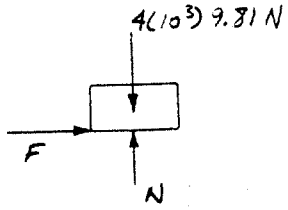
***R1-31.** The rocket sled has a mass of 4 Mg and travels from rest along the smooth horizontal track such that it maintains a constant power output of 450 kW. Neglect the loss of fuel mass and air resistance, and determine how far it must travel to reach a speed of $v = 60$ m/s.





$$\stackrel{+}{\rightarrow} \Sigma F_x = m a_x; \quad F = m a = m\left(\frac{v\,dv}{ds}\right)$$

$$P = F v = m\left(\frac{v^2\,dv}{ds}\right)$$

$$\int P\,ds = m\int v^2\,dv$$

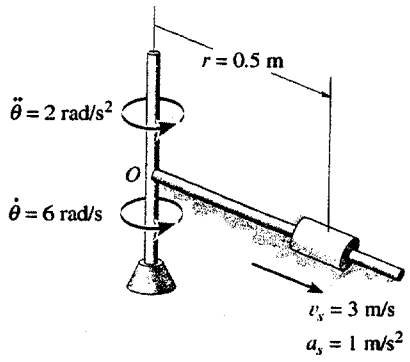
$$P\int_0^s ds = m\int_0^v v^2\,dv$$

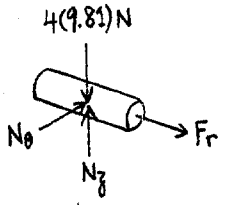
$$Ps = \frac{m v^3}{3}$$

$$s = \frac{m v^3}{3P}$$

$$s = \frac{4(10^3)(60)^3}{3(450)(10^3)} = 640 \text{ m} \qquad \textbf{Ans}$$

***R1-32.** The spool, which has a mass of 4 kg, slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $\dot{\theta} = 6$ rad/s and this rotation is increasing at $\ddot{\theta} = 2$ rad/s^2. At this same instant, the spool has a velocity of 3 m/s and an acceleration of 1 m/s^2, both measured relative to the rod and directed away from the center O when $r = 0.5$ m. Determine the radial frictional force and the normal force, both exerted by the rod on the spool at this instant.





$r = 0.5$ m

$\dot{r} = 3$ m/s $\qquad \dot{\theta} = 6$ rad/s

$\ddot{r} = 1$ m/s^2 $\qquad \ddot{\theta} = 2$ rad/s

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1 - 0.5(6)^2 = -17$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(2) + 2(3)(6) = 37$$

$$\Sigma F_r = ma_r; \quad F_r = 4(-17) = -68 \text{ N}$$

$$\Sigma F_\theta = ma_\theta; \quad N_\theta = 4(37) = 148 \text{ N}$$

$$\Sigma F_z = ma_z; \quad N_z - 4(9.81) = 0$$

$$N_z = 39.24 \text{ N}$$

$$F_r = -68 \text{ N} \qquad \textbf{Ans}$$

$$N_C = \sqrt{(148)^2 + (39.24)^2} = 153 \text{ N} \qquad \textbf{Ans}$$

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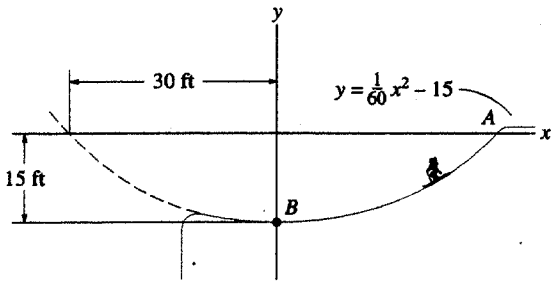
R1-33. A skier starts from rest at A (30 ft, 0) and descends the smooth slope, which may be approximated by a parabola. If she has a weight of 120 lb, determine the normal force she exerts on the ground at the instant she arrives at point B.



$T_A + V_A = T_B + V_B$

$0 + (120)(15) = \frac{1}{2}(\frac{120}{32.2})v_B^2 + 0$

$v_B = 31.08$ ft/s

$y = \frac{1}{60}x^2 - 15$

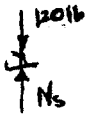
$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{1}{30}x\right|_{x=0} = 0$

$\frac{d^2y}{dx^2} = \frac{1}{30}$

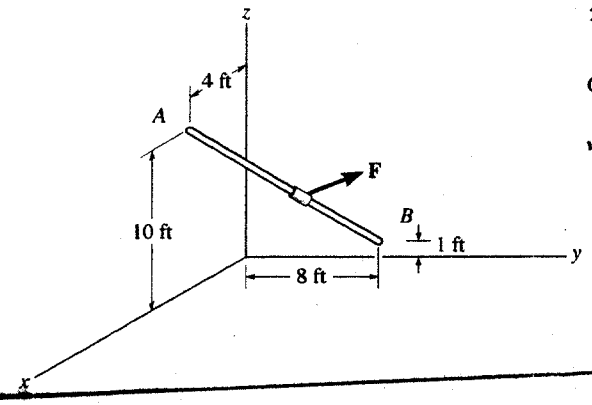
$\rho = \left|\frac{[1+(\frac{dy}{dx})^2]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \frac{(1+0)^{3/2}}{\frac{1}{30}} = 30$ ft

$+\uparrow \Sigma F_n = ma_n; \quad N_s - 120 = \frac{120}{32.2}[\frac{(31.08)^2}{30}]$

$N_s = 240$ lb Ans



R1-34. The small 2-lb collar starting from rest at A slides down along the smooth rod. During the motion, the collar is acted upon by a force $\mathbf{F} = \{10\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k}\}$ lb, where x, y, z are in feet. Determine the collar's speed when it strikes the wall at B.



$r_{AB} = r_B - r_A = -4\mathbf{i} + 8\mathbf{j} - 9\mathbf{k}$

$T_1 + \Sigma\int F ds = T_2$

$0 + 2(10-1) + \int_4^0 10dx + \int_0^8 6y\,dy + \int_{10}^1 2z\,dz = \frac{1}{2}(\frac{2}{32.2})v_B^2$

$v_B = 47.8$ ft/s Ans

R1-35. A ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

Just before impact

$T_1 + T_2 = T_2 + V_2$

$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)(v_2)^2 + 0$

$v_2 = 2.80$ m/s

Just after impact

$T_3 + V_3 = T_4 + V_4$

$\frac{1}{2}(0.2)(v_3)^2 + 0 = 0 + 0.2(9.81)(0.325)$

$v_3 = 2.53$ m/s

$e = \frac{(v_{rel})_2}{(v_{rel})_1} = \frac{2.53}{2.80} = 0.901$ Ans

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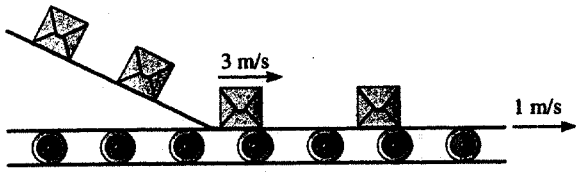
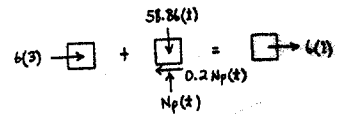
***R1-36.** Packages having a mass of 6 kg slide down a smooth chute and land horizontally with a speed of 3 m/s on the surface of a conveyor belt. If the coefficient of kinetic friction between the belt and a package is $\mu_k = 0.2$, determine the time needed to bring the package to rest on the belt if the belt is moving in the same direction as the package with a speed $v = 1$ m/s.





$(+\uparrow)$ $m(v_1)_y + \Sigma \int F_y\,dt = m(v_2)_y$

$0 + N_p(t) - 58.86(t) = 0$

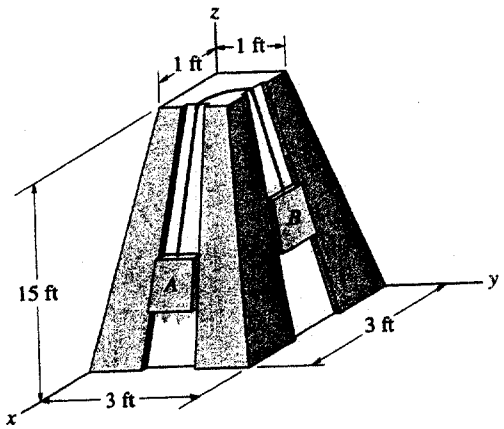
$N_p = 58.86$ N

$(\overset{+}{\rightarrow})$ $m(v_1)_x + \Sigma \int F_x\,dt = m(v_2)_x$

$6(3) - 0.2(58.86)(t) = 6(1)$

$t = 1.02$ s **Ans**

R1-37. The blocks A and B weigh 10 and 30 lb, respectively. They are connected together by a light cord and ride in the frictionless grooves. Determine the speed of each block after block A moves 6 ft up along the plane. The blocks are released from rest.



$$\frac{6}{z} = \frac{\sqrt{15^2 + 2^2}}{15}$$

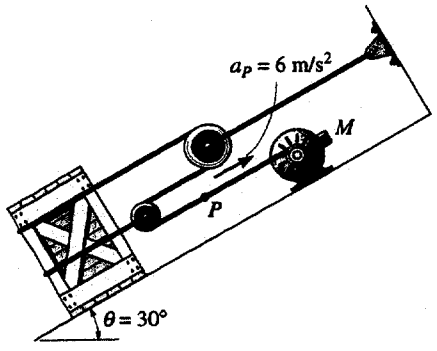
$z = 5.95$ ft

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}\left(\frac{10}{32.2}\right)v_2^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)v_2^2 + 10(5.95) - 30(5.95)$$

$v_2 = 13.8$ ft/s **Ans**

R1-38. The motor M pulls in its attached rope with an acceleration $a_p = 6$ m/s^2. Determine the towing force exerted by M on the rope in order to move the 50-kg crate up the inclined plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and rope.



$\nwarrow + \Sigma F_y = ma_y$; $N_C - 50(9.81)\cos 30° = 0$

$N_C = 424.79$

$\nearrow + \Sigma F_x = ma_x$; $3T - 0.3(424.79) - 50(9.81)\sin 30° = 50a_C$ (1)

Kinematics $2s_C + (s_C - s_P) = l$

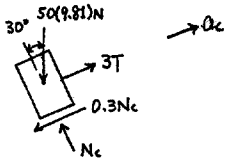
Taking two time derivatives, yields

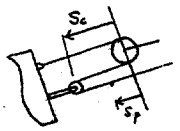
$3a_C = a_P$

Thus, $a_C = \frac{6}{3} = 2$

Substituting into Eq.(1) and solving,

$T = 158$ N **Ans**





R1-39. If a particle has an initial velocity $v_0 = 12$ ft/s to the right, and a constant acceleration of 2 ft/s^2 to the left, determine the particle's displacement in 10 s. Originally $s_0 = 0$.

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 12(10) + \frac{1}{2}(-2)(10)^2$

$s = 20.0$ ft **Ans**

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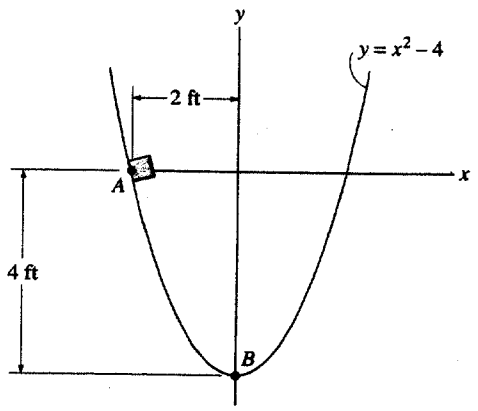
***R1-40.** A 3-lb block, initially at rest at point A, slides along the smooth parabolic surface. Determine the normal force acting on the block when it reaches B. Neglect the size of the block.



$T_1 + V_1 = T_2 + V_2$

$0 + 3(4) = \frac{1}{2}(\frac{3}{32.2})v_2^2 + 0$

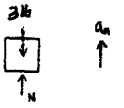
$v_2 = 16.05$ ft/s

$\rho = \frac{[1+(\frac{dy}{dx})^2]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1+(2x)^2]^{3/2}}{2}$

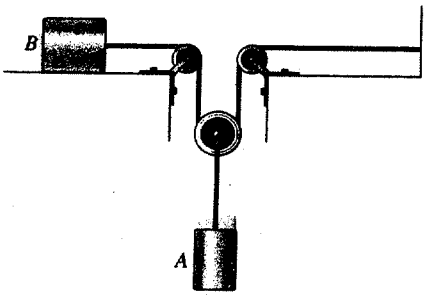
At $x = 0$, $\rho = 0.5$ ft

$+\uparrow \Sigma F_n = m a_n$; $N - 3 = \frac{3}{32.2}[\frac{(16.05)^2}{0.5}]$

$N = 51.0$ lb Ans



R1-41. At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



Block A:

$+\downarrow \Sigma F_y = m a_y$; $10 - 2T = \frac{10}{32.2} a_A$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = m a_x$; $-T + 0.2(4) = \frac{4}{32.2} a_B$

Kinematics:

$2s_A + s_B = l$

$2a_A = -a_B$

Solving,

$T = 3.38$ lb

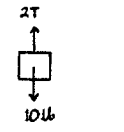
$a_A = 10.40$ ft/s^2

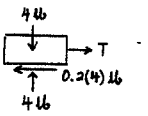
$a_B = -20.81$ ft/s^2

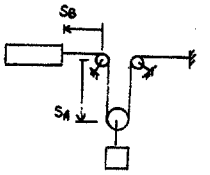
$(+\downarrow)$ $v_A = (v_A)_0 + a_A t$

$v_A = 6 + 10.40(2)$

$v_A = 26.8$ ft/s $\downarrow$ Ans







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R1-42. A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v-t graph for the motion.

$0 \le t \le t'$

$a_c = 0.5$ ft/s^2

$(\overset{+}{\rightarrow}) v = v_0 + a_c t$

$v' = 0 + 0.5t'$ (1)

$(\overset{+}{\rightarrow}) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$x = 0 + 0 + \frac{1}{2}(0.5)(t')^2$ (2)

$t' < t < 160$ s

$(\overset{+}{\rightarrow}) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

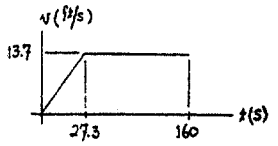
$2000 = x + v'(160 - t') + 0$ (3)

Solving Eqs. (1)–(3) for $t' < 160$ s yields

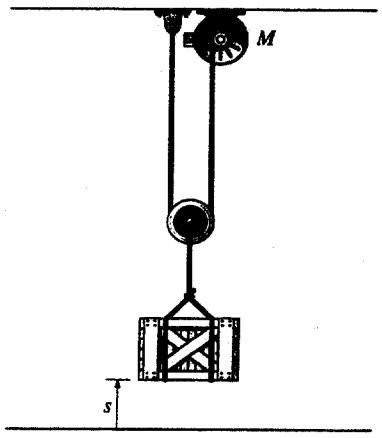
$t' = 27.3$ s Ans

$v' = 13.7$ ft/s

$x = 187$ ft



R1-43. The crate, having a weight of 50 lb, is hoisted by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 12 ft/s after rising 10 ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\epsilon = 0.74$.



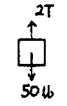
$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$(12)^2 = 0 + 2a_c(10 - 0)$

$a_c = 7.20$ ft/s^2

$+\uparrow \Sigma F_y = ma_y$; $2T - 50 = \frac{50}{32.2}(7.20)$

$T = 30.6$ lb



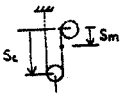
$s_C + (s_C - s_M) = l$

$v_M = 2v_C$

$v_M = 2(12) = 24$ ft/s

$P_0 = \mathbf{T} \cdot \mathbf{v} = 30.6(24) = 734.2$ lb·ft/s

$P_i = \frac{734.2}{0.74} = 992.1$ lb·ft/s $= 1.80$ hp Ans



***R1-44.** An automobile is traveling with a *constant speed* along a horizontal circular curve that has a radius $\rho = 750$ ft. If the magnitude of acceleration is $a = 8$ ft/s^2, determine the speed at which the automobile is traveling.

$a = a_n = 8 = \frac{v^2}{\rho}$

$8 = \frac{v^2}{750}$

$v = 77.4$ ft/s Ans

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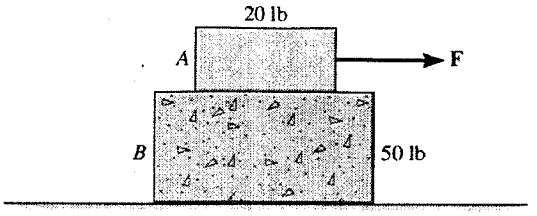
R1-45. Block B rests on a smooth surface. If the coefficients of friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, determine the acceleration of each block if (a) $F = 6$ lb, and (b) $F = 50$ lb.



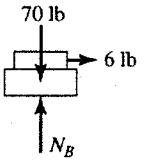
a) The maximum friction force between blocks A and B is

$F_{max} = 0.4(20) = 8 \text{ lb} > 6 \text{ lb}$

Thus, both blocks move together.

$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 6 = \frac{70}{32.2}a$

$a_B = a_A = a = 2.76 \text{ ft/s}^2$ **Ans**

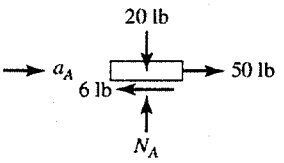


b) In this case $8 \text{ lb} < F = 50 \text{ lb}$

Block A:

$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 20(0.3) = \frac{20}{32.2}a_A$

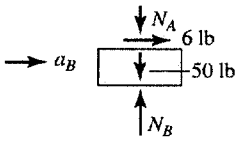
$a_A = 70.8 \text{ ft/s}^2$ **Ans**



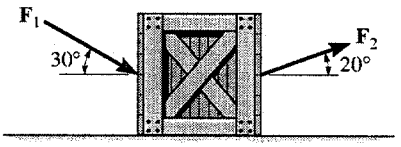
Block B:

$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 20(0.3) = \frac{50}{32.2}a_B$

$a_B = 3.86 \text{ ft/s}^2$ **Ans**

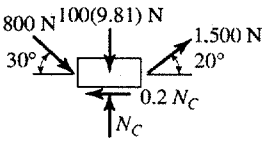


R1-46. The 100-kg crate is subjected to the action of two forces, $F_1 = 800$ N and $F_2 = 1.5$ kN, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



$+\uparrow \sum F_y = 0; \quad N_C - 800 \sin 30° - 100(9.81) + 1500 \sin 20° = 0$

$N_C = 867.97 \text{ N}$



$T_1 + \sum U_{1-2} = T_2$

$0 + 800 \cos 30°(s) - 0.2(867.97)(s) + 1500 \cos 20°(s) = \frac{1}{2}(100)(6)^2$

$s(1928.7) = 1800$

$s = 0.933 \text{ m}$ **Ans**

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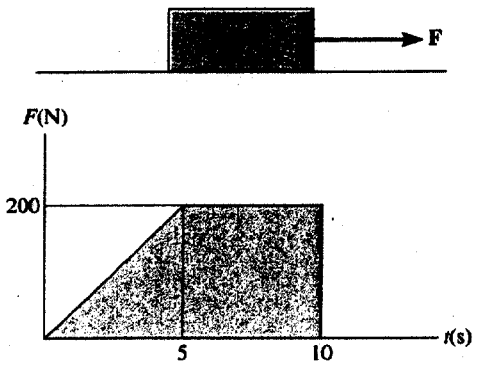
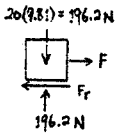
R1-47. A 20-kg block is originally at rest on a horizontal surface for which the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.5$. If a horizontal force F is applied such that it varies with time as shown, determine the speed of the block in 10 s. *Hint:* First determine the time needed to overcome friction and start the block moving.





The crate starts moving when

$F = F_r = 0.6(196.2) = 117.72$ N

From the graph since

$$F = \frac{200}{5}t, \quad 0 \le t \le 5 \text{ s}$$

The time needed for the crate to start moving is

$$t = \frac{5}{200}(117.72) = 2.943 \text{ s}$$

Hence, the impulse due to F is equal to the area under the curve from 2.943 s $\le t \le$ 10 s

$$\overset{+}{\rightarrow} \quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

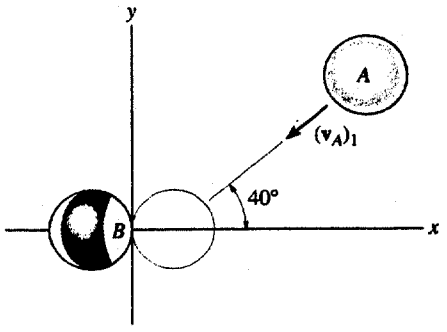
$$0 + \int_{2.943}^{5} \frac{200}{5}t\,dt + \int_{5}^{10} 200\,dt - (0.5)196.2(10-2.943) = 20v_2$$

$$40\left(\frac{1}{2}t^2\right)\Big|_{2.943}^{5} + 200(10-5) - 692.292 = 20v_2$$

$$634.483 = 20v_2$$

$$v_2 = 31.7 \text{ m/s} \qquad \text{Ans}$$

***R1-48.** Two smooth billiard balls A and B have an equal mass of $m = 200$ g. If A strikes B with a velocity of $(v_A)_1 = 2$ m/s as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.75$.



$$(v_A)_{x_1} = -2\cos 40° = -1.532 \text{ m/s}$$

$$(v_A)_{y_1} = -2\sin 40° = -1.285 \text{ m/s}$$

$$(\overset{+}{\rightarrow}) \quad m_A(v_A)_{x_1} + m_B(v_B)_{x_1} = m_A(v_A)_{x_2} + m_B(v_B)_{x_2}$$

$$-2(1.532) + 0 = 0.2(v_A)_{x_2} + 0.2(v_B)_{x_2} \qquad (1)$$

$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_{rel})_2}{(v_{rel})_1}$$

$$0.75 = \frac{(v_A)_{x_2} - (v_B)_{x_2}}{1.532} \qquad (2)$$

Solving Eqs.(1) and (2)

$$(v_A)_{x_2} = -0.1915 \text{ m/s}$$

$$(v_B)_{x_2} = -1.3405 \text{ m/s}$$

For A :

$$(+\downarrow) \quad m_A(v_A)_{y_1} = m_A(v_A)_{y_2}$$

$$(v_A)_{y_2} = 1.285 \text{ m/s}$$

For B :

$$(+\uparrow) \quad m_B(v_B)_{y_1} = m_B(v_B)_{y_2}$$

$$(v_B)_{y_2} = 0$$

Hence $(v_B)_2 = (v_B)_{x_2} = 1.34$ m/s $\leftarrow$ Ans

$$(v_A)_2 = \sqrt{(-0.1915)^2 + (1.285)^2} = 1.30 \text{ m/s} \qquad \text{Ans}$$

$$(\theta_A)_2 = \tan^{-1}\left(\frac{0.1915}{1.285}\right) = 8.47° \qquad \text{Ans}$$

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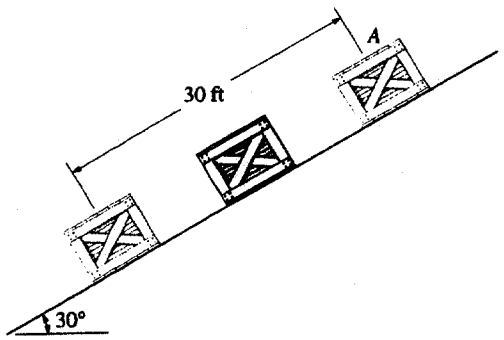
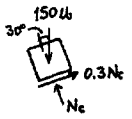
R1-49. If a 150-lb crate is released from rest at A, determine its speed after it slides 30 ft down the plane. The coefficient of kinetic friction between the crate and plane is $\mu_k = 0.3$.



$+\nwarrow \Sigma F_y = 0; \quad N_C - 150\cos 30° = 0$

$N_C = 129.9$ lb

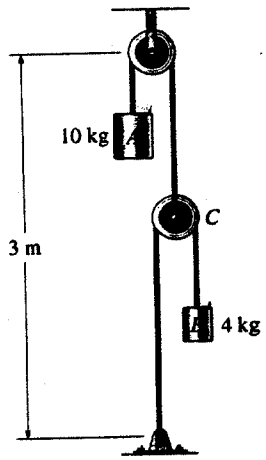


$T_1 + \Sigma U_{1-2} = T_2$

$0 + 150\sin 30°(30) - (0.3)129.9(30) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_2^2$

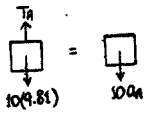
$v_2 = 21.5$ ft/s **Ans**

R1-50. Determine the tension developed in the two cords and the acceleration of each block. Neglect the mass of the pulleys and cords. *Hint:* Since the system consists of *two* cords, relate the motion of block A to C, and of block B to C. Then, by elimination, relate the motion of A to B.



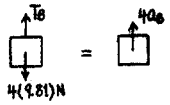
Block A :

$+\downarrow \Sigma F_y = ma_y; \quad 10(9.81) - T_A = 10a_A$ (1)



Block B :

$+\uparrow \Sigma F_y = ma_y; \quad T_B - 4(9.81) = 4a_B$ (2)



Pulley C :

$+\uparrow \Sigma F_y = 0; \quad T_A - 2T_B = 0$ (3)

Kinematics:

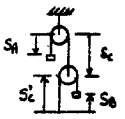
$s_A + s_C = l$

Taking the two time derivatives :

$a_A = -a_C$

Also,

$s_C' + (s_C' - s_B) = l'$



So that $2a_C' = a_B$

Since $a_C' = -a_C$,

$a_B = 2a_A$ (4)

Solving Eqs.(1) – (4),

$a_A = 0.755$ m/s^2 **Ans**

$a_B = 1.51$ m/s^2 **Ans**

$T_A = 90.6$ N **Ans**

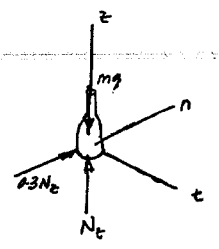
$T_B = 45.3$ N **Ans**

R1-51. The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.

$\Sigma F_z = ma_z; \quad N_z - mg = 0 \quad N_z = mg$

$\Sigma F_n = ma_n; \quad 0.3(mg) = m\left(\frac{v^2}{r}\right)$

$v = \sqrt{0.3gr} = \sqrt{0.3(32.2)(3)} = 5.38$ ft/s **Ans**



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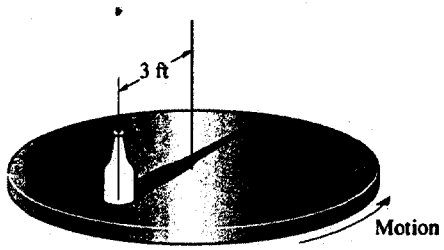
***R1-52.** Work Prob. R1–51 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.

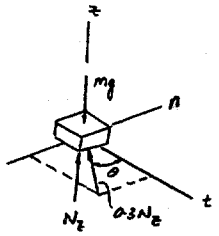


$\Sigma F_z = ma_z$; $N_z - mg = 0$ $N_z = mg$

$\Sigma F_t = ma_t$; $-0.3(mg)\cos\theta = -m(2)$ $\theta = 78.05°$

$\Sigma F_n = ma_n$; $0.3(mg)\sin 78.05° = m\left(\frac{v^2}{3}\right)$

$v = 5.32$ ft/s **Ans**



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16-1. A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(15)^2 = (10)^2 + 2(3)(\theta - 0)$$

$$\theta = 20.83 \text{ rad} = 20.83\left(\frac{1}{2\pi}\right) = 3.32 \text{ rev.} \quad \text{Ans}$$

$$\omega = \omega_0 + \alpha_c t$$

$$15 = 10 + 3t$$

$$t = 1.67 \text{ s} \quad \text{Ans}$$

16-2. A flywheel has its angular speed increased uniformly from 15 rad/s to 60 rad/s. in 80s. If the diameter of the wheel is 2 ft, determine the magnitudes of the normal and tangential components of acceleration of a point on the rim of the wheel when $t = 80$ s, and the total distance the point travels during the time period.

$$\omega = \omega_0 + \alpha_c t$$

$$60 = 15 + \alpha_c(80)$$

$$\alpha_c = 0.5625 \text{ rad/s}^2$$

$$a_t = \alpha r = (0.5625)(1) = 0.562 \text{ ft/s}^2 \quad \text{Ans}$$

$$a_n = \omega^2 r = (60)^2(1) = 3600 \text{ ft/s}^2 \quad \text{Ans}$$

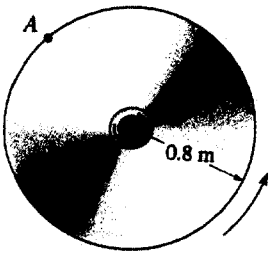
$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(60)^2 = (15)^2 + 2(0.5625)(\theta - 0)$$

$$\theta = 3000 \text{ rad}$$

$$s = \theta r = 3000(1) = 3000 \text{ ft} \quad \text{Ans}$$

16-3. The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.



$$\omega = (5t^2 + 2) \text{ rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10t$$

$$t = 0.5 \text{ s}$$

$$\omega = 3.25 \text{ rad/s}$$

$$\alpha = 5 \text{ rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60 \text{ m/s} \quad \text{Ans}$$

$$a_t = \alpha r = 5(0.8) = 4 \text{ m/s}^2$$

$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45 \text{ m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35 \text{ m/s}^2 \quad \text{Ans}$$

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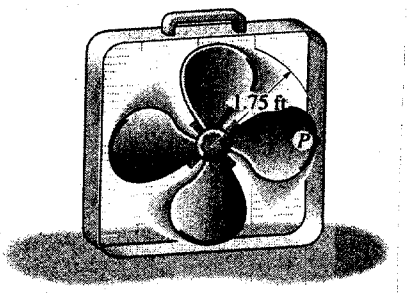
***16-4.** Just after the fan is turned on, the motor gives the blade an angular acceleration $\alpha = (20e^{-0.6t})$ rad/s^2, where t is in seconds. Determine the speed of the tip P of one of the blades when $t = 3$ s. How many revolutions has the blade turned in 3 s? When $t = 0$ the blade is at rest.



$d\omega = \alpha\, dt$

$$\int_0^{\omega} d\omega = \int_0^{t} 20e^{-0.6t}\, dt$$

$$\omega = -\frac{20}{0.6}e^{-0.6t}\Big|_0^{t} = 33.3\left(1 - e^{-0.6t}\right)$$

$\omega = 27.82$ rad/s

$v_P = \omega r = 27.82(1.75) = 48.7$ ft/s **Ans**

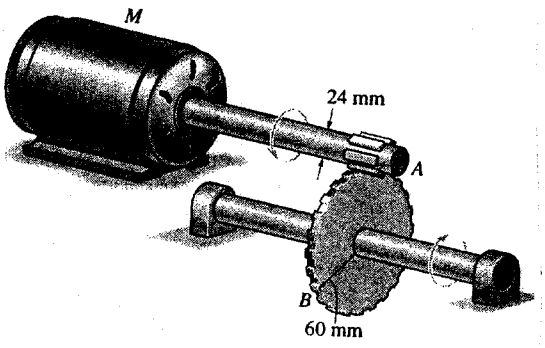
$d\theta = \omega\, dt$

$$\int_0^{\theta} d\theta = \int_0^{t} 33.3\left(1 - e^{-0.6t}\right) dt$$

$$\theta = 33.3\left(t + \left(\frac{1}{0.6}\right)e^{-0.6t}\right)\Big|_0^{3} = 33.3\left[3 + \left(\frac{1}{0.6}\right)\left(e^{-0.6(3)} - 1\right)\right]$$

$\theta = 53.63$ rad $= 8.54$ rev **Ans**

16-5. Due to an increase in power, the motor M rotates the shaft A with an angular acceleration of $\alpha = (0.06\theta^2)$ rad/s^2, where θ is in radians. If the shaft is initially turning at $\omega_0 = 50$ rad/s, determine the angular velocity of gear B after the shaft undergoes an angular displacement $\Delta\theta = 10$ rev.



$\omega\, d\omega = \alpha\, d\theta$

$$\int_{50}^{\omega} \omega\, d\omega = \int_0^{2\pi(10)} 0.06\theta^2\, d\theta$$

$$\frac{1}{2}\omega^2\Big|_{50}^{\omega} = 0.02\theta^3\Big|_0^{2\pi(10)}$$

$0.5\omega^2 - 1250 = 4961$

$\omega = 111.45$ rad/s

$\omega_A r_A = \omega_B r_B$

$(111.45)(12) = \omega_B(60)$

$\omega_B = 22.3$ rad/s **Ans**

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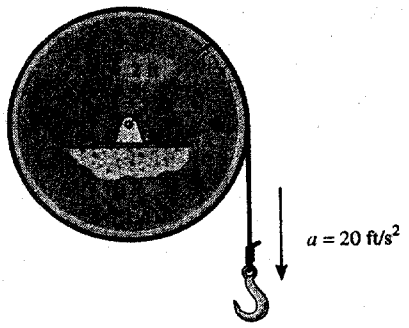
16-6. The hook moves from rest with an acceleration of 20 ft/s^2. If it is attached to a cord which is wound around the drum, determine the angular acceleration of the drum and its angular velocity after the drum has completed 10 rev. How many more revolutions will the drum turn after it has first completed 10 rev and the hook continues to move downward for 4 s?



Angular Motion : The angular acceleration of the drum can be determine by applying Eq. 16 – 11.

$$a_t = \alpha r, \qquad 20 = \alpha(2) \qquad \alpha = 10.0 \text{ rad/s}^2 \qquad \textbf{Ans}$$

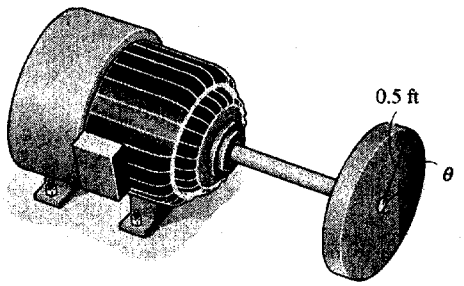
Applying Eq. 16 – 7 with $\alpha_c = \alpha = 10.0$ rad/s^2 and $\theta = (10 \text{ rev}) \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ $= 20\pi$ rad, we have

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$
$$\omega^2 = 0 + 2(10.0)(20\pi - 0)$$
$$\omega = 35.45 \text{ rad/s} = 35.4 \text{ rad/s} \qquad \textbf{Ans}$$

The angular displacement of the drum 4 s after it has completed 10 revolutions can be determine by applying Eq. 16 – 6 with $\omega_0 = 35.45$ rad/s.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$
$$= 0 + 35.45(4) + \frac{1}{2}(10.0)\left(4^2\right)$$
$$= (221.79 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 35.3 \text{ rev} \qquad \textbf{Ans}$$

16-7. The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



Angular Displacement : At $t = 90$ s,

$$\theta = 20(90) + 4\left(90^2\right) = (34200 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev} \qquad \textbf{Ans}$$

Angular Velocity : Applying Eq. 16 – 1, we have

$$\omega = \frac{d\theta}{dt} = 20 + 8t\Big|_{t=90s} = 740 \text{ rad/s} \qquad \textbf{Ans}$$

Angular Acceleration : Applying Eq. 16 – 2, we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2 \qquad \textbf{Ans}$$

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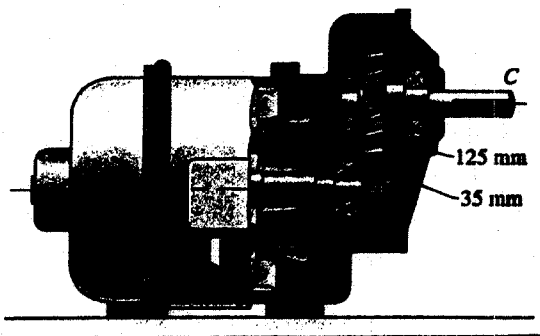
***16-8.** The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3\ \text{rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2$ s starting from rest. The shaft is fixed to B and turns with it.



$\omega = \omega_0 + \alpha_c t$

$\omega_A = 0 + 3(2) = 6\ \text{rad/s}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

$\theta_A = 0 + 0 + \frac{1}{2}(3)(2)^2$

$\theta_A = 6\ \text{rad}$

$\omega_A r_A = \omega_B r_B$

$6(35) = \omega_B(125)$

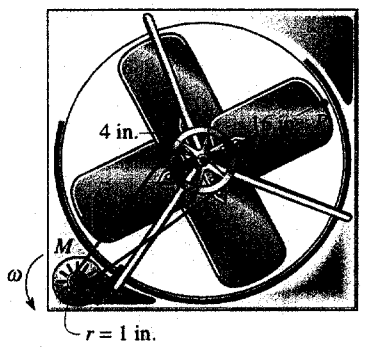
$\omega_C = \omega_B = 1.68\ \text{rad/s}$ **Ans**

$\theta_A r_A = \theta_B r_B$

$6(35) = \theta_B(125)$

$\theta_C = \theta_B = 1.68\ \text{rad}$ **Ans**

16-9. The motor M begins rotating at $\omega = 4(1 - e^{-t})$ rad/s, where t is in seconds. If the pulleys and fan have the radii shown, determine the magnitudes of the velocity and acceleration of point P on the fan blade when $t = 0.5$ s. Also, what is the maximum speed of this point?



$\omega_m = 4(1 - e^{-t})$

$\alpha_m = \frac{d\omega}{dt} = 4e^{-t}$

When $t = 0.5$ s

$\omega_m = 1.57388\ \text{rad/s}$

$\alpha_m = 2.4261\ \text{rad/s}$

$\omega_m r_m = \omega_p r_p$

$1.57388(1) = \omega_p(4)$

$\omega_p = 0.39347\ \text{rad/s}$

$v_p = 0.39347(16) = 6.30\ \text{in./s}$ **Ans**

$\alpha_m r_m = \alpha_p r_p$

$2.4261(1) = \alpha_p(4)$

$\alpha_p = 0.606525\ \text{rad/s}^2$

$a_t = \alpha r = 0.606525(16) = 9.7044\ \text{in./s}^2$

$a_n = \omega^2 r = (0.39347)^2(16) = 2.4771\ \text{in./s}^2$

$a_p = \sqrt{(9.7044)^2 + (2.4771)^2} = 10.0\ \text{in./s}^2$ **Ans**

As $t \to \infty$

$\omega_m = 4\ \text{rad/s}$

$4(1) = \omega_p(4)$

$\omega_p = 1\ \text{rad/s}$

$v_p = 1(16) = 16.0\ \text{in./s}$ **Ans**

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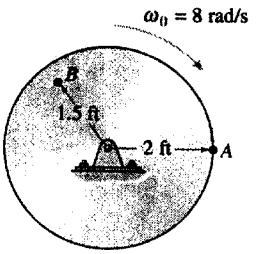
16-10. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 0.5$ s.



$\omega = \omega_0 + \alpha_c t$

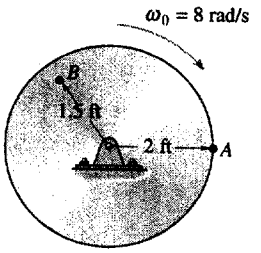
$\omega = 8 + 6(0.5) = 11$ rad/s

$v = r\omega;$ $\quad v_A = 2(11) = 22$ ft/s **Ans**

$a_t = r\alpha;$ $\quad (a_A)_t = 2(6) = 12.0$ ft/s^2 **Ans**

$a_n = \omega^2 r;$ $\quad (a_A)_n = (11)^2(2) = 242$ ft/s^2 **Ans**

16-11. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point B just after the wheel undergoes 2 revolutions.



$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

$\omega^2 = (8)^2 + 2(6)[2(2\pi) - 0]$

$\omega = 14.66$ rad/s

$v_B = \omega r = 14.66(1.5) = 22.0$ ft/s **Ans**

$(a_B)_t = \alpha r = 6(1.5) = 9.00$ ft/s^2 **Ans**

$(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322$ ft/s^2 **Ans**

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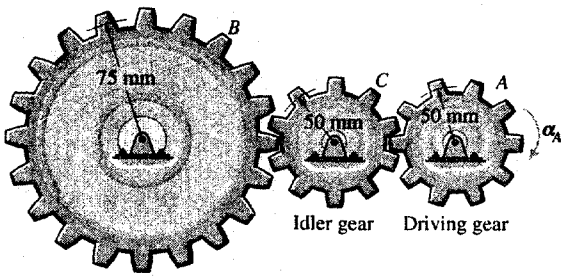
***16-12.** When only two gears are in mesh, the driving gear A and the driven gear B will always turn in opposite directions. In order to get them to turn in the *same direction* an idler gear C is used. In the case shown, determine the angular velocity of gear B when $t = 5$ s, if gear A starts from rest and has an angular acceleration of $\alpha_A = (3t + 2)$ rad/s^2, where t is in seconds.



$d\omega = \alpha\, dt$

$\int_0^{\omega_A} d\omega_A = \int_0^t (3t + 2)\, dt$

$\omega_A = 1.5t^2 + 2t\big|_{t=5} = 47.5$ rad/s

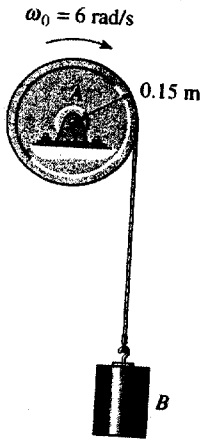
$(47.5)(50) = \omega_C(50)$

$\omega_C = 47.5$ rad/s

$\omega_B(75) = 47.5(50)$

$\omega_B = 31.7$ rad/s **Ans**

16-13. A motor gives disk A an angular acceleration of $\alpha_A = (0.6t^2 + 0.75)$ rad/s^2, where t is in seconds. If the initial angular velocity of the disk is $\omega_0 = 6$ rad/s, determine the magnitudes of the velocity and acceleration of block B when $t = 2$ s.



$d\omega = \alpha\, dt$

$\int_6^{\omega} d\omega = \int_0^2 (0.6t^2 + 0.75)\, dt$

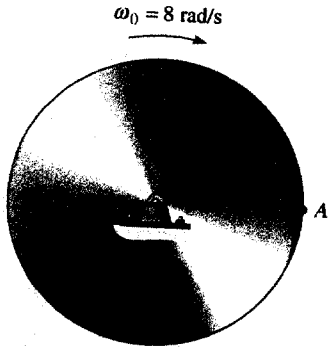
$\omega - 6 = (0.2t^3 + 0.75t)\big|_0^2$

$\omega = 9.10$ rad/s

$v_B = \omega r = 9.10(0.15) = 1.37$ m/s **Ans**

$a_B = a_t = \alpha r = [0.6(2)^2 + 0.75](0.15) = 0.472$ m/s^2 **Ans**

16-14. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration $\alpha_c = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 3$ s.



$\omega = \omega_0 + \alpha_c t$

$\omega = 8 + 6(3) = 26$ rad/s

$v = \omega r = 26(2) = 52.0$ ft/s **Ans**

$a_t = \alpha r = 6(2) = 12.0$ ft/s^2 **Ans**

$a_n = \omega^2 r = (26)^2(2) = 1.35(10^3)$ ft/s^2 **Ans**

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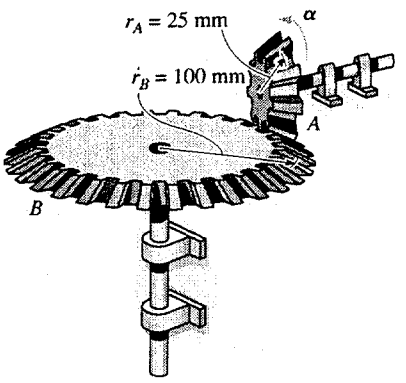
16-15. Gear A is in mesh with gear B as shown. If A starts from rest and has a constant angular acceleration of $\alpha_A = 2$ rad/s^2, determine the time needed for B to attain an angular velocity of $\omega_B = 50$ rad/s.



Angular Motion: The angular acceleration of gear B must be determined first. Here, $\alpha_A r_A = \alpha_B r_B$. Then,

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{25}{100}\right)(2) = 0.5 \text{ rad/s}^2$$

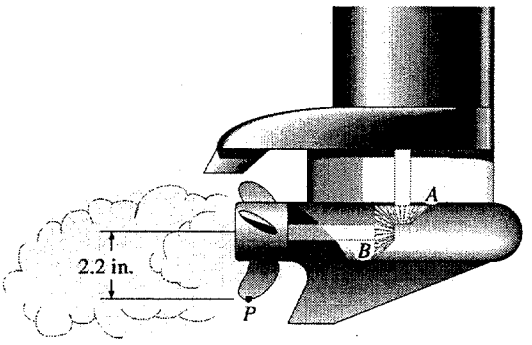
The time for gear B to attain an angular velocity of $\omega_B = 50$ rad/s can be obtained by applying Eq. 16-5.

$$\omega_B = (\omega_0)_B + \alpha_B t$$

$$50 = 0 + 0.5t$$

$$t = 100 \text{ s} \qquad \textbf{Ans}$$

***16-16.** The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.7$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.4$ in. Determine the angular velocity of the propeller in $t = 1.3$ s if the drive shaft rotates with an angular acceleration $\alpha = (300\sqrt{t})$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



$$\alpha_A r_A = \alpha_B r_B$$

$$(300\sqrt{t})(0.7) = \alpha_P(1.4)$$

$$\alpha_P = 150\sqrt{t}$$

$$d\omega = \alpha\, dt$$

$$\int_0^{\omega} d\omega = \int_0^t 150\sqrt{t}\, dt$$

$$\omega = 100t^{3/2}\Big|_{t=1.3} = 148 \text{ rad/s} \qquad \textbf{Ans}$$

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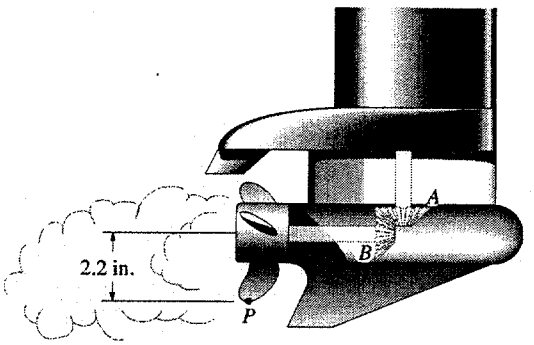
16-17. For the outboard motor in Prob. 16-16, determine the magnitudes of the velocity and acceleration of a point P located on the tip of the propeller at the instant $t = 0.75$ s.



$(a_P)_t = 285.8 \text{ in./s}^2$

$(a_P)_n = \omega^2 r = [100(0.75)^{3/2}]^2(2.2)$

$(a_P)_n = 9281 \text{ in./s}^2$

$a_P = \sqrt{(285.8)^2 + (9281)^2}$

$= 9285.6 = 9.29 \times 10^3 \text{ in./s}^2$ **Ans**

$\alpha_A r_A = \alpha_P r_B$

$(300\sqrt{t})(0.7) = \alpha_P(1.4)$

$\alpha_P = 150\sqrt{t}$

$d\omega = \alpha\, dt$

$\int_0^{\omega} d\omega = \int_0^t 150\sqrt{t}\, dt$

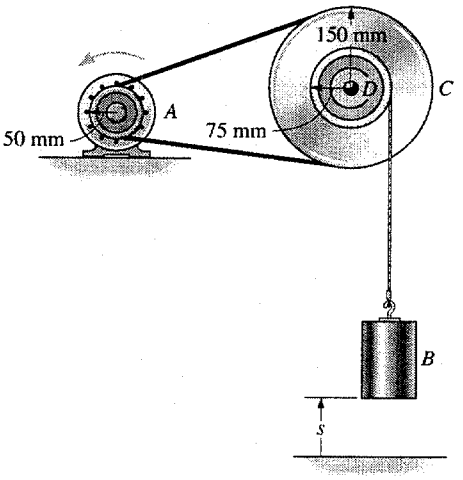
$\omega = 100t^{3/2}$

$v_P = \omega r = [100(0.75)^{3/2}](2.2)$

$v_P = 143 \text{ in./s}$ **Ans**

$(\alpha_P)_t = \alpha r = [150\sqrt{0.75}](2.2)$

16-18. Starting from rest when $s = 0$, pulley A is given an angular acceleration $\alpha = (6\theta)$ rad/s^2, where θ is in radians. Determine the speed of block B when it has risen $s = 6$ m. The pulley has an inner hub D which is fixed to C and turns with it.



$\alpha_A = 6\theta_A$

$\theta_C = \dfrac{6}{0.075} = 80 \text{ rad}$

$\theta_A(0.05) = 80(0.15)$

$\theta_A = 240 \text{ rad}$

$\alpha\, d\theta = \omega\, d\omega$

$\int_0^{240} 6\theta_A\, d\theta_A = \int_0^{\omega_A} \omega_A\, d\omega_A$

$\omega_A = [6(240)^2]^{1/2} = 587.88 \text{ rad/s}$

$(587.88)(0.05) = \omega_C(0.15)$

$\omega_C = 195.96$

$v_B = 195.96(0.075) = 14.7 \text{ m/s}$ **Ans**

Also.

$\alpha_A = 6\theta_A$

But $\alpha_A(50) = 150\alpha_C$

$\alpha_A = 3\alpha_C$

$3\alpha_C = 6\theta_A$

$\alpha_C = 2\theta_A$

But $\theta_A(50) = 150(\theta_C)$

$\theta_A = 3\theta_C$

Thus, $\alpha_C = 6\theta_C$

$\int_0^{\theta_C} 6\theta_C\, d\theta_C = \int_0^{\omega_C} \omega_C\, d\omega_C$

$6\theta_C^2 = \omega_C^2$

$\omega_C = \dfrac{6}{0.075} = 80 \text{ rad}$

$\omega_C = \sqrt{6}(80) = 195.96$

$v_B = (195.96)(0.075) = 14.7 \text{ m/s}$ **Ans**

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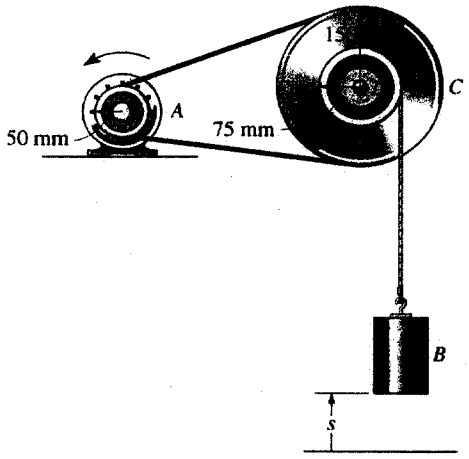
16-19. Starting from rest when $s = 0$, pulley A is given a constant angular acceleration $\alpha_c = 6$ rad/s^2. Determine the speed of block B when it has risen $s = 6$ m. The pulley has an inner hub D which is fixed to C and turns with it.



$\alpha_A r_A = \alpha_C r_C$

$6(50) = \alpha_C(150)$

$\alpha_C = 2$ rad/s^2

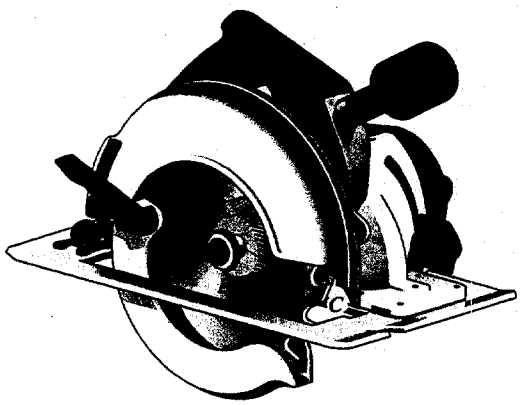
$a_B = \alpha_C r_B = 2(0.075) = 0.15$ m/s^2

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(0.15)(6-0)$

$v = 1.34$ m/s **Ans**

***16-20.** Initially the motor on the circular saw turns its drive shaft at $\omega = (20t^{2/3})$ rad/s, where t is in seconds. If the radii of gears A and B are 0.25 in. and 1 in., respectively, determine the magnitudes of the velocity and acceleration of a tooth C on the saw blade after the drive shaft rotates $\theta = 5$ rad starting from rest.



$\omega = 20t^{2/3}$

$\alpha = \frac{d\omega}{dt} = \frac{40}{3}t^{-1/3}$

$d\theta = \omega\, dt$

$\int_0^\theta d\theta = \int_0^t 20t^{2/3}dt$

$\theta = 20(\frac{3}{5})t^{5/3}$

When $\theta = 5$ rad,

$t = 0.59139$ s

$\alpha = 15.885$ rad/s^2

$\omega = 14.091$ rad/s

$\omega_A r_A = \omega_B r_B$

$14.091(0.25) = \omega_B(1)$

$\omega_B = 3.523$ rad/s

$v_C = \omega_B r = 3.523(2.5) = 8.81$ in./s **Ans**

$\alpha_A r_A = \alpha_B r_B$

$15.885(0.25) = \alpha_B(1)$

$\alpha_B = 3.9712$ rad/s^2

$(a_C)_t = \alpha_B r = 3.9712(2.5) = 9.928$ in./s^2

$(a_C)_n = \omega_B^2 r = (3.523)^2(2.5) = 31.025$ in./s^2

$a_C = \sqrt{(9.928)^2 + (31.025)^2}$

$= 32.6$ in./s^2 **Ans**

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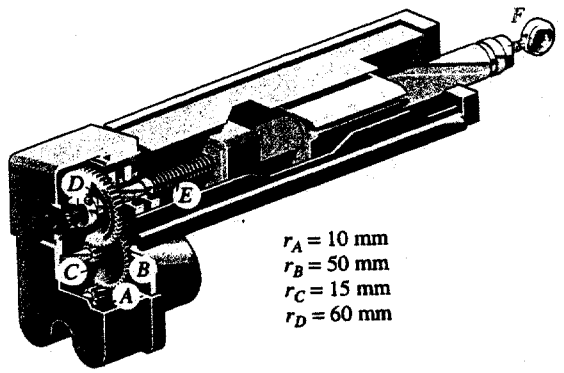
16-21. Due to the screw at E, the actuator provides linear motion to the arm at F when the motor turns the gear at A. If the gears have the radii listed in the figure, and the screw at E has a pitch $p = 2$ mm, determine the speed at F when the motor turns A at $\omega_A = 20$ rad/s. *Hint:* The screw pitch indicates the amount of advance of the screw for each full revolution.



$r_A = 10$ mm
$r_B = 50$ mm
$r_C = 15$ mm
$r_D = 60$ mm

$\omega_A r_A = \omega_B r_B$

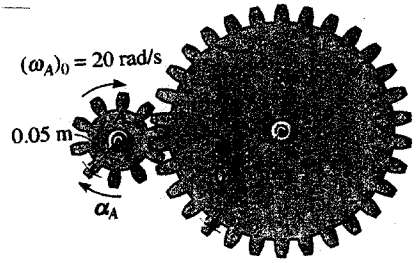
$\omega_C r_C = \omega_D r_D$

Thus,

$$\omega_D = \frac{r_A}{r_B}\frac{r_C}{r_D}\omega_A = \frac{10}{50}\frac{15}{60}20 = 1 \text{ rad/s}$$

$$v_F = \frac{1 \text{ rad/s } 1 \text{ rev}}{2\pi \text{ rad}}(2 \text{ mm}) = 0.318 \text{ mm/s} \quad \textbf{Ans}$$

16-22. A motor gives gear A an angular acceleration of $\alpha_A = (0.25\theta^3 + 0.5)$ rad/s^2, where θ is in radians. If this gear is initially turning at $(\omega_A)_0 = 20$ rad/s, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



$\alpha_A = 0.25\theta^3 + 0.5$

$\alpha\, d\theta = \omega\, d\omega$

$$\int_0^{20\pi}(0.25\theta^3 + 0.5)d\theta_A = \int_{20}^{\omega_A}\omega_A\, d\omega_A$$

$$(0.0625\theta^4 + 0.5\theta)\Big|_0^{20\pi} = \frac{1}{2}(\omega_A)^2\Big|_{20}^{\omega_A}$$

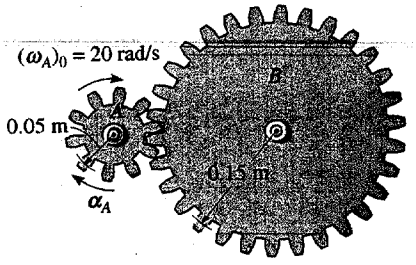
$\omega_A = 1395.94$ rad/s

$\omega_A r_A = \omega_B r_B$

$1395.94(0.05) = \omega_B(0.15)$

$\omega_B = 465$ rad/s **Ans**

16-23. A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)$ rad/s^2, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20$ rad/s, determine the angular velocity of gear B when $t = 2$ s.



$\alpha_A = 4t^3$

$d\omega = \alpha\, dt$

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A\, dt = \int_0^t 4t^3\, dt$$

$\omega_A = t^4 + 20$

When $t = 2$ s,

$\omega_A = 36$ rad/s

$\omega_A r_A = \omega_B r_B$

$36(0.05) = \omega_B(0.15)$

$\omega_B = 12$ rad/s **Ans**

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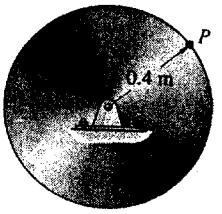
***16-24.** The disk starts from rest and is given an angular acceleration $\alpha = (10\theta^{1/3})$ rad/s^2, where θ is in radians. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.



$\alpha = 10\theta^{\frac{1}{3}}$

$\omega\, d\omega = \alpha\, d\theta$

$\int_0^{\omega} \omega\, d\omega = \int_0^{\theta} 10\theta^{\frac{1}{3}}\, d\theta$

$\frac{1}{2}\omega^2 = 10\left(\frac{3}{4}\theta^{\frac{4}{3}}\right) = 7.5\theta^{\frac{4}{3}}$

$\omega = \frac{d\theta}{dt} = \sqrt{15}\,\theta^{\frac{2}{3}}$

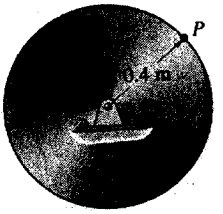
$\int_0^{\theta} \theta^{-\frac{2}{3}}\, d\theta = \int_0^{t} \sqrt{15}\, dt$

$3\theta^{\frac{1}{3}} = \sqrt{15}\, t$

$\theta = 2.152t^3\big|_{t=4} = 138 \text{ rad}$ **Ans**

$\omega = \frac{d\theta}{dt} = 6.455t^2\Big|_{t=4} = 103 \text{ rad/s}$ **Ans**

16-25. The disk starts from rest and is given an angular acceleration $\alpha = (10\theta^{1/3})$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 4$ s.



$\alpha = 10\theta^{\frac{1}{3}}$

$\omega\, d\omega = \alpha\, d\theta$

$\int_0^{\omega} \omega\, d\omega = \int_0^{\theta} 10\theta^{\frac{1}{3}}\, d\theta$

$\frac{1}{2}\omega^2 = 10\left(\frac{3}{4}\theta^{\frac{4}{3}}\right) = 7.5\theta^{\frac{4}{3}}$

$\omega = \frac{d\theta}{dt} = \sqrt{15}\,\theta^{\frac{2}{3}}$

$\int_0^{\theta} \theta^{-\frac{2}{3}}\, d\theta = \int_0^{t} \sqrt{15}\, dt$

$3\theta^{\frac{1}{3}} = \sqrt{15}\, t$

$\theta = 2.152t^3\big|_{t=4} = 137.71$

$\omega = \frac{d\theta}{dt} = 6.455t^2\Big|_{t=4} = 103.28$

$(a_P)_n = \omega^2 r = (103.28)^2(0.4) = 4267 \text{ m/s}^2$ **Ans**

$(a_P)_t = \alpha r = \left(10(137.71)^{\frac{1}{3}}\right)(0.4) = 20.7 \text{ m/s}^2$ **Ans**

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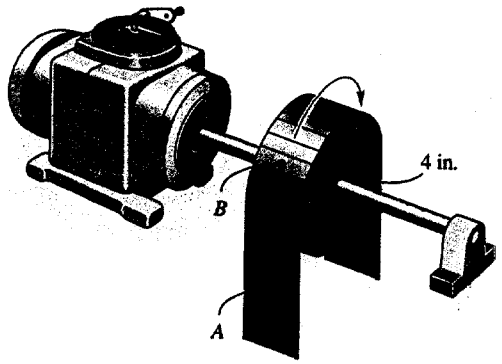
16-26. If the angular velocity of the drum is increased uniformly from 6 rad/s when $t = 0$ to 12 rad/s when $t = 5$ s, determine the magnitudes of the velocity and acceleration of points A and B on the belt when $t = 1$ s. At this instant the points are located as shown.



$\omega = \omega_0 + \alpha_c t$

$12 = 6 + \alpha(5) \qquad \alpha = 1.2 \text{ rad/s}^2$

At $t = 1$ s,

$\omega = 6 + 1.2(1) = 7.2 \text{ rad/s}$

$v_A = v_B = \omega r = 7.2\left(\frac{4}{12}\right) = 2.4 \text{ ft/s}$ Ans

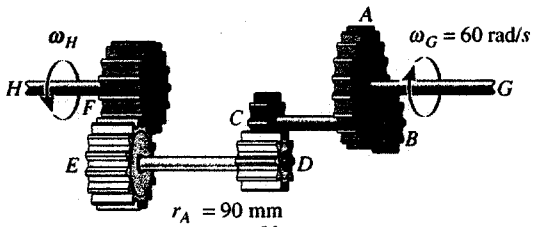
$a_A = \alpha r = 1.2\left(\frac{4}{12}\right) = 0.4 \text{ ft/s}^2$ Ans

$(a_B)_t = \alpha r = 1.2\left(\frac{4}{12}\right) = 0.4 \text{ ft/s}^2$

$(a_B)_n = \omega^2 r = (7.2)^2\left(\frac{4}{12}\right) = 17.28 \text{ ft/s}^2$

$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{0.4^2 + 17.28^2} = 17.3 \text{ ft/s}^2$ Ans

16-27. The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the crank shaft G is turning with an angular speed of 60 rad/s, determine the angular speed of the drive shaft H. Each of the gears rotates about a fixed axis. Note that gears A and B, C and D, E and F are in mesh. The radii of each of these gears are reported in the figure.



$r_A = 90$ mm
$r_B = r_C = 30$ mm
$r_D = 50$ mm
$r_E = 70$ mm
$r_F = 60$ mm

$60(90) = \omega_{BC}(30)$

$\omega_{BC} = 180 \text{ rad/s}$

$180(30) = 50(\omega_{DE})$

$\omega_{DE} = 108 \text{ rad/s}$

$108(70) = (60)(\omega_H)$

$\omega_H = 126 \text{ rad/s}$ Ans

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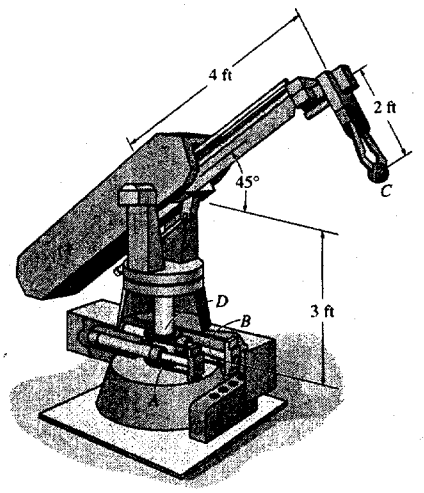
***16-28.** Rotation of the robotic arm occurs due to liner movement of the hydraulic cylinders A and B. If cylinder A is extending at the constant rate 0.5 ft/s while B is held fixed, determine the magnitude of velocity and acceleration of the part C held in the grips of the arm. The gear at D has a radius of 0.10 ft.



Angular Motion : The angular velocity of gear D must be determined first. Applying Eq. 16 – 8, we have

$$v = \omega_D r_D; \quad 0.5 = \omega_D (0.10) \quad \omega_D = 5.00 \text{ rad/s}$$

Motion of Part C : Since the shaft that turns the robot's arm is attached to gear D, then the angular velocity of the robot's arm $\omega_R = \omega_D = 5.00$ rad/s. The distance of part C from the rotating shaft is $r_C = 4\cos 45° + 2\sin 45° = 4.243$ ft. The magnitude of the velocity of part C can be determined using Eq. 16 – 8.

$$v_C = \omega_R r_C = 5.00(4.243) = 21.2 \text{ ft/s} \qquad \textbf{Ans}$$

The tangential and normal components of the acceleration of part C can be determined using Eqs. 16 – 11 and 16 – 12 respectively.

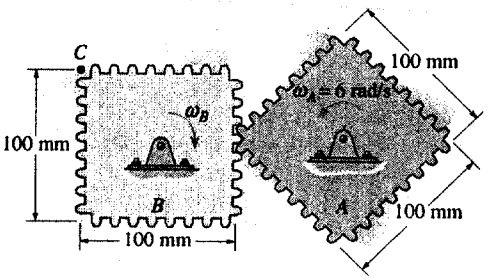
$$a_t = \alpha r_C = 0$$

$$a_n = \omega_R^2 r_C = \left(5.00^2\right)(4.243) = 106.07 \text{ ft/s}^2$$

The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 106.07^2} = 106 \text{ ft/s}^2 \qquad \textbf{Ans}$$

16-29. At the instant shown, gear A is rotating with a constant angular velocity of $\omega_A = 6$ rad/s. Determine the largest angular velocity of gear B and the maximum speed of point C.



$(r_B)_{max} = (r_A)_{max} = 50\sqrt{2}$ mm

$(r_B)_{min} = (r_A)_{min} = 50$ mm

When r_A is max., r_B is min.

$\omega_B (r_B) = \omega_A r_A$

$$(\omega_B)_{max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$(\omega_B)_{max} = 8.49$ rad/s **Ans**

$v_C = (\omega_B)_{max} r_C = 8.49\left(0.05\sqrt{2}\right)$

$v_C = 0.6$ m/s **Ans**

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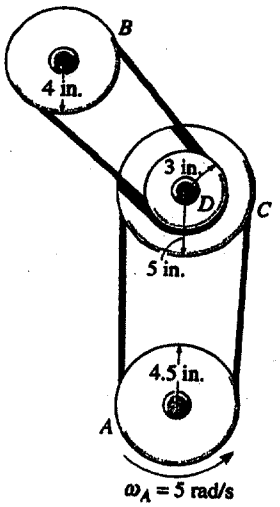
16-30. A mill in a textile plant uses the belt-and-pulley arrangement shown to transmit power. When $t = 0$ an electric motor is turning pulley A with an angular velocity of $\omega_A = 5$ rad/s. If this pulley is subjected to a constant angular acceleration 2 rad/s^2, determine the angular velocity of pulley B after B turns 6 revolutions. The hub at D is rigidly *connected* to pulley C and turns with it.



When $\theta_B = 6$ rev;

$4(6) = 3\,\theta_C$

$\theta_C = 8$ rev

$8(5) = 4.5(\theta_A)$

$\theta_A = 8.889$ rev

$(\omega_A)_2^2 = (\omega_A)_1^2 + 2\alpha_C[(\theta_A)_2 - (\theta_A)_1]$

$(\omega_A)_2^2 = (5)^2 + 2(2)[(8.889)(2\pi) - 0]$

$(\omega_A)_2 = 15.76$ rad/s

$15.76(4.5) = 5\omega_C$

$\omega_C = 14.18$ rad/s

$14.18(3) = 4(\omega_B)_2$

$(\omega_B)_2 = 10.6$ rad/s Ans

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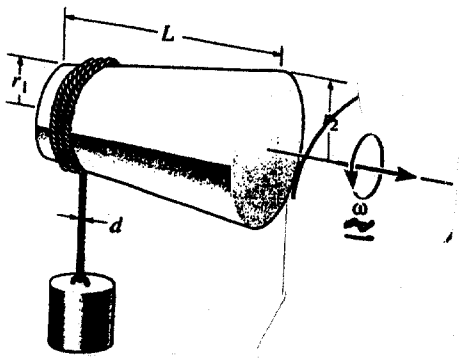
16-31. The rope of diameter d is wrapped around the tapered drum which has the dimensions shown. If the drum is rotating at a constant rate of ω, determine the upward acceleration of the block. Neglect the small horizontal displacement of the block.



$$v = \omega r$$

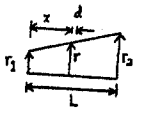
$$a = \frac{d(\omega r)}{dt}$$

$$= \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

$$= \omega(\frac{dr}{dt})$$

$$r = r_1 + (\frac{r_2 - r_1}{L})x$$

$$dr = (\frac{r_2 - r_1}{L})dx$$



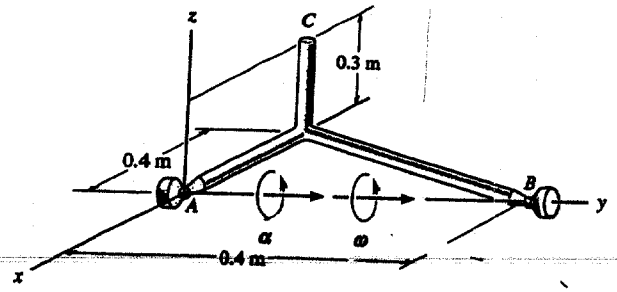
But $\frac{dx}{dt} = \frac{d\theta}{2\pi}$

Thus $\frac{dr}{dt} = \frac{1}{2\pi}(\frac{r_2 - r_1}{L})d(\frac{d\theta}{dt})$

$$= \frac{1}{2\pi}(\frac{r_2 - r_1}{L})d\omega$$

Thus, $a = \frac{\omega^2}{2\pi}(\frac{r_2 - r_1}{L})d$ **Ans**

***16-32.** The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5$ rad/s and has an angular acceleration $\alpha = 8$ rad/s^2. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{v}_C = 5\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) = \{1.5\mathbf{i} + 2\mathbf{k}\} \text{ m/s}$$

$$v_C = \sqrt{1.5^2 + 2^2} = 2.50 \text{ m/s}$$ **Ans**

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}$$

$$= 8\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) - 5^2(-0.4\mathbf{i} + 0.3\mathbf{k})$$

$$= \{12.4\mathbf{i} - 4.3\mathbf{k}\} \text{ m/s}^2$$

$$a_C = \sqrt{12.4^2 + (-4.3)^2} = 13.1 \text{ m/s}^2$$ **Ans**

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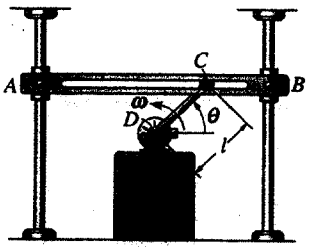
16-33. The bar DC rotates uniformly about the shaft at D with a constant angular velocity ω. Determine the velocity and acceleration of the bar AB, which is confined by the guides to move vertically.



$y = l\sin\theta$

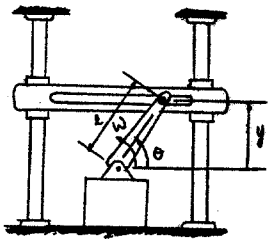
$\dot{y} = v_y = l\cos\theta\dot{\theta}$

$\ddot{y} = a_y = l(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2)$

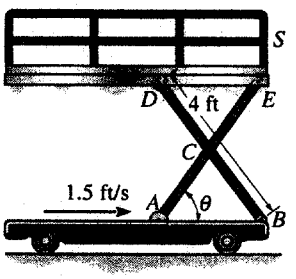
Here $v_y = v_{AB}$, $a_y = a_{AB}$, and $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha = 0$.

$v_{AB} = l\cos\theta(\omega) = \omega\, l\cos\theta$ **Ans**

$a_{AB} = l[\cos\theta(0) - \sin\theta(\omega)^2] = -\omega^2 l\sin\theta$ **Ans**



16-34. The scaffold S is raised hydraulically by moving the roller at A toward the pin at B. If A is approaching B with a speed of 1.5 ft/s, determine the speed at which the platform is rising as a function of θ. The 4-ft links are pin-connected at their midpoint.



Position coordinate equation :

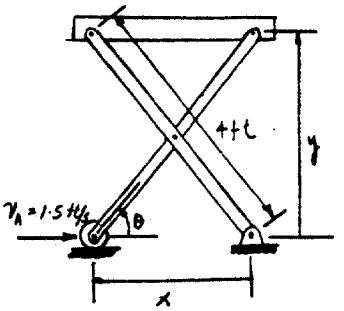
$$x = 4\cos\theta \qquad y = 4\sin\theta$$

Time derivatives :

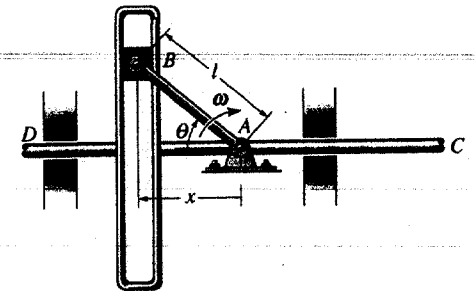
$$\dot{x} = -4\sin\theta\dot{\theta} \qquad \text{However, } \dot{x} = -v_A = -1.5 \text{ ft/s}$$

$$-1.5 = -4\sin\theta\dot{\theta} \qquad \dot{\theta} = \frac{0.375}{\sin\theta}$$

$$\dot{y} = v_y = 4\cos\theta\dot{\theta} = 4\cos\theta\left(\frac{0.375}{\sin\theta}\right) = 1.5\cot\theta$$ **Ans**



16-35. The mechanism is used to convert the constant circular motion ω of rod AB into translating motion of rod CD. Determine the velocity and acceleration of CD for any angle θ of AB.



$x = l\cos\theta$

$\dot{x} = v_x = -l\sin\theta\dot{\theta}$

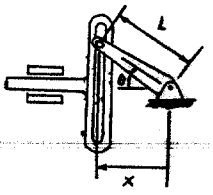
$\ddot{x} = a_x = -l(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)$

Here $v_x = v_{CD}$, $a_x = a_{CD}$, and $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha = 0$.

$v_{CD} = -l\sin\theta(\omega) = -\omega\, l\sin\theta$ **Ans**

$a_{CD} = -l[\sin\theta(0) + \cos\theta(\omega)^2] = -\omega^2 l\cos\theta$ **Ans**

Negative signs indicate that both v_{CD} and a_{CD} are directed opposite to positive x.



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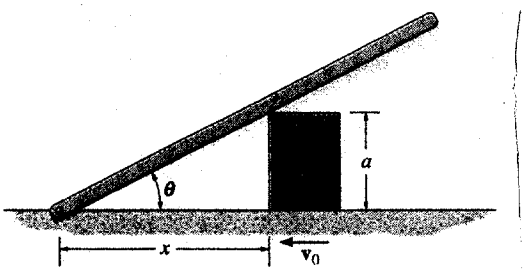
***16-36.** The block moves to the left with a constant velocity v_0. Determine the angular velocity and angular acceleration of the bar as a function of θ.

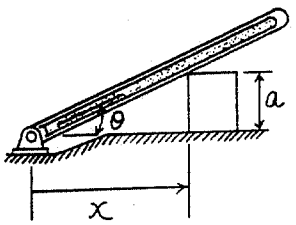


Position Coordinate Equation : From the geometry,

$$x = \frac{a}{\tan\theta} = a\cot\theta \qquad [1]$$

Time Derivatives : Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = -a\csc^2\theta\frac{d\theta}{dt} \qquad [2]$$



Since v_0 is directed toward negative x, then $\frac{dx}{dt} = -v_0$. Also, $\frac{d\theta}{dt} = \omega$. From Eq. [2],

$$-v_0 = -a\csc^2\theta(\omega)$$

$$\omega = \frac{v_0}{a\csc^2\theta} = \frac{v_0}{a}\sin^2\theta \qquad \textbf{Ans}$$

Here, $\alpha = \frac{d\omega}{dt}$. Then from the above expression

$$\alpha = \frac{v_0}{a}(2\sin\theta\cos\theta)\frac{d\theta}{dt} \qquad [3]$$

However, $2\sin\theta\cos\theta = \sin 2\theta$ and $\omega = \frac{d\theta}{dt} = \frac{v_0}{a}\sin^2\theta$. Substitute these values into Eq. [3] yields

$$\alpha = \frac{v_0}{a}\sin 2\theta\left(\frac{v_0}{a}\sin^2\theta\right) = \left(\frac{v_0}{a}\right)^2\sin 2\theta\sin^2\theta \qquad \textbf{Ans}$$

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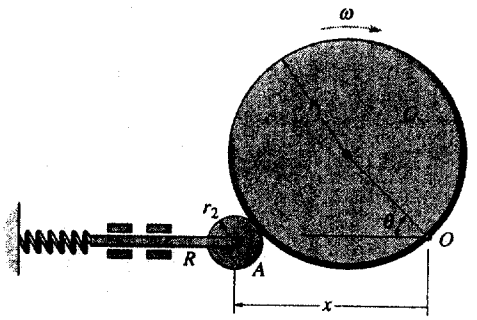
16-37. Determine the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of A on C.



Position Coordinate Equation : Using law of cosine,

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x\cos\theta \qquad [1]$$

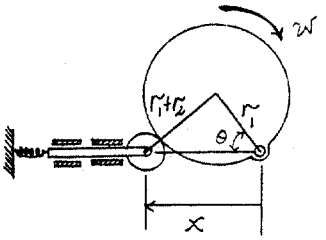
Time Derivatives : Taking the time derivative of Eq. [1], we have

$$0 = 2x\frac{dx}{dt} - 2r_1\left(-x\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dx}{dt}\right) \qquad [2]$$

However $v = \frac{dx}{dt}$ and $\omega = \frac{d\theta}{dt}$. From Eq. [2],

$$0 = xv - r_1(v\cos\theta - x\omega\sin\theta)$$

$$v = \frac{r_1 x\omega\sin\theta}{r_1\cos\theta - x} \qquad [3]$$



However, the positive root of Eq. [1] is

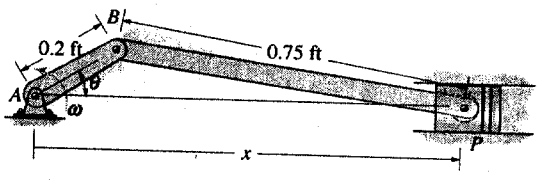
$$x = r_1\cos\theta + \sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1 r_2}$$

Substitute into Eq. [3], we have

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1 r_2}} + r_1\omega\sin\theta\right) \qquad \textbf{Ans}$$

Note : Negative sign indicates that v is directed in the opposite direction to that of positive x.

16-38. The crankshaft AB is rotating at a constant angular velocity of $\omega = 150$ rad/s. Determine the velocity of the piston P at the instant $\theta = 30°$.



$$x = 0.2\cos\theta + \sqrt{(0.75)^2 - (0.2\sin\theta)^2}$$

$$\dot{x} = -0.2\sin\theta\dot{\theta} + \frac{1}{2}\left[(0.75)^2 - (0.2\sin\theta)^2\right]^{-\frac{1}{2}}(-2)(0.2\sin\theta)(0.2\cos\theta)\dot{\theta}$$

$$v_P = -0.2\omega\sin\theta - \left(\frac{1}{2}\right)\frac{(0.2)^2\omega\sin 2\theta}{\sqrt{(0.75)^2 - (0.2\sin\theta)^2}}$$

At $\theta = 30°$, $\omega = 150$ rad/s

$$v_P = -0.2(150)\sin 30° - \left(\frac{1}{2}\right)\frac{(0.2)^2(150)\sin 60°}{\sqrt{(0.75)^2 - (0.2\sin 30°)^2}}$$

$v_P = -18.5$ ft/s $= 18.5$ ft/s $\leftarrow$ **Ans**

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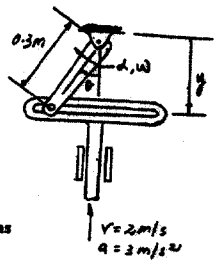
16-39. At the instant $\theta = 50°$, the slotted guide is moving upward with an acceleration of 3 m/s^2 and a velocity of 2 m/s. Determine the angular acceleration and angular velocity of link AB at this instant. *Note:* The upward motion of the guide is in the negative y direction.



$y = 0.3\cos\theta$

$\dot{y} = v_y = -0.3\sin\theta\dot{\theta}$

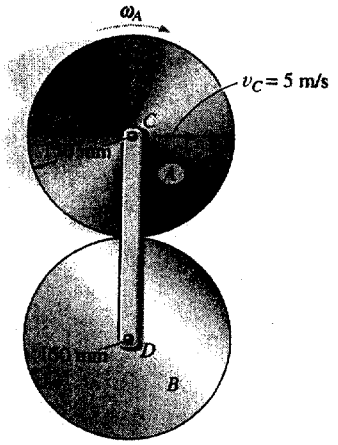
$\ddot{y} = a_y = -0.3(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)$

Here $v_y = -2$ m/s, $a_y = -3$ m/s^2, and $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha$, $\theta = 50°$.

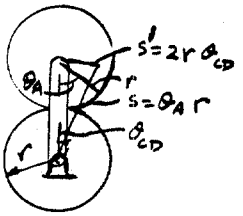
$-2 = -0.3\sin 50°(\omega)$ $\omega = 8.70$ rad/s Ans

$-3 = -0.3[\sin 50°(\alpha) + \cos 50°(8.70)^2]$ $\alpha = -50.5$ rad/s^2 Ans

***16-40.** Disk A rolls without slipping over the surface of the *fixed* cylinder B. Determine the angular velocity of A if its center C has a speed $v_C = 5$ m/s. How many revolutions will A have made about its center just after link DC completes one revolution?



As shown by the construction, as A rolls through the arc $s = \theta_A r$, the center of the disk moves through the same distance $s' = s$. Hence,



$s = \theta_A r$

$\dot{s} = \dot{\theta}_A r$

$5 = \omega_A(0.15)$

$\omega_A = 33.3$ rad/s Ans

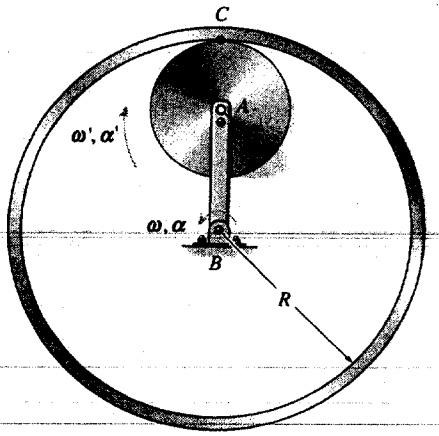
Link :

$s' = 2r\theta_{CD} = s = \theta_A r$

$2\theta_{CD} = \theta_A$

Thus, A makes 2 revolutions for each revolution of CD. Ans

16-41. Arm AB has an angular velocity of ω and an angular acceleration of α. If no slipping occurs between the disk and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.

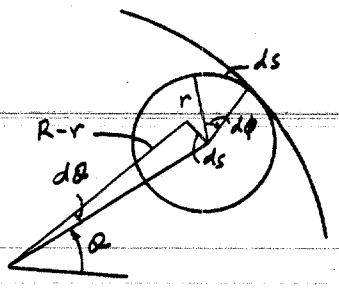


$ds = (R - r)\,d\theta = -r\,d\phi$

$(R - r)\left(\frac{d\theta}{dt}\right) = -r\left(\frac{d\phi}{dt}\right)$

$\omega' = -\frac{(R - r)\omega}{r}$ Ans

$\alpha' = -\frac{(R - r)\alpha}{r}$ Ans



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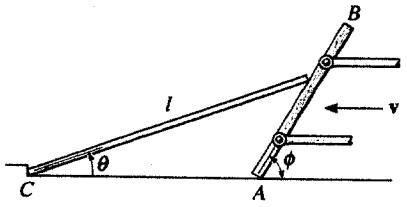
16-42. The inclined plate moves to the left with a constant velocity v. Determine the angular velocity and angular acceleration of the slender rod of length l. The rod pivots about the step at C as it slides on the plate.



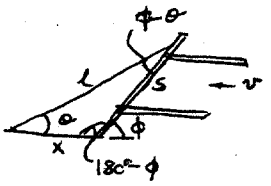
$$\frac{x}{\sin(\phi-\theta)} = \frac{l}{\sin(180^\circ-\phi)} = \frac{l}{\sin\phi}$$

$$x\sin\phi = l\sin(\phi-\theta)$$

$$\dot{x}\sin\phi = -l\cos(\phi-\theta)\,\dot{\theta}$$

Thus

$$\omega = \frac{-v(\sin\phi)}{l\cos(\phi-\theta)} \qquad \text{Ans}$$



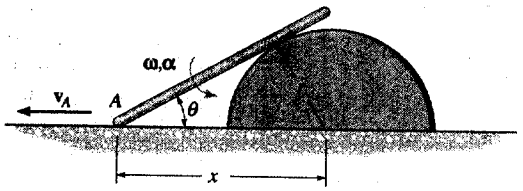
$$\ddot{x}\sin\phi = -l\cos(\phi-\theta)\ddot{\theta} - l\sin(\phi-\theta)(\dot{\theta})^2$$

$$0 = -\cos(\phi-\theta)\alpha - \sin(\phi-\theta)\,\omega^2$$

$$\alpha = \frac{-\sin(\phi-\theta)}{\cos(\phi-\theta)}\left(\frac{v^2\sin^2\phi}{l^2\cos^2(\phi-\theta)}\right)$$

$$\alpha = \frac{-v^2\sin^2\phi\sin(\phi-\theta)}{l^2\cos^3(\phi-\theta)} \qquad \text{Ans}$$

16-43. The end A of the bar is moving to the left with a constant velocity v_A. Determine the angular velocity ω and angular acceleration α of the bar as a function of its position x.



Position Coordinate Equation : From the geometry,

$$x = \frac{r}{\sin\theta} \qquad [1]$$

Time Derivatives : Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = -\frac{r\cos\theta\,d\theta}{\sin^2\theta\,dt} \qquad [2]$$

Since v_0 is directed toward positive x, then $\frac{dx}{dt} = v_A$. Also, $\frac{d\theta}{dt} = \omega$. From the geometry, $\sin\theta = \frac{r}{x}$ and $\cos\theta = \frac{\sqrt{x^2-r^2}}{x}$. Substitute these values into Eq. [2], we have

$$v_A = -\left(\frac{r\left(\sqrt{x^2-r^2}/x\right)}{(r/x)^2}\right)\omega$$

$$\omega = -\left(\frac{r}{x\sqrt{x^2-r^2}}\right)v_A \qquad \text{Ans}$$

Taking the time derivative of Eq. [2], we have

$$\frac{d^2x}{dt^2} = \frac{r}{\sin^2\theta}\left[\left(\frac{1+\cos^2\theta}{\sin\theta}\right)\left(\frac{d\theta}{dt}\right)^2 - \cos\theta\frac{d^2\theta}{dt^2}\right] \qquad [3]$$

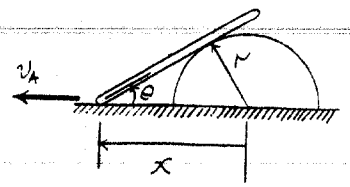
Here, $\frac{d^2x}{dt^2} = a = 0$ and $\frac{d^2\theta}{dt^2} = \alpha$. Substitute into Eq. [3], we have

$$0 = \frac{r}{\sin^2\theta}\left[\left(\frac{1+\cos^2\theta}{\sin\theta}\right)\omega^2 - \alpha\cos\theta\right]$$

$$\alpha = \left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right)\omega^2$$

However, $\sin\theta = \frac{r}{x}$, $\cos\theta = \frac{\sqrt{x^2-r^2}}{x}$ and $\omega = -\left(\frac{r}{x\sqrt{x^2-r^2}}\right)v_A$. Substitute these values into Eq. [4] yields

$$\alpha = \left[\frac{r(2x^2-r^2)}{x^2(x^2-r^2)^{3/2}}\right]v_A^2 \qquad \text{Ans}$$



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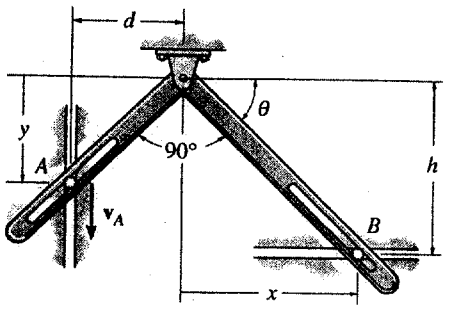
***16-44.** The pins at A and B are confined to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at v_A, determine the velocity of B at the instant shown.



Position coordinate equation :

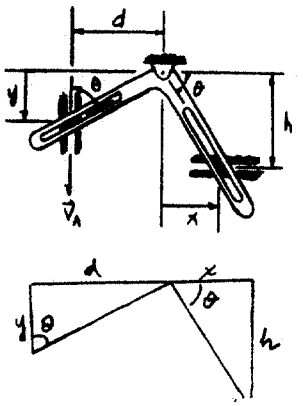
$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$

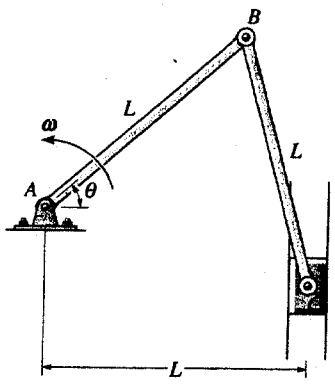
Time derivatives :

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

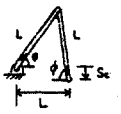
$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans}$$



16-45. Bar AB rotates uniformly about the fixed pin A with a constant angular velocity ω. Determine the velocity and acceleration of block C, at the instant $\theta = 60°$.



$$L\cos\theta + L\cos\phi = L$$



$$\cos\theta + \cos\phi = 1$$

$$\sin\theta\,\dot{\theta} + \sin\phi\,\dot{\phi} = 0 \qquad (1)$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\,\ddot{\theta} + \sin\phi\,\ddot{\phi} + \cos\phi\,(\dot{\phi})^2 = 0 \qquad (2)$$

When $\theta = 60°$, $\phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from Eq.(1))

$$\ddot{\theta} = 0$$

$$\ddot{\phi} = -1.155\omega^2 \quad \text{(from Eq.(2))}$$

Also, $s_C = L\sin\phi - L\sin\theta$

$$v_C = L\cos\phi\,\dot{\phi} - L\cos\theta\,\dot{\theta}$$

$$a_C = -L\sin\phi(\dot{\phi})^2 + L\cos\phi(\ddot{\phi}) - L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°$, $\phi = 60°$

$$s_C = 0$$

$$v_C = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega = L\omega\uparrow \qquad \textbf{Ans}$$

$$a_C = -L\sin 60°(-\omega)^2 + L\cos 60°(-1.155\omega^2) + 0 + L\sin 60°(\omega)^2$$

$$a_C = -0.577L\omega^2 = 0.577L\omega^2\uparrow \qquad \textbf{Ans}$$

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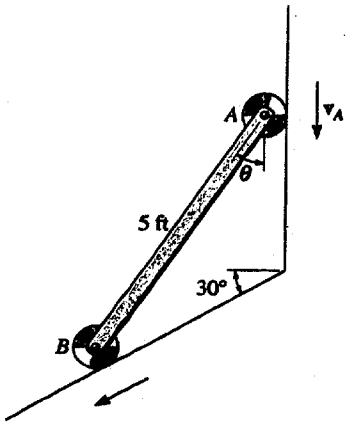
16-46. The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at A is $v_A = 6$ ft/s when $\theta = 45°$, determine the bar's angular velocity and the velocity of roller B at this instant



$s_B \cos 30° = 5 \sin\theta$

$s_B = 5.774 \sin\theta$

$\dot{s}_B = 5.774 \cos\theta\ \dot{\theta}$ (1)

$5 \cos\theta = s_A + s_B \sin 30°$

$-5 \sin\theta\ \dot{\theta} = \dot{s}_A + \dot{s}_B \sin 30°$ (2)

Combine Eqs.(1) and (2) :

$-5 \sin\theta\ \dot{\theta} = -6 + 5.774 \cos\theta(\dot{\theta})(\sin 30°)$

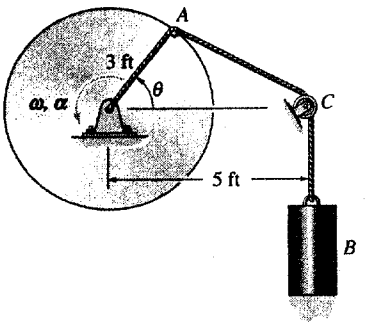
$-3.536\dot{\theta} = -6 + 2.041\dot{\theta}$

$\omega = \dot{\theta} = 1.08$ rad/s **Ans**

From Eq.(1) :

$v_B = \dot{s}_B = 5.774\cos 45°(1.076) = 4.39$ ft/s **Ans**

16-47. The disk is rotating with an angular velocity of ω and has an angular acceleration of α. Determine the velocity and acceleration of cylinder B. Neglect the size of the pulley at C.



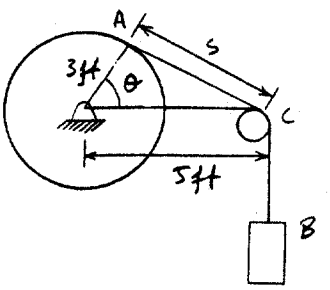
$s = \sqrt{3^2 + 5^2 - 2(3)(5)\cos\theta}$

$v_B = \dot{s} = \frac{1}{2}(34 - 30\cos\theta)^{-\frac{1}{2}}(30\sin\theta)\,\dot{\theta}$

$v_B = \dfrac{15\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}}$ **Ans**

$a_B = \ddot{s} = \dfrac{15\omega\cos\theta\dot{\theta} + 15\dot{\omega}\sin\theta}{\sqrt{34 - 30\cos\theta}} + \dfrac{\left(-\frac{1}{2}\right)(15\omega\sin\theta)\left(30\sin\theta\dot{\theta}\right)}{(34 - 30\cos\theta)^{\frac{3}{2}}}$

$= \dfrac{15(\omega^2\cos\theta + \alpha\sin\theta)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \dfrac{225\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}}$ **Ans**



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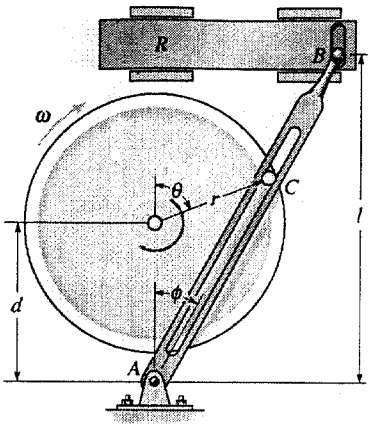
***16-48.** The slotted yoke is pinned at A while end B is used to move the ram R horizontally. If the disk rotates with a constant angular velocity ω, determine the velocity and acceleration of the ram. The crank pin C is fixed to the disk and turns with it.



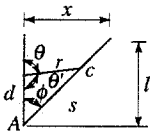
$$x = l\tan\phi \qquad [1]$$

However $$\frac{r}{\sin\phi} = \frac{s}{\sin(180° - \theta)} = \frac{s}{\sin\theta} \qquad \sin\phi = \frac{r}{s}\sin\theta$$

$$d = s\cos\phi - r\cos\theta \qquad \cos\phi = \frac{d + r\cos\theta}{s}$$

From Eq. [1] $$x = l\left(\frac{\sin\phi}{\cos\phi}\right) = l\left(\frac{\frac{r}{s}\sin\theta}{\frac{d + r\cos\theta}{s}}\right) = \frac{lr\sin\theta}{d + r\cos\theta}$$

$$\dot{x} = v = \frac{(d + r\cos\theta)(lr\cos\theta\dot{\theta}) - (lr\sin\theta)(-r\sin\theta\dot{\theta})}{(d + r\cos\theta)^2} \qquad \text{Where } \dot{\theta} = \omega$$

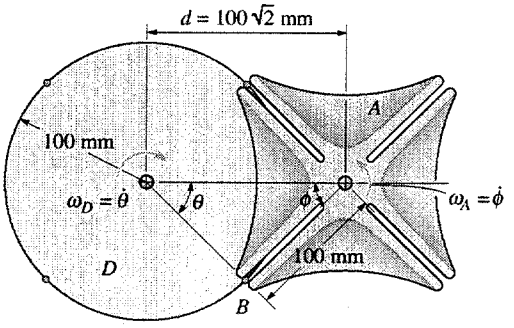


$$= \frac{lr(r + d\cos\theta)}{(d + r\cos\theta)^2}\omega \qquad \textbf{Ans}$$

$$\ddot{x} = a = lr\omega\left[\frac{(d + r\cos\theta)^2(-d\sin\theta\dot{\theta}) - (r + d\cos\theta)(2)(d + r\cos\theta)(-r\sin\theta\dot{\theta})}{(d + r\cos\theta)^4}\right]$$

$$= \frac{lr\sin\theta(2r^2 - d^2 + rd\cos\theta)}{(d + r\cos\theta)^3}\omega^2 \qquad \textbf{Ans}$$

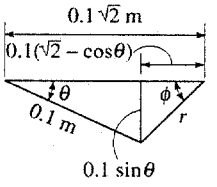
16-49. The Geneva wheel A provides intermittent rotary motion ω_A for continuous motion $\omega_D = 2$ rad/s of disk D. By choosing $d = 100\sqrt{2}$ mm, the wheel has zero angular velocity at the instant pin B enters or leaves one of the four slots. Determine the magnitude of the angular velocity ω_A of the Geneva wheel at any angle θ for which pin B is in contact with the slot.



$$\tan\phi = \frac{0.1\sin\theta}{0.1(\sqrt{2} - \cos\theta)} = \frac{\sin\theta}{\sqrt{2} - \cos\theta}$$

$$\sec^2\phi\dot{\phi} = \frac{(\sqrt{2} - \cos\theta)(\cos\theta\dot{\theta}) - \sin\theta(\sin\theta\dot{\theta})}{(\sqrt{2} - \cos\theta)^2} = \frac{\sqrt{2}\cos\theta - 1}{(\sqrt{2} - \cos\theta)^2}\dot{\theta} \qquad [1]$$

From the geometry:



$$r^2 = (0.1\sin\theta)^2 + [0.1(\sqrt{2} - \cos\theta)]^2 = 0.01(3 - 2\sqrt{2}\cos\theta)$$

$$\sec^2\phi = \frac{r^2}{[0.1(\sqrt{2} - \cos\theta)]^2} = \frac{0.01(3 - 2\sqrt{2}\cos\theta)}{[0.1(\sqrt{2} - \cos\theta)]^2} = \frac{(3 - 2\sqrt{2}\cos\theta)}{(\sqrt{2} - \cos\theta)^2}$$

From Eq. [1]

$$\frac{(3 - 2\sqrt{2}\cos\theta)}{(\sqrt{2} - \cos\theta)^2}\dot{\phi} = \frac{\sqrt{2}\cos\theta - 1}{(\sqrt{2} - \cos\theta)^2}\dot{\theta}$$

$$\dot{\phi} = \frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\dot{\theta} \qquad \text{Here } \dot{\phi} = \omega_A \text{ and } \dot{\theta} = \omega_D = 2 \text{ rad/s}$$

$$\omega_A = 2\left(\frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\right) \qquad \textbf{Ans}$$

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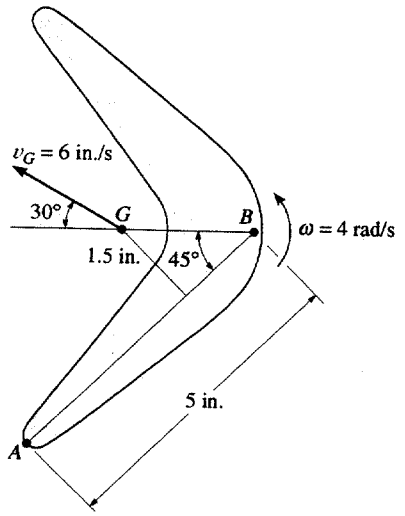
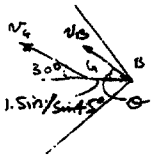
16-50. At the instant shown the boomerang has an angular velocity $\omega = 4$ rad/s, and its mass center G has a velocity $v_G = 6$ in./s. Determine the velocity of point B at this instant.



$$\mathbf{v}_B = \mathbf{v}_G + \mathbf{v}_{B/G}$$

$$\mathbf{v}_B = 6 + [4(1.5/\sin 45°) = 8.4852]$$
$$30° \nwarrow \qquad \uparrow$$



$$(\overset{+}{\leftarrow})\ (v_B)_x = 6\cos 30° + 0 = 5.196 \text{ in./s}$$

$$(+\uparrow)(v_B)_y = 6\sin 30° + 8.4852 = 11.485 \text{ in./s}$$

$$v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.6 \text{ in./s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\frac{11.485}{5.196} = 65.7°$$

Also;

$$\mathbf{v}_B = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{B/G}$$

$$(v_B)_x\mathbf{i} + (v_B)_y\mathbf{j} = (-6\cos 30°\mathbf{i} + 6\sin 30°\mathbf{j}) + (4\mathbf{k}) \times (1.5/\sin 45°)\mathbf{i}$$

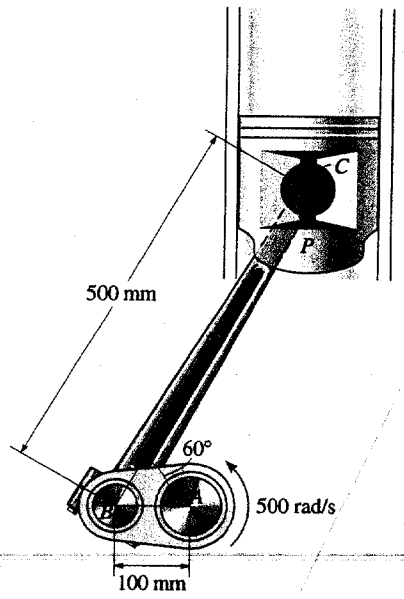
$$(v_B)_x = -6\cos 30° = -5.196 \text{ in./s}$$

$$(v_B)_y = 6\sin 30° + 8.4853 = 11.485 \text{ in./s}$$

$$v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.6 \text{ in./s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\frac{11.485}{5.196} = 65.7°$$

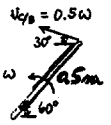
16-51. The crankshaft AB is rotating at 500 rad/s about a fixed axis passing through A. Determine the speed of the piston P at the instant it is in the position shown.



$$v_B = 500(0.1) = 50 \text{ m/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$\mathbf{v}_C = 50 + 0.5\omega$$
$$\downarrow \quad \downarrow\ 30° \nwarrow$$



$$\overset{+}{\rightarrow} \quad 0 = -0.5\omega\cos 30°, \qquad \omega = 0$$

$$+\downarrow \quad v_C = 50 \text{ m/s} \qquad \text{Ans}$$

Also; $\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$$-v_C\mathbf{j} = -50\mathbf{j} + \{\omega\mathbf{k}\} \times \{0.5\cos 60°\mathbf{i} + 0.5\sin 60°\mathbf{j}\}$$

$$\overset{+}{\rightarrow} \quad 0 = 0 - \omega(0.5\sin 60°)$$

$$\omega = 0$$

$$+\uparrow \quad -v_C = -50 + 0$$

$$v_C = 50 \text{ m/s} \qquad \text{Ans}$$

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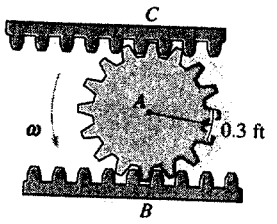
***16-52.** The pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 4$ rad/s. Determine the velocity of the gear rack C.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$(\overset{+}{\leftarrow}) \quad v_C = 0 + 4(0.6)$$

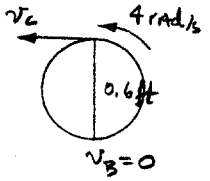
$$v_C = 2.40 \text{ ft/s} \qquad \textbf{Ans}$$

Also:

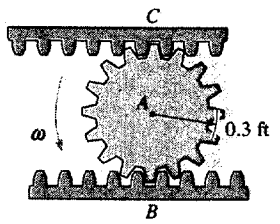
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-v_C\mathbf{i} = 0 + (4\mathbf{k}) \times (0.6\mathbf{j})$$

$$v_C = 2.40 \text{ ft/s} \qquad \textbf{Ans}$$



16-53. The pinion gear rolls on the gear racks. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$(\overset{+}{\rightarrow}) \quad -4 = 8 - 0.6(\omega)$$

$$\omega = 20 \text{ rad/s} \qquad \textbf{Ans}$$

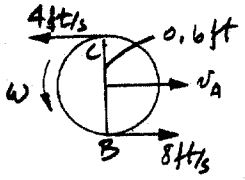
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$(\overset{+}{\rightarrow}) \quad v_A = 8 - 20(0.3)$$

$$v_A = 2 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

Also:

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$



$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \text{ rad/s} \qquad \textbf{Ans}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$$v_A = 2 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

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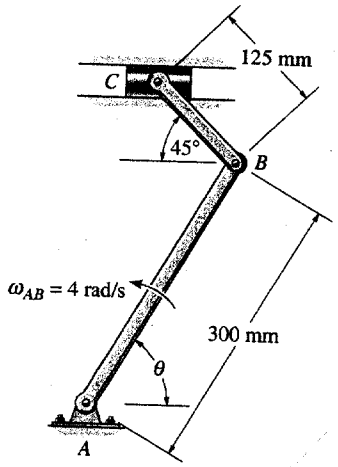
16-54. The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant $\theta = 60°$, if link AB is rotating at 4 rad/s.



$v_C = v_B + \omega \times r_{C/B}$

$-v_C\mathbf{i} = -4(0.3)\sin 30°\mathbf{i} + 4(0.3)\cos 30°\mathbf{j} + \omega\mathbf{k} \times (-0.125\cos 45°\mathbf{i} + 0.125\sin 45°\mathbf{j})$

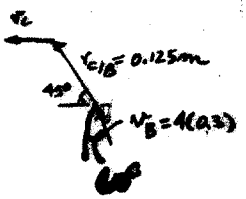
$-v_C = -1.0392 - 0.008839\omega$

$0 = 0.6 - 0.08839\omega$

Solving,

$\omega = 6.79$ rad/s

$v_C = 1.64$ m/s Ans



16-55. Determine the velocity of the slider block at C at the instant $\theta = 45°$, if link AB is rotating at 4 rad/s.

$v_C = v_B + \omega \times r_{C/B}$

$-v_C\mathbf{i} = -4(0.3)\cos 45°\mathbf{i} + 4(0.3)\sin 45°\mathbf{j} + \omega\mathbf{k} \times (-0.125\cos 45°\mathbf{i} + 0.125\sin 45°\mathbf{j})$

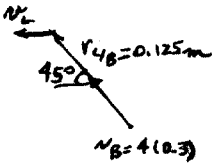
$-v_C = -0.8485 - 0.08839\omega$

$0 = 0.8485 - 0.08839\omega$

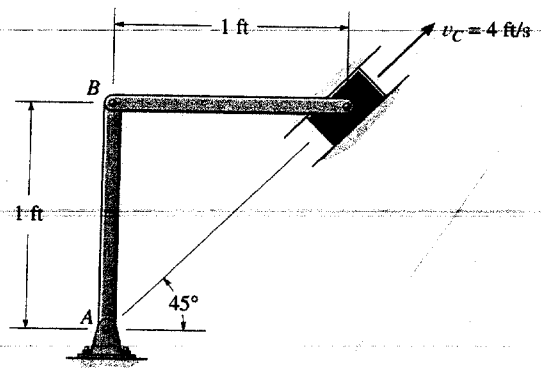
Solving,

$\omega = 9.60$ rad/s

$v_C = 1.70$ m/s Ans



***16-56.** The velocity of the slider block C is 4 ft/s up the inclined groove. Determine the angular velocity of links AB and BC and the velocity of point B at the instant shown.



For link BC

$v_C = \{-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}\}$ ft/s $\quad v_B = -v_B\mathbf{i} \quad \omega = \omega_{BC}\mathbf{k}$

$r_{A/B} = \{1\mathbf{i}\}$ ft

$v_C = v_B + \omega \times r_{C/B}$

$-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j} = -v_B\mathbf{i} + (\omega_{BC}\mathbf{k}) \times (1\mathbf{i})$

$-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j} = -v_B\mathbf{i} + \omega_{BC}\mathbf{j}$

Equating the i and j components yields :

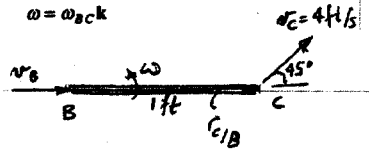
$-4\cos 45° = -v_B \quad v_B = 2.83$ ft/s Ans

$4\sin 45° = \omega_{BC} \quad \omega_{BC} = 2.83$ rad/s Ans

For link AB : Link AB rotates about the fixed point A. Hence

$v_B = \omega_{AB} r_{AB}$

$2.83 = \omega_{AB}(1) \quad \omega_{AB} = 2.83$ rad/s Ans



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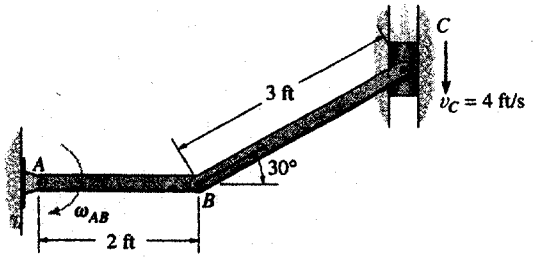
16-57. If the block at C is moving downward at 4 ft/s, determine the angular velocity of bar AB at the instant shown.



Kinematic Diagram : Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 2\omega_{AB}$. At the instant shown, $\mathbf{v}_B$ is directed towards the *negative* y axis. Also, block C is moving downward vertically due to the constraint of the guide. Then $\mathbf{v}_C$ is directed toward *negative* y axis.

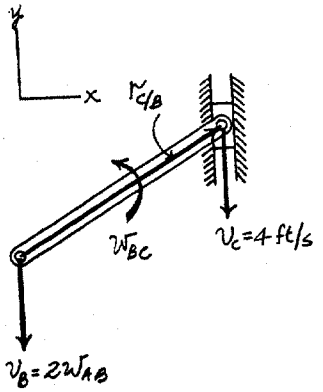
Velocity Equation : Here, $\mathbf{r}_{C/A} = \{3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j}\}$ ft $= \{2.598\mathbf{i} + 1.50\mathbf{j}\}$ ft. Applying Eq. 16 – 16, we have

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$
$$-4\mathbf{j} = -2\omega_{AB}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (2.598\mathbf{i} + 1.50\mathbf{j})$$
$$-4\mathbf{j} = -1.50\omega_{BC}\mathbf{i} + (2.598\omega_{BC} - 2\omega_{AB})\mathbf{j}$$

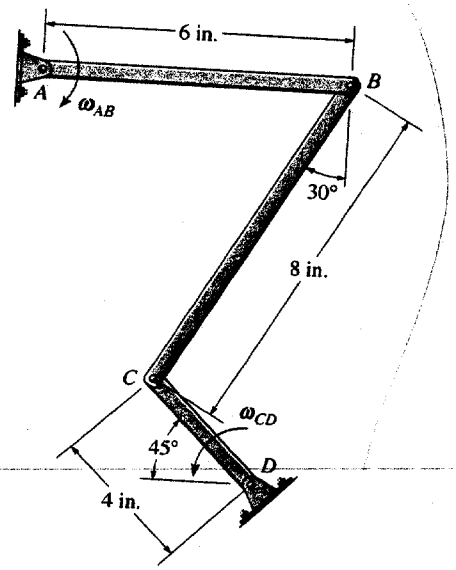
Equating **i** and **j** components gives

$$0 = -1.50\omega_{BC} \qquad \omega_{BC} = 0$$

$$-4 = 2.598(0) - 2\omega_{AB} \qquad \omega_{AB} = 2.00 \text{ rad/s} \qquad \textbf{Ans}$$



16-58. If link AB is rotating at $\omega_{AB} = 3$ rad/s, determine the angular velocity of link CD at the instant shown.



$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D}$

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$

$(\omega_{CD}\mathbf{k}) \times (-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}) = (-3\mathbf{k}) \times (6\mathbf{i}) + (\omega_{BC}\mathbf{k}) \times (-8\sin 30°\mathbf{i} - 8\cos 30°\mathbf{j})$

$-2.828\omega_{CD} = 0 + 6.928\omega_{BC}$

$-2.828\omega_{CD} = -18 - 4\omega_{BC}$

Solving,

$\omega_{BC} = -1.65$ rad/s

$\omega_{CD} = 4.03$ rad/s **Ans**

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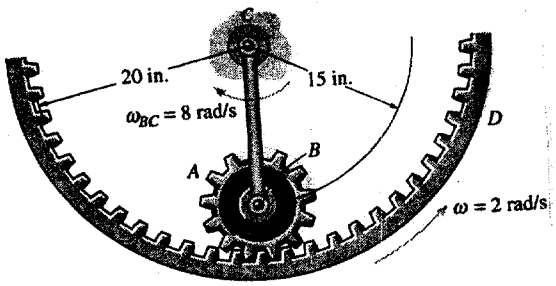
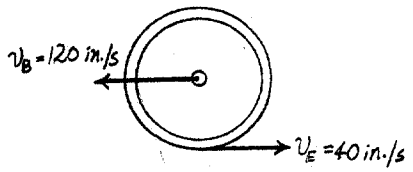
16-59. The planetary gear A is pinned at B. Link BC rotates clockwise with an angular velocity of 8 rad/s, while the outer gear rack rotates counterclockwise with an angular velocity of 2 rad/s. Determine the angular velocity of gear A.





Kinematic Diagram : Since link BC is rotating about fixed point C, then $\mathbf{v}_B$ is always directed perpendicular to link BC and its magnitude is $v_B = \omega_{BC} r_{BC} = 8(15) = 120$ in./s. At the instant shown, $\mathbf{v}_B$ is directed to the left. Also, at the same instant, point E is moving to the right with a speed of $v_E = \omega_D r_{CE} = 2(20) = 40$ in./s.

Velocity Equation : Here, $v_{B/E} = \omega_A r_{B/E} = 5\omega_A$ which is directed to the left. Applying Eq. 16 – 15, we have

$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E}$$

$$\left[\begin{matrix}120\\ \leftarrow\end{matrix}\right] = \left[\begin{matrix}40\\ \rightarrow\end{matrix}\right] + \left[\begin{matrix}5\omega_A\\ \leftarrow\end{matrix}\right]$$

$$(\overset{+}{\rightarrow}) \qquad -120 = 40 - 5\omega_A$$

$$\omega_A = 32.0 \text{ rad/s} \qquad \textbf{Ans}$$

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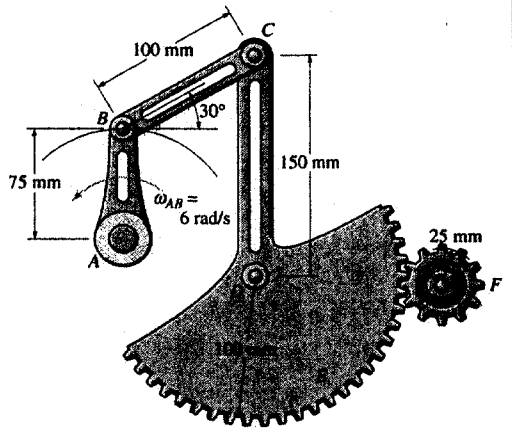
***16-60.** The rotation of link AB creates an oscillating movement of gear F. If AB has an angular velocity of $\omega_{AB} = 6$ rad/s, determine the angular velocity of gear F at the instant shown. Gear E is rigidly attached to arm CD and pinned at D to a fixed point.



Kinematic Diagram : Since link AB and arm CD are rotating about the fixed points A and D respectively, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular their their respective arms with the magnitude of $v_B = \omega_{AB} r_{AB} = 6(0.075)$ $= 0.450$ m/s and $v_C = \omega_{CD} r_{CD} = 0.15\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ and $\mathbf{v}_C$ are directed toward *negative* x axis.

Velocity Equation : Here, $\mathbf{r}_{B/C} = \{0.1\cos 30°\mathbf{i} + 0.1\sin 30°\mathbf{j}\}$ m $= \{0.08660\mathbf{i} + 0.05\mathbf{j}\}$ m. Applying Eq. 16 – 16, we have

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$
$$-0.450\mathbf{i} = -0.15\omega_{CD}\mathbf{i} + (\omega_{BC}\mathbf{k}) \times (0.08660\mathbf{i} + 0.05\mathbf{j})$$
$$-0.450\mathbf{i} = -(0.05\omega_{BC} + 0.15\omega_{CD})\mathbf{i} + 0.08660\omega_{BC}\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components gives

$$0 = 0.08660\omega_{BC} \qquad \omega_{BC} = 0$$
$$-0.450 = -[0.05(0) + 0.15\omega_{CD}] \qquad \omega_{CD} = 3.00 \text{ rad/s}$$

Angular Motion About a Fixed Point : The angular velocity of gear E is the same with arm CD since they are attached together. Then, $\omega_E = \omega_{CD}$ $= 3.00$ rad/s. Here, $\omega_E r_E = \omega_F r_F$ where ω_F is the angular velocity of gear F.

$$\omega_F = \frac{r_E}{r_F}\omega_E = \left(\frac{100}{25}\right)(3.00) = 12.0 \text{ rad/s} \qquad \textbf{Ans}$$

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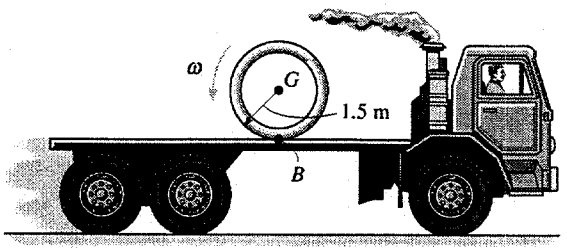
16-61. At the instant shown, the truck is traveling to the right at 3 m/s, while the pipe is rolling counterclockwise at $\omega = 8$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$

$\underset{\rightarrow}{v_G} = \underset{\rightarrow}{3} + \underset{\leftarrow}{12}$

$v_G = 9 \text{ m/s} \leftarrow$ **Ans**

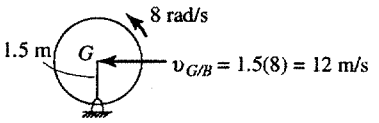
Also:

$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$

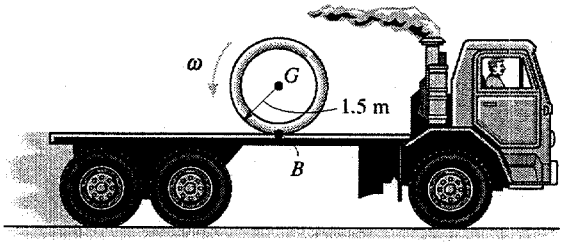
$v_G\mathbf{i} = 3\mathbf{i} + (8\mathbf{k}) \times (1.5\mathbf{j})$

$v_G = 3 - 12$

$v_G = -9 \text{ m/s} = 9 \text{ m/s} \leftarrow$ **Ans**



16-62. At the instant shown, the truck is traveling to the right at 8 m/s. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary.



$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$

$0 = \underset{\rightarrow}{8} + \underset{\leftarrow}{1.5\omega}$

$\omega = \dfrac{8}{1.5} = 5.33 \text{ rad/s}$ **Ans**

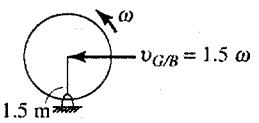
Also:

$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$

$0\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (1.5\mathbf{j})$

$0 = 8 - 1.5\omega$

$\omega = \dfrac{8}{1.5} = 5.33 \text{ rad/s}$ **Ans**



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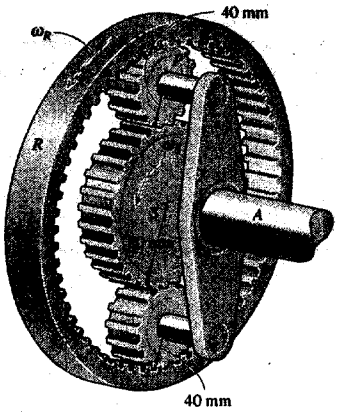
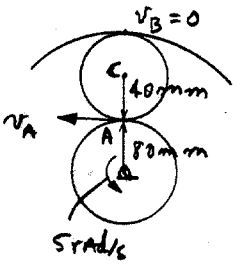
16-63. The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.





$v_A = 5(80) = 400$ mm/s ←

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p \mathbf{k}) \times (80\mathbf{j})$

$0 = -400\mathbf{i} - 80\omega_p$

$\omega_p = -5$ rad/s $= 5$ rad/s **Ans**

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

$\omega_A = \dfrac{200}{120} = 1.67$ rad/s **Ans**

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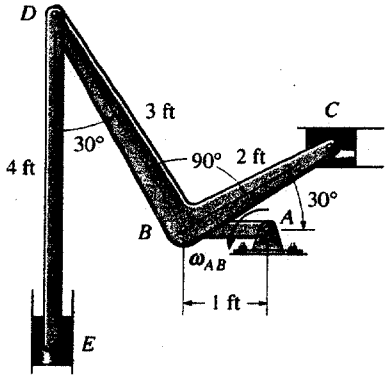
***16-64.** * If the link AB is rotating about the pin at A with an angular velocity $\omega_{AB} = 5$ rad/s, determine the velocities of blocks C and E at the instant shown.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$v_C = 5 + \omega(2)$$
$$\leftarrow \quad \downarrow \quad \nwarrow 30°$$

$$\stackrel{+}{\leftarrow} \quad v_C = 0 + \omega(2)\sin 30°$$

$$+\downarrow \quad 0 = 5 - \omega(2)\cos 30°$$

$$\omega = 2.887 \text{ rad/s}$$

$$v_C = 2.89 \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$\mathbf{v}_D = 5 + 2.887(3) + \omega_{DE}(4)$$
$$\downarrow \quad \nwarrow 60° \quad \rightarrow$$

$$\mathbf{v}_E = \mathbf{v}_D + \mathbf{v}_{E/D}$$

$$v_E = 5 + 2.887(3) + \omega_{DE}(4)$$
$$\downarrow \quad \downarrow \quad \nwarrow 60° \quad \rightarrow$$

$$\stackrel{+}{\rightarrow} \quad 0 = 0 - 7.500 + \omega_{DE}(4)$$

$$\omega_{DE} = 1.875 \text{ rad/s}$$

$$+\downarrow \quad v_E = 5 + 4.330$$

$$v_E = 9.33 \text{ ft/s} \downarrow \qquad \textbf{Ans}$$

Also:

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-v_C\mathbf{i} = -5\mathbf{j} + (\omega\mathbf{k}) \times (2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$-v_C = 0 - \omega$$

$$0 = -5 + 1.732\omega$$

$$\omega = 2.887 \text{ rad/s}$$

$$v_C = 2.89 \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{D/B}$$

$$\mathbf{v}_D = -5\mathbf{j} + (2.887\mathbf{k}) \times (-3\sin 30°\mathbf{i} + 3\cos 30°\mathbf{j})$$

$$\mathbf{v}_D = \{-7.5\mathbf{i} - 9.33\mathbf{j}\}$$

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

$$-v_E\mathbf{j} = (-7.5\mathbf{i} - 9.33\mathbf{j}) + (\omega_{DE}\mathbf{k}) \times (-4\mathbf{j})$$

$$0 = -7.5 + 4\omega_{DE}$$

$$-v_E = -9.33$$

$$\omega_{DE} = 1.875 \text{ rad/s}$$

$$v_E = 9.33 \text{ ft/s} \qquad \textbf{Ans}$$

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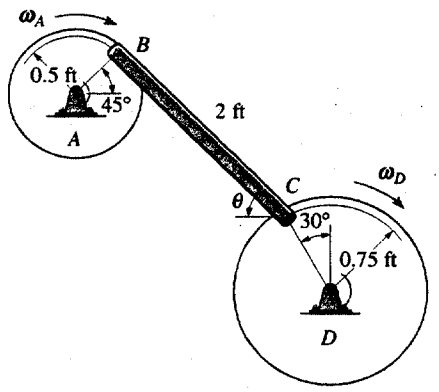
16-65. If disk D has a constant angular velocity $\omega_D = 2$ rad/s, determine the angular velocity of disk A at the instant $\theta = 60°$.



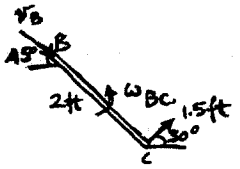
$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$

$v_B = 1.5 + 2\omega_{BC}$

$\searrow 45° \quad \angle 30° \nwarrow 60°$

$(\overset{+}{\rightarrow}) \quad v_B \cos 45° = 1.5\cos 30° - 2\omega_{BC} \sin 60°$

$(+\downarrow) \quad v_B \sin 45° = -1.5\sin 30° + 2\omega_{BC} \cos 60°$



$\omega_{BC} = 0.75$ rad/s

$v_B = 0$

$\omega_A = \dfrac{0}{0.5} = 0$ **Ans**

Also:

$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$

$v_B \cos 45°\mathbf{i} - v_B \sin 45°\mathbf{j} = 1.5\cos 30°\mathbf{i} + 1.5\sin 30°\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (2\cos 60°\mathbf{i} - 2\sin 60°\mathbf{j})$

$v_B \cos 45° = 1.5\cos 30° + \omega_{BC}(2\sin 60°)$

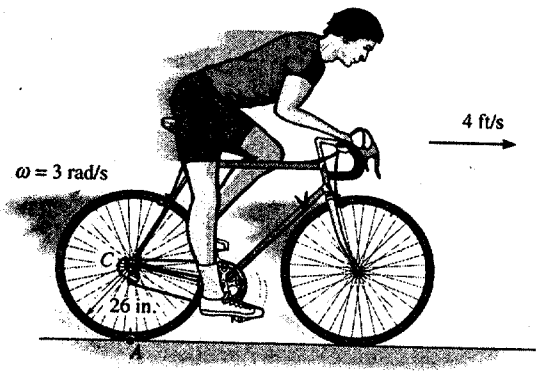
$-v_B \sin 45° = 1.5\sin 30° + \omega_{BC}(2\cos 60°)$

$\omega_{BC} = 0.75$ rad/s

$v_B = 0$

$\omega_A = \dfrac{0}{0.5} = 0$ **Ans**

16-66. The bicycle has a velocity $v = 4$ ft/s, and at the same instant the rear wheel has a clockwise angular velocity $\omega = 3$ rad/s, which causes it to slip at its contact point A. Determine the velocity of point A.



$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$

$\left[\underset{\leftarrow}{v_A}\right] = \left[\underset{\rightarrow}{4}\right] + \left[\left(\dfrac{26}{12}\right)\underset{\leftarrow}{(3)}\right]$

$v_A = 2.5$ ft/s **Ans**

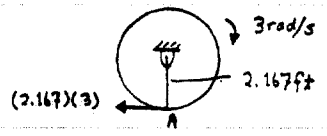
Also,

$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C}$

$\mathbf{v}_A = 4\mathbf{i} + (-3\mathbf{k}) \times \left(-\dfrac{26}{12}\mathbf{j}\right)$

$\mathbf{v}_A = 4\mathbf{i} - 6.5\mathbf{i} = -2.5\mathbf{i}$

$v_A = 2.5$ ft/s $\leftarrow$ **Ans**



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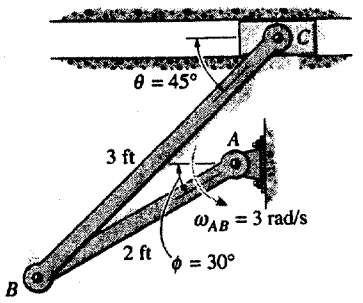
16-67. If the angular velocity of link AB is $\omega_{AB} = 3$ rad/s, determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant $\theta = 45°$ and $\phi = 30°$.



$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

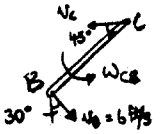
$$\left[\begin{array}{c} v_C \\ \leftarrow \end{array}\right] = \left[\begin{array}{c} 6 \\ 30° \end{array}\right] + \left[\begin{array}{c} \omega_{CB}(3) \\ 45° \end{array}\right]$$

$\left(\overset{+}{\rightarrow}\right)$ $-v_C = 6\sin 30° - \omega_{CB}(3)\cos 45°$

$(+\uparrow)$ $0 = -6\cos 30° + \omega_{CB}(3)\sin 45°$

$\omega_{CB} = 2.45$ rad/s ↻ **Ans**

$v_C = 2.20$ ft/s ← **Ans**



Also,

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}) + (\omega_{CB}\mathbf{k}) \times (3\cos 45°\mathbf{i} + 3\sin 45°\mathbf{j})$

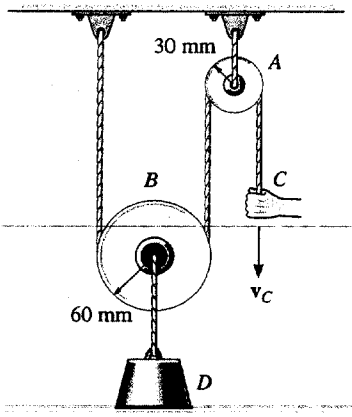
$\left(\overset{+}{\rightarrow}\right)$ $-v_C = 3 - 2.12\omega_{CB}$

$(+\uparrow)$ $0 = -5.196 + 2.12\omega_{CB}$

$\omega_{CB} = 2.45$ rad/s ↻ **Ans**

$v_C = 2.20$ ft/s ← **Ans**

***16-68.** If the end of the cord is pulled downward with a speed $v_C = 120$ mm/s, determine the angular velocities of pulleys A and B and the speed of block D. Assume that the cord does not slip on the pulleys.

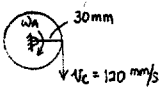


For pulley A : Motion about a fixed axis through the center applies.

$v_C = r_A \omega_A$

$120 = 30\omega_A$

$\omega_A = 4$ rad/s **Ans**



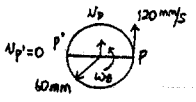
For pulley B : Point P' is at rest during the instant considered.

Thus,

$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/P'}$

$(+\uparrow)$ $120 = 0 + 120(\omega_B)$

$\omega_B = 1$ rad/s **Ans**



$\mathbf{v}_D = \mathbf{v}_{P'} + \mathbf{v}_{D/P'}$

$(+\uparrow)$ $v_D = 0 + 60(1)$

$v_D = 60$ mm/s **Ans**

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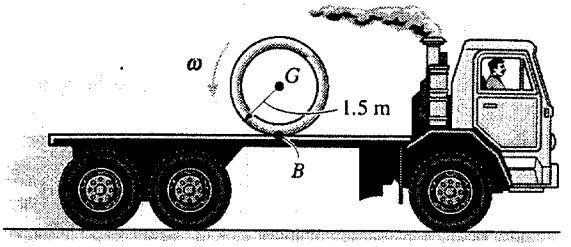
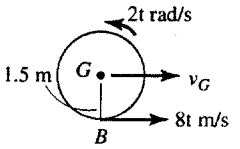
16-69. At the instant shown, the truck is traveling to the right at $8t$ m/s, while the pipe is rolling counterclockwise at $\omega = 2t$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$

$\left[\underset{\rightarrow}{v_G}\right] = \left[\underset{\rightarrow}{8t}\right] + \left[\underset{\leftarrow}{1.5(2t)}\right]$



$v_G = 5t$ m/s → **Ans**

Also:

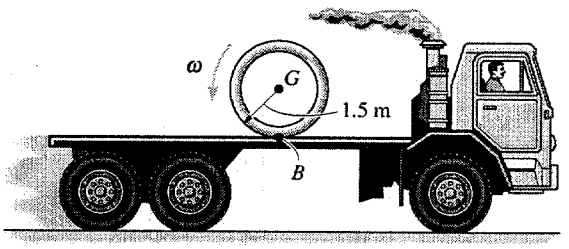
$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$

$v_G\mathbf{i} = (8t)\mathbf{i} + (2t)\mathbf{k} \times (1.5)\mathbf{j}$

$v_G = 8t - 3t$

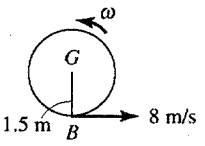
$v_G = 5t$ m/s → **Ans**

16-70. At the instant shown, the truck is traveling to the right at 12 m/s. If the pipe does not slip at B, determine its angular velocity if its mass center G appears to an observer on the ground to be moving to the right at 3 m/s.



$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$

$\left[\underset{\rightarrow}{3}\right] = \left[\underset{\rightarrow}{12}\right] + \left[\underset{\leftarrow}{1.5\omega}\right]$



$\omega = \frac{9}{1.5} = 6$ rad/s ↺ **Ans**

Also:

$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$

$3\mathbf{i} = 12\mathbf{i} + (\omega\mathbf{k}) \times (1.5\mathbf{j})$

$3 = 12 - 1.5\omega$

$\omega = \frac{9}{1.5} = 6$ rad/s ↺ **Ans**

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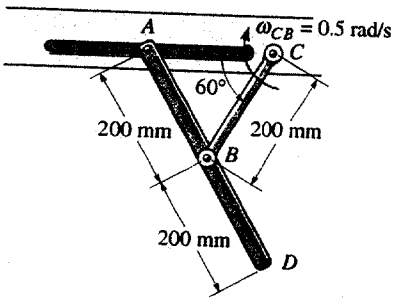
16-71. The top view of an automatic service window at a fast-food restaurant is shown in the figure. During operation, a motor drives the pin-connected link CB with an angular velocity $\omega_{CB} = 0.5$ rad/s. Determine the velocity at the instant shown of the end A, which moves along the slotted guide.



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

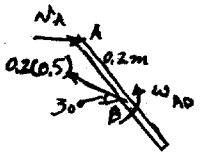
$$v_A = (0.2)(0.5) + \omega_{AD}(0.2)$$
$$\rightarrow \quad 30^\circ \nwarrow \quad 30^\circ \nearrow$$

$$(\overset{+}{\rightarrow})\ v_A = -(0.2)(0.5)\cos 30^\circ - \omega_{AD}(0.2)\cos 30^\circ$$

$$(+\uparrow)\ 0 = 0.2(0.5)\sin 30^\circ - \omega_{AD}(0.2)\sin 30^\circ$$

$$\omega_{AD} = 0.5 \text{ rad/s}$$

$$v_A = -0.17321 \text{ m/s} = -0.173 \text{ m/s} \qquad \textbf{Ans}$$



Also;

$$\mathbf{v}_A = \mathbf{v}_B + \omega_{AD} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = -0.2(0.5)\cos 30^\circ\mathbf{i} + 0.2(0.5)\sin 30^\circ\mathbf{j} + (\omega_{AD}\mathbf{k}) \times (-0.2\cos 60^\circ\mathbf{i} + 0.2\sin 60^\circ\mathbf{j})$$

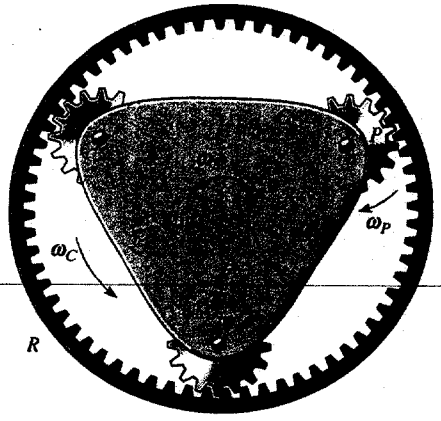
$$v_A = -(0.2)(0.5)\cos 30^\circ - \omega_{AD}(0.2)\cos 30^\circ$$

$$0 = 0.2(0.5)\sin 30^\circ - \omega_{AD}(0.2)\sin 30^\circ$$

$$\omega_{AD} = 0.5 \text{ rad/s}$$

$$v_A = -0.17321 \text{ m/s} = -0.173 \text{ m/s} \qquad \textbf{Ans}$$

***16–72.** Part of an automatic transmission consists of a *fixed* ring gear R, three equal planet gears P, the sun gear S, and the planet carrier C, which is shaded. If the sun gear is rotating at $\omega_S = 6$ rad/s, determine the angular velocity ω_C of the *planet carrier*. Note that C is pin-connected to the center of each of the planet gears.



$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A}$$

$$24 = 0 + 4(\omega_P)$$
$$\swarrow \qquad \searrow$$

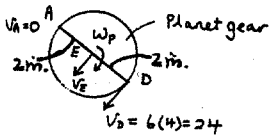
$$\omega_P = 6 \text{ rad/s}$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A}$$

$$v_E = 0 + 6(2)$$
$$\swarrow \qquad \swarrow$$

$$v_E = 12 \text{ in./s}$$

$$\omega_C = \frac{12}{6} = 2 \text{ rad/s} \qquad \textbf{Ans}$$



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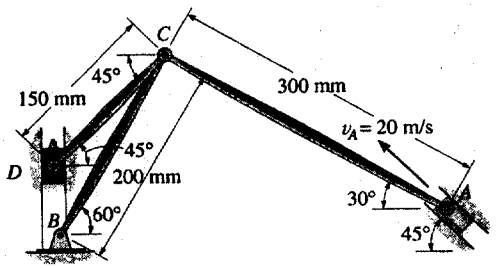
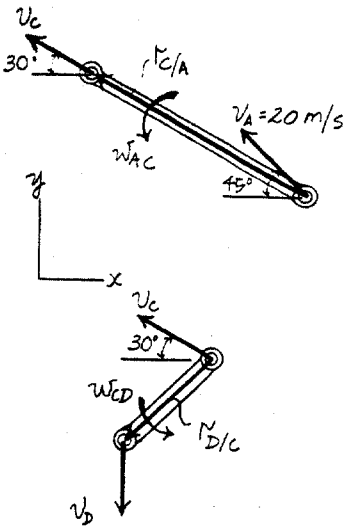
16-73. The mechanism shown is used in a riveting machine. It consists of a driving piston A, three links, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at $v_A = 20$ m/s.



Kinematic Diagram : Since link BC is rotating about fixed point B, then $\mathbf{v}_C$ is always directed perpendicular to link BC. At the instant shown, $\mathbf{v}_C = -v_C\cos 30°\mathbf{i} + v_C\sin 30°\mathbf{j} = -0.8660v_C\mathbf{i} + 0.500v_C\mathbf{j}$. Also, block D is moving towards the *negative* y axis due to the constraint of the guide. Then, $\mathbf{v}_D = -v_D\mathbf{j}$.

Velocity Equation : Here, $\mathbf{v}_A = \{-20\cos 45°\mathbf{i} + 20\sin 45°\mathbf{j}\}$ m/s $= \{-14.14\mathbf{i} + 14.14\mathbf{j}\}$ m/s and $\mathbf{r}_{C/A} = \{-0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j}\}$ m $= \{-0.2598\mathbf{i} + 0.150\mathbf{j}\}$ m. Applying Eq. 16 – 16 to link AC, we have

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A}$$
$$-0.8660v_C\mathbf{i} + 0.500v_C\mathbf{j} = -14.14\mathbf{i} + 14.14\mathbf{j} + (\omega_{AC}\mathbf{k}) \times (-0.2598\mathbf{i} + 0.150\mathbf{j})$$
$$-0.8660v_C\mathbf{i} + 0.500v_C\mathbf{j} = -(14.14 + 0.150\omega_{AC})\mathbf{i} + (14.14 - 0.2598\omega_{AC})\mathbf{j}$$



Equating i and j components gives

$$-0.8660v_C = -(14.14 + 0.150\omega_{AC}) \qquad [1]$$

$$0.500v_C = 14.14 - 0.2598\omega_{AC} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\omega_{AC} = 17.25 \text{ rad/s} \qquad v_C = 19.32 \text{ m/s}$$

Thus, $\mathbf{v}_C = \{-19.32\cos 30°\mathbf{i} + 19.32\sin 30°\mathbf{j}\}$ m/s $= \{-16.73\mathbf{i} + 9.659\mathbf{j}\}$ m/s and $\mathbf{r}_{D/C} = \{-0.15\cos 45°\mathbf{i} - 0.15\sin 45°\mathbf{j}\}$ m $= \{-0.1061\mathbf{i} - 0.1061\mathbf{j}\}$ m. Applying Eq. 16 – 16 to link CD, we have

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$
$$-v_D\mathbf{j} = -16.73\mathbf{i} + 9.659\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-0.1061\mathbf{i} - 0.1061\mathbf{j})$$
$$-v_D\mathbf{j} = (0.1061\omega_{CD} - 16.73)\mathbf{i} + (9.659 - 0.1061\omega_{CD})\mathbf{j}$$

Equating i and j components gives

$$0 = 0.1061\omega_{CD} - 16.73 \qquad [3]$$

$$-v_D = 9.659 - 0.1061\omega_{CD} \qquad [4]$$

Solving Eqs. [3] and [4] yields

$$\omega_{CD} = 157.74 \text{ rad/s}$$
$$v_D = 7.07 \text{ m/s} \qquad \textbf{Ans}$$

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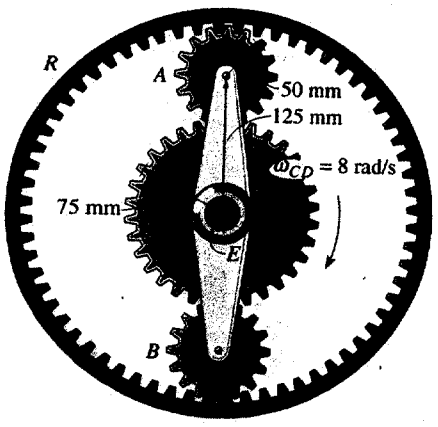
16-74. In an automobile transmission the planet pinions A and B rotate on shafts that are mounted on the planet-pinion carrier CD. As shown, CD is attached to a shaft at E which is aligned with the center of the *fixed* sun-gear S. This shaft is not attached to the sun gear. If CD is rotating at $\omega_{CD} = 8$ rad/s, determine the angular velocity of the ring gear R.



$$\mathbf{v}_C = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{C/P}$$

$$1\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times (0.05\mathbf{j})$$

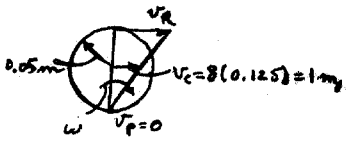
$$\omega = 20 \text{ rad/s}$$

$$\mathbf{v}_R = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{R/P}$$

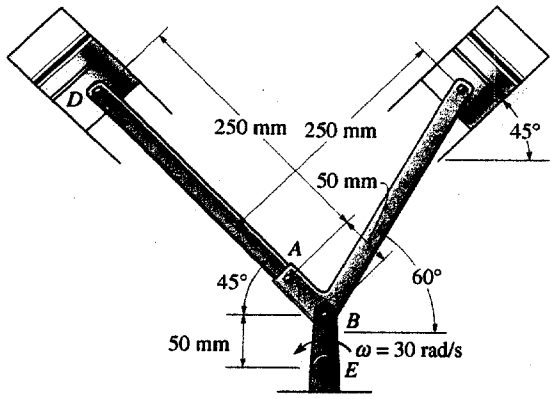
$$\mathbf{v}_R = \mathbf{0} + (-20\mathbf{k}) \times (0.1\mathbf{j})$$

$$\mathbf{v}_R = 2\mathbf{i}$$

$$\omega_R = \frac{v_R}{r} = \frac{2}{(0.125 + 0.05)} = 11.4 \text{ rad/s}$$ Ans



16-75. The two-cylinder engine is designed so that the pistons are connected to the crankshaft BE using a master rod ABC and articulated rod AD. If the crankshaft is rotating at $\omega = 30$ rad/s, determine the velocities of the pistons C and D at the instant shown.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

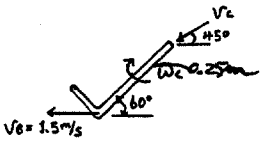
$$\underset{45^\circ \swarrow}{\mathbf{v}_C} = \underset{\leftarrow}{1.5} + \underset{\searrow 30^\circ}{(0.25)\omega}$$

$$(\overset{+}{\leftarrow}) \quad v_C \cos 45^\circ = 1.5 - \omega_C(0.25)(\cos 30^\circ)$$

$$(+\downarrow) \quad v_C \sin 45^\circ = 0 + \omega_C(0.25)(\sin 30^\circ)$$

$$v_C = 0.776 \text{ m/s}$$ Ans

$$\omega_C = 4.39 \text{ rad/s}$$



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

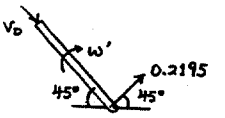
$$\underset{\rightarrow}{\mathbf{v}_A} = \underset{\leftarrow}{1.5} + \underset{\nearrow 45^\circ}{[0.05(4.39) = 0.2195]}$$

$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A}$$

$$\underset{45^\circ \nwarrow}{\mathbf{v}_D} = \underset{45^\circ \swarrow}{1.5} + \underset{\nearrow}{0.2195} + \underset{\nearrow}{\omega'(0.25)}$$

$$(\overset{+}{\searrow}) \quad v_D = -1.5 \sin 45^\circ$$

$$v_D = 1.06 \text{ m/s} \ \nwarrow$$ Ans



Also:

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/E}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$-v_C \cos 45^\circ \mathbf{i} - v_C \sin 45^\circ \mathbf{j} = (30\mathbf{k}) \times (0.05\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (0.25\cos 60^\circ \mathbf{i} + 0.25 \sin 60^\circ \mathbf{j})$$

$$-v_C \cos 45^\circ = -1.5 - \omega_{BC}(0.2165)$$

$$-v_C \sin 45^\circ = 0.125\omega_{BC}$$

$$v_C = 0.776 \text{ m/s}$$ Ans

$$\omega_{BC} = 4.39 \text{ rad/s}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{A/B}$$

$$\mathbf{v}_D = \mathbf{v}_A + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{D/A}$$

$$v_D \cos 45^\circ \mathbf{i} - v_D \sin 45^\circ \mathbf{j} = (30\mathbf{k}) \times (0.05\mathbf{j}) + (-4.39\mathbf{k}) \times (0.05\cos 45^\circ \mathbf{i} + 0.05 \sin 45^\circ \mathbf{j}) + (\omega_{AD}\mathbf{k}) \times (-0.25\cos 45^\circ \mathbf{i} + 0.25 \sin 45^\circ \mathbf{j})$$

$$v_D \cos 45^\circ = -1.5 + 0.1552 - \omega_{AD}(0.1768)$$

$$-v_D \sin 45^\circ = 0.1552 - 0.1768\omega_{AD}$$

$$\omega_{AD} = 3.36 \text{ rad/s}$$

$$v_D = 1.06 \text{ m/s}$$ Ans

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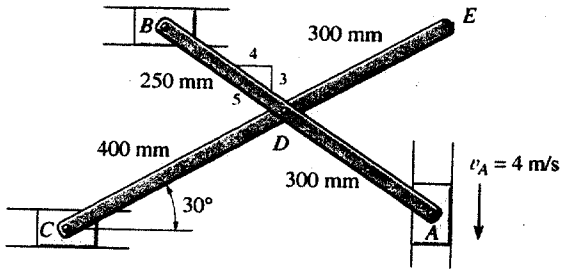
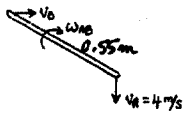
16-76. If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocities of blocks B and C at the instant shown.



$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$\overset{v_B}{\rightarrow} = 4\downarrow + \omega_{AB}(0.55)$

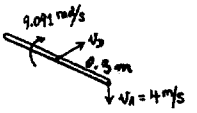


$(\overset{+}{\rightarrow})\quad v_B = 0 + \omega_{AB}(0.55)(\frac{3}{5})$

$(+\uparrow)\quad 0 = -4 + \omega_{AB}(0.55)(\frac{4}{5})$

Solving,

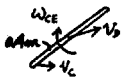
$\omega_{AB} = 9.091$ rad/s



$v_B = 3.00$ m/s **Ans**

$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A}$

$\underset{\downarrow}{\mathbf{v}_D} = \underset{\downarrow}{4} + \underset{\overset{3}{\nearrow}_5}{[(0.3)(9.091)}\ \underset{\overset{4}{\nearrow}}{=\ 2.727]}$



$\mathbf{v}_C = \mathbf{v}_D + \mathbf{v}_{C/D}$

$\underset{\rightarrow}{v_C} = \underset{\downarrow}{4} + \underset{\overset{5}{\nearrow}_4\ 3}{2.727} + \underset{\nwarrow\ 30°}{\omega_{CE}(0.4)}$

$(\overset{+}{\rightarrow})\quad v_C = 0 + 2.727(\frac{3}{5}) - \omega_{CE}(0.4)(\sin 30°)$

$(+\uparrow)\quad 0 = -4 + 2.727(\frac{4}{5}) + \omega_{CE}(0.4)(\cos 30°)$

$\omega_{CE} = 5.249$ rad/s

$v_C = 0.587$ m/s **Ans**

Also:

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$

$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \{\frac{-4}{5}(0.55)\mathbf{i} + \frac{3}{5}(0.55)\mathbf{j}\}$

$v_B = \omega_{AB}(0.33)$

$0 = -4 + 0.44\omega_{AB}$

$\omega_{AB} = 9.091$ rad/s

$v_B = 3.00$ m/s **Ans**

$\mathbf{v}_D = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/A}$

$\mathbf{v}_D = -4\mathbf{j} + (-9.091\mathbf{k}) \times \{\frac{-4}{5}(0.3)\mathbf{i} + \frac{3}{5}(0.3)\mathbf{j}\}$

$\mathbf{v}_D = \{1.636\mathbf{i} - 1.818\mathbf{j}\}$ m/s

$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{C/D}$

$v_C\mathbf{i} = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-\omega_{CE}\mathbf{k}) \times (-0.4\cos 30°\mathbf{i} - 0.4\sin 30°\mathbf{j})$

$v_C = 1.636 - 0.2\omega_{CE}$

$0 = -1.818 - 0.3464\omega_{CE}$

$\omega_{CE} = 5.25$ rad/s

$v_C = 0.587$ m/s **Ans**

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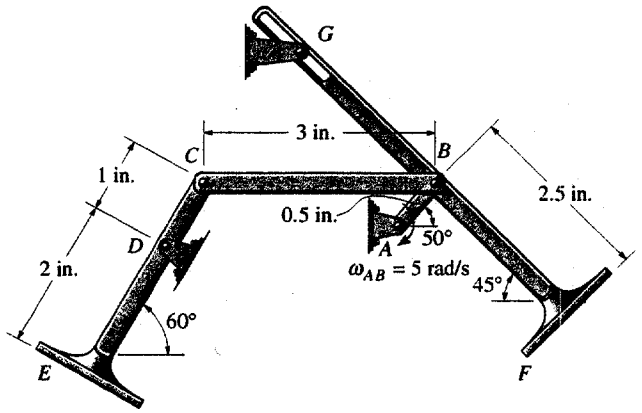
16-77. Mechanical toy animals often use a walking mechanism as shown idealized in the figure. If the driving crank AB is propelled by a spring motor such that $\omega_{AB} = 5$ rad/s, determine the velocity of the rear foot E at the instant shown. Although not part of this problem, the upper end of the foreleg has a slotted guide which is constrained by the fixed pin at G.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$v_C = 2.5 + 3\omega$$

$$(\overset{+}{\rightarrow}) \quad v_C \cos 30° = 2.5 \sin 50° + 0$$

$$v_C = 2.21 \text{ in./s}$$

$$\omega_{BC} = \frac{2.21}{1} = 2.21 \text{ rad/s}$$

$$v_E = (2.21)(2) = 4.42 \text{ in./s} \qquad \textbf{Ans}$$

Also:

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_C = \omega_{BC} \times \mathbf{r}_{C/D}$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$(\omega_{BC}\mathbf{k}) \times (\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) = (-5\mathbf{k}) \times (0.5\cos 50°\mathbf{i} + 0.5\sin 50°\mathbf{j}) + (\omega\mathbf{k}) \times (-3\mathbf{i})$$

$$-0.866\omega_{BC} = 1.915$$

$$0.5\omega_{BC} = -1.607 - 3\omega$$

$$\omega_{BC} = -2.21 \text{ rad/s}$$

$$\omega = -0.167 \text{ rad/s}$$

$$v_E = 2(2.21) = 4.42 \text{ in./s} \qquad \textbf{Ans}$$

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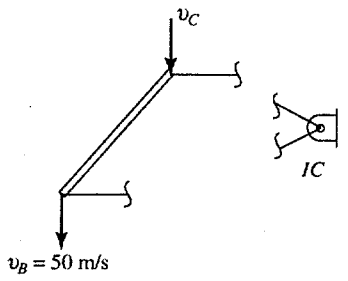
16-78. Solve Prob. 16-51 using the method of instantaneous center of zero velocity.

$v_B = 500(0.1) = 50$ m/s

Since IC is at ∞

$\omega = 0$

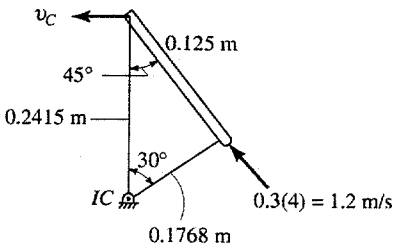
$v_C = v_B = 50$ m/s **Ans**



16-79. Solve Prob. 16-54 using the method of instantaneous center of zero velocity.

$$\omega = \frac{1.2}{0.1768} = 6.79 \text{ rad/s}$$

$v_C = 6.79(0.2415) = 1.64$ m/s ← **Ans**



***16-80.** Solve Prob. 16-62 using the method of instantaneous center of zero velocity.

Instantaneous center for link BC is at infinity.

$v_C = v_B = 6(0.075) = 0.45$ m/s

$$\omega_{CD} = \frac{0.45}{0.15} = 3 \text{ rad/s}$$

$$\omega_F = 3\left(\frac{100}{25}\right) = 12.0 \text{ rad/s}$$ **Ans**

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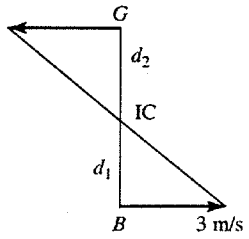
16-81. Solve Prob. 16-61 using the method of instantaneous center of zero velocity.

$v_B = 3$ m/s

$d_1 = \frac{3}{8} = 0.375$ m

$d_2 = 1.5 - 0.375 = 1.125$ m

$v_G = 8(1.125)$ m $= 9$ m/s $\leftarrow$ **Ans**

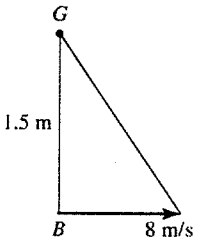


16-82. Solve Prob. 16-62 using the method of instantaneous center of zero velocity.

Mass center G is the instantaneous center.

$r\omega = v_B$

$\omega = \frac{v_B}{r} = \frac{8}{1.5} = 5.33$ rad/s **Ans**



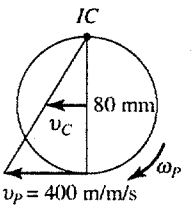
16-83. Solve Prob. 16-63 using the method of instantaneous center of zero velocity.

$v_P = (80)(5) = 40$ m/s

$\omega_P = \frac{400}{80} = 5$ rad/s ↻ **Ans**

$v_C = (5)(40) = 200$ mm/s

$\omega_A = \frac{200}{(80 + 40)} = 1.67$ rad/s ↺ **Ans**



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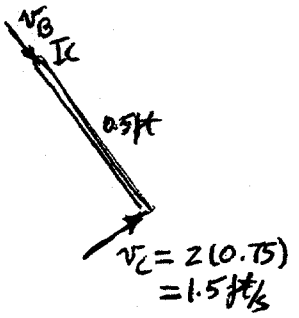
***16-84.** Solve Prob. 16–65 using the method of instantaneous center of zero velocity.

When radial lines extend from B and C, they intersect at B, thus the IC is at B and so

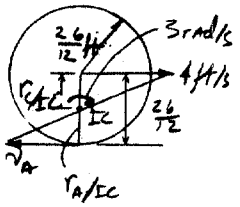
$$v_B = 0 \qquad \textbf{Ans}$$

Hence

$$\omega_A = \frac{0}{0.5 \text{ ft}} = 0 \qquad \textbf{Ans}$$



16-85. Solve Prob. 16–66 using the method of instantaneous center of zero velocity.



$$r_{C/IC} = \frac{4}{3} = 1.33 \text{ ft}$$

$$r_{A/IC} = \frac{26}{12} - 1.33 \text{ ft} = 0.833 \text{ ft}$$

$$v_A = 3(0.833) = 2.5 \text{ ft/s} \leftarrow \qquad \textbf{Ans}$$

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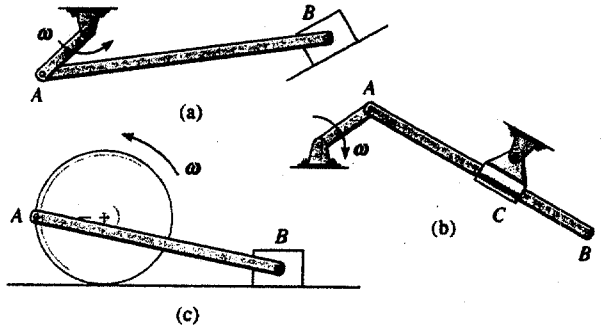
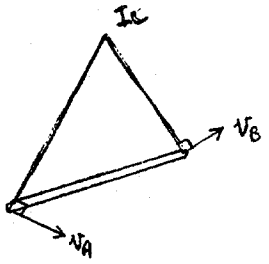
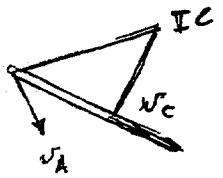
16-86. In each case show graphically how to locate the instantaneous center of zero velocity of link AB. Assume the geometry is known.



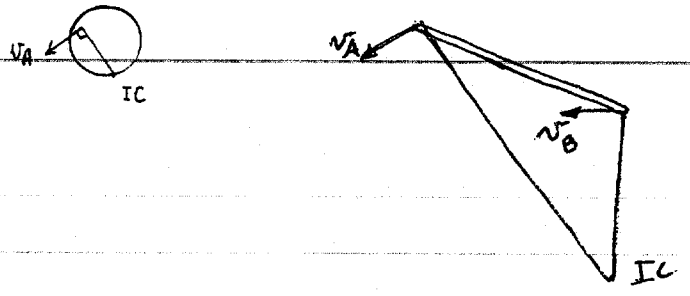
a)



b)



c)



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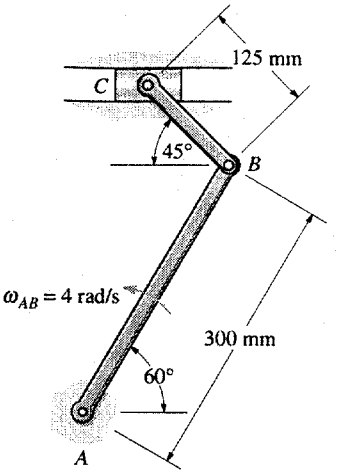
16-87. The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the angular velocity of link CB at the instant shown, if the link AB is rotating at 4 rad/s.



Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 4(0.3) = 1.20$ m/s. At the instant shown, $\mathbf{v}_B$ is directed at an angle 30° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.

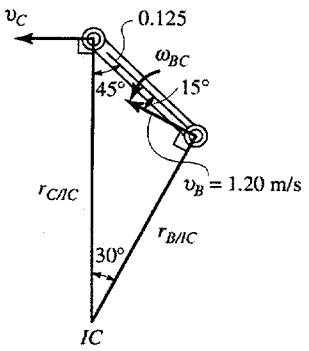
Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 45°} = \frac{0.125}{\sin 30°} \qquad r_{B/IC} = 0.1768 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105°} = \frac{0.125}{\sin 30°} \qquad r_{C/IC} = 0.2415 \text{ m}$$

The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.1768} = 6.79 \text{ rad/s} \qquad \textbf{Ans}$$



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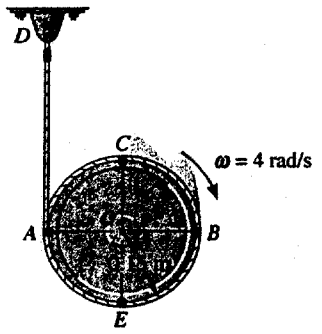
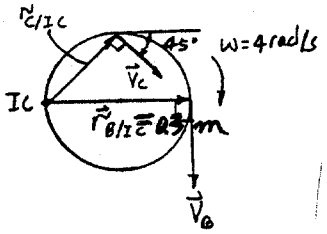
***16-88.** At the instant shown, the disk is rotating at $\omega = 4$ rad/s. Determine the velocities of points A, B, and C.





The instantaneous center is located at point A. Hence, $v_A = 0$ **Ans**

$r_{C/IC} = \sqrt{0.15^2 + 0.15^2} = 0.2121$ m $\quad r_{B/IC} = 0.3$ m

$v_B = \omega\, r_{B/IC} = 4(0.3) = 1.2$ m/s **Ans**

$v_C = \omega\, r_{C/IC} = 4(0.2121) = 0.849$ m/s $\searrow 45°$ **Ans**

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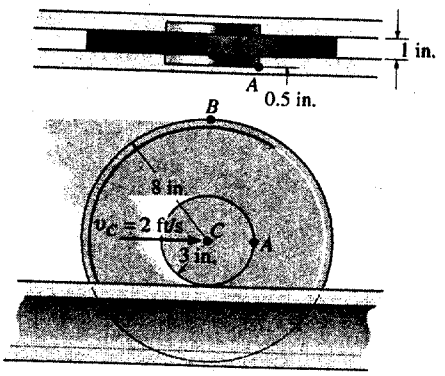
16-89. The wheel rolls on its hub without slipping on the horizontal surface. If the velocity of the center of the wheel is $v_C = 2$ ft/s to the right, determine the velocities of points A and B at the instant shown.



$v_C = \omega r_{C/IC}$

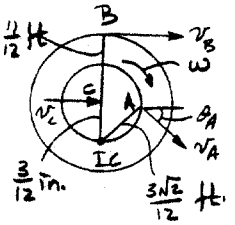
$2 = \omega\left(\frac{3}{12}\right)$

$\omega = 8$ rad/s

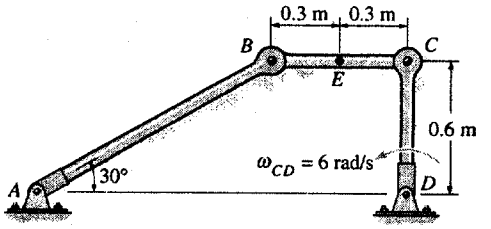
$v_B = \omega r_{B/IC} = 8\left(\frac{11}{12}\right) = 7.33$ ft/s $\rightarrow$ **Ans**

$v_A = \omega r_{A/IC} = 8\left(\frac{3\sqrt{2}}{12}\right) = 2.83$ ft/s **Ans**

$\theta_A = \tan^{-1}\left(\frac{3}{3}\right) = 45°$ $\nwarrow$ **Ans**



16-90. If link CD has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point E on link BC and the angular velocity of link AB at the instant shown.



$v_C = \omega_{CD}(r_{CD}) = (6)(0.6) = 3.60$ m/s

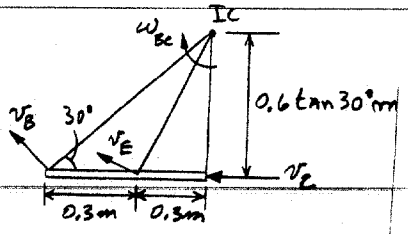
$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3.60}{0.6\tan 30°} = 10.39$ rad/s

$v_B = \omega_{BC} r_{B/IC} = (10.39)\left(\frac{0.6}{\cos 30°}\right) = 7.20$ m/s

$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{7.20}{\left(\frac{0.6}{\sin 30°}\right)} = 6$ rad/s $\circlearrowleft$ **Ans**

$v_E = \omega_{BC} r_{E/IC} = 10.39\sqrt{(0.6\tan 30°)^2 + (0.3)^2} = 4.76$ m/s **Ans**

$\theta = \tan^{-1}\left(\frac{0.3}{0.6\tan 30°}\right) = 40.9°$ $\searrow$ **Ans**



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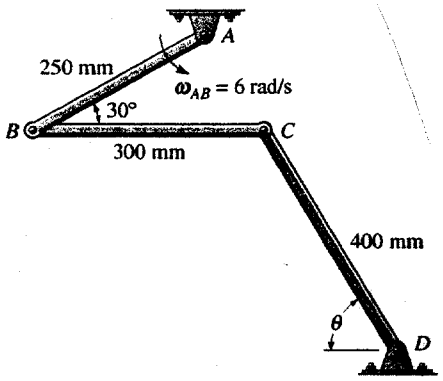
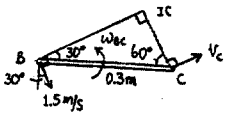
16-91. If link AB is rotating at $\omega_{AB} = 6$ rad/s, determine the angular velocities of links BC and CD at the instant $\theta = 60°$.



$r_{IC-B} = 0.3\cos 30° = 0.2598$ m

$r_{IC-C} = 0.3\cos 60° = 0.1500$ m

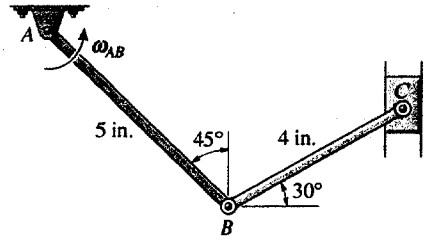
$\omega_{BC} = \dfrac{1.5}{0.2598} = 5.774 = 5.77$ rad/s **Ans**



$v_C = 5.774(0.15) = 0.8661$ m/s

$\omega_{CD} = \dfrac{0.8661}{0.4} = 2.17$ rad/s **Ans**

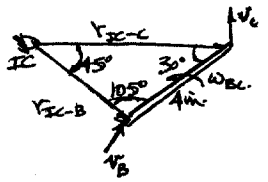
***16-92.** Determine the angular velocity of link AB at the instant shown if block C is moving upward at 12 in./s.



$\dfrac{4}{\sin 45°} = \dfrac{r_{IC-B}}{\sin 30°} = \dfrac{r_{IC-C}}{\sin 105°}$

$r_{IC-C} = 5.464$ in.

$r_{IC-B} = 2.828$ in.



$v_C = \omega_{BC}(r_{IC-C})$

$12 = \omega_{BC}(5.464)$

$\omega_{BC} = 2.1962$ rad/s

$v_B = \omega_{BC}(r_{IC-B})$

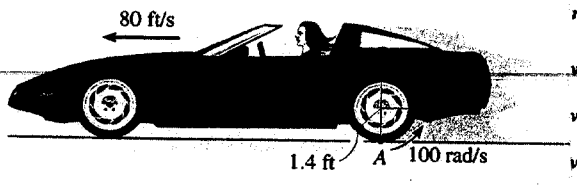
$= 2.1962(2.828) = 6.211$ in./s

$v_B = \omega_{AB} r_{AB}$

$6.211 = \omega_{AB}(5)$

$\omega_{AB} = 1.24$ rad/s **Ans**

16-93. As the car travels forward at 80 ft/s on a wet road, due to slipping, the rear wheels have an angular velocity $\omega = 100$ rad/s. Determine the speeds of points A, B, and C caused by the motion.

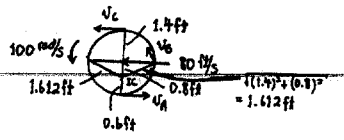


$r = \dfrac{80}{100} = 0.8$ ft

$v_A = 0.6(100) = 60.0$ ft/s → **Ans**

$v_C = 2.2(100) = 220$ ft/s ← **Ans**

$v_B = 1.612(100) = 161$ ft/s $^{60.3°}\nwarrow$ **Ans**



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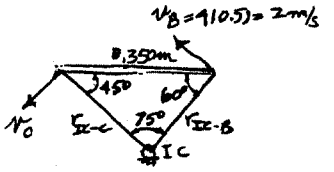
16-94. Knowing that the angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.



$$\frac{0.350}{\sin 75°} = \frac{r_{IC-B}}{\sin 45°} = \frac{r_{IC-C}}{\sin 60°}$$

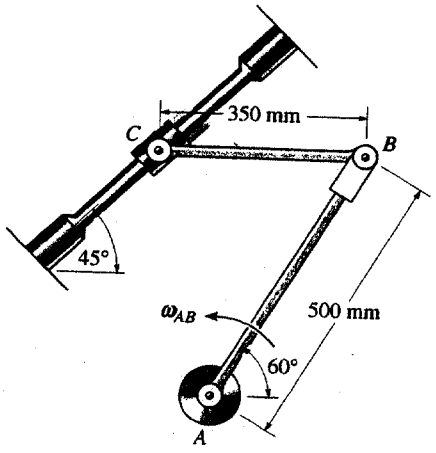
$r_{IC-B} = 0.2562$ m

$r_{IC-C} = 0.3138$ m

$\omega_{CB} = \dfrac{2}{0.2562} = 7.8059 = 7.81$ rad/s **Ans**

$v_C = 7.8059(0.3138) = 2.45$ m/s **Ans**

16-95. If the collar at C is moving downward to the left at $v_C = 8$ m/s, determine the angular velocity of link AB at the instant shown.



$$\frac{0.350}{\sin 75°} = \frac{r_{IC-B}}{\sin 45°} = \frac{r_{IC-C}}{\sin 60°}$$

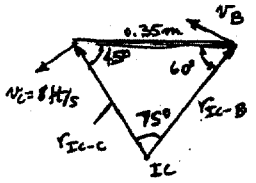
$r_{IC-B} = 0.2562$ m

$r_{IC-C} = 0.3138$ m

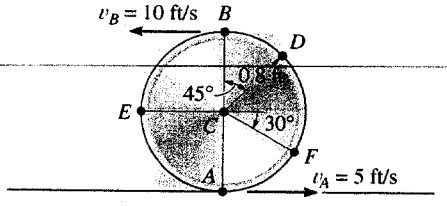
$\omega_{CB} = \dfrac{8}{0.3138} = 25.494$ rad/s

$v_B = 25.494(0.2562) = 6.5315$ m/s

$\omega_{AB} = \dfrac{6.5315}{0.5} = 13.1$ rad/s **Ans**



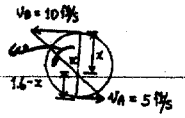
***16-96.** Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point D at this instant.



$$\frac{1.6 - x}{5} = \frac{x}{10}$$

$5x = 16 - 10x$

$x = 1.06667$ ft

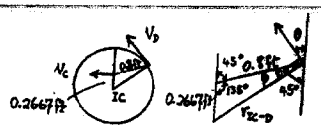


$\omega = \dfrac{10}{1.06667} = 9.375$ rad/s

$r_{IC-D} = \sqrt{(0.2667)^2 + (0.8)^2 - 2(0.2667)(0.8)\cos 135°} = 1.006$ ft

$$\frac{\sin\phi}{0.2667} = \frac{\sin 135°}{1.006}$$

$\phi = 10.80°$



$v_C = 0.2667(9.375) = 2.50$ ft/s **Ans**

$v_D = 1.006(9.375) = 9.43$ ft/s **Ans**

$\theta = 45° + 10.80° = 55.8°$

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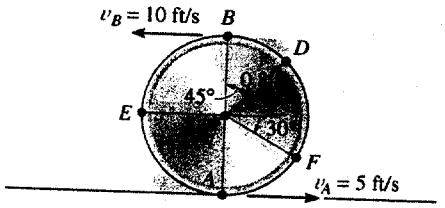
16-97. Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point E at this instant.



$$\frac{1.6-x}{5} = \frac{x}{10}$$

$$5x = 16 - 10x$$

$$x = 1.06667 \text{ ft}$$

$$\omega = \frac{10}{1.06667} = 9.375 \text{ rad/s}$$

$$v_C = \omega(r_{IC-C})$$

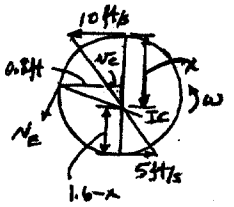
$$= 9.375(1.06667 - 0.8)$$

$$= 2.50 \text{ ft/s} \quad \textbf{Ans}$$

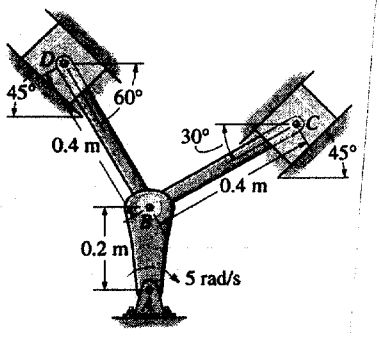
$$v_E = \omega(r_{IC-E})$$

$$= 9.375\sqrt{(0.8)^2 + (0.26667)^2}$$

$$= 7.91 \text{ ft/s} \quad \textbf{Ans}$$



16-98. The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$$v_B = 0.2(5) = 1 \text{ m/s} \rightarrow$$

Member BC:

$$\frac{r_{C/IC}}{\sin 60^\circ} = \frac{0.4}{\sin 45^\circ}$$

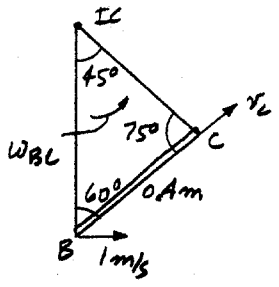
$$r_{C/IC} = 0.4899 \text{ m}$$

$$\frac{r_{B/IC}}{\sin 75^\circ} = \frac{0.4}{\sin 45^\circ}$$

$$r_{B/IC} = 0.5464 \text{ m}$$

$$\omega_{BC} = \frac{1}{0.5464} = 1.830 \text{ rad/s}$$

$$v_C = 0.4899(1.830) = 0.897 \text{ m/s} \quad \textbf{Ans}$$



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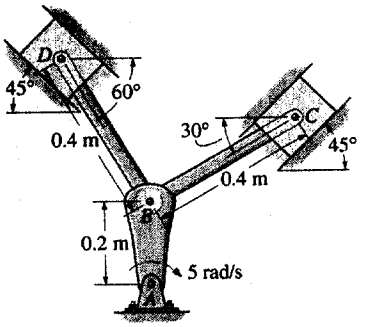
16-99. The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at D the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$v_B = 0.2(5) = 1$ m/s $\rightarrow$

Member BD :

$$\frac{r_{B/IC}}{\sin 105°} = \frac{0.4}{\sin 45°}$$

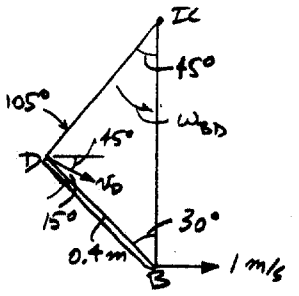
$r_{B/IC} = 0.54641$ m

$$\frac{r_{D/IC}}{\sin 30°} = \frac{0.4}{\sin 45°}$$

$r_{D/IC} = 0.28284$ m

$$\omega_{BD} = \frac{1}{0.54641} = 1.830 \text{ rad/s}$$

$v_D = 1.830(0.28284) = 0.518$ m/s $\searrow$ **Ans**



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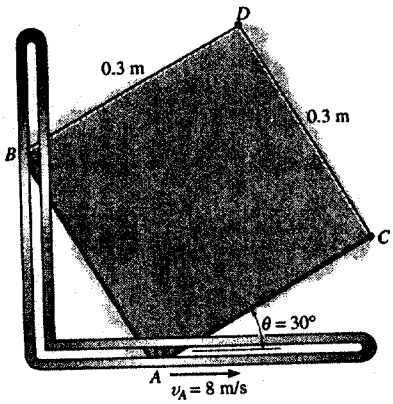
***16-100.** The square plate is confined within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point C at this instant.



$r_{A/IC} = 0.3\cos30° = 0.2598$ m

$\omega = \dfrac{8}{0.2598} = 30.792$ rad/s

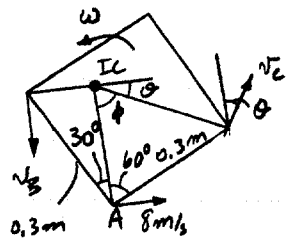
$r_{C/IC} = \sqrt{(0.2598)^2 + (0.3)^2 - 2(0.2598)(0.3)\cos60°} = 0.2821$ m

$v_C = (0.2821)(30.792) = 8.69$ m/s **Ans**

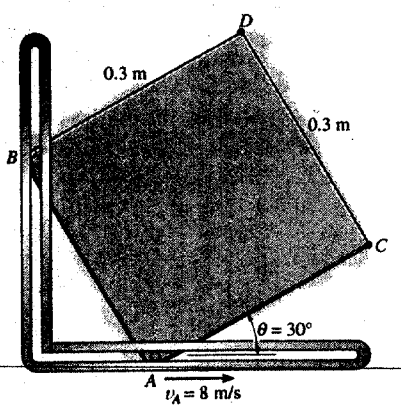
$\dfrac{\sin\phi}{0.3} = \dfrac{\sin60°}{0.2821}$

$\phi = 67.09°$

$\theta = 90° - 67.09° = 22.9°$ **Ans**



16-101. The square plate is confined within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point D at this instant.



$r_{A/IC} = 0.3\cos30° = 0.2598$ m

$\omega = \dfrac{8}{0.2598} = 30.792$ rad/s

$r_{B/IC} = 0.3\sin30° = 0.15$ m

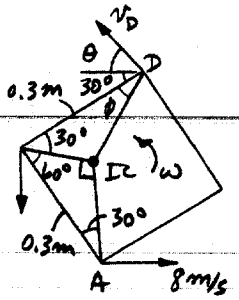
$r_{D/IC} = \sqrt{(0.3)^2 + (0.15)^2 - 2(0.3)(0.15)\cos30°} = 0.1859$ m

$v_D = (30.792)(0.1859) = 5.72$ m/s **Ans**

$\dfrac{\sin\phi}{0.15} = \dfrac{\sin30°}{0.1859}$

$\phi = 23.794°$

$\theta = 90° - 30° - 23.794° = 36.2°$ **Ans**



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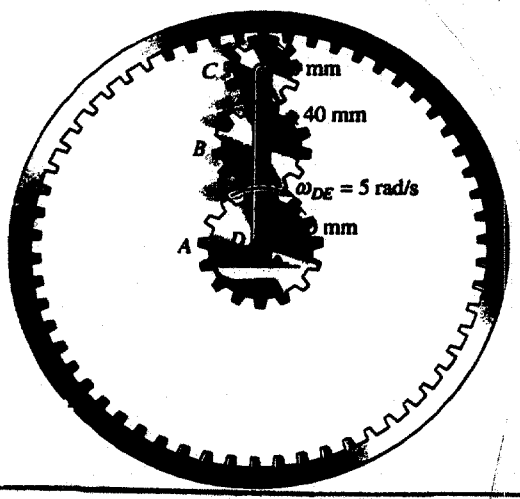
16-102. The epicyclic gear train is driven by the rotating link DE, which has an angular velocity $\omega_{DE} = 5$ rad/s. If the ring gear F is fixed, determine the angular velocities of gears A, B, and C.



$v_E = 0.16(5) = 0.8$ m/s

$\omega_C = \dfrac{0.8}{0.03} = 26.7$ rad/s **Ans**

$v_P = (0.06)(26.7) = 1.6$ m/s

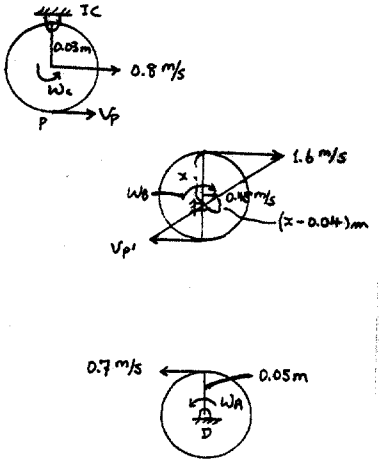
$\dfrac{1.6}{x} = \dfrac{0.45}{x-0.04}$

$x = 0.05565$ m

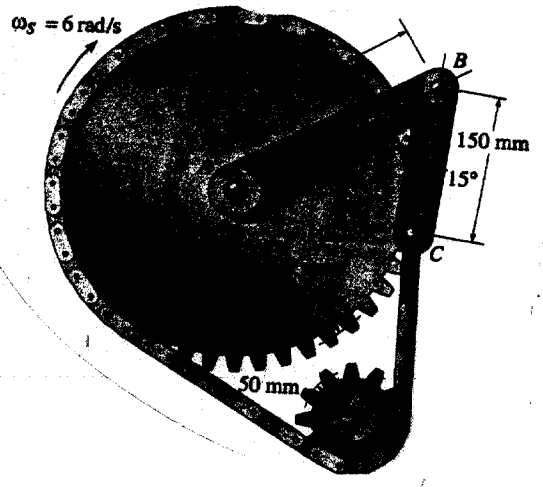
$\omega_B = \dfrac{1.6}{0.05565} = 28.75$ rad/s **Ans**

$v_{P'} = 28.75(0.08-0.05565) = 0.700$ m/s ←

$\omega_A = \dfrac{0.700}{0.05} = 14.0$ rad/s **Ans**



16-103. The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular velocity of $\omega_S = 6$ rad/s, determine the angular velocity of link AB at this instant. The sprocket S is mounted on a shaft which is separate from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links.



Kinematic Diagram : Since link AB is rotating about the fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 0.2\omega_{AB}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle 60° with the horizontal. Since point C is attached to the chain, at the instant shown, it moves vertically with a speed of $v_C = \omega_S r_S = 6(0.175) = 1.05$ m/s.

Instantaneous Center : The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 105°} = \frac{0.15}{\sin 30°} \qquad r_{B/IC} = 0.2898 \text{ m}$$

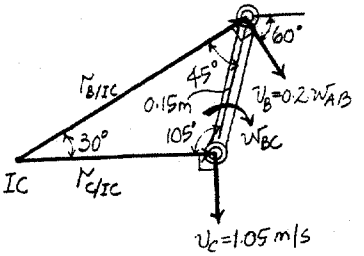
$$\frac{r_{C/IC}}{\sin 45°} = \frac{0.15}{\sin 30°} \qquad r_{C/IC} = 0.2121 \text{ m}$$

The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{1.05}{0.2121} = 4.950 \text{ rad/s}$$

Thus, the angular velocity of link AB is given by

$$v_B = \omega_{BC} r_{B/IC}$$
$$0.2\omega_{AB} = 4.950(0.2898)$$
$$\omega_{AB} = 7.17 \text{ rad/s} \qquad \textbf{Ans}$$



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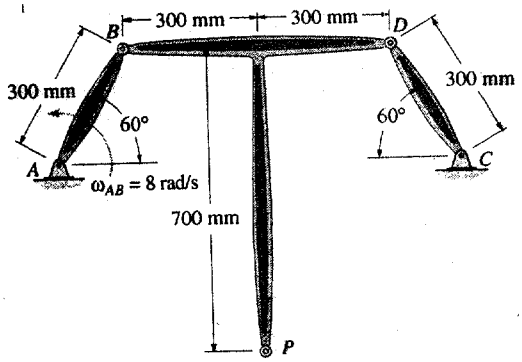
***16-104.** The similar links AB and CD rotate about the fixed pins at A and C. If AB has an angular velocity $\omega_{AB} = 8$ rad/s, determine the angular velocity of BDP and the velocity of point P.



Kinematic Diagram : Since link AB and CD is rotating about fixed points A and C, then $\mathbf{v}_B$ and $\mathbf{v}_D$ are always directed perpendicular to link AB and CD respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_D$ are $v_B = \omega_{AB} r_{AB} = 8(0.3)$ = 2.40 m/s and $v_D = \omega_{CD} r_{CD} = 0.3\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ and $\mathbf{v}_D$ are directed at 30° with the horizontal.

Instantaneous Center : The instantaneous center of zero velocity of link BDP at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_D$. From the geometry.

$$r_{B/IC} = \frac{0.3}{\cos 60°} = 0.600 \text{ m}$$

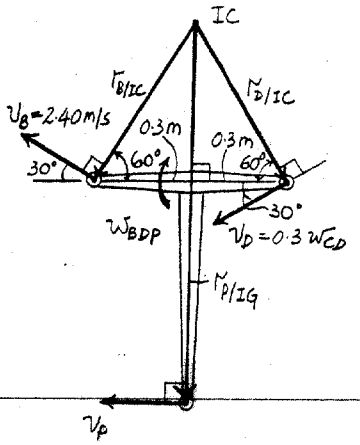
$$r_{P/IC} = 0.3\tan 60° + 0.7 = 1.220 \text{ m}$$

The angular velocity of link BDP is given by

$$\omega_{BDP} = \frac{v_B}{r_{B/IC}} = \frac{2.40}{0.600} = 4.00 \text{ rad/s} \qquad \textbf{Ans}$$

Thus, the velocity of point P is given by

$$v_P = \omega_{BDP} r_{P/IC} = 4.00(1.220) = 4.88 \text{ m/s} \leftarrow \qquad \textbf{Ans}$$



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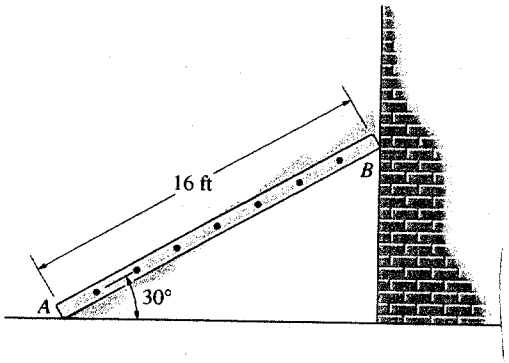
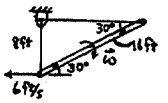
16-105. At a given instant the bottom A of the ladder has an acceleration $a_A = 4$ ft/s^2 and velocity $v_A = 6$ ft/s, both acting to the left. Determine the acceleration of the top of the ladder, B, and the ladder's angular acceleration at this same instant.

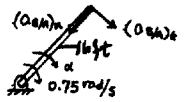




$$\omega = \frac{6}{8} = 0.75 \text{ rad/s}$$

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$a_B = 4 + (0.75)^2(16) + \alpha(16)$$
$$\downarrow \quad \leftarrow \quad 30^\circ \swarrow \quad 30^\circ \nwarrow$$



$$(\overset{+}{\leftarrow}) \quad 0 = 4 + (0.75)^2(16)\cos 30^\circ - \alpha(16)\sin 30^\circ$$

$$(+\downarrow) \quad a_B = 0 + (0.75)^2(16)\sin 30^\circ + \alpha(16)\cos 30^\circ$$

Solving,

$$\alpha = 1.47 \text{ rad/s}^2 \qquad \text{Ans}$$

$$a_B = 24.9 \text{ ft/s}^2 \downarrow \qquad \text{Ans}$$

Also:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-a_B\mathbf{j} = -4\mathbf{i} + (\alpha\mathbf{k}) \times (16\cos 30^\circ \mathbf{i} + 16\sin 30^\circ \mathbf{j}) - (0.75)^2(16\cos 30^\circ \mathbf{i} + 16\sin 30^\circ \mathbf{j})$$

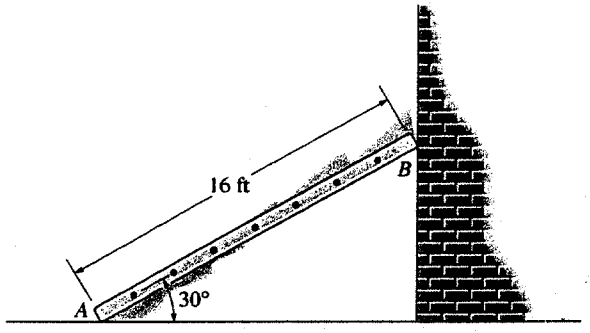
$$0 = -4 - 8\alpha - 7.794$$

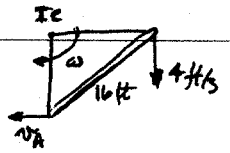
$$-a_B = 13.856\alpha - 4.5$$

$$\alpha = 1.47 \text{ rad/s}^2 \qquad \text{Ans}$$

$$a_B = 24.9 \text{ ft/s}^2 \downarrow \qquad \text{Ans}$$

16-106. At a given instant the top B of the ladder has an acceleration $a_B = 2$ ft/s^2 and a velocity of $v_B = 4$ ft/s, both acting downward. Determine the acceleration of the bottom A of the ladder, and the ladder's angular acceleration at this instant.





$$\omega = \frac{4}{16\cos 30^\circ} = 0.288675 \text{ rad/s}$$

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

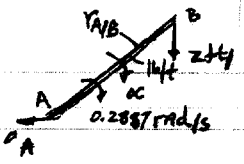
$$-a_A\mathbf{i} = -2\mathbf{j} + (\alpha\mathbf{k}) \times (-16\cos 30^\circ \mathbf{i} - 16\sin 30^\circ \mathbf{j}) - (0.288675)^2(-16\cos 30^\circ \mathbf{i} - 16\sin 30^\circ$$

$$-a_A = 8\alpha + 1.1547$$

$$0 = -2 - 13.856\,\alpha + 0.6667$$

$$\alpha = -0.0962 \text{ rad/s}^2 = 0.0962 \text{ rad/s}^2 \,) \qquad \text{Ans}$$

$$a_A = -0.385 \text{ ft/s}^2 = 0.385 \text{ ft/s}^2 \rightarrow \qquad \text{Ans}$$



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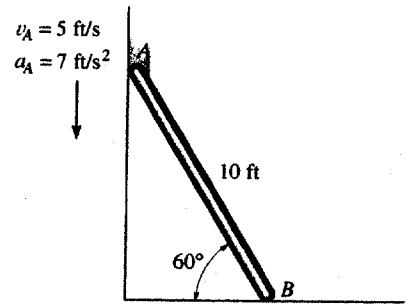
16-107. At a given instant the top end A of the bar has the velocity and acceleration shown. Determine the acceleration of the bottom B and the bar's angular acceleration at this instant.



$$\omega = \frac{5}{5} = 1.00 \text{ rad/s}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

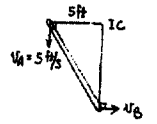
$$a_B = 7 + 10 + \alpha(10)$$
$$\rightarrow \quad \downarrow \quad \nwarrow 30° \quad \angle 30°$$

$$(\overset{+}{\rightarrow}) \quad a_B = 0 - 10\sin 30° + \alpha(10)\cos 30°$$

$$(+\uparrow) \quad 0 = -7 + 10\cos 30° + \alpha(10)\sin 30°$$

$$\alpha = -0.3321 \text{ rad/s}^2 = 0.332 \text{ rad/s}^2 \qquad \textbf{Ans}$$

$$a_B = -7.875 \text{ ft/s}^2 = 7.88 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans}$$



Also:

$$\mathbf{a}_B = \mathbf{a}_A - \omega^2 \mathbf{r}_{B/A} + \alpha \times \mathbf{r}_{B/A}$$

$$a_B\mathbf{i} = -7\mathbf{j} - (1)^2(10\cos 60°\mathbf{i} - 10\sin 60°\mathbf{j}) + (\alpha\mathbf{k}) \times (10\cos 60°\mathbf{i} - 10\sin 60°\mathbf{j})$$

$$\overset{+}{\rightarrow} \quad a_B = -10\cos 60° + \alpha(10\sin 60°)$$

$$+\uparrow \quad 0 = -7 + 10\sin 60° + \alpha(10\cos 60°)$$

$$\alpha = -0.3321 \text{ rad/s}^2 = 0.332 \text{ rad/s}^2 \qquad \textbf{Ans}$$

$$a_B = -7.875 \text{ ft/s}^2 = 7.88 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans}$$

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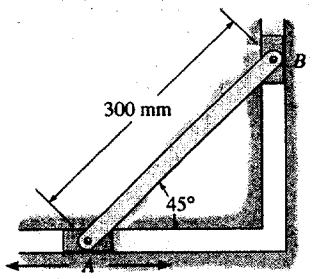
***16-108.** At a given instant, the slider block A has the velocity and deceleration shown. Determine the acceleration of block B and the angular acceleration of the link at this instant.



$v_A = 1.5$ m/s $\quad a_A = 16$ m/s^2

$$\omega_{AB} = \frac{v_B}{r_{A/IC}} = \frac{1.5}{0.3\cos 45°} = 7.07 \text{ rad/s}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-a_B\mathbf{j} = 16\mathbf{i} + (\alpha\mathbf{k}) \times (0.3\cos 45°\mathbf{i} + 0.3\sin 45°\mathbf{j}) - (7.07)^2 (0.3\cos 45°\mathbf{i} + 0.3\sin 45°\mathbf{j})$$

$$\left(\overset{+}{\rightarrow}\right) \quad 0 = 16 - \alpha(0.3)\sin 45° - (7.07)^2(0.3)\cos 45°$$

$$(+\downarrow) \quad a_B = 0 - \alpha(0.3)\cos 45° + (7.07)^2(0.3)\sin 45°$$

Solving :

$\alpha_{AB} = 25.4$ rad/s^2 ↻ **Ans**

$a_B = 5.21$ m/s^2 ↓ **Ans**

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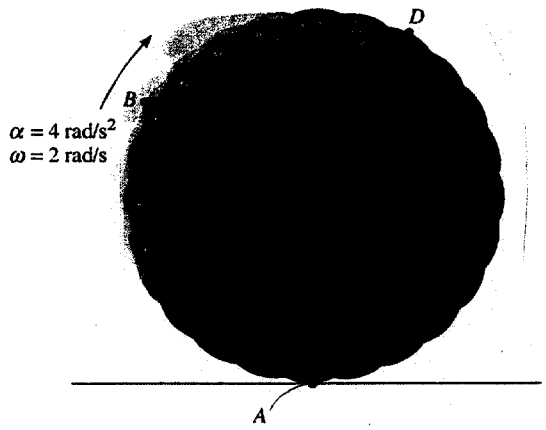
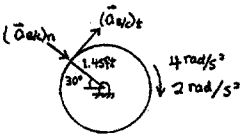
16-109. The wheel is moving to the right such that it has an angular velocity $\omega = 2$ rad/s and angular acceleration $\alpha = 4$ rad/s^2 at the instant shown. If it does not slip at A, determine the acceleration of point B.



Since no slipping

$a_C = \alpha r = 4(1.45) = 5.80$ ft/s^2



$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$

$\mathbf{a}_B = 5.80 + (2)^2(1.45) \quad + 4(1.45)$
$\rightarrow \qquad \searrow 30° \qquad \nearrow 30°$

$(\overset{+}{\rightarrow}) \quad (a_B)_x = 5.80 + 5.02 + 2.9 = 13.72$

$(+\uparrow) \quad (a_B)_y = 0 - 2.9 + 5.02 = 2.12$

$a_B = \sqrt{(13.72)^2 + (2.12)^2} = 13.9$ ft/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{2.123}{13.72}\right) = 8.80°$ ∠θ **Ans**

Also:

$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$

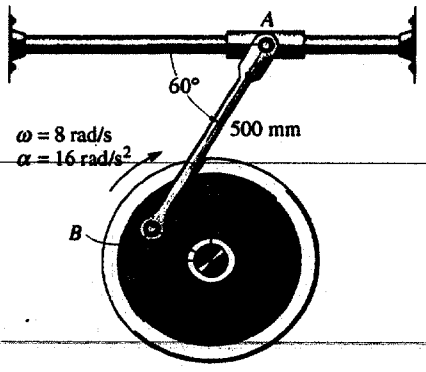
$\mathbf{a}_B = 5.80\mathbf{i} + (-4\mathbf{k}) \times (-1.45\cos 30°\mathbf{i} + 1.45\sin 30°\mathbf{j}) - (2)^2(-1.45\cos 30°\mathbf{i} + 1.45\sin 30°\mathbf{j})$

$\mathbf{a}_B = \{13.72\mathbf{i} + 2.123\mathbf{j}\}$ ft/s^2

$a_B = \sqrt{(13.72)^2 + (2.123)^2} = 13.9$ ft/s^2 **Ans**

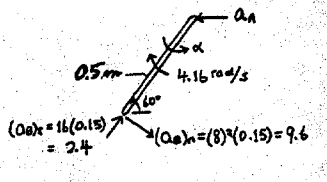
$\theta = \tan^{-1}\left(\frac{2.123}{13.72}\right) = 8.80°$ ∠θ **Ans**

16-110. At a given instant the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.



See Prob. 16–59.

$\omega = 4.16$ rad/s



$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

$a_A \quad = 2.4 \quad + 9.6 \quad + (4.16)^2(0.5) \quad + \alpha(0.5)$
$\leftarrow \quad \nearrow 60° \quad \searrow 30° \quad 60°\swarrow \qquad \nwarrow 60°$

$(\overset{+}{\rightarrow}) \quad -a_A = 2.4\cos 60° + 9.6\cos 30° - 8.65\cos 60° - \alpha(0.5)\sin 60°$

$(+\uparrow) \quad 0 = 2.4\sin 60° - 9.6\sin 30° - 8.65\sin 60° + \alpha(0.5)\cos 60°$

$\alpha = 40.8$ rad/s^2 ↻

$a_A = 12.5$ m/s^2 ← **Ans**

Also:

$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$

$a_A\mathbf{i} = (8)^2(0.15)(\cos 30°)\mathbf{i} - (8)^2(0.15)\sin 30°\mathbf{j} + (16)(0.15)\sin 30°\mathbf{i} + (16)(0.15)\cos 30°\mathbf{j}$
$+ (\alpha\mathbf{k}) \times (0.5\cos 60°\mathbf{i} + 0.5\sin 60°\mathbf{j}) - (4.16)^2(0.5\cos 60°\mathbf{i} + 0.5\sin 60°\mathbf{j})$

$a_A = 8.314 + 1.200 - 0.433\alpha - 4.326$

$0 = -4.800 + 2.0785 + 0.25\alpha - 7.4935$

$\alpha = 40.8$ rad/s^2 ↻

$a_A = 12.5$ m/s^2 ← **Ans**

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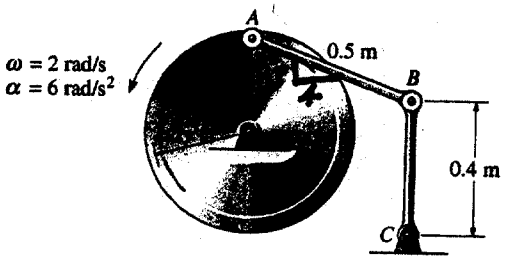
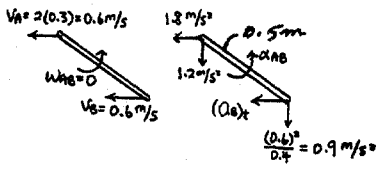
16-111. The flywheel rotates with an angular velocity $\omega = 2$ rad/s and an angular acceleration $\alpha = 6$ rad/s^2. Determine the angular acceleration of links AB and BC at this instant.



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(a_B)_t + 0.9 = 1.8 + 1.2 + \alpha_{AB}(0.5)$$
$$\leftarrow \quad \downarrow \quad \leftarrow \quad \downarrow \quad \tfrac{5}{3}\ 4$$



$$(\overset{+}{\leftarrow}) \quad (a_B)_t = 1.8 - \frac{3}{5}(\alpha_{AB})(0.5)$$

$$(+\downarrow) \quad 0.9 = 1.2 - \frac{4}{5}(\alpha_{AB})(0.5)$$

$$\alpha_{AB} = 0.75 \text{ rad/s}^2 \curvearrowleft \qquad \textbf{Ans}$$

$$(a_B)_t = 1.575 \text{ m/s}^2$$

$$\alpha_{BC} = \frac{1.575}{0.4} = 3.94 \text{ rad/s}^2 \curvearrowleft \qquad \textbf{Ans}$$

Also:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-(a_B)_t\mathbf{i} - \frac{(0.6)^2}{0.4}\mathbf{j} = -6(0.3)\mathbf{i} - (2)^2(0.3)\mathbf{j} + (\alpha_{AB}\mathbf{k}) \times (0.4\mathbf{i} - 0.3\mathbf{j}) - 0$$

$$-(a_B)_t = -1.8 + 0.3\alpha_{AB}$$

$$-0.9 = -1.2 + 0.4\alpha_{AB}$$

$$\alpha_{AB} = 0.75 \text{ rad/s}^2 \curvearrowleft \qquad \textbf{Ans}$$

$$(a_B)_t = 1.575 \text{ m/s}^2$$

$$\alpha_{BC} = \frac{1.575}{0.4} = 3.94 \text{ rad/s}^2 \curvearrowleft \qquad \textbf{Ans}$$

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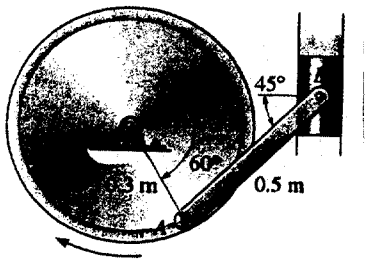
***16-112.** At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block B at this instant.



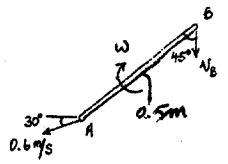
$\omega = 2$ rad/s
$\alpha = 6$ rad/s^2

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$v_B = 0.6 \quad + \omega(0.5)$
$\downarrow \ 30° \nearrow \qquad \searrow \ 45°$

$\stackrel{+}{\rightarrow} \quad 0 = -0.6\cos 30° + \omega(0.5)\cos 45°$

$\omega = 1.470$ rad/s



$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$a_B = 1.8 \quad + 1.2 \quad + (1.470)^2(0.5) \quad + \alpha(0.5)$
$\downarrow \ 30° \nearrow \quad 30° \nwarrow \quad 45° \nearrow \quad \searrow \ 45°$

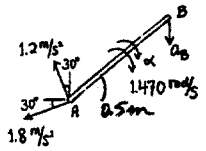
$(\stackrel{+}{\rightarrow}) \quad 0 = -1.8\cos 30° - 1.2\sin 30° - 1.08\sin 45° + \alpha(0.5)\cos 45°$

$(+\downarrow) \quad a_B = 1.8\sin 30° - 1.2\cos 30° + 1.08\sin 45° + \alpha(0.5)\sin 45°$

$\alpha = 8.266$ rad/s^2

$a_B = 3.55$ m/s^2 **Ans**

Also:



$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$-v_B\mathbf{i} = (-0.6\cos 30°\mathbf{i} - 0.6\sin 30°\mathbf{j}) + (-\omega\mathbf{k}) \times (0.5\cos 45°\mathbf{i} + 0.5\sin 45°\mathbf{j})$

$0 = -0.6\cos 30° + \omega(0.5\sin 45°)$

$\omega = 1.470$ rad/s

$\mathbf{a}_B = \mathbf{a}_A - \omega^2\mathbf{r}_{B/A} + \boldsymbol{\alpha} \times \mathbf{r}_{B/A}$

$-a_B\mathbf{j} = (-1.2\sin 30°\mathbf{i} + 1.2\cos 30°\mathbf{j}) + (-1.8\cos 30°\mathbf{i} - 1.8\sin 30°\mathbf{j})$
$+ (-\alpha\mathbf{k}) \times (0.5\cos 45°\mathbf{i} + 0.5\sin 45°\mathbf{j})$

$0 = -1.2\sin 30° - 1.8\cos 30° - (1.470)^2(0.5\cos 45°) + \alpha(0.5\sin 45°)$

$-a_B = 1.2\cos 30° - 1.8\sin 30° - (1.470)^2(0.5\sin 45°) - \alpha(0.5\sin 45°)$

$\alpha = 8.266$ rad/s

$a_B = 3.55$ m/s^2 **Ans**

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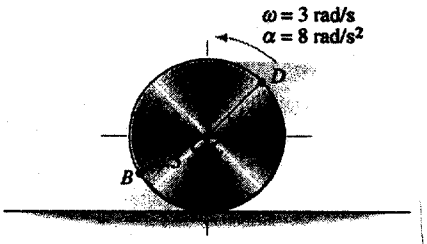
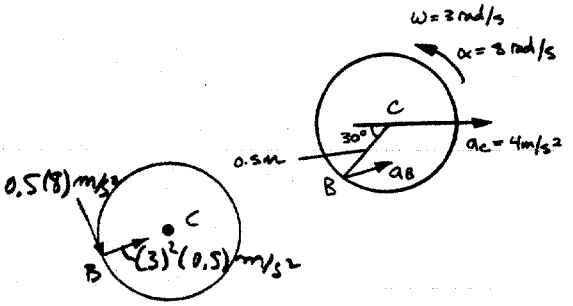
16-113. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



$a_C = 0.5(8) = 4\ \text{m/s}^2$

$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$

$$\mathbf{a}_B = \left[\begin{matrix}4\\ \leftarrow\end{matrix}\right] + \left[\begin{matrix}(3)^2(0.5)\\ \measuredangle 30^\circ\end{matrix}\right] + \left[\begin{matrix}(0.5)(8)\\ \measuredangle 30^\circ\end{matrix}\right]$$



$$\left(\overset{+}{\rightarrow}\right) \quad (a_B)_x = -4 + 4.5\cos 30^\circ + 4\sin 30^\circ = 1.897\ \text{m/s}^2$$

$$(+\uparrow) \quad (a_B)_y = 0 + 4.5\sin 30^\circ - 4\cos 30^\circ = -1.214\ \text{m/s}^2$$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \ \searrow \qquad \textbf{Ans}$$

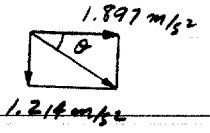
Also,

$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$

$(a_B)_x\,\mathbf{i} + (a_B)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (-0.5\cos 30^\circ \mathbf{i} - 0.5\sin 30^\circ \mathbf{j}) - (3)^2(-0.5\cos 30^\circ \mathbf{i} - 0.5\sin 30^\circ \mathbf{j})$

$$\left(\overset{+}{\rightarrow}\right) \quad (a_B)_x = -4 + 8(0.5\sin 30^\circ) + (3)^2(0.5\cos 30^\circ) = 1.897\ \text{m/s}^2$$

$$(+\uparrow) \quad (a_B)_y = 0 - 8(0.5\cos 30^\circ) + (3)^2(0.5\sin 30^\circ) = -1.214\ \text{m/s}^2$$



$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \ \searrow \qquad \textbf{Ans}$$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans}$$

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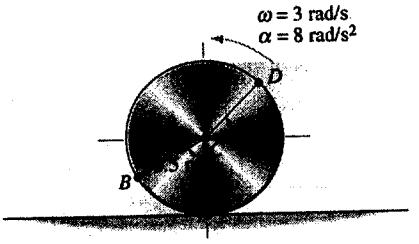
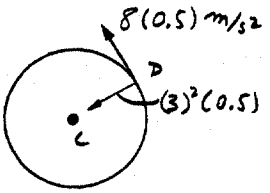
16-114. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point D.

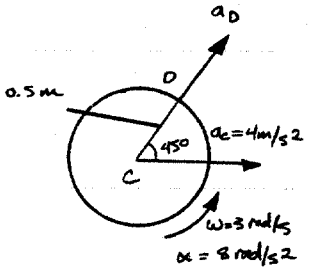




$a_C = 0.5(8) = 4\ \text{m/s}^2$

$\mathbf{a}_D = \mathbf{a}_C + \mathbf{a}_{D/C}$

$$\mathbf{a}_D = \left[\begin{matrix}4\\ \leftarrow\end{matrix}\right] + \left[\begin{matrix}(3)^2(0.5)\\ \nearrow 45^\circ\end{matrix}\right] + \left[\begin{matrix}8(0.5)\\ 45^\circ\ \nwarrow\end{matrix}\right]$$

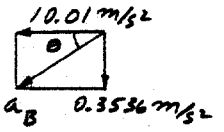


$\left(\overset{+}{\rightarrow}\right)\quad (a_D)_x = -4 - 4.5\sin 45^\circ - 4\cos 45^\circ = -10.01\ \text{m/s}^2$

$(+\uparrow)\quad (a_D)_y = 0 - 4.5\cos 45^\circ + 4\sin 45^\circ = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \qquad \textbf{Ans}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \nearrow \qquad \textbf{Ans}$$



Also,

$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{D/C} - \omega^2 \mathbf{r}_{D/C}$

$(a_D)_x\,\mathbf{i} + (a_D)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (0.5\cos 45^\circ \mathbf{i} + 0.5\sin 45^\circ \mathbf{j}) - (3)^2(0.5\cos 45^\circ \mathbf{i} + 0.5\sin 45^\circ \mathbf{j})$

$\left(\overset{+}{\rightarrow}\right)\quad (a_D)_x = -4 - 8(0.5\sin 45^\circ) - (3)^2(0.5\cos 45^\circ) = -10.01\ \text{m/s}^2$

$(+\uparrow)\quad (a_D)_y = +8(0.5\cos 45^\circ) - (3)^2(0.5\sin 45^\circ) = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \nearrow \qquad \textbf{Ans}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \qquad \textbf{Ans}$$

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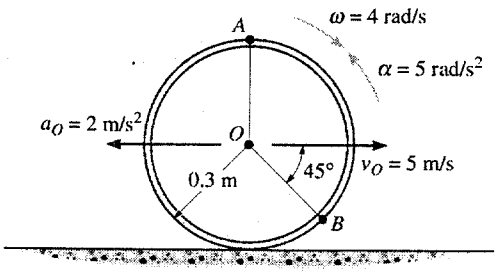
16-115. The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point A at this instant.



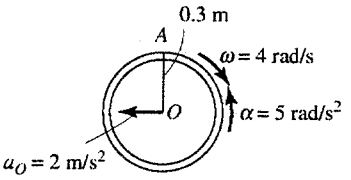
$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$

$\mathbf{a}_A = \left[\underset{\leftarrow}{2}\right] + \left[\underset{\downarrow}{(4)^2(0.3)}\right] + \left[\underset{\longleftarrow}{5(0.3)}\right]$

$\mathbf{a}_A = \left[\underset{\leftarrow}{3.5}\right] + \left[\underset{\downarrow}{4.8}\right]$

$a_A = 5.94$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{4.8}{3.5}\right) = 53.9°$ ↙ **Ans**



Also:

$\mathbf{a}_A = \mathbf{a}_O - \omega^2\mathbf{r}_{A/O} + \alpha \times \mathbf{r}_{A/O}$

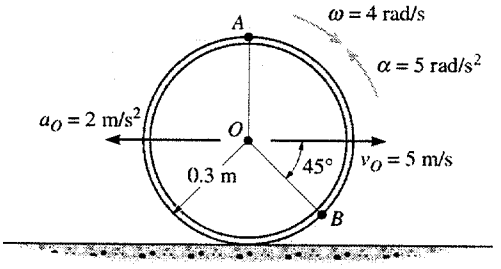
$\mathbf{a}_A = -2\mathbf{i} - (4)^2(0.3\mathbf{j}) + 5\mathbf{k} \times (0.3\mathbf{j})$

$\mathbf{a}_A = \{-3.5\mathbf{i} - 4.8\mathbf{j}\}$ m/s^2

$a_A = 5.94$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{4.8}{3.5}\right) = 53.9°$ ↙ **Ans**

***16-116.** The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity of $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point B at this instant.



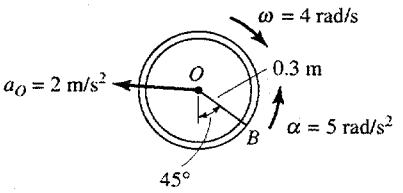
$\mathbf{a}_B = \mathbf{a}_O + \mathbf{a}_{B/O}$

$\mathbf{a}_B = \left[\underset{\leftarrow}{2}\right] + \left[\underset{\swarrow}{5(0.3)}\right] + \left[\underset{\nwarrow}{(4)^2(0.3)}\right]$

$\mathbf{a}_B = \left[\underset{\leftarrow}{4.333}\right] + \left[\underset{\uparrow}{4.455}\right]$

$a_B = 6.21$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{4.455}{4.333}\right) = 45.8°$ ↖



Also:

$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2\mathbf{r}_{B/O}$

$\mathbf{a}_B = -2\mathbf{i} + 5\mathbf{k} \times (0.3\cos 45°\mathbf{i} - 0.3\sin 45°\mathbf{j}) - (4)^2(0.3\cos 45°\mathbf{i} - 0.3\sin 45°\mathbf{j})$

$\mathbf{a}_B = \{-4.333\mathbf{i} + 4.455\mathbf{j}\}$ m/s^2

$a_B = 6.21$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{4.455}{4.333}\right) = 45.8°$ ↖

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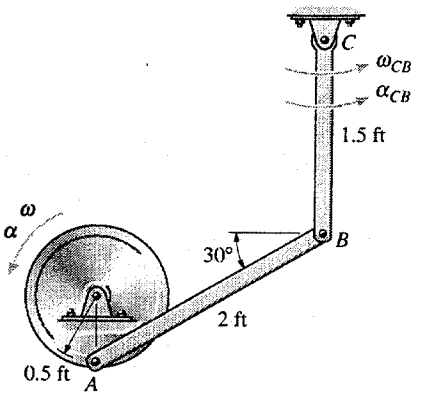
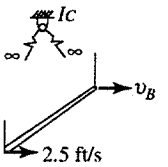
16-117. The disk rotates with an angular velocity $\omega = 5$ rad/s and an angular acceleration $\alpha = 6$ rad/s^2. Determine the angular acceleration of link CB at this instant.



The IC is at ∞. Thus



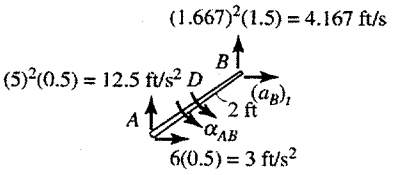
$\omega = 0$

$v_A = v_B = 2.5$ ft/s

$\omega_{BC} = \dfrac{2.5}{1.5} = 1.667$ rad/s

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$$\underset{\rightarrow}{(a_B)_t} + \underset{\uparrow}{4.167} = \underset{\rightarrow}{3} + \underset{\uparrow}{12.5} + \underset{30^\circ}{2\alpha_{AB}}$$



$(\overset{+}{\leftarrow})$ $(a_B)_t = 3 - 2\alpha_{AB} \sin 30°$

$(+\uparrow)$ $4.167 = 12.5 + 2\alpha_{AB} \cos 30°$

$\alpha_{AB} = -4.81$ rad/s^2 $= 4.81$ rad/s^2 ↻

$(a_B)_t = 7.81$ ft/s^2

$\alpha_{BC} = \dfrac{7.81}{1.5} = 5.21$ rad/s^2 ↺ **Ans**

Also:

$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

$(a_B)_t \mathbf{i} + 4.167\mathbf{j} = (3\mathbf{i} + 12.5\mathbf{j}) - 0 + (\alpha_{AB}\mathbf{k}) \times (2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$

$(a_B)_t = 3 - 2\alpha_{AB} \sin 30°$

$4.167 = 12.5 + 2\alpha_{AB} \cos 30°$

$\alpha_{AB} = -4.81$ rad/s^2 $= 4.81$ rad/s^2 ↻

$(a_B)_t = 7.81$ ft/s^2

$\alpha_{BC} = \dfrac{7.81}{1.5} = 5.21$ rad/s^2 ↺ **Ans**

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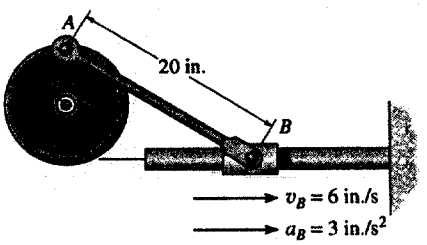
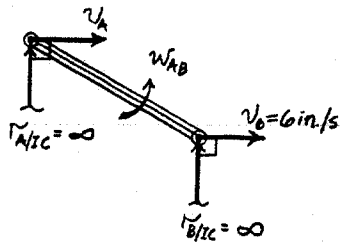
16-118. At a given instant, the slider block B is traveling to the right with the velocity and acceleration shown. Determine the angular acceleration of the wheel at this instant.



Velocity Analysis : The angular velocity of link AB can be obtained by using the method of instantaneous center of zero velocity. Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, $r_{A/IC} = r_{B/IC} = \infty$. Thus, $\omega_{AB} = 0$. Since $\omega_{AB} = 0$, $v_A = v_B = 6$ in./s. Thus, the angular velocity of the wheel is $\omega_W = \dfrac{v_A}{r_{OA}} = \dfrac{6}{5} = 1.20$ rad/s.

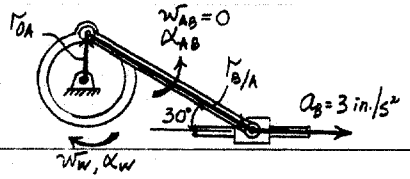


Acceleration Equation : The acceleration of point A can be obtained by analyzing the angular motion of link OA about point O. Here, $\mathbf{r}_{OA} = \{5\mathbf{j}\}$ in.

$$\begin{aligned}\mathbf{a}_A &= \boldsymbol{\alpha}_W \times \mathbf{r}_{OA} - \omega_W^2 \mathbf{r}_{OA} \\ &= (-\alpha_W \mathbf{k}) \times (5\mathbf{j}) - 1.20^2 (5\mathbf{j}) \\ &= \{5\alpha_W \mathbf{i} - 7.20\mathbf{j}\} \text{ in./s}^2\end{aligned}$$

Link AB is subjected to general plane motion. Applying Eq. 16 – 18 with $\mathbf{r}_{B/A} = \{20\cos 30°\mathbf{i} - 20\sin 30°\mathbf{j}\}$ in. $= \{17.32\mathbf{i} - 10.0\mathbf{j}\}$ in., we have

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ 3\mathbf{i} &= 5\alpha_W \mathbf{i} - 7.20\mathbf{j} + \alpha_{AB}\mathbf{k} \times (17.32\mathbf{i} - 10.0\mathbf{j}) - 0 \\ 3\mathbf{i} &= (10.0\alpha_{AB} + 5\alpha_W)\mathbf{i} + (17.32\alpha_{AB} - 7.20)\mathbf{j}\end{aligned}$$



Equating **i** and **j** component, we have

$$3 = 10.0\alpha_{AB} + 5\alpha_W \qquad [1]$$
$$0 = 17.32\alpha_{AB} - 7.20 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\alpha_{AB} = 0.4157 \text{ rad/s}^2$$
$$\alpha_W = -0.2314 \text{ rad/s}^2 = 0.231 \text{ rad/s}^2 \qquad \textbf{Ans}$$

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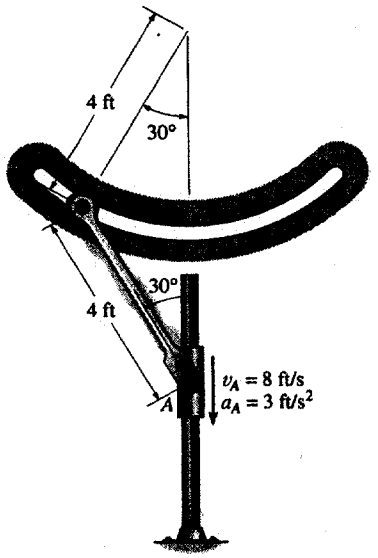
16-119. The ends of the bar AB are confined to move along the paths shown. At a given instant, A has a velocity of 8 ft/s and an acceleration of 3 ft/s^2. Determine the angular velocity and angular acceleration of AB at this instant.



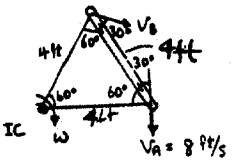
$\omega = \frac{8}{4} = 2$ rad/s) **Ans**

$v_B = 4(2) = 8$ ft/s

$(a_B)_n = \frac{(8)^2}{4} = 16$ ft/s^2

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$$\begin{bmatrix} 16 \\ 30^\circ \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ 30^\circ \end{bmatrix} = \begin{bmatrix} -3 \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha(4) \\ 60^\circ \end{bmatrix} + \begin{bmatrix} (2)^2(4) \\ 60^\circ \end{bmatrix}$$

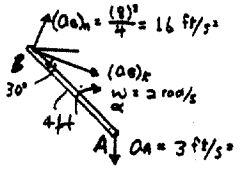


$(\overset{+}{\rightarrow})$ $16\sin 30^\circ + (a_B)_t \cos 30^\circ = 0 + \alpha(4)\sin 60^\circ + 16\cos 60^\circ$

$(+\uparrow)$ $16\cos 30^\circ - (a_B)_t \sin 30^\circ = -3 + \alpha(4)\cos 60^\circ - 16\sin 60^\circ$

$\alpha = 7.68$ rad/s^2) **Ans**

$(a_B)_t = 30.7$ ft/s^2



Also:

$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

$(a_B)_t \cos 30^\circ \mathbf{i} - (a_B)_t \sin 30^\circ \mathbf{j} + (\frac{(8)^2}{4})\sin 30^\circ \mathbf{i} + (\frac{(8)^2}{4})\cos 30^\circ \mathbf{j} = -3\mathbf{j}$
$+(\alpha \mathbf{k}) \times (-4\sin 30^\circ \mathbf{i} + 4\cos 30^\circ \mathbf{j}) - (2)^2 (-4\sin 30^\circ \mathbf{i} + 4\cos 30^\circ \mathbf{j})$

$(\overset{+}{\rightarrow})$ $(a_B)_t \cos 30^\circ + 8 = -3.464\alpha + 8$

$(+\uparrow)$ $-(a_B)_t \sin 30^\circ + 13.8564 = -3 - 2\alpha - 13.8564$

$\alpha = 7.68$ rad/s^2) **Ans**

$(a_B)_t = 30.7$ ft/s^2

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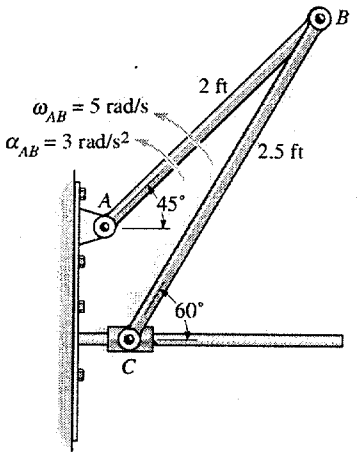
***16-120.** Rod AB has the angular motion shown. Determine the acceleration of the collar C at this instant.



$$\frac{r_{B/IC}}{\sin 30°} = \frac{2.5}{\sin 135°}$$

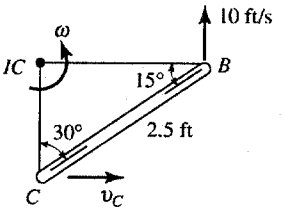
$$r_{B/IC} = 1.7678 \text{ ft}$$

$$\omega = \frac{10}{1.7678} = 5.66 \text{ rad/s} \curvearrowright$$

$$(a_B)_n = 5^2(2) = 50 \text{ ft/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

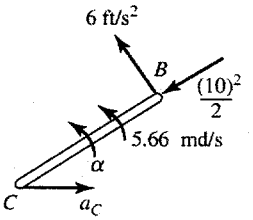
$$\left[\begin{matrix} a_C \\ \rightarrow \end{matrix}\right] = \left[\begin{matrix} 6 \\ 45° \nwarrow \end{matrix}\right] + \left[\begin{matrix} 50 \\ 45° \swarrow \end{matrix}\right] + \left[\begin{matrix} (5.66)^2(2.5) \\ \measuredangle 60° \end{matrix}\right] + \left[\begin{matrix} \alpha(2.5) \\ \measuredangle 30° \end{matrix}\right]$$



$$(\overset{+}{\rightarrow}) \quad a_C = -6\cos 45° - 50\cos 45° + 80\cos 60° + \alpha(2.5)\cos 30°$$

$$(+\uparrow) \quad 0 = 6\sin 45° - 50\sin 45° + 80\sin 60° - \alpha(2.5)\sin 30°$$

$$\alpha = 30.5 \text{ rad/s}^2 \curvearrowleft$$

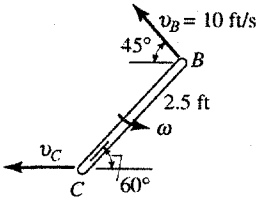


$$a_C = 66.5 \text{ ft/s}^2 \rightarrow \qquad \textbf{Ans}$$

Also,

$$v_B = 5(2) = 10 \text{ ft/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$



$$-v_C\mathbf{i} = -10\cos 45°\mathbf{i} + 10\sin 45°\mathbf{j} + \omega\mathbf{k} \times (-2.5\sin 30°\mathbf{i} - 2.5\cos 30°\mathbf{j})$$

$$(+\uparrow) \quad 0 = 10\sin 45° - 2.5\omega\sin 30°$$

$$\omega = 5.66 \text{ rad/s}$$

$$\mathbf{a}_C = \mathbf{a}_B + \alpha \times \mathbf{r}_{C/B} - \omega^2\mathbf{r}_{C/B}$$

$$a_C\mathbf{i} = -\frac{(10)^2}{2}\cos 45°\mathbf{i} - \frac{(10)^2}{2}\sin 45°\mathbf{j} - 6\cos 45°\mathbf{i} + 6\sin 45°\mathbf{j}$$

$$+ (\alpha\mathbf{k}) \times (-2.5\cos 60°\mathbf{i} - 2.5\sin 60°\mathbf{j}) - (5.66)^2(-2.5\cos 60°\mathbf{i} - 2.5\sin 60°\mathbf{j})$$

$$(\overset{+}{\rightarrow}) \quad a_C = -35.355 - 4.243 + 2.165\alpha + 40$$

$$(+\uparrow) \quad 0 = -35.355 + 4.243 - 1.25\alpha + 69.282$$

$$\alpha = 30.5 \text{ rad/s}^2 \curvearrowleft$$

$$a_C = 66.5 \text{ ft/s}^2 \rightarrow \qquad \textbf{Ans}$$

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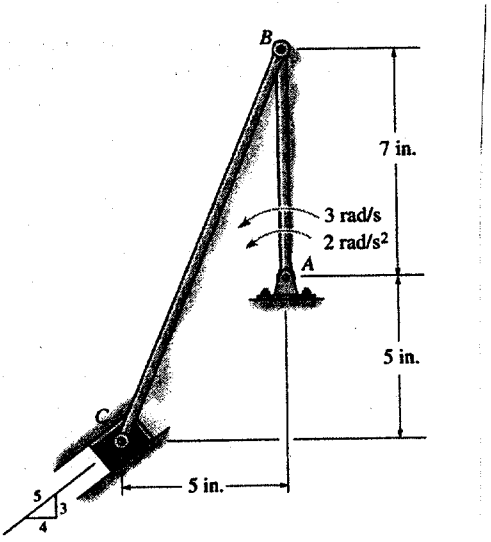
16-121. At the given instant member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



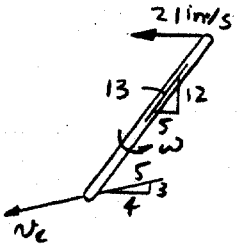
$v_B = 3(7) = 21$ in./s ←

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$-v_C\left(\frac{4}{5}\right)\mathbf{i} - v_C\left(\frac{3}{5}\right)\mathbf{j} = -21\mathbf{i} + \omega\mathbf{k} \times (-5\mathbf{i} - 12\mathbf{j})$

$\left(\overset{+}{\rightarrow}\right) \quad -0.8v_C = -21 + 12\omega$

$(+\uparrow) \quad -0.6v_C = -5\omega$



Solving :

$\omega = 1.125$ rad/s

$v_C = 9.375$ in./s $= 9.38$ in./s ↙ **Ans**

$(a_B)_n = (3)^2(7) = 63$ in./s² ↓

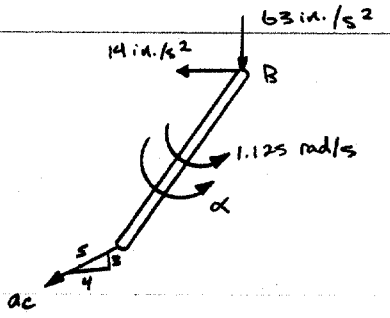
$(a_B)_t = (2)(7) = 14$ in./s² ←

$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$

$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = -14\mathbf{i} - 63\mathbf{j} + (\alpha\mathbf{k}) \times (-5\mathbf{i} - 12\mathbf{j}) - (1.125)^2(-5\mathbf{i} - 12\mathbf{j})$

$\left(\overset{+}{\rightarrow}\right) \quad -0.8a_C = -14 + 12\alpha + 6.328$

$(+\uparrow) \quad -0.6a_C = -63 - 5\alpha + 15.1875$



$a_C = 54.7$ in./s² ↙ **Ans**

$\alpha = -3.00$ rad/s²

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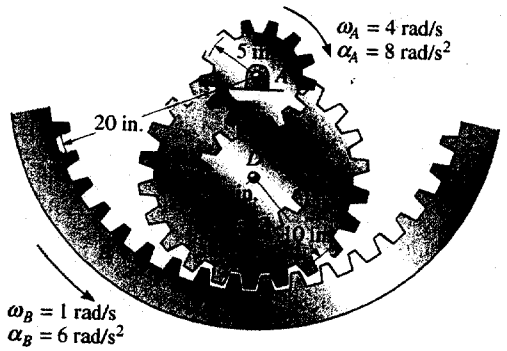
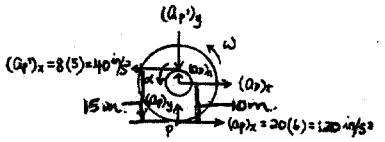
16-122. At a given instant gears A and B have the angular motions shown. Determine the angular acceleration of gear C and the acceleration of its center point D at this instant. Note that the inner hub of gear C is in mesh with gear A and its outer rim is in mesh with gear B.



$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/P'}$

$(\overset{+}{\rightarrow})$ $120 = -40 + \alpha(15)$

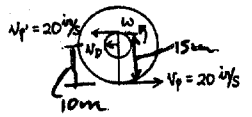
$\alpha = 10.67$ rad/s^2 ↻ **Ans**



$\mathbf{a}_P = \mathbf{a}_D + \mathbf{a}_{P/D}$

$(\overset{+}{\rightarrow})$ $120 = (a_D)_t + (10.67)(10)$

$(a_D)_t = 13.3$ in./s^2 →



$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/P'}$

$(\overset{+}{\rightarrow})$ $20 = -20 + \omega(15)$

$\omega = 2.667$ rad/s

$\mathbf{v}_D = \mathbf{v}_P + \mathbf{v}_{D/P}$

$(\overset{+}{\leftarrow})$ $v_D = -20 + 10(2.667)$

$v_D = 6.67$ in./s

$(a_D)_n = \dfrac{(6.67)^2}{10} = 4.44$ in./s^2 ↑

$\theta = \tan^{-1}\left(\dfrac{4.44}{13.3}\right) = 18.4°$

$a_D = \sqrt{(4.44)^2 + (13.3)^2} = 14.1$ in./s^2 **Ans**

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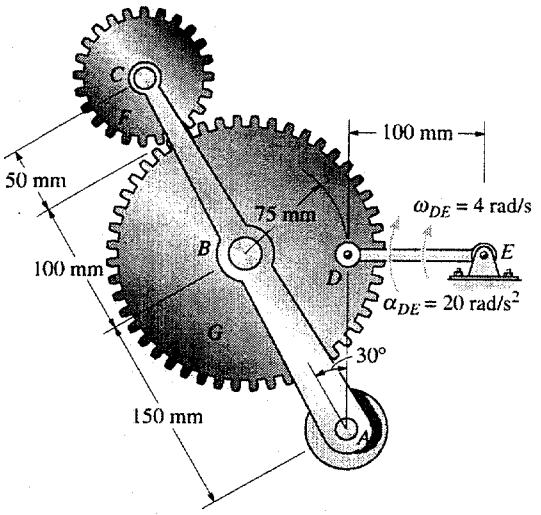
16-123. The tied crank and gear mechanism gives rocking motion to crank AC, necessary for the operation of a printing press. If link DE has the angular motion shown, determine the respective angular velocities of gear F and crank AC at this instant, and the angular acceleration of crank AC.



Velocity analysis:

$v_D = \omega_{DE} r_{D/E} = 4(0.1) = 0.4$ m/s ↑

$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$

$$\underset{\angle 30^\circ}{v_B} = \underset{\uparrow}{0.4} + \underset{\downarrow}{(\omega_G)(0.075)}$$

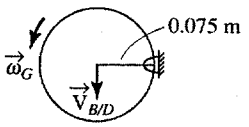
$(\overset{+}{\rightarrow})$ $v_B \cos 30^\circ = 0, \quad v_B = 0$

$(+\uparrow)$ $\omega_G = 5.33$ rad/s

Since $v_B = 0, \quad v_C = 0, \quad \omega_{AC} = 0$ **Ans**

$\omega_F r_F = \omega_G r_G$

$$\omega_F = 5.33\left(\frac{100}{50}\right) = 10.7 \text{ rad/s}$$ **Ans**



Acceleration analysis:

$(a_D)_n = (4)^2(0.1) = 1.6$ m/s^2 →

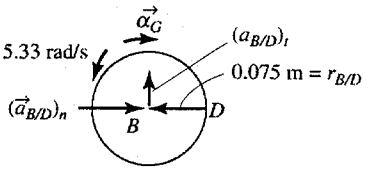
$(a_D)_t = (20)(0.1) = 2$ m/s^2 ↑

$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (\mathbf{a}_D)_n + (\mathbf{a}_D)_t + (\mathbf{a}_{B/D})_n + (\mathbf{a}_{B/D})_t$

$$\underset{\angle 30^\circ}{0} + \underset{\rightarrow}{(a_B)_t} = \underset{\rightarrow}{1.6} + \underset{\uparrow}{2} + \underset{\rightarrow}{(5.33)^2(0.075)} + \underset{\uparrow}{\alpha_G(0.075)}$$

$(+\uparrow)$ $(a_B)_t \sin 30^\circ = 0 + 2 + 0 + \alpha_G(0.075)$

$(\overset{+}{\rightarrow})$ $(a_B)_t \cos 30^\circ = 1.6 + 0 + (5.33)^2(0.075) + 0$



Solving.

$(a_B)_t = 4.31$ m/s^2, $\quad \alpha_G = 2.052$ rad/s^2 ↻

Hence,

$$\alpha_{AC} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.31}{0.15} = 28.7 \text{ rad/s}^2 \circlearrowright$$ **Ans**

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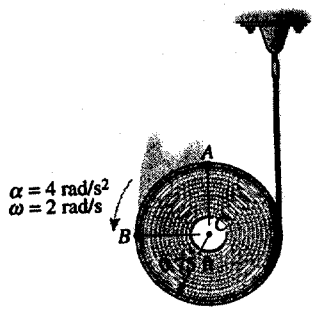
***16-124.** As the cord unravels from the cylinder, the cylinder has an angular acceleration of $\alpha = 4$ rad/s^2 and an angular velocity of $\omega = 2$ rad/s at the instant shown. Determine the accelerations of points A and B at this instant.



$a_C = 4(0.75) = 3$ ft/s^2 ↓

$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$

$\mathbf{a}_A = -3\mathbf{j} + 4\mathbf{k} \times (0.75\mathbf{j}) - (2)^2 (0.75\mathbf{j})$

$\mathbf{a}_A = \{-3\mathbf{i} - 6\mathbf{j}\}$ ft/s^2

$a_A = \sqrt{(-3)^2 + (-6)^2} = 6.71$ ft/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{6}{3}\right) = 63.4°$ ↗ **Ans**

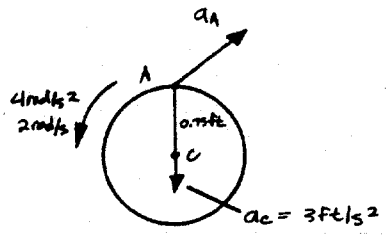
$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$

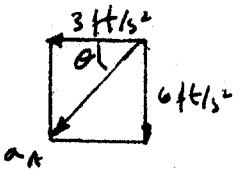
$\mathbf{a}_B = -3\mathbf{j} + 4\mathbf{k} \times (-0.75\mathbf{i}) - (2)^2 (-0.75\mathbf{i})$

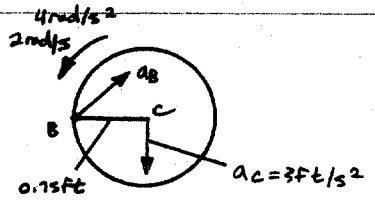
$\mathbf{a}_B = \{3\mathbf{i} - 6\mathbf{j}\}$ ft/s^2

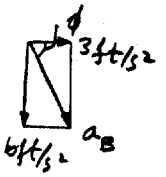
$a_B = \sqrt{(3)^2 + (-6)^2} = 6.71$ ft/s^2 **Ans**

$\phi = \tan^{-1}\left(\frac{6}{3}\right) = 63.4°$ ↖ **Ans**









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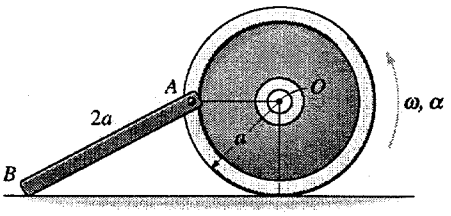
16-125. The wheel rolls without slipping such that at the instant shown it has an angular velocity ω and angular acceleration α. Determine the velocity and acceleration of point B on the rod at this instant.



Velocity:

$$(\overset{+}{\leftarrow}) \quad v_B = \left(\frac{1}{\sqrt{2}}\right)(\sqrt{2}a\omega + \frac{\omega}{\sqrt{3}}(2a)\left(\frac{1}{2}\right)$$

$$v_B = 1.58\,\omega a \qquad \textbf{Ans}$$

Acceleration:

$$(\overset{+}{\leftarrow}) \quad a_B = -\omega^2 a + \alpha a - \frac{\omega^2}{3}(2a)\cos 30° + \left(\frac{\alpha}{\sqrt{3}} - \frac{\omega^2}{3\sqrt{3}}\right)(2a)\sin 30°$$

$$a_B = 1.58\,\alpha a - 1.77\,\omega^2 a \qquad \textbf{Ans}$$

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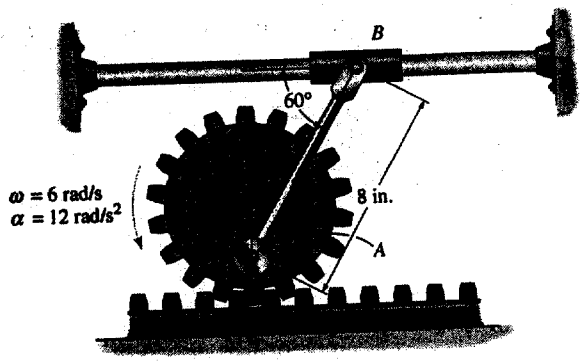
16-126. At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



For the gear

$v_A = \omega r_{A/IC} = 6(1) = 6$ in./s

$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\}$ in./s^2 $\quad \mathbf{r}_{A/O} = \{-2\mathbf{j}\}$ in. $\quad \boldsymbol{\alpha} = \{12\mathbf{k}\}$ rad/s^2

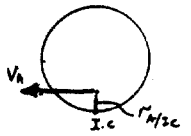
$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

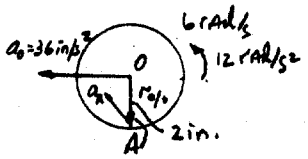
$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$

$= \{-12\mathbf{i} + 72\mathbf{j}\}$ in./s^2

$a_A = \sqrt{(-12)^2 + 72^2} = 73.0$ in./s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{72}{12}\right) = 80.5°$ ↘ **Ans**





For link AB

The IC is at ∞, so $\omega_{AB} = 0$, i.e.,

$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{\infty} = 0$

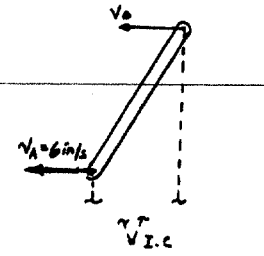
$\mathbf{a}_B = a_B\mathbf{i} \quad \boldsymbol{\alpha}_{AB} = -\alpha_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = \{8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}\}$ in.

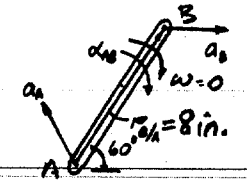
$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

$a_B\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\mathbf{k}) \times (8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}) - 0$

$(\stackrel{+}{\rightarrow}) \quad a_B = -12 + 8\sin 60°(18) = 113$ in./s^2 → **Ans**

$(+\uparrow) \quad 0 = 72 - 8\cos 60°\alpha_{AB} \quad \alpha_{AB} = 18$ rad/s^2 ↻ **Ans**





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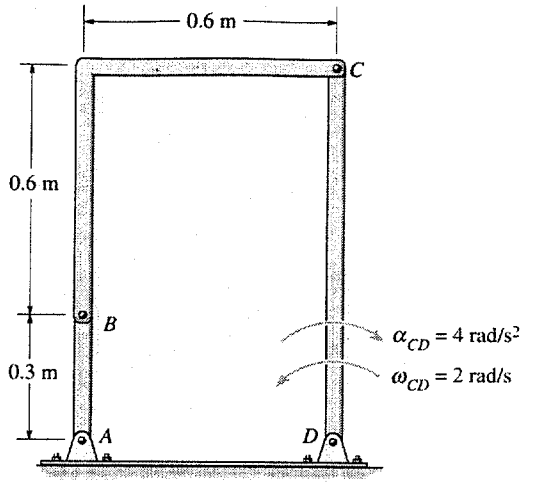
16-127. Determine the angular acceleration of link AB if link CD has the angular velocity and angular deceleration shown.



IC is at ∞, thus

$\omega_{BC} = 0$

$v_B = v_C = (0.9)(2) = 1.8$ m/s

$(a_C)_n = (2)^2(0.9) = 3.6$ m/s² ↓

$(a_C)_t = 4(0.9) = 3.6$ m/s² →

$(a_B)_n = \dfrac{(1.8)^2}{0.3} = 10.8$ m/s² ↓

$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$

$(a_B)_t\mathbf{i} - 10.8\mathbf{j} = 3.6\mathbf{i} - 3.6\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (-0.6\mathbf{i} - 0.6\mathbf{j}) - \mathbf{0}$

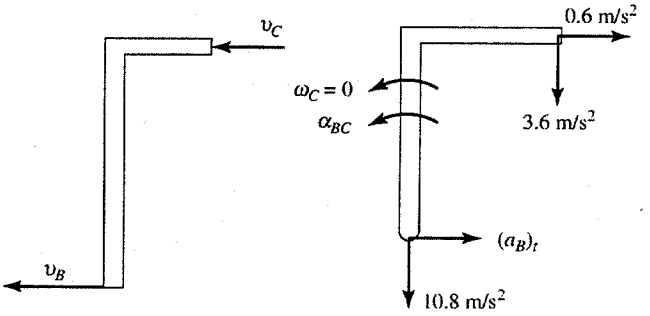
$(\overset{+}{\rightarrow})$ $(a_B)_t = 3.6 + 0.6\alpha_{BC}$

$(+\uparrow)$ $-10.8 = -3.6 - 0.6\alpha_{BC}$

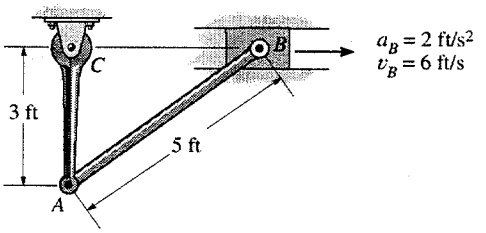
$\alpha_{BC} = 12$ rad/s²

$(a_B)_t = 10.8$ m/s²

$\alpha_{AB} = \dfrac{10.8}{0.3} = 36$ rad/s² ↻ **Ans**

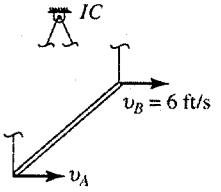


***16-128.** The slider block B is moving to the right with an acceleration of 2 ft/s². At the instant shown, its velocity is 6 ft/s. Determine the angular acceleration of link AB and the acceleration of point A at this instant.



$\omega_{AB} = \dfrac{v_B}{r_{B/IC}} = \dfrac{6}{\infty} = 0$ $v_A = v_B = 6$ ft/s

$\omega_{AC} = \dfrac{v_A}{r_{AC}} = \dfrac{6}{3} = 2$ rad/s



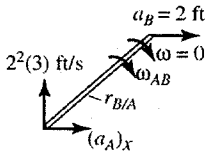
$\mathbf{a}_B = \{2\mathbf{i}\}$ ft/s $\mathbf{a}_A = (a_A)_x\mathbf{i} + (2)^2(3)\mathbf{j} = (a_A)_x\mathbf{i} + 12\mathbf{j}$

$\boldsymbol{\alpha}_{AB} = -\alpha_{AB}\mathbf{k}$ $\mathbf{r}_{B/A} = \{4\mathbf{i} + 3\mathbf{j}\}$ ft

$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$

$2\mathbf{i} = [(a_A)_x\mathbf{i} + 12\mathbf{j}] + (-\alpha_{AB}\mathbf{k}) \times (4\mathbf{i} + 3\mathbf{j}) - \mathbf{0}$

$2\mathbf{i} = [(a_A)_x + 3\alpha_{AB}]\mathbf{i} + (12 - 4\alpha_{AB})\mathbf{j}$



$0 = 12 - 4\alpha_{AB}$ $\alpha_{AB} = 3$ rad/s² ↻ **Ans**

$2 = (a_A)_x + 3(3)$ $(a_A)_x = -7$ ft/s²

$\mathbf{a}_A = \{-7\mathbf{i} + 12\mathbf{j}\}$ ft/s²

$a_A = \sqrt{(-7)^2 + 12^2} = 13.9$ ft/s² **Ans**

$\theta = \tan^{-1}\dfrac{12}{7} = 59.7°$ ↖θ **Ans**

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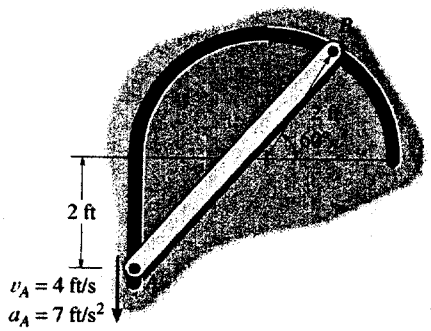
16-129. The ends of the bar AB are confined to move along the paths shown. At a given instant, A has a velocity of $v_A = 4$ ft/s and an acceleration of $a_A = 7$ ft/s^2. Determine the angular velocity and angular acceleration of AB at this instant.



$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

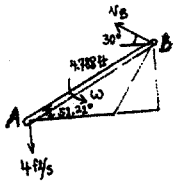
v_B (30° ↖) $= 4 + \omega(4.788)$ (↓ ↘ 51.21°)

$(\stackrel{+}{\rightarrow})$ $-v_B\cos 30° = 0 - \omega(4.788)\sin 51.21°$

$(+\uparrow)$ $v_B\sin 30° = -4 + \omega(4.788)\cos 51.21°$

$v_B = 20.39$ ft/s 30° ↖

$\omega = 4.73$ rad/s ↻ Ans



$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

a_t (30° ↖) $+ 207.9$ (60° ↗) $= 7 + 107.2 + 4.788(\alpha)$ (↓ ↗ 51.21° ↘ 51.21°)

$(\stackrel{+}{\leftarrow})$ $a_t\cos 30° + 207.9\cos 60° = 0 + 107.2\cos 51.21° + 4.788\alpha(\sin 51.21°)$

$(+\uparrow)$ $a_t\sin 30° - 207.9\sin 60° = -7 - 107.2\sin 51.21° + 4.788\alpha(\cos 51.21°)$

$a_t(0.866) - 3.732\alpha = -36.78$

$a_t(0.5) - 3\alpha = 89.49$

$a_t = -607$ ft/s^2

$\alpha = -131$ rad/s^2 $= 131$ rad/s^2 ↺ Ans

Also:

$\mathbf{v}_B = \mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

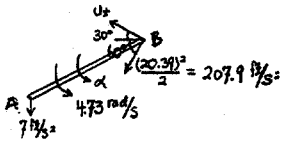
$-v_B\cos 30°\mathbf{i} + v_B\sin 30°\mathbf{j} = -4\mathbf{j} + (\omega\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-v_B\cos 30° = -\omega(3.732)$

$v_B\sin 30° = -4 + \omega(3)$

$\omega = 4.73$ rad/s ↻ Ans

$v_B = 20.39$ ft/s



$\mathbf{a}_B = \mathbf{a}_A - \omega^2\mathbf{r}_{B/A} + \alpha \times \mathbf{r}_{B/A}$

$(-a_t\cos 30°\mathbf{i} + a_t\sin 30°\mathbf{j}) + (-207.9\cos 60°\mathbf{i} - 207.9\sin 60°\mathbf{j}) = -7\mathbf{j} - (4.732)^2(3\mathbf{i} + 3.732\mathbf{j}) + (\alpha\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-a_t\cos 30° - 207.9\cos 60° = -(4.732)^2(3) - \alpha(3.732)$

$a_t\sin 30° - 207.9\sin 60° = -7 - (4.732)^2(3.732) + \alpha(3)$

$a_t = -607$ ft/s^2

$\alpha = -131$ rad/s^2 $= 131$ rad/s^2 ↺ Ans

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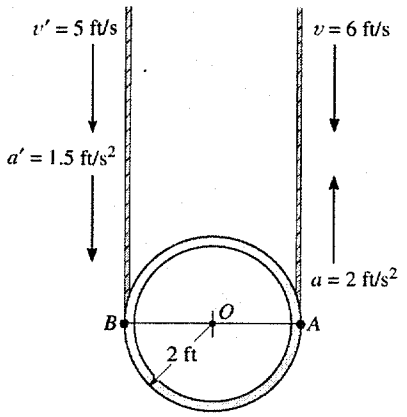
16-130. At a given instant, the cables supporting the pipe have the motions shown. Determine the angular velocity and angular acceleration of the pipe and the velocity and acceleration of point B located on the pipe.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$(+\downarrow)$ $5 = 6 - \omega(4)$

$\omega = 0.25$ rad/s ↻ **Ans**

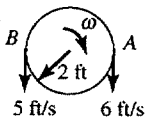
$v_B = 5.00$ ft/s ↓ **Ans**

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$1.5 + (a_B)_x = 2 + (a_A)_x + (\alpha)(4) + (0.25)^2(4)$$
$$\downarrow \quad \rightarrow \quad \uparrow \quad \leftarrow \quad \downarrow \quad \rightarrow$$

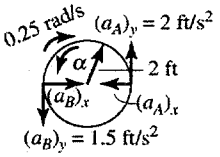
$(+\downarrow)$ $1.5 = -2 + \alpha(4)$

$\alpha = 0.875$ rad/s² ↺ **Ans**



$$\mathbf{a}_B = \mathbf{a}_O + \mathbf{a}_{B/O}$$

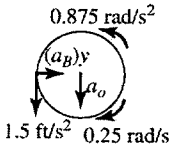
$$1.5 + (a_B)_x = \alpha_O + 0.875(2) + (0.25)^2(2)$$
$$\downarrow \quad \rightarrow \quad \downarrow \quad \downarrow \quad \rightarrow$$



$(\overset{+}{\rightarrow})$ $(a_B)_x = (0.25)^2(2) = 0.125$ ft/s²

$a_B = \sqrt{(1.5)^2 + (0.125)^2} = 1.51$ ft/s² **Ans**

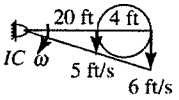
$\theta = \tan^{-1}\left(\dfrac{1.5}{0.125}\right) = 85.2°$ ∠θ **Ans**



Also:

$\omega = \dfrac{5}{20} = 0.25$ rad/s **Ans**

$v_B = 5.00$ ft/s ↓ **Ans**



$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-1.5\mathbf{j} + (a_B)_x\mathbf{i} = 2\mathbf{j} - (a_A)_x\mathbf{i} + (\alpha\mathbf{k}) \times (-4\mathbf{i}) - (0.25)^2(-4\mathbf{i})$$

$-1.5 = 2 - 4\alpha$

$\alpha = 0.875$ rad/s² ↺ **Ans**

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$-1.5\mathbf{j} + (a_B)_x\mathbf{i} = -a_O\mathbf{j} + (\alpha\mathbf{k}) \times (-2\mathbf{i}) - (0.25)^2(-2\mathbf{i})$$

$(a_B)_x = (0.25)^2(2) = 0.125$

$a_B = \sqrt{(1.5)^2 + (0.125)^2} = 1.51$ ft/s² **Ans**

$\theta = \tan^{-1}\left(\dfrac{1.5}{0.125}\right) = 85.2°$ ∠θ **Ans**

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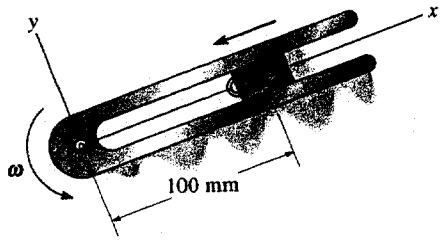
16-131. Block A, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at O with an acceleration of $4\ m/s^2$ and its velocity is $2\ m/s$. Determine the acceleration of the block at this instant. The rod rotates about O with a constant angular velocity $\omega = 4$ rad/s.



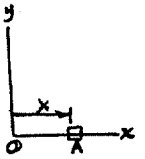
Motion of moving reference.

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = 4\mathbf{k}$

$\dot{\Omega} = \mathbf{0}$



Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.1\mathbf{i}$

$\mathbf{v}_{A/O} = -2\mathbf{i}$

$\mathbf{a}_{A/O} = -4\mathbf{i}$

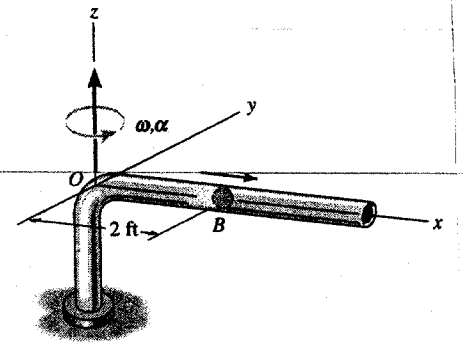
Thus,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (4\mathbf{k}) \times (4\mathbf{k} \times 0.1\mathbf{i}) + 2(4\mathbf{k} \times (-2\mathbf{i})) - 4\mathbf{i}$$

$$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ m/s^2 \qquad \text{Ans}$$

***16-132.** The ball B of negligible size rolls through the tube such that at the instant shown it has a velocity of 5 ft/s and an acceleration of $3\ ft/s^2$, measured relative to the tube. If the tube has an angular velocity of $\omega = 3$ rad/s and an angular acceleration of $\alpha = 5\ rad/s^2$ at this same instant, determine the velocity and acceleration of the ball.



Kinematic Equations :

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz} \qquad (1)$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad (2)$$

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

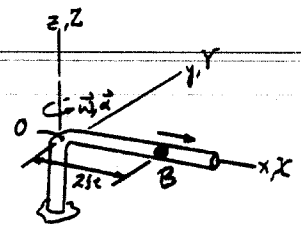
$\Omega = \{3\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{5\mathbf{k}\}\ rad/s^2$

$\mathbf{r}_{B/O} = \{2\mathbf{i}\}$ ft

$(\mathbf{v}_{B/O})_{xyz} = \{5\mathbf{i}\}$ ft/s

$(\mathbf{a}_{B/O})_{xyz} = \{3\mathbf{i}\}\ ft/s^2$



Substitute the data into Eqs. (1) and (2) yields :

$$\mathbf{v}_B = \mathbf{0} + (3\mathbf{k}) \times (2\mathbf{i}) + (5\mathbf{i}) = \{5\mathbf{i} + 6\mathbf{j}\}\ ft/s \qquad \text{Ans}$$

$$\mathbf{a}_B = \mathbf{0} + (5\mathbf{k}) \times (2\mathbf{i}) + (3\mathbf{k}) \times \left[(3\mathbf{k}) \times (2\mathbf{i})\right] + 2(3\mathbf{k}) \times (5\mathbf{i}) + (3\mathbf{i})$$

$$= \{-15\mathbf{i} + 40\mathbf{j}\}\ ft/s^2 \qquad \text{Ans}$$

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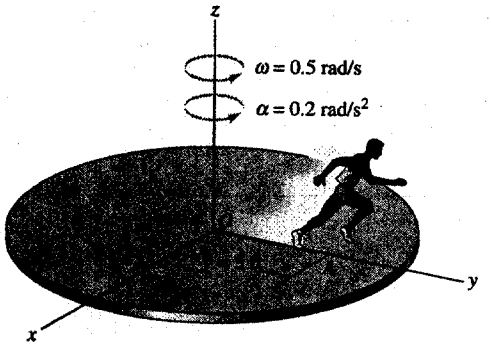
16-133. The man stands on the platform at O and runs out toward the edge such that when he is at A, $y = 5$ ft, his mass center has a velocity of 2 ft/s and an acceleration of 3 ft/s^2, both measured with respect to the platform and directed along the y axis. If the platform has the angular motions shown, determine the velocity and acceleration of his mass center at this instant.



$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$

$\mathbf{v}_A = \mathbf{0} + (0.5\mathbf{k}) \times (5\mathbf{j}) + 2\mathbf{j}$

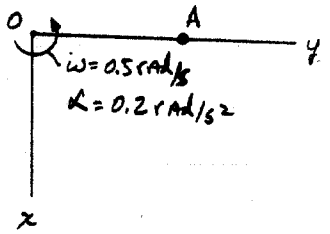
$\mathbf{v}_A = \{-2.50\mathbf{i} + 2.00\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/O}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$

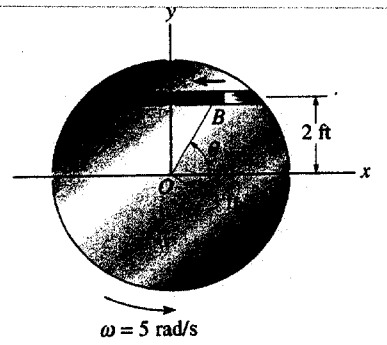
$\mathbf{a}_A = \mathbf{0} + (0.2\mathbf{k}) \times (5\mathbf{j}) + (0.5\mathbf{k}) \times (0.5\mathbf{k} \times 5\mathbf{j}) + 2(0.5\mathbf{k}) \times (2\mathbf{j}) + 3\mathbf{j}$

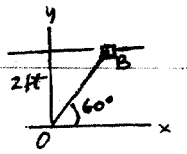
$\mathbf{a}_A = -1\mathbf{i} - 1.25\mathbf{j} - 2\mathbf{i} + 3\mathbf{j}$

$\mathbf{a}_A = \{-3.00\mathbf{i} + 1.75\mathbf{j}\}$ ft/s^2 **Ans**



16-134. Block B moves along the slot in the platform with a constant speed of 2 ft/s, measured relative to the platform in the direction shown. If the platform is rotating at a constant rate of $\omega = 5$ rad/s, determine the velocity and acceleration of the block at the instant $\theta = 60°$.





$\mathbf{r}_{B/O} = \frac{2}{\tan 60°}\mathbf{i} + 2\mathbf{j} = \{1.55\mathbf{i} + 2\mathbf{j}\}$ ft

$\mathbf{v}_B = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$

$\mathbf{v}_B = 0 + 5\mathbf{k} \times (1.155\mathbf{i} + 2\mathbf{j}) - 2\mathbf{i}$

$\mathbf{v}_B = \{-12.0\mathbf{i} + 5.77\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_B = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/O}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$

$\mathbf{a}_B = 0 + 0 + 5\mathbf{k} \times [(5\mathbf{k}) \times (1.155\mathbf{i} + 2\mathbf{j})] + 2(5\mathbf{k}) \times (-2\mathbf{i}) + 0$

$\mathbf{a}_B = 0 + 0 - 28.87\mathbf{i} - 50\mathbf{j} - 20\mathbf{j}$

$\mathbf{a}_B = \{-28.9\mathbf{i} - 70.0\mathbf{j}\}$ ft/s^2 **Ans**

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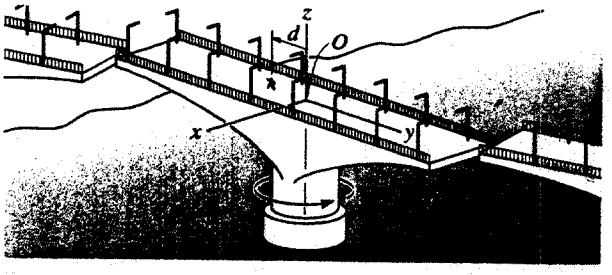
16-135. While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway at a constant speed of 5 ft/s relative to the roadway. Determine his velocity and acceleration at the instant $d = 15$ ft.



$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = 0$

$\mathbf{r}_{m/O} = \{-15\,\mathbf{j}\}$ ft

$(\mathbf{v}_{m/O})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(\mathbf{a}_{m/O})_{xyz} = 0$

$\mathbf{v}_m = \mathbf{v}_O + \Omega \times \mathbf{r}_{m/O} + (\mathbf{v}_{m/O})_{xyz}$

$\mathbf{v}_m = 0 + (0.5\mathbf{k}) \times (-15\mathbf{j}) - 5\mathbf{j}$

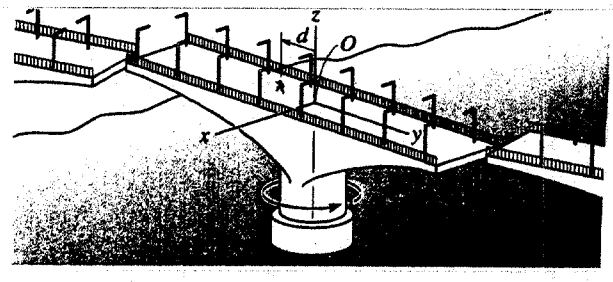
$\mathbf{v}_m = \{7.5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_m = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\,\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

$\mathbf{a}_m = 0 + 0 + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-15\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) + 0$

$\mathbf{a}_m = \{5\mathbf{i} + 3.75\mathbf{j}\}$ ft/s^2 **Ans**

***16-136.** While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway such that when $d = 10$ ft he is running outward from the center at 5 ft/s with an acceleration of 2 ft/s^2, both measured relative to the roadway. Determine his velocity and acceleration at this instant.



$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = 0$

$\mathbf{r}_{m/O} = \{-10\,\mathbf{j}\}$ ft

$(\mathbf{v}_{m/O})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(\mathbf{a}_{m/O})_{xyz} = \{-2\mathbf{j}\}$ ft/s^2

$\mathbf{v}_m = \mathbf{v}_O + \Omega \times \mathbf{r}_{m/O} + (\mathbf{v}_{m/O})_{xyz}$

$\mathbf{v}_m = 0 + (0.5\mathbf{k}) \times (-10\mathbf{j}) - 5\mathbf{j}$

$\mathbf{v}_m = \{5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_m = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\,\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

$\mathbf{a}_m = 0 + 0 + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-10\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) - 2\mathbf{j}$

$\mathbf{a}_m = \{5\mathbf{i} + 0.5\mathbf{j}\}$ ft/s^2 **Ans**

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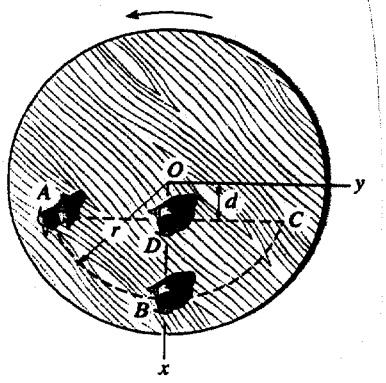
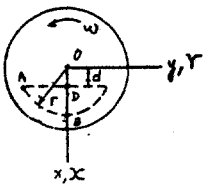
16-137. A girl stands at A on a platform which is rotating with a constant angular velocity $\omega = 0.5$ rad/s. If she walks at a constant speed of $v = 0.75$ m/s measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.



(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{D/O} + \Omega\times(\Omega\times\mathbf{r}_{D/O}) + 2\Omega\times(\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \qquad [1]$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \mathbf{0}$	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$



Substitute the data into Eq.[1]:

$$\mathbf{a}_D = \mathbf{0} + (\mathbf{0})\times(1\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(1\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{B/O} + \Omega\times(\Omega\times\mathbf{r}_{B/O}) + 2\Omega\times(\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad [2]$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \mathbf{0}$	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\mathbf{i} + (a_{B/O})_t\mathbf{j}$
	$= -\left(\frac{0.75^2}{3}\right)\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

Substitute the data into Eq.[2]:

$$\mathbf{a}_B = \mathbf{0} + (\mathbf{0})\times(3\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(3\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

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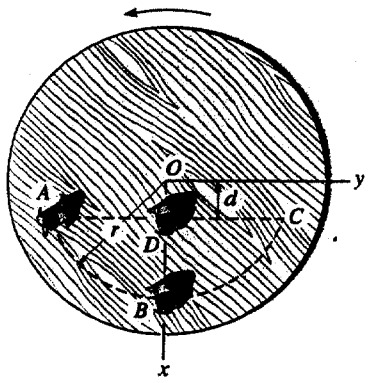
16-138. A girl stands at A on a platform which is rotating with an angular acceleration $\alpha = 0.2\ \text{rad/s}^2$ and at the instant shown has an angular velocity $\omega = 0.5$ rad/s. If she walks at a constant speed $v = 0.75$ m/s measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.



(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{D/O} + \Omega\times(\Omega\times\mathbf{r}_{D/O}) + 2\Omega\times(\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \qquad [1]$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.[1]:

$$\mathbf{a}_B = \mathbf{0} + (0.2\mathbf{k})\times(1\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(1\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{B/O} + \Omega\times(\Omega\times\mathbf{r}_{B/O}) + 2\Omega\times(\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad [2]$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\,\mathbf{i} + (a_{B/O})_t\,\mathbf{j}$
	$= -\left(\frac{0.75^2}{3}\right)\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

Substitute the data into Eq.[2]:

$$\mathbf{a}_B = \mathbf{0} + (0.2\mathbf{k})\times(3\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(3\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

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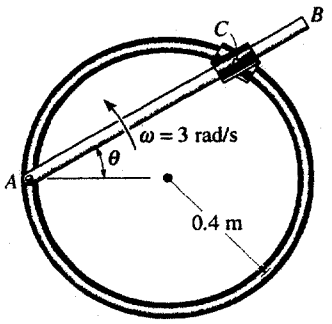
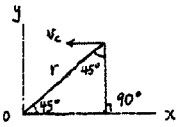
16-139. Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity and acceleration of point C located on the double collar when $\theta = 45°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.





$\mathbf{r}_{C/A} = \{0.400\mathbf{i} + 0.400\mathbf{j}\}$

$\mathbf{v}_C = -v_C\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-v_C\mathbf{i} = 0 + (3\mathbf{k}) \times (0.400\mathbf{i} + 0.400\mathbf{j}) + (v_{C/A}\cos 45°\mathbf{i} + v_{C/A}\sin 45°\mathbf{j})$

$-v_C\mathbf{i} = 0 - 1.20\mathbf{i} + 1.20\mathbf{j} + 0.707v_{C/A}\mathbf{i} + 0.707v_{C/A}\mathbf{j}$

$-v_C = -1.20 + 0.707v_{C/A}$

$0 = 1.20 + 0.707v_{C/A}$

$v_C = 2.40$ m/s **Ans**

$v_{C/A} = -1.697$ m/s

$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\Omega} \times \mathbf{r}_{C/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$$-(a_C)_t\mathbf{i} - \frac{(2.40)^2}{0.4}\mathbf{j} = 0 + 0 + 3\mathbf{k} \times [3\mathbf{k} \times (0.4\mathbf{i} + 0.4\mathbf{j})] + 2(3\mathbf{k}) \times [0.707(-1.697)\mathbf{i} + 0.707(-1.697)\mathbf{j}] + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$$

$-(a_C)_t\mathbf{i} - 14.40\mathbf{j} = 0 + 0 - 3.60\mathbf{i} - 3.60\mathbf{j} + 7.20\mathbf{i} - 7.20\mathbf{j} + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$

$-(a_C)_t = -3.60 + 7.20 + 0.707\,a_{C/A}$

$-14.40 = -3.60 - 7.20 + 0.707a_{C/A}$

$a_{C/A} = -5.09$ m/s^2

$(a_C)_t = 0$

Thus,

$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4 \text{ m/s}^2$$

$\mathbf{a}_C = \{-14.4\mathbf{j}\}$ m/s^2 **Ans**

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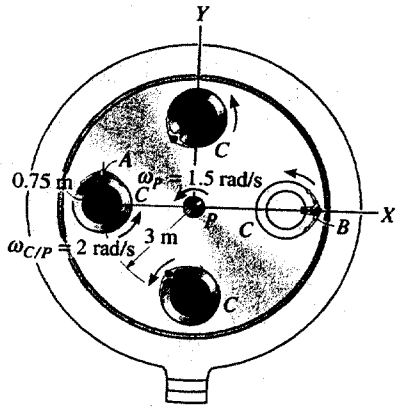
***16-140.** A ride in an amusement park consists of a rotating platform P, having a constant angular velocity $\omega_P = 1.5$ rad/s, and four cars, C, mounted on the platform, which have constant angular velocities $\omega_{C/P} = 2$ rad/s measured relative to the platform. Determine the velocity and acceleration of the passenger at B at the instant shown.



Motion of moving reference.

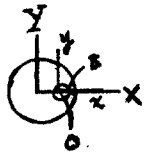
Fix the x,y,z axes to the platform with the origin at O.

$\mathbf{v}_O = (1.5)(3)\mathbf{j} = 4.5\mathbf{j}$

$\mathbf{a}_O = (a_C)_n = -(1.5)^2(3)\mathbf{i} = -6.75\mathbf{i}$

$\Omega = 1.5\mathbf{k}$

$\dot{\Omega} = 0$



Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.75\mathbf{j}$

$(\mathbf{v}_{A/O})_{xyz} = 2(0.75)\mathbf{j} = 1.5\mathbf{j}$

$(\mathbf{a}_{A/O})_{xyz} = (a_{A/O})_n = -(2)^2(0.75)\mathbf{i} = -3\mathbf{i}$

$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$

$= 4.5\mathbf{j} + (1.5\mathbf{k}) \times (0.75\mathbf{j}) + 1.5\mathbf{j}$

$\mathbf{v}_B = \{7.12\mathbf{j}\}$ m/s **Ans**

$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$

$= -6.75\mathbf{i} + 0 + 1.5\mathbf{k} \times [(1.5\mathbf{k}) \times (0.75\mathbf{i})] + 2(1.5\mathbf{k}) \times (1.5\mathbf{j})] - 3\mathbf{i}$

$\mathbf{a}_B = \{-15.9\mathbf{i}\}$ m/s^2 **Ans**

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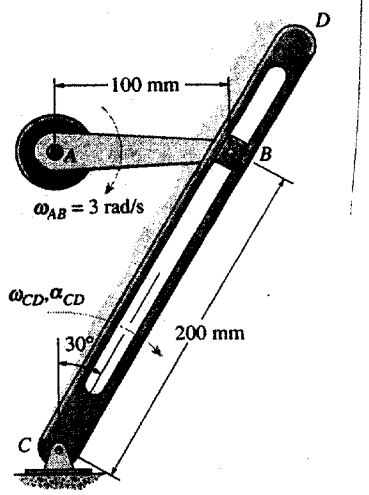
16-141. Block B of the mechanism is confined to move within the slot member CD. If AB is rotating at a constant rate of $\omega_{AB} = 3$ rad/s, determine the angular velocity and angular acceleration of member CD at the instant shown.



Coordinate Axes : The origin of both the fixed and moving frames of reference are located at point C. The x, y, z moving frame is attached to and rotates with rod CD since peg B slides along the slot in member CD.

Kinematic Equation : Applying Eqs. 16 – 24 and 16 – 27, we have

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz} \qquad [1]$$

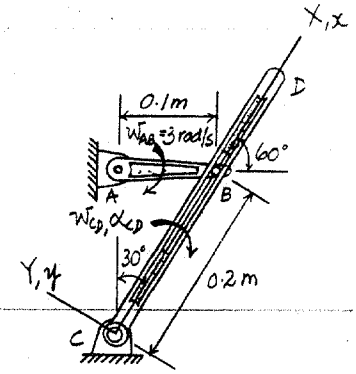
$$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz} \qquad [2]$$

Motion of moving reference	*Motion of C with respect to moving reference*
$\mathbf{v}_C = \mathbf{0}$	$\mathbf{r}_{B/C} = \{0.2\mathbf{i}\}$ m
$\mathbf{a}_C = \mathbf{0}$	$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\mathbf{i}$
$\Omega = -\omega_{CD}\mathbf{k}$	$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\mathbf{i}$
$\dot{\Omega} = -\alpha_{CD}\mathbf{k}$	

The velocity and acceleration of peg B can be determined using Eqs. 16 – 9 and 16 – 14 with $\mathbf{r}_{B/A} = \{0.1\cos 60°\mathbf{i} - 0.1\sin 60°\mathbf{j}\}$ m $= \{0.05\mathbf{i} - 0.08660\mathbf{j}\}$ m.

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = -3\mathbf{k} \times (0.05\mathbf{i} - 0.08660\mathbf{j}) = \{-0.2598\mathbf{i} - 0.150\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} = \mathbf{0} - 3^2(0.05\mathbf{i} - 0.08660\mathbf{j}) = \{-0.450\mathbf{i} + 0.7794\mathbf{j}\} \text{ m/s}$$



Substitute the above data into Eq. [1] yields

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$$
$$-0.2598\mathbf{i} - 0.150\mathbf{j} = \mathbf{0} + (-\omega_{CD}\mathbf{k}) \times 0.2\mathbf{i} + (v_{B/C})_{xyz}\mathbf{i}$$
$$-0.2598\mathbf{i} - 0.150\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} - 0.2\omega_{CD}\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$(v_{B/C})_{xyz} = -0.2598 \text{ m/s}$$
$$\omega_{CD} = 0.750 \text{ rad/s} \qquad \textbf{Ans}$$

Substitute the above data into Eq. [2] yields

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$$
$$-0.450\mathbf{i} + 0.7794\mathbf{j} = \mathbf{0} + (-\alpha_{CD}\mathbf{k}) \times 0.2\mathbf{i} + (-0.750\mathbf{k}) \times [(-0.750\mathbf{k}) \times 0.2\mathbf{i}] + 2(-0.750\mathbf{k}) \times (-0.2598\mathbf{i}) + (a_{B/C})_{xyz}\mathbf{i}$$
$$-0.450\mathbf{i} + 0.7794\mathbf{j} = \left[(a_{B/C})_{xyz} - 0.1125\right]\mathbf{i} + (0.3897 - 0.2\alpha_{CD})\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$(a_{B/C})_{xyz} = -0.3375 \text{ m/s}^2$$
$$\alpha_{CD} = -1.95 \text{ rad/s}^2 = 1.95 \text{ rad/s}^2 \qquad \textbf{Ans}$$

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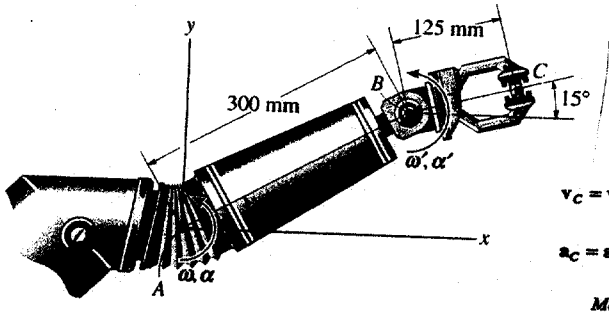
16-142. At the instant shown, the robotic arm AB is rotating counterclockwise at $\omega = 5$ rad/s and has an angular acceleration $\alpha = 2$ rad/s^2. Simultaneously, the grip BC is rotating counterclockwise at $\omega' = 6$ rad/s and $\alpha' = 2$ rad/s^2, both measured relative to a *fixed* reference. Determine the velocity and acceleration of the object held at the grip C.



$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} \qquad [1]$$

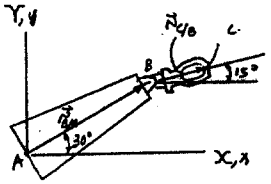
$$\mathbf{a}_C = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz} \qquad [2]$$

Motion of moving reference — *Motion of C with respect to moving reference*

$$\mathbf{r}_{C/B} = \{0.125\cos 15°\mathbf{i} + 0.125\sin 15°\mathbf{j}\} \text{ m}$$

$\boldsymbol{\Omega} = \{6\mathbf{k}\}$ rad/s $\qquad (\mathbf{v}_{C/B})_{xyz} = \mathbf{0}$

$\dot{\boldsymbol{\Omega}} = \{2\mathbf{k}\}$ rad/s^2 $\qquad (\mathbf{a}_{C/B})_{xyz} = \mathbf{0}$



Motion of B :

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$= (5\mathbf{k}) \times (0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j})$$

$$= \{-0.75\mathbf{i} + 1.2990\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$= (2\mathbf{k}) \times (0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j}) - (5)^2 (0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j})$$

$$= \{-6.7952\mathbf{i} - 3.2304\mathbf{j}\} \text{ m/s}^2$$

Substitute the data into Eqs.[1] and [2] yields :

$$\mathbf{v}_C = (-0.75\mathbf{i} + 1.2990\mathbf{j}) + (6\mathbf{k}) \times (0.125\cos 15°\mathbf{i} + 0.125\sin 15°\mathbf{j}) + \mathbf{0}$$

$$= \{-0.944\mathbf{i} + 2.02\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = (-6.79527\mathbf{i} - 3.2304\mathbf{j}) + (2\mathbf{k}) \times (0.125\cos 15°\mathbf{i} + 0.125\sin 15°\mathbf{j}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.125\cos 15°\mathbf{i} + 0.125\sin 15°\mathbf{j})] + \mathbf{0} + \mathbf{0}$$

$$= \{-11.2\mathbf{i} - 4.15\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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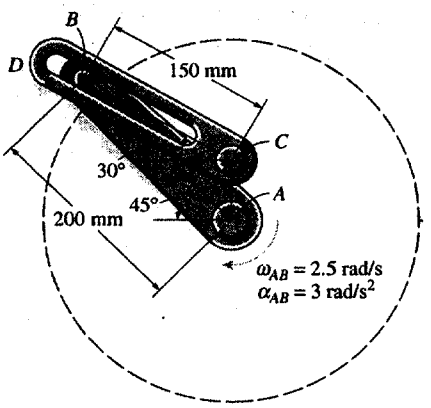
16-143. The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s and an angular acceleration of $\alpha_{AB} = 3$ rad/s^2, determine the angular velocity and angular acceleration of CD (output) at this instant.



$\mathbf{v}_C = \mathbf{0}$

$\mathbf{a}_C = \mathbf{0}$

$\Omega = -\omega_{DC}\mathbf{k}$

$\dot{\Omega} = -\alpha_{DC}\mathbf{k}$

$\mathbf{r}_{B/C} = \{-0.15\mathbf{i}\}\,\text{m}$

$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\,\mathbf{i}$

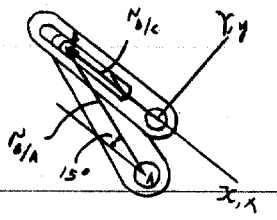
$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$= \{0.1294\mathbf{i} + 0.4830\mathbf{j}\}$ m/s

$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$

$= (-3\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j}) - (2.5)^2(-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$= \{1.3627\mathbf{i} + 0.2560\mathbf{j}\}$ m/s^2



$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\,\mathbf{i}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = (v_{B/C})_{xyz}\,\mathbf{i} + 0.15\omega_{DC}\mathbf{j}$

Solving:

$(v_{B/C})_{xyz} = 0.1294$ m/s

$\omega_{DC} = 3.22$ rad/s) **Ans**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

$1.3627\mathbf{i} + 0.2560\mathbf{j} = \mathbf{0} + (-\alpha_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (-3.22\mathbf{k}) \times [(-3.22\mathbf{k}) \times (-0.15\mathbf{i})]$

$+ 2(-3.22\mathbf{k}) \times (0.1294\mathbf{i}) + (a_{B/C})_{xyz}\,\mathbf{i}$

$1.3627\mathbf{i} + 0.2560\mathbf{j} = [1.5550 + (a_{B/C})_{xyz}]\mathbf{i} + (0.15\alpha_{DC} - 0.8333)\mathbf{j}$

Solving:

$(a_{B/A})_{xyz} = -0.1923$ m/s^2

$\alpha_{DC} = 7.26$ rad/s^2) **Ans**

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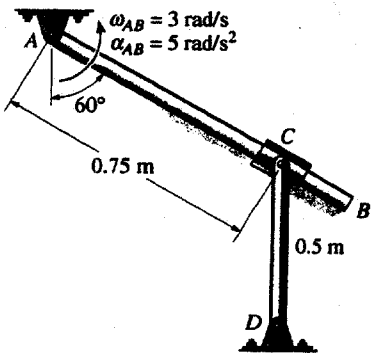
***16-144.** At the instant shown rod AB has an angular velocity $\omega_{AB} = 3$ rad/s and an angular acceleration $\alpha_{AB} = 5$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin-connected to CD and slides over AB.



$\mathbf{r}_{C/A} = (0.75\sin 60°)\mathbf{i} - (0.75\cos 60°)\mathbf{j}$

$\mathbf{r}_{C/A} = \{0.6495\mathbf{i} - 0.375\mathbf{j}\}$ m

$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D}$

$= (\omega_{CD}\mathbf{k}) \times (0.5\mathbf{j})$

$= \{-0.5\omega_{CD}\mathbf{i}\}$ m/s

$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$

$= (\alpha_{CD}\mathbf{k}) \times (0.5\mathbf{j}) - \omega_{CD}^2(0.5\mathbf{j})$

$\mathbf{a}_C = \{-0.5\alpha_{CD}\mathbf{i} - \omega_{CD}^2(0.5)\mathbf{j}\}$ m/s^2

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-0.5\omega_{CD}\mathbf{i} = 0 + (3\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j}) + v_{C/A}\sin 60°\mathbf{i} - v_{C/A}\cos 60°\mathbf{j}$

$-0.5\omega_{CD} = 1.125 + 0.866 v_{C/A}$

$0 = 1.9485 - 0.5 v_{C/A}$

$v_{C/A} = 3.897$ m/s

$\omega_{CD} = -9.00$ rad/s $= 9.00$ rad/s $\circlearrowright$ **Ans**

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$\mathbf{a}_C = 0 + (5\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j}) + (3\mathbf{k}) \times [(3\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j})]$
$+ 2(3\mathbf{k}) \times [3.897(0.866)\mathbf{i} - 0.5(3.897)\mathbf{j}] + 0.866 a_{C/A}\mathbf{i} - 0.5 a_{C/A}\mathbf{j}$

$0.5\alpha_{CD}\mathbf{i} - (-9.00)^2(0.5)\mathbf{j} = 0 + 1.875\mathbf{i} + 3.2475\mathbf{j} - 5.8455\mathbf{i} + 3.375\mathbf{j} + 11.6910\mathbf{i}$
$+ 20.2488\mathbf{j} + 0.866 a_{C/A}\mathbf{i} - 0.5 a_{C/A}\mathbf{j}$

$0.5\alpha_{CD} = 7.7205 + 0.866 a_{C/A}$

$-40.5 = 26.8713 - 0.5 a_{C/A}$

$a_{C/A} = 134.7$ m/s^2

$\alpha_{CD} = 249$ rad/s^2 $\circlearrowright$ **Ans**

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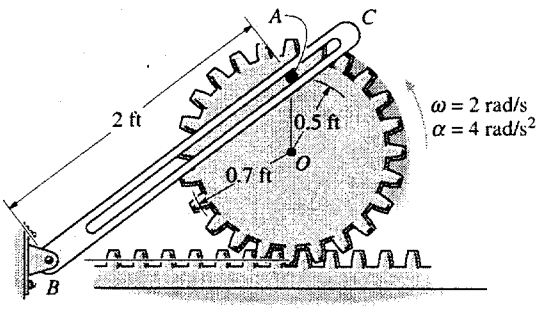
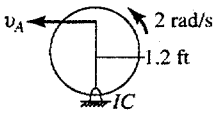
16-145. The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg at A is fixed to the gear.



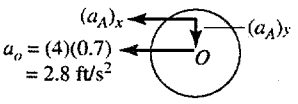
$v_A = (1.2)(2) = 2.4$ ft/s ←



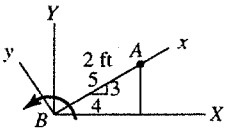
$a_O = 4(0.7) = 2.8$ ft/s^2

$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$

$\mathbf{a}_A = 2.8 + 4(0.5) + (2)^2(0.5)$
$\quad\leftarrow \quad\leftarrow \quad\downarrow$



$\mathbf{a}_A = 4.8 + 2$
$\quad\leftarrow \quad\downarrow$



$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$

$$-2.4\mathbf{i} = 0 + (\Omega\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + v_{A/B}\left(\frac{4}{5}\right)\mathbf{i} + v_{A/B}\left(\frac{3}{5}\right)\mathbf{j}$$

$$-2.4\mathbf{i} = 1.6\Omega\mathbf{j} - 1.2\Omega\mathbf{i} + 0.8v_{A/B}\mathbf{i} + 0.6v_{A/B}\mathbf{j}$$

$$-2.4 = -1.2\Omega + 0.8v_{A/B}$$

$$0 = 1.6\Omega + 0.6v_{A/B}$$

Solving,

$\omega_{BC} = \Omega = 0.720$ rad/s ↻ **Ans**

$v_{A/B} = -1.92$ ft/s

$\mathbf{a}_A = \mathbf{a}_B + \Omega \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$

$$-4.8\mathbf{i} - 2\mathbf{j} = 0 + (\dot{\Omega}\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + (0.72\mathbf{k}) \times (0.72\mathbf{k} \times (1.6\mathbf{i} + 1.2\mathbf{j}))$$
$$+ 2(0.72\mathbf{k}) \times [-(0.8)(1.92)\mathbf{i} - 0.6(1.92)\mathbf{j}] + 0.8a_{B/A}\mathbf{i} + 0.6a_{B/A}\mathbf{j}$$

$$-4.8\mathbf{i} - 2\mathbf{j} = 1.6\dot{\Omega}\mathbf{j} - 1.2\dot{\Omega}\mathbf{i} - 0.8294\mathbf{i} - 0.6221\mathbf{j} - 2.2118\mathbf{j} + 1.6589\mathbf{i} + 0.8a_{B/A}\mathbf{i} + 0.6a_{B/A}.$$

$$-4.8 = -1.2\dot{\Omega} - 0.8294 + 1.6589 + 0.8a_{B/A}$$

$$-2 = 1.6\dot{\Omega} - 0.6221 - 2.2118 + 0.6a_{B/A}$$

$$-4.6913 = -\dot{\Omega} + 0.667a_{B/A}$$

$$0.5212 = \dot{\Omega} + 0.357a_{B/A}$$

$\alpha_{BC} = \dot{\Omega} = 2.02$ rad/s^2 ↻ **Ans**

$a_{B/A} = -4.00$ ft/s^2

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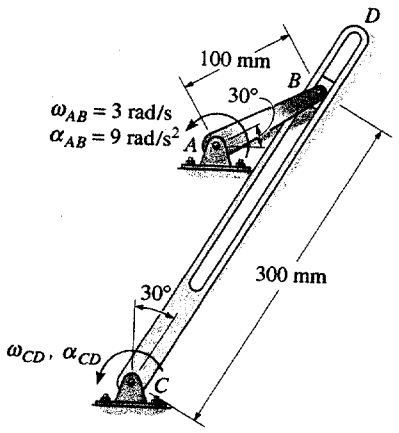
16-146. The quick-return mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9$ m/s^2

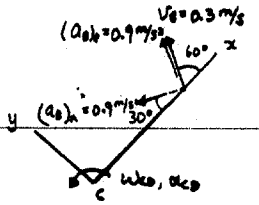
$(a_B)_n = (3)^2(0.1) = 0.9$ m/s^2

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3\cos 60°\mathbf{i} + 0.3\sin 60°\mathbf{j} = 0 + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s ↻ Ans



$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

$0.9\cos 60°\mathbf{i} - 0.9\cos 30°\mathbf{i} + 0.9\sin 60°\mathbf{j} + 0.9\sin 30°\mathbf{j} = 0 + (\alpha_{CD}\mathbf{k}) \times (0.3\mathbf{i})$

$+ (0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\mathbf{i}$

$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\mathbf{i}$

$a_{B/C} = -0.104$ m/s^2

$\alpha_{CD} = 3.23$ rad/s^2 ↻ Ans

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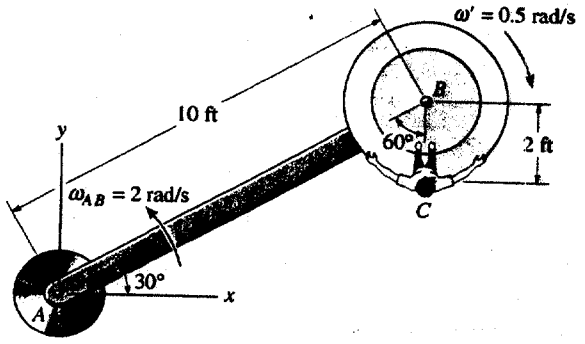
16-147. A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



$\mathbf{r}_{B/A} = (10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}$ ft

$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}$ ft/s

$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$

$= 0 - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-34.64\mathbf{i} - 20\mathbf{j}\}$ ft/s^2

$\boldsymbol{\Omega} = (2 - 0.5)\mathbf{k} = 1.5\mathbf{k}$

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$

$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$

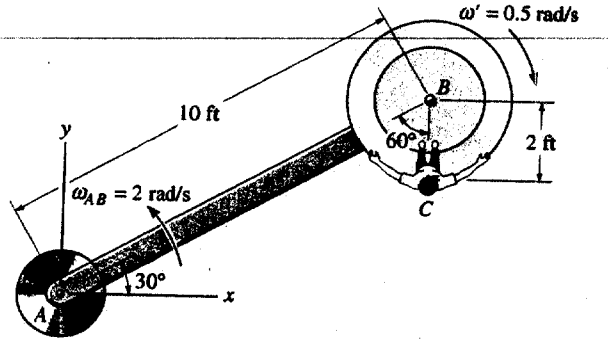
$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_C = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$

$= -34.64\mathbf{i} - 20\mathbf{j} + 0 + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$

$= \{-34.6\mathbf{i} - 15.5\mathbf{j}\}$ ft/s^2 **Ans**

***16-148.** A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1$ rad/s^2 when $\omega_{AB} = 2$ rad/s at the instant shown. Also at this instant the car mounted at the end of the arm has a relative angular acceleration of $\boldsymbol{\alpha}' = \{-0.6\mathbf{k}\}$ rad/s^2 when $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s. Determine the velocity and acceleration of the passenger C at this instant.



$\mathbf{r}_{B/A} = (10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}$ ft

$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}$ ft/s

$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$

$= (1\mathbf{k}) \times (8.66\mathbf{i} + 5\mathbf{j}) - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-39.64\mathbf{i} - 11.34\mathbf{j}\}$ ft/s^2

$\boldsymbol{\Omega} = (2 - 0.5)\mathbf{k} = 1.5\mathbf{k}$

$\dot{\boldsymbol{\Omega}} = (1 - 0.6)\mathbf{k} = 0.4\mathbf{k}$

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$

$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$

$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}$ ft/s **Ans**

$\mathbf{a}_C = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$

$= -39.64\mathbf{i} - 11.34\mathbf{j} + (0.4\mathbf{k}) \times (-2\mathbf{j}) + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$

$= \{-38.8\mathbf{i} - 6.84\mathbf{j}\}$ ft/s^2 **Ans**

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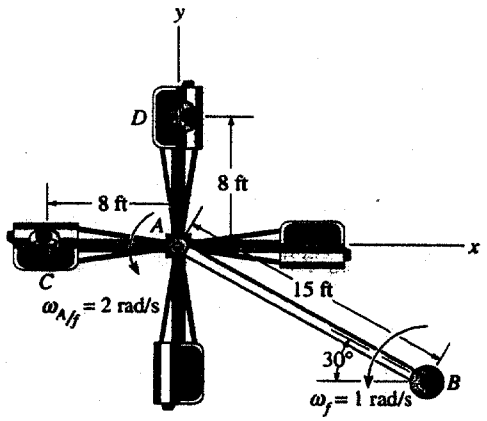
16-149. The cars on the amusement-park ride rotate around the axle at A with a constant angular velocity $\omega_{A/f} = 2$ rad/s, measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with a constant angular velocity $\omega_f = 1$ rad/s. Determine the velocity and acceleration of the passenger at C at the instant shown.



$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad [1]$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad [2]$$

Motion of moving reference

$\Omega = \{3\mathbf{k}\}$ rad/s

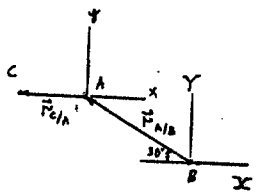
$\dot{\Omega} = 0$

Motion of C with respect to moving reference

$\mathbf{r}_{C/A} = \{-8\mathbf{i}\}$ ft

$(\mathbf{v}_{C/A})_{xyz} = 0$

$(\mathbf{a}_{C/A})_{xyz} = 0$



Motion of A :

$$\mathbf{v}_A = \omega \times \mathbf{r}_{A/B}$$

$$= (1\mathbf{k}) \times (-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{-7.5\mathbf{i} - 12.99\mathbf{j}\}\ \text{ft/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

$$= 0 - (1)^2(-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{12.99\mathbf{i} - 7.5\mathbf{j}\}\ \text{ft/s}^2$$

Substitute the data into Eqs.[1] and [2] yields :

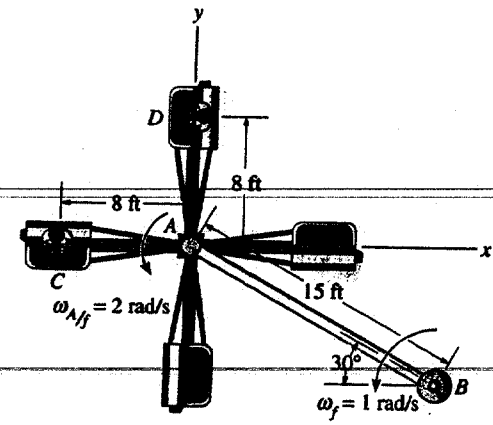
$$\mathbf{v}_C = (-7.5\mathbf{i} - 12.99\mathbf{j}) + (3\mathbf{k}) \times (-8\mathbf{i}) + 0$$

$$= \{-7.5\mathbf{i} - 37.0\mathbf{j}\}\ \text{ft/s}$$ Ans

$$\mathbf{a}_C = (12.99\mathbf{i} - 7.5\mathbf{j}) + 0 + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-8\mathbf{i})] + 0 + 0$$

$$= \{85.0\mathbf{i} - 7.5\mathbf{j}\}\ \text{ft/s}^2$$ Ans

16-150. The cars on the amusement-park ride rotate around the axle at A with a constant angular velocity $\omega_{A/f} = 2$ rad/s, measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with a constant angular velocity $\omega_f = 1$ rad/s. Determine the velocity and acceleration of the passenger at D at the instant shown.



$$\mathbf{v}_D = \mathbf{v}_A + \Omega \times \mathbf{r}_{D/A} + (\mathbf{v}_{D/A})_{xyz} \qquad [1]$$

$$\mathbf{a}_D = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{D/A} + \Omega \times (\Omega \times \mathbf{r}_{D/A}) + 2\Omega \times (\mathbf{v}_{D/A})_{xyz} + (\mathbf{a}_{D/A})_{xyz} \qquad [2]$$

Motion of moving reference

$\Omega = \{3\mathbf{k}\}$ rad/s

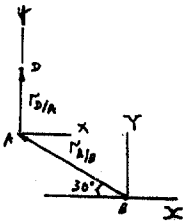
$\dot{\Omega} = 0$

Motion of D with respect to moving reference

$\mathbf{r}_{D/A} = \{8\mathbf{j}\}$ ft

$(\mathbf{v}_{D/A})_{xyz} = 0$

$(\mathbf{a}_{D/A})_{xyz} = 0$



Motion of A :

$$\mathbf{v}_A = \omega \times \mathbf{r}_{A/B}$$

$$= (1\mathbf{k}) \times (-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{-7.5\mathbf{i} - 12.99\mathbf{j}\}\ \text{ft/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

$$= 0 - (1)^2(-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{12.99\mathbf{i} - 7.5\mathbf{j}\}\ \text{ft/s}^2$$

Substitute the data into Eqs.[1] and [2] yields :

$$\mathbf{v}_D = (-7.5\mathbf{i} - 12.99\mathbf{j}) + (3\mathbf{k}) \times (8\mathbf{j}) + 0$$

$$= \{-31.5\mathbf{i} - 13.0\mathbf{j}\}\ \text{ft/s}$$ Ans

$$\mathbf{a}_D = (12.99\mathbf{i} - 7.5\mathbf{j}) + 0 + (3\mathbf{k}) \times [(3\mathbf{k}) \times (8\mathbf{j})] + 0 + 0$$

$$= \{13.0\mathbf{i} - 79.5\mathbf{j}\}\ \text{ft/s}^2$$ Ans

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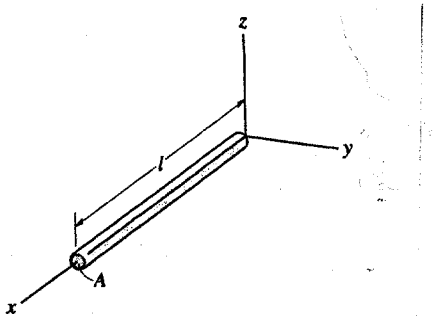
17-1. Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



$$I_y = \int_M x^2\, dm$$

$$= \int_0^l x^2\, (\rho A\, dx)$$

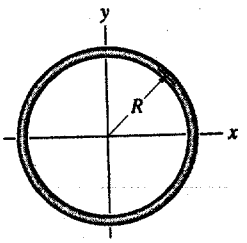
$$= \frac{1}{3}\rho A\, l^3$$

$$m = \rho A\, l$$

Thus,

$$I_y = \frac{1}{3} m\, l^2 \qquad \textbf{Ans}$$

17-2. Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



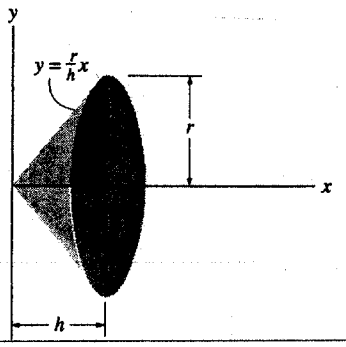
$$I_z = \int_0^{2\pi} \rho A\, (R\, d\theta)\, R^2 = 2\pi \rho A R^3$$

$$m = \int_0^{2\pi} \rho A\, R\, d\theta = 2\pi \rho A R$$

Thus,

$$I_z = m R^2 \qquad \textbf{Ans}$$

17-3. The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



$$dm = \rho\, dV = \rho\left(\pi y^2\, dx\right)$$

$$m = \int_0^h \rho(\pi)\left(\frac{r^2}{h^2}\right)x^2\, dx = \rho\pi\left(\frac{r^2}{h^2}\right)\left(\frac{1}{3}\right)h^3 = \frac{1}{3}\rho\pi r^2 h$$

$$dI_x = \frac{1}{2}y^2\, dm$$

$$= \frac{1}{2}y^2\left(\rho\pi y^2\, dx\right)$$

$$= \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx$$

$$I_x = \int_0^h \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx = \frac{1}{10}\rho\pi r^4 h$$

Thus

$$I_x = \frac{3}{10} m r^2 \qquad \textbf{Ans}$$

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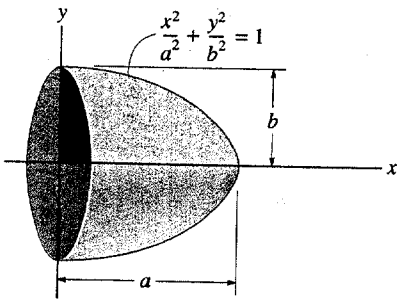
***17-4.** A semiellipsoid is formed by rotating the shaded area about the x axis. Determine the moment of inertia of this solid with respect to the x axis and express the result in terms of the mass m of the solid. The material has a constant density ρ.



$$dI_x = \frac{y^2 dm}{2}$$

$$m = \int_v \rho dV$$

$$= \int_0^a \rho\pi b^2\left(1-\frac{x^2}{a^2}\right)dx$$

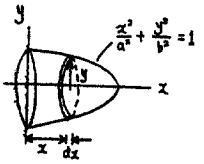
$$= \frac{2}{3}\rho\pi ab^2$$

$$I_x = \frac{1}{2}\rho\pi\int_0^a b^4\left(1-\frac{x^2}{a^2}\right)^2 dx$$

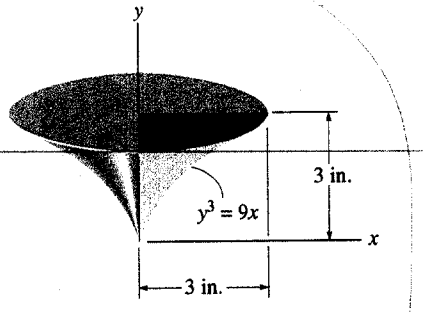
$$= \frac{4}{15}\rho\pi ab^4$$

Thus,

$$I_x = \frac{2}{5}mb^2 \qquad \text{Ans}$$



17-5. The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is $\gamma = 380$ lb/ft^3.



The moment of inertia of the solid : The mass of the disk element $dm = \rho\pi x^2 dy = \frac{1}{81}\rho\pi y^6 dy$.

$$dI_y = \frac{1}{2}dmx^2$$

$$= \frac{1}{2}(\rho\pi x^2 dy)x^2$$

$$= \frac{1}{2}\rho\pi x^4 dy = \frac{1}{2(9^4)}\rho\pi y^{12} dy$$

$$I_y = \int dI_y = \frac{1}{2(9^4)}\rho\pi\int_0^3 y^{12} dy$$

$$= 29.632\rho$$

The mass of the solid :

$$m = \int_m dm = \frac{1}{81}\rho\pi\int_0^3 y^6 dy = 12.117\rho$$

$$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.632\rho}{12.117\rho}} = 1.56 \text{ in.} \qquad \text{Ans}$$

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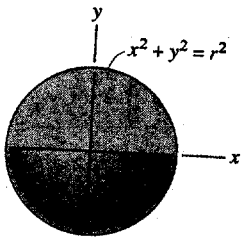
17-6. The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\pi(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho r^5$$

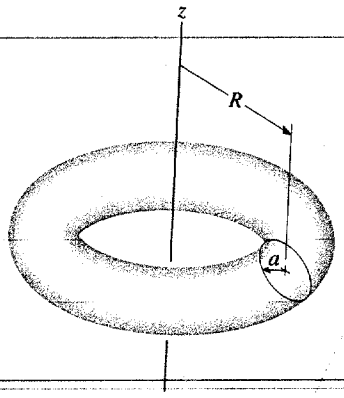
$$m = \int_{-r}^{r} \rho\pi(r^2 - x^2)\,dx$$

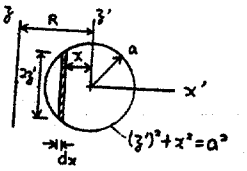
$$= \frac{4}{3}\rho\pi r^3$$

Thus,

$$I_x = \frac{2}{5}m r^2 \qquad \textbf{Ans}$$

17-7. Determine the moment of inertia I_z of the torus. The mass of the torus is m and the density ρ is constant. *Suggestion:* Use a shell element.





$$dm = 2\pi(R - x)(2z'\rho\,dx)$$

$$dI_z = (R - x)^2\,dm$$

$$= 4\pi\rho[(R^3 - 3R^2x + 3Rx^2 - x^3)\sqrt{a^2 - x^2}\,dx]$$

$$I_z = 4\pi\rho\left[R^3\int_{-a}^{a}\sqrt{a^2 - x^2}\,dx - 3R^2\int_{-a}^{a}x^3\sqrt{a^2 - x^2}\,dx + 3R\int_{-a}^{a}\sqrt{a^2 - x^2}\,dx - \int_{-a}^{a}x^3\sqrt{a^2 - x^2}\,dx\right]$$

$$= 2\pi^2\rho R a^2\left(R^2 + \frac{3}{4}a^2\right)$$

Since $m = \rho V = 2\pi R\rho\pi a^2$

$$I_z = m\left(R^2 + \frac{3}{4}a^2\right) \qquad \textbf{Ans}$$

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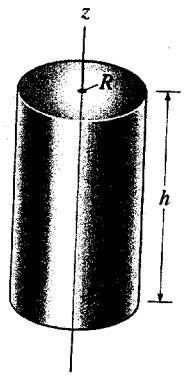
***17-8.** The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



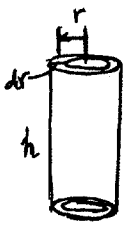
Consider a shell element of radius r and mass

$$dm = \rho\, dV = \rho(2\pi r\, dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi r\, dr)h$$

$$m = 2\pi h\left(\frac{kR^2}{2} + \frac{aR^4}{4}\right)$$

$$m = \pi h R^2\left(k + \frac{aR^2}{2}\right) \qquad \textbf{Ans}$$



$$dI = r^2 dm = r^2(\rho)(2\pi r\, dr)h$$

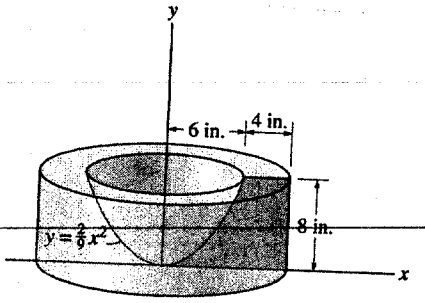
$$I_z = \int_0^R r^2(k + ar^2)(2\pi r\, dr)\, h$$

$$I_z = 2\pi h\int_0^R (k r^3 + a r^5)\, dr$$

$$I_z = 2\pi h\left[\frac{k R^4}{4} + \frac{aR^6}{6}\right]$$

$$I_z = \frac{\pi h R^4}{2}\left[k + \frac{2aR^2}{3}\right] \qquad \textbf{Ans}$$

17-9. The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is $\gamma = 150\ \text{lb/ft}^3$.



$$dI_y = \frac{1}{2}(dm)(10)^2 - \frac{1}{2}(dm)x^2$$

$$= \frac{1}{2}\{\pi\rho\,(10)^2\, dy\}(10)^2 - \frac{1}{2}\pi\rho\, x^2\, dy\, x^2$$

$$I_y = \frac{1}{2}\pi\rho\left[\int_0^8 (10)^4\, dy - \int_0^8 \left(\frac{9}{2}\right)^2 y^2\, dy\right]$$

$$= \frac{\frac{1}{2}\pi(150)}{32.2(12)^3}\left[(10)^4(8) - \left(\frac{9}{2}\right)^2\left(\frac{1}{3}\right)(8)^3\right]$$

$$= 324.1\ \text{slug}\cdot\text{in}^2$$

$$I_y = 2.25\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

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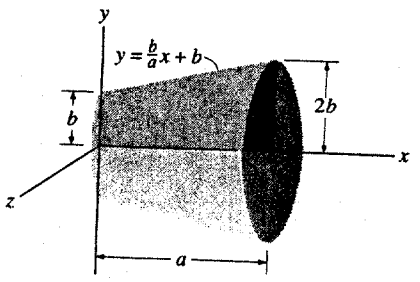
17-10. The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density.



$$dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\tfrac{b^2}{a^2}x^2 + \tfrac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \tfrac{1}{2}dm y^2 = \tfrac{1}{2}\rho\pi y^4 dx$$

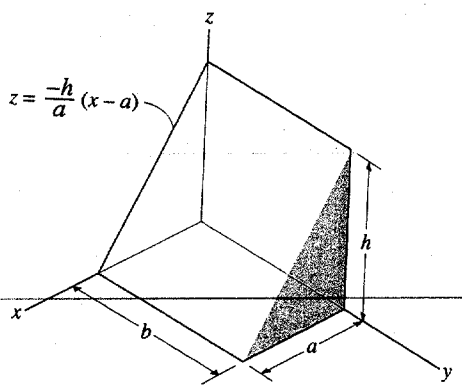
$$dI_x = \tfrac{1}{2}\rho\pi\left(\tfrac{b^4}{a^4}x^4 + \tfrac{4b^4}{a^3}x^3 + \tfrac{6b^4}{a^2}x^2 + \tfrac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \tfrac{1}{2}\rho\pi\int_0^a \left(\tfrac{b^4}{a^4}x^4 + \tfrac{4b^4}{a^3}x^3 + \tfrac{6b^4}{a^2}x^2 + \tfrac{4b^4}{a}x + b^4\right)dx$$

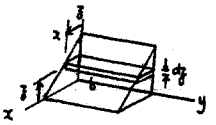
$$= \tfrac{31}{10}\rho\pi ab^4$$

$$m = \int_m dm = \rho\pi\int_0^a \left(\tfrac{b^2}{a^2}x^2 + \tfrac{2b^2}{a}x + b^2\right)dx = \tfrac{7}{3}\rho\pi ab^2 \qquad I_x = \tfrac{93}{70}mb^2 \qquad \textbf{Ans}$$

17-11. Determine the moment of inertia of the homogeneous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint:* For integration, use thin plate elements parallel to the x–y plane and having a thickness dz.



$$dV = bx\,dz = b(a)(1 - \tfrac{z}{h})dz$$



$$dI_y = d\bar{I}_y + (dm)[(\tfrac{x}{2})^2 + z^2]$$

$$= \frac{1}{12}dm(x^2) + dm(\frac{x^2}{4}) + dmz^2$$

$$= dm(\frac{x^2}{3} + z^2)$$

$$= [b(a)(1 - \tfrac{z}{h})dz](\rho)[\tfrac{a^2}{3}(1 - \tfrac{z}{h})^2 + z^2]$$

$$I_y = ab\rho\int_0^h [\tfrac{a^3}{3}(\tfrac{h-z}{h})^3 + z^2(1 - \tfrac{z}{h})]dz$$

$$= ab\rho[\frac{a^2}{3h^3}(h^4 - \frac{3}{2}h^4 + h^4 - \frac{1}{4}h^4) + \frac{1}{h}(\frac{1}{3}h^4 - \frac{1}{4}h^4)]$$

$$= \frac{1}{12}abh\rho(a^2 + h^2)$$

$$m = \rho V = \frac{1}{2}abh\rho$$

Thus,

$$I_y = \frac{m}{6}(a^2 + h^2) \qquad \textbf{Ans}$$

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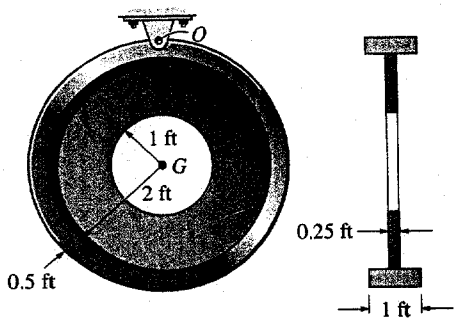
***17-12.** Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through point O. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.



$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$

$$+\frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$

$$= 117.72\ \text{slug}\cdot\text{ft}^2$$

$$I_O = I_G + md^2$$

$$m = \left(\frac{90}{32.2}\right)\pi(2^2 - 1^2)(0.25) + \left(\frac{90}{32.2}\right)\pi(2.5^2 - 2^2)(1) = 26.343\ \text{slug}$$

$$I_O = 117.72 + 26.343(2.5)^2 = 282\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-13. The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through O?

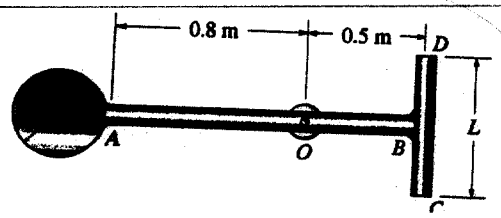
Measured from the right side,

$$\bar{y} = \frac{6(1.5) + 2(1.3)(0.65)}{6 + 1.3(2) + L(2)} = 0.5$$

$$L = 6.39\ \text{m} \qquad \textbf{Ans}$$

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(6.39)(6.39)^2 + 2(6.39)(0.5)^2$$

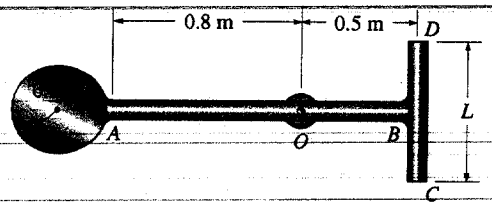
$$I_O = 55.2\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$



17-14. The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. If $L = 0.75$ m, determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through O.

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(0.75)(0.75)^2$$
$$+ 2(0.75)(0.5)^2$$

$$I_O = 6.99\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$



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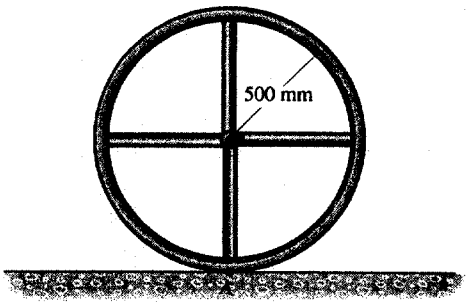
17-15. The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods and each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.

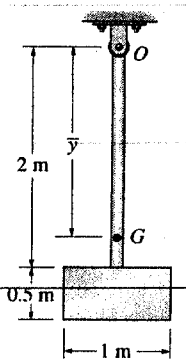


$$I_A = I_O + md^2$$

$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg} \cdot \text{m}^2 \quad \textbf{Ans}$$

***17-16.** The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



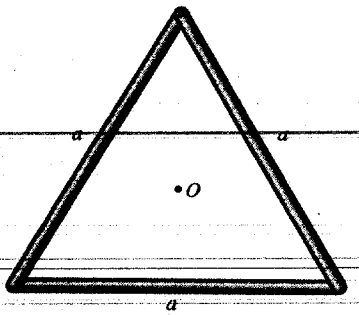
$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3+5} = 1.781 \text{ m} = 1.78 \text{ m} \quad \textbf{Ans}$$

$$I_G = \Sigma \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2 \quad \textbf{Ans}$$

17-17. Each of the three rods has a mass m. Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center point O.



$$I_O = 3\left[\frac{1}{12}ma^2 + m\left(\frac{a\sin 60°}{3}\right)^2\right] = \frac{1}{2}ma^2 \quad \textbf{Ans}$$

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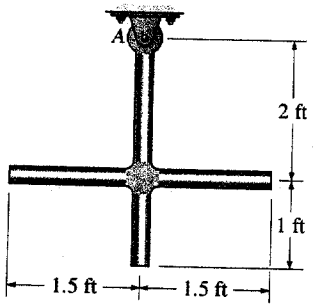
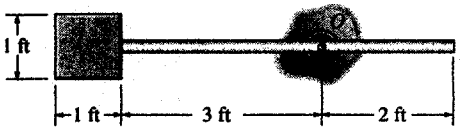
17-18. The slender rods have a weight of 3 lb/ft. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through the pin at A.



$$I_A = \left[\frac{1}{3}\left(\frac{3(3)}{32.2}\right)(3)^2\right] + \left[\frac{1}{12}\left(\frac{3(3)}{32.2}\right)(3)^2 + \left(\frac{3(3)}{32.2}\right)(2)^2\right]$$

$$= 2.17 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-19. The pendulum consists of a plate having a weight of 12 lb and a slender rod having a weight of 4 lb. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



$$I_O = \Sigma I_G + md^2$$

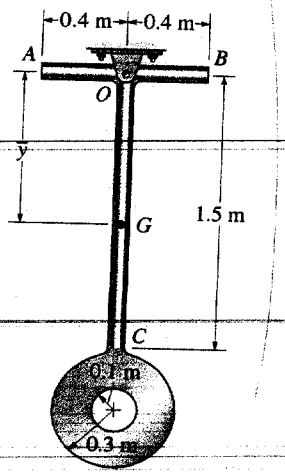
$$= \frac{1}{12}\left(\frac{4}{32.2}\right)(5)^2 + \left(\frac{4}{32.2}\right)(0.5)^2 + \frac{1}{12}\left(\frac{12}{32.2}\right)(1^2 + 1^2) + \left(\frac{12}{32.2}\right)(3.5)^2$$

$$= 4.917 \text{ slug}\cdot\text{ft}^2$$

$$m = \left(\frac{4}{32.2}\right) + \left(\frac{12}{32.2}\right) = 0.4969 \text{ slug}$$

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{4.917}{0.4969}} = 3.15 \text{ ft} \qquad \textbf{Ans}$$

***17-20.** The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



$$\bar{y} = \frac{0 + 0.75(3)(1.5) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{3(0.8) + 3(1.5) + \pi(0.3)^2(12) - \pi(0.1)^2(12)}$$

$$= 0.8878 = 0.888 \text{ m} \qquad \textbf{Ans}$$

$$I_O = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 \text{ kg}\cdot\text{m}^2$$

$$I_O = I_G + (9.916)(0.8878)^2$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

Also:

$$I_G = \frac{1}{12}[3(0.8)](0.8)^2 + 3(0.8)(0.8878)^2 + \frac{1}{12}[3(1.5)](1.5)^2 + 3(1.5)(0.8878 - 0.75)^2$$

$$+ \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2 + [12(\pi)(0.3)^2](1.8 - 0.8878)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2$$

$$- [12(\pi)(0.1)^2](1.8 - 0.8878)^2$$

$$= 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

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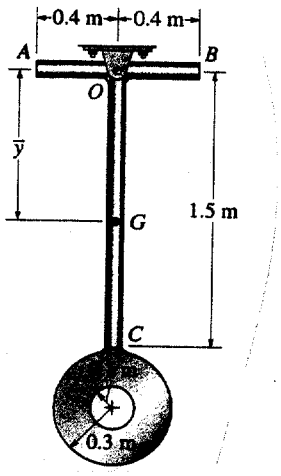
17-21. The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the moment of inertia of the pendulum about an axis perpendicular to the page and passing through the pin at O.



$$I_O = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 = 13.4 \text{ kg}\cdot\text{m}^2 \quad \text{Ans}$$

Also, from the solution to Prob. 17–16,

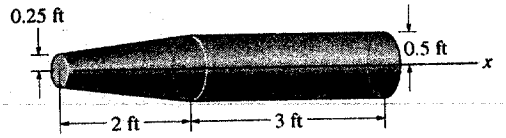
$$m = 3(0.8+1.5) + 12[\pi(0.3)^2 - \pi(0.1)^2] = 9.916 \text{ kg}$$

$$I_O = I_G + m d^2$$

$$= 5.61 + 9.916(0.8878)^2$$

$$= 13.4 \text{ kg}\cdot\text{m}^2 \quad \text{Ans}$$

17-22. Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490$ lb/ft^3.

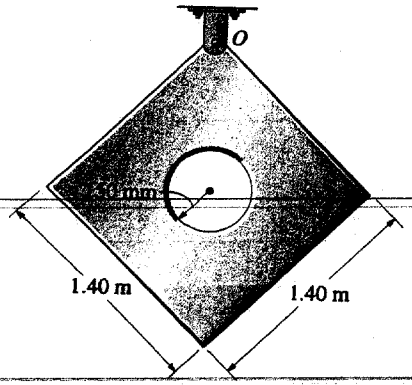


$$I_x = \frac{1}{2}m_1(0.5)^2 + \frac{3}{10}m_2(0.5)^2 - \frac{3}{10}m_3(0.25)^2$$

$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64 \text{ slug}\cdot\text{ft}^2 \quad \text{Ans}$$

17-23. Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50$ kg/m^3.



$$m_p = 50(1.40)^2(0.050) = 4.90 \text{ kg}$$

$$m_h = (50)(\pi)(0.15)^2(0.050) = 0.1767 \text{ kg}$$

$$I_O = \frac{1}{12}(4.90)[(1.40)^2 + (1.40)^2] + 4.90(0.990)^2 - [\frac{1}{2}(0.1767)(0.15)^2 + (0.1767)(0.990)^2]$$

$$I_O = 6.23 \text{ kg}\cdot\text{m}^2 \quad \text{Ans}$$

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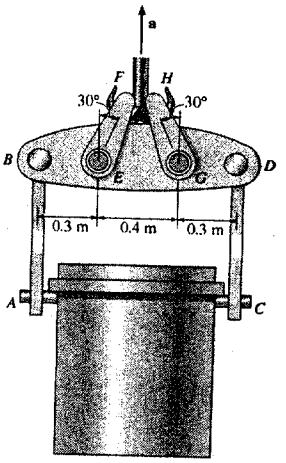
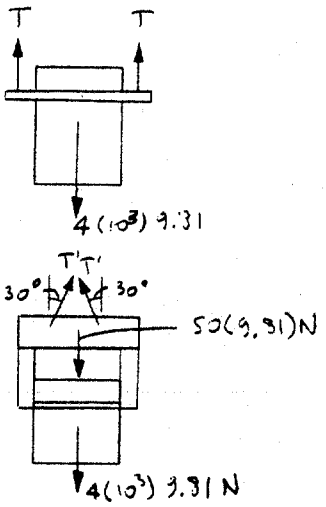
***17-24.** The 4-Mg canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.





Canister :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2T - 4(10^3)(9.81) = 4(10^3)(2)$

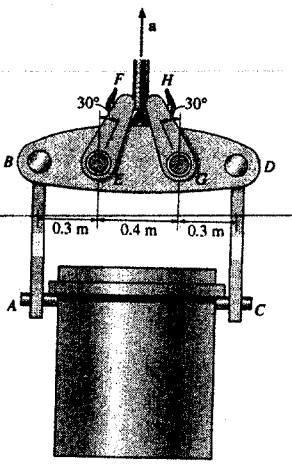
$T_{AB} = T_{CD} = T = 23.6\ \text{kN}$ **Ans**

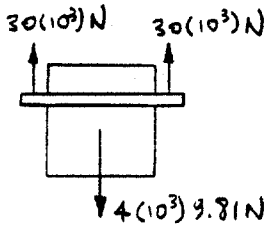
System :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2T'\cos 30° - 4050(9.81) = 4050(2)$

$T_{EF} = T_{GH} = T' = 27.6\ \text{kN}$ **Ans**

17-25. The 4-Mg canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration $\mathbf{a}$ of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.





Canister :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2(30)(10^3) - 4(10^3)(9.81) = 4(10^3)a$

$a = 5.19\ \text{m/s}^2$

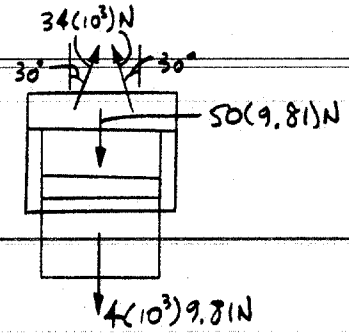
System :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2[34(10^3)\cos 30°] - 4050(9.81) = 4050a$

$a = 4.73\ \text{m/s}^2$

Thus,

$a_{max} = 4.73\ \text{m/s}^2$ **Ans**



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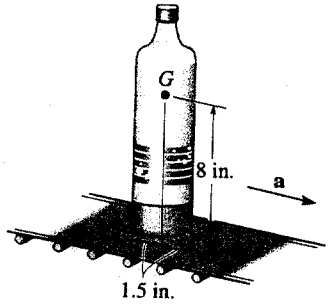
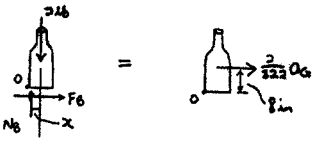
17-26. The 2-lb bottle rests on the check-out conveyor at a grocery store. If the coefficient of static friction is $\mu_s = 0.2$, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at G.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad F_B = \frac{2}{32.2} a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_B - 2 = 0$

$\zeta + \Sigma M_O = \Sigma (M_K)_O;$ $\quad 2x = \frac{2}{32.2} a_G (8)$



Assume bottle is about to slip.

$N_B = 2$ lb, $\quad F_B = 0.2(2) = 0.4$ lb

$a_G = 6.44$ ft/s^2, $\quad x = 1.6$ in. > 1.5 in.

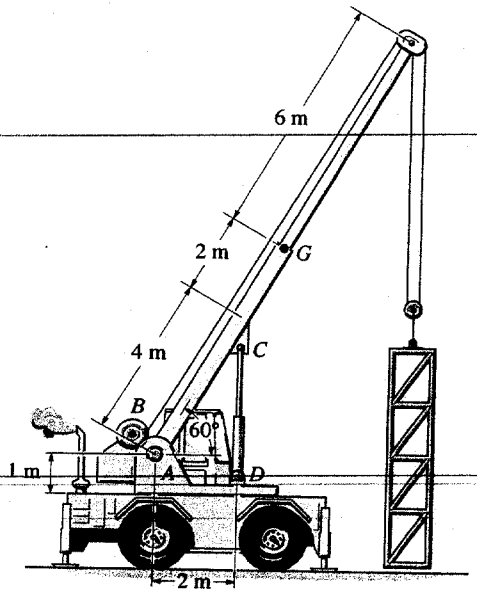
Bottle will tip before slipping.

Set $x = 1.5$ in.

$N_B = 2$ lb, $\quad a_G = 6.04$ ft/s^2 **Ans**

$F_B = 0.375$ lb < 0.4 lb **(O.K!)**

17-27. The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at B draws in the cable with an acceleration of 2 m/s^2, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at G.

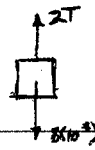


$s_B + 2s_L = l$

$a_B = -2a_L$

$2 = -2a_L$

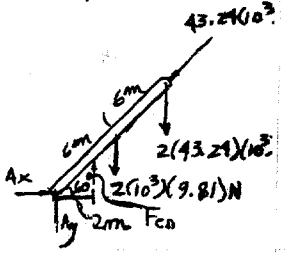
$a_L = -1$ m/s^2



Assembly:

$+\uparrow \Sigma F_y = m a_y;$ $\quad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24$ kN



Boom:

$(+\Sigma M_A = 0;$ $\quad F_{CD}(2) - 2(10^3)(9.81)(6\cos 60°) - 2(43.24)(10^3)(12\cos 60°) = 0$

$F_{CD} = 289$ kN **Ans**

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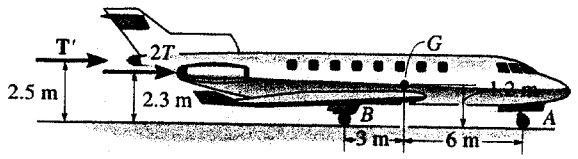
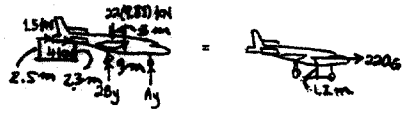
***17-28.** The jet aircraft has a total mass of 22 Mg and a center of mass at G. Initially at take-off the engines provide a thrust $2T = 4$ kN and $T' = 1.5$ kN. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.





$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x$; $1.5 + 4 = 22a_G$

$+\uparrow \Sigma F_y = 0$; $2B_y + A_y - 22(9.81) = 0$

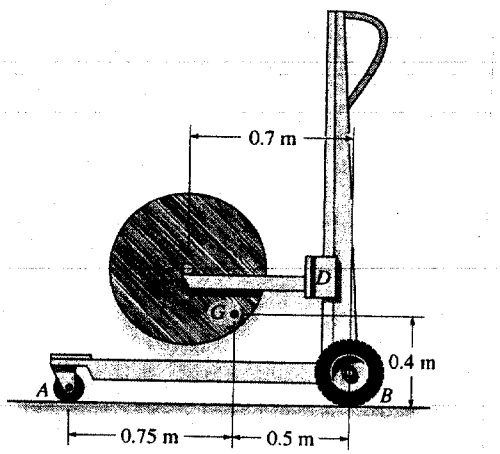
$\zeta + \Sigma M_B = \Sigma (M_K)_B$; $-4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22a_G(1.2)$

$A_y = 72.6$ kN **Ans**

$B_y = 71.6$ kN **Ans**

$a_G = 0.250$ m/s^2 **Ans**

17-29. The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of 3 m/s^2, determine the reactions of each of the four wheels on the ground. The loading is symmetric. Neglect the mass of the movable arm CD.

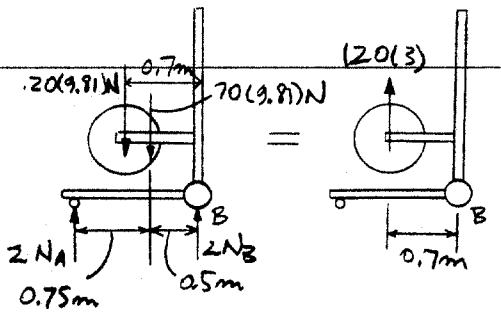


$\zeta + \Sigma M_B = \Sigma (M_k)_B$; $70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25) = -120(3)(0.7)$

$N_A = 567.76$ N $= 568$ N **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y$; $2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$

$N_B = 544$ N **Ans**



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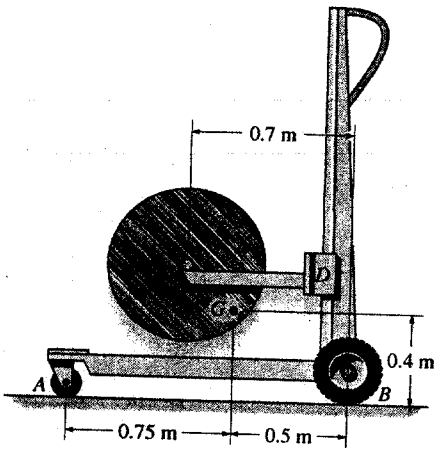
17-30. The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction of the wheels on the ground exceeds 600 N.



Assume $N_A = 600$ N,

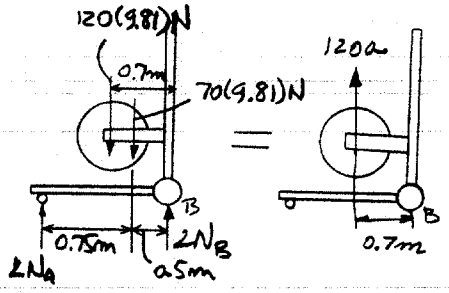
$(+\Sigma M_B = \Sigma (M_k)_B;$ $\quad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$a = 3.960 \text{ m/s}^2$

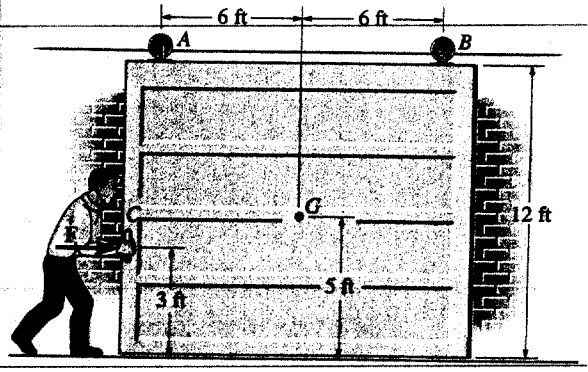
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600\text{N}$ OK

Thus $a = 3.96 \text{ m/s}^2$ **Ans**

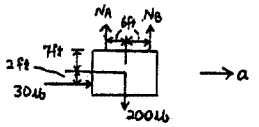


17-31. The door has a weight of 200 lb and a center of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad 30 = (\frac{200}{32.2})a_G$

$a_G = 4.83 \text{ ft/s}^2$



$(+\Sigma M_A = \Sigma (M_k)_A;$ $\quad N_B(12) - 200(6) + 30(9) = (\frac{200}{32.2})(4.83)(7)$

$N_B = 95.0 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A + 95.0 - 200 = 0$

$N_A = 105 \text{ lb}$ **Ans**

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_G t^2$

$s = 0 + 0 + \frac{1}{2}(4.83)(2)^2 = 9.66 \text{ ft}$ **Ans**

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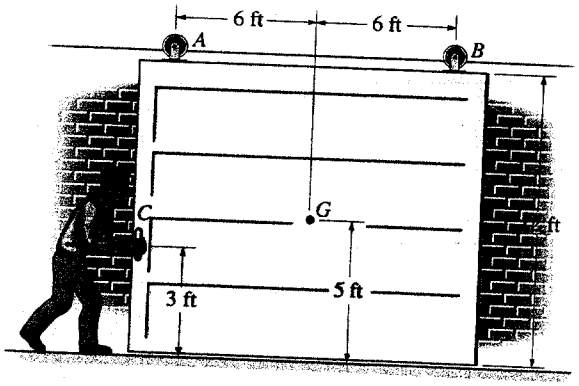
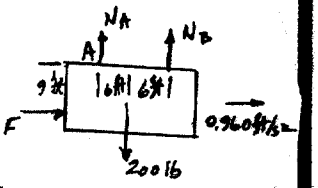
***17-32.** The door has a weight of 200 lb and a center of gravity at G. Determine the constant force F that must be applied to the door to push it open 12 ft to the right in 5 s, starting from rest. Also, find the vertical reactions at the rollers A and B.



$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$12 = 0 + 0 + \frac{1}{2}a_c(5)^2$$

$$a_c = 0.960\ \text{ft/s}^2$$



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\quad F = \frac{200}{32.2}(0.960)$$

$$F = 5.9627\ \text{lb} = 5.96\ \text{lb} \qquad \textbf{Ans}$$

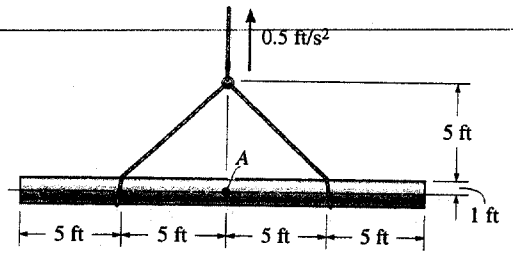
$$+\circlearrowleft \Sigma M_A = \Sigma(M_k)_A;\quad N_B(12) - 200(6) + 5.9627(9) = \frac{200}{32.2}(0.960)(7)$$

$$N_B = 99.0\ \text{lb} \qquad \textbf{Ans}$$

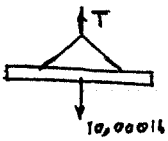
$$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + 99.0 - 200 = 0$$

$$N_A = 101\ \text{lb} \qquad \textbf{Ans}$$

17-33. The uniform pipe has a weight of 500 lb/ft and diameter of 2 ft. If it is hoisted as shown with an acceleration of 0.5 ft/s^2, determine the internal moment at the center A of the pipe due to the lift.



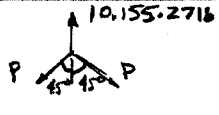
Pipe:



$$+\uparrow \Sigma F_y = m a_y;\quad T - 10\,000 = \frac{10\,000}{32.2}(0.5)$$

$$T = 10\,155.27\ \text{lb}$$

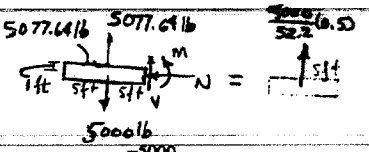
Cables:



$$+\uparrow \Sigma F_y = 0;\quad 10\,155.27 - 2P\cos 45° = 0$$

$$P = 7\,180.867\ \text{lb}$$

$$P_x = P_y = 7\,180.867\left(\frac{1}{\sqrt{2}}\right) = 5\,077.64\ \text{lb}$$



$$+\circlearrowleft \Sigma M_A = \Sigma(M_k)_A;\quad M_A + 5000(5) - 5077.64(5) - 5077.64(1) = \frac{-5000}{32.2}(0.5)(5)$$

$$M_A = 5077.6\ \text{lb·ft} = 5.08(10^3)\ \text{lb·ft} \qquad \textbf{Ans}$$

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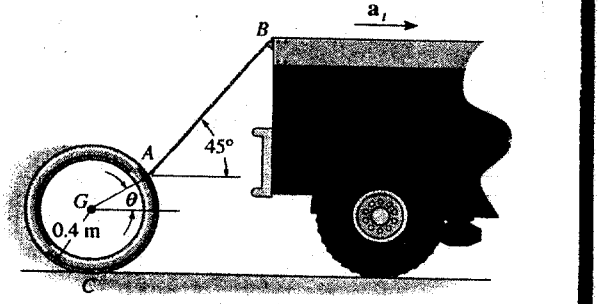
17-34. The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



$\rightarrow \Sigma F_x = ma_x;\quad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y;\quad N_C - 800(9.81) + T\sin 45° = 0$

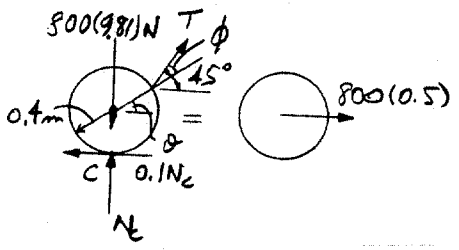
$\circlearrowleft + \Sigma M_G = 0;\quad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$

$N_C = 6770.9\ \text{N}$

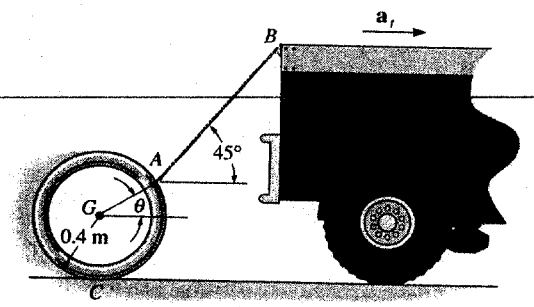
$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans**

$\sin\phi = \dfrac{0.1(6770.9)}{1523.24} \qquad \phi = 26.39°$

$\theta = 45° - \phi = 18.6°$ **Ans**



17-35. The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



$\rightarrow \Sigma F_x = ma_x;\quad T\cos 45° - 0.1N_C = 800a$

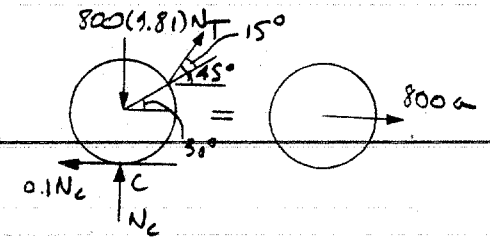
$+\uparrow \Sigma F_y = ma_y;\quad N_C - 800(9.81) + T\sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0;\quad T\sin 15°(0.4) - 0.1N_C(0.4) = 0$

$N_C = 6164\ \text{N}$

$T = 2382\ \text{N} = 2.38\ \text{kN}$ **Ans**

$a = 1.33\ \text{m/s}^2$ **Ans**



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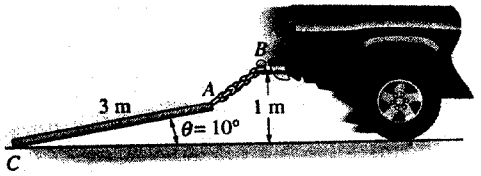
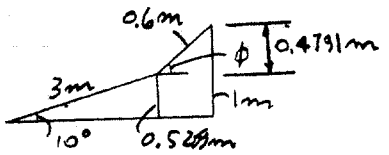
***17-36.** The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is $\mu_k = 0.4$, determine the acceleration of the truck if the angle $\theta = 10°$ with the road as shown.





$$\phi = \sin^{-1}\left(\frac{0.4791}{0.6}\right) = 52.98°$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad T\cos 52.98° - 0.4N_C = 500a_G$$

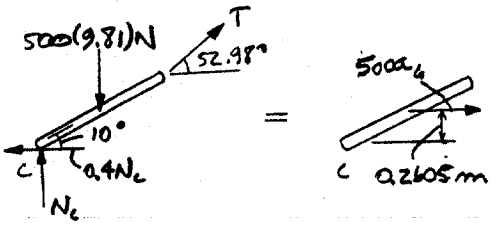
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_C - 500(9.81) + T\sin 52.98° = 0$$

$$\left(+\Sigma M_C = \Sigma (M_k)_C; \quad -500(9.81)(1.5\cos 10°) + T\sin(52.98° - 10°)(3) = -500a_G(0.2605)\right.$$

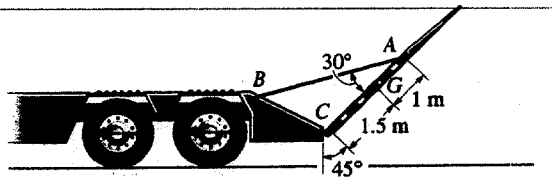
$$T = 3.39 \text{ kN}$$

$$N_C = 2.19 \text{ kN}$$

$$a_G = 2.33 \text{ m/s}^2 \quad \textbf{Ans}$$



17-37. The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at 5 m/s^2. Also, what are the horizontal and vertical components of reaction at the hinge C?

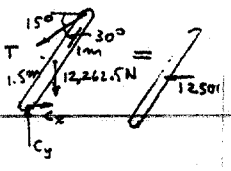


$$\left(+\Sigma M_C = \Sigma (M_k)_C; \quad T\sin 30°(2.5) - 12\,262.5(1.5\cos 45°) = 1250(5)(1.5\sin 45°)\right.$$

$$T = 15\,708.4 \text{ N} = 15.7 \text{ kN} \quad \textbf{Ans}$$

$$\stackrel{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad -C_x + 15\,708.4\cos 15° = 1250(5)$$

$$C_x = 8.92 \text{ kN} \quad \textbf{Ans}$$



$$+\uparrow \Sigma F_y = m(a_G)_y; \quad C_y - 12\,262.5 - 15\,708.4\sin 15° = 0$$

$$C_y = 16.3 \text{ kN} \quad \textbf{Ans}$$

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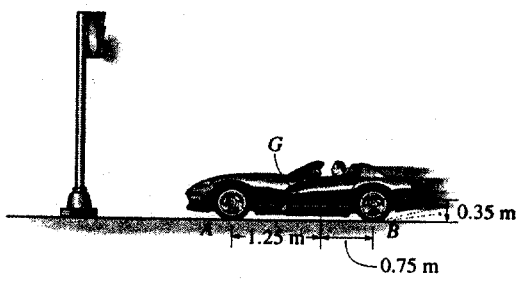
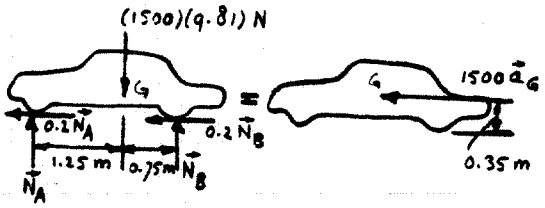
17-38. The sports car has a mass of 1.5 Mg and a center of mass at G. Determine the shortest time it takes for it to reach a speed of 80 km/h, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is $\mu_s = 0.2$. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of 80 km/h?



$\stackrel{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad 0.2N_A + 0.2N_B = 1500a_G \quad (1)$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A + N_B - 1500(9.81) = 0 \quad (2)$

$(+\Sigma M_G = 0;$ $\quad -N_A(1.25) + N_B(0.75) - (0.2N_A + 0.2N_B)(0.35) = 0 \quad (3)$



For rear-wheel drive:

Set the friction force $0.2N_A = 0$ in Eqs. (1) and (3)

Solving yields:

$N_A = 5.18 \text{ kN} > 0$ (OK); $\quad N_B = 9.53 \text{ kN}; \quad a_G = 1.271 \text{ m/s}^2$

Since $v = 80 \text{ km/h} = 22.22 \text{ m/s}$, then

$\left(\stackrel{+}{\rightarrow}\right) \quad v = v_0 + a_G t$

$22.22 = 0 + 1.271t$

$t = 17.5 \text{ s}$ **Ans**

For 4-wheel drive:

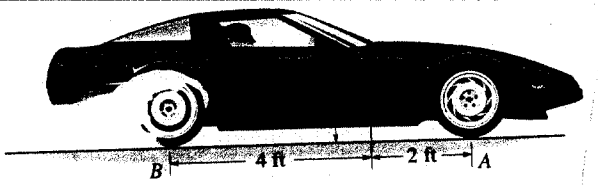
$N_A = 5.00 \text{ kN} > 0$ (OK); $\quad N_B = 9.71 \text{ kN}; \quad a_G = 1.962 \text{ m/s}^2$

Since $v_2 = 80 \text{ km/h} = 22.22 \text{ m/s}$, then

$v_2 = v_1 + a_G t; \quad 22.22 = 0 + 1.962t$

$t = 11.3 \text{ s}$ **Ans**

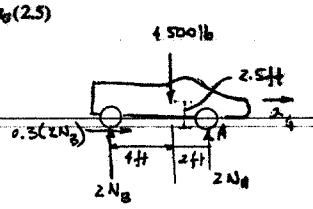
17-39. The sports car has a weight of 4500 lb and center of gravity at G. If it starts from rest it causes the rear wheels to slip as it accelerates. Determine how long it takes for it to reach a speed of 10 ft/s. Also, what are the normal reactions at *each* of the four wheels on the road? The coefficients of static and kinetic friction at the road are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Neglect the mass of the wheels.



$(+\Sigma M_A = \Sigma(M_k)_A; \quad -2N_B(6) + 4500(2) = \frac{-4500}{32.2} a_G(2.5)$

$\stackrel{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.3(2N_B) = \frac{4500}{32.2} a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad 2N_B + 2N_A - 4500 = 0$



Solving,

$N_A = 1393 \text{ lb}$ **Ans**

$N_B = 857 \text{ lb}$ **Ans**

$a_G = 3.68 \text{ ft/s}^2$

$\left(\stackrel{+}{\rightarrow}\right) \quad v = v_0 + a_c t$

$10 = 0 + 3.68t$

$t = 2.72 \text{ s}$ **Ans**

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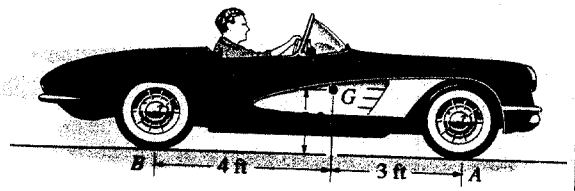
***17-40.** The car accelerates uniformly from rest to 88 ft/s in 15 seconds. If it has a weight of 3800 lb and a center of gravity at G, determine the normal reaction of *each wheel* on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be $\mu_s = 0.4$ and $\mu_k = 0.2$, respectively.

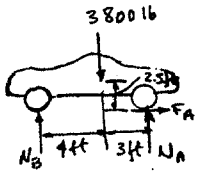


$v = v_0 + a_c t$

$88 = 0 + a_c(15)$

$a_c = 5.867 \text{ ft/s}^2$

Assume no slipping



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad F_A = \frac{3800}{32.2}(5.867)$

$F_A = 692 \text{ lb}$

$\zeta + \Sigma M_B = \Sigma(M_K)_B; \quad 3800(4) - N_A(7) = \frac{3800}{32.2}(5.867)(2.5)$

$N_A = 1924.2 \text{ lb}$

$(F_A)_{max} = 1924.2(0.4) = 770 \text{ lb} > 692 \text{ lb} \quad \text{(O.K!)}$

$+\uparrow \Sigma F_y = 0; \quad N_B - 3800 + 1924.2 = 0$

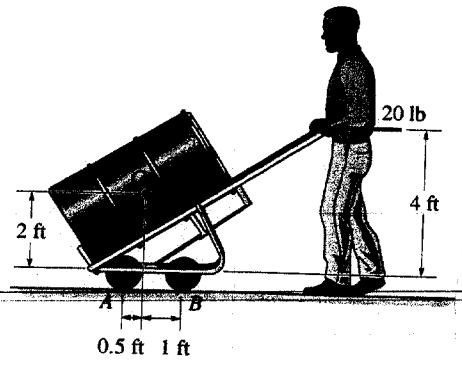
$N_B = 1875.8 \text{ lb}$

Normal reactions are

$N_A' = \frac{1924.2}{2} = 962 \text{ lb}$ Ans

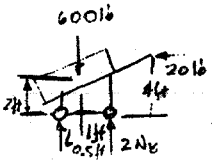
$N_B' = \frac{1875.8}{2} = 938 \text{ lb}$ Ans

17-41. The drum truck supports the 600-lb drum that has a center of gravity at G. If the operator pushes it forward with a horizontal force of 20 lb, determine the acceleration of the truck and the normal reactions at each of the four wheels. Neglect the mass of the wheels.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad 20 = (\frac{600}{32.2})a_G$

$a_G = 1.0733 \text{ ft/s}^2 = 1.07 \text{ ft/s}^2$ Ans



$\zeta + \Sigma M_A = \Sigma(M_K)_A; \quad 20(4) - 600(0.5) + 2N_B(1.5) = \frac{600}{32.2}(1.0733)(2)$

$N_B = 86.7 \text{ lb}$ Ans

$+\uparrow \Sigma F_y = m(a_G)_y; \quad 2N_A + 2(86.7) - 600 = 0$

$N_A = 213 \text{ lb}$ Ans

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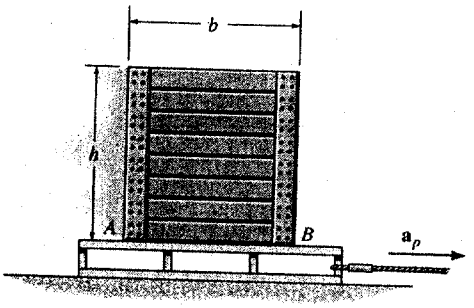
17-42. The uniform crate has a mass m and rests on a rough pallet for which the coefficient of static friction between the crate and pallet is μ_s. If the pallet is given an acceleration of a_p, show that the crate will tip and slip at the same time provided $\mu_s = b/h$.



$$\stackrel{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad \mu_s N_c = ma_p$$

$$(+\Sigma M_A = \Sigma (M_k)_A; \quad mg\left(\frac{b}{2}\right) = ma_p\left(\frac{h}{2}\right)$$

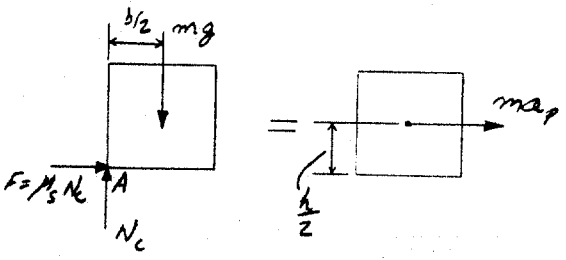
$$a_p = g\left(\frac{b}{h}\right)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_c - mg = 0$$

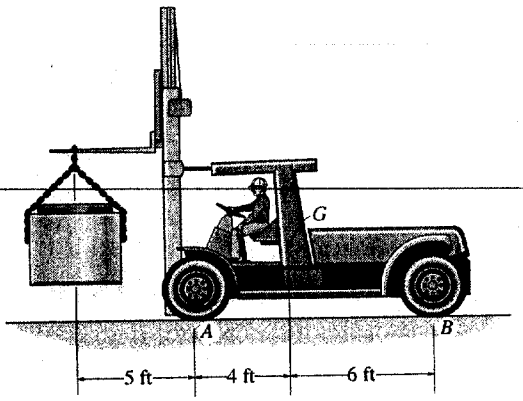
$$N_c = mg$$

$$\mu_s (mg) = m\left(g\frac{b}{h}\right)$$

$$\mu_s = \frac{b}{h} \qquad \textbf{Q.E.D.}$$



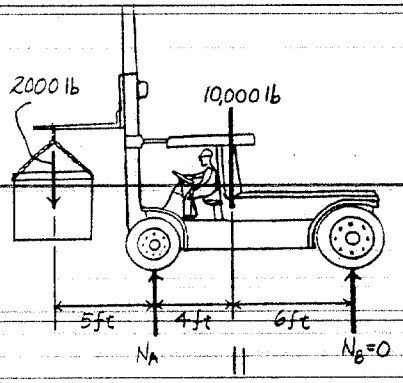
17-43. The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.

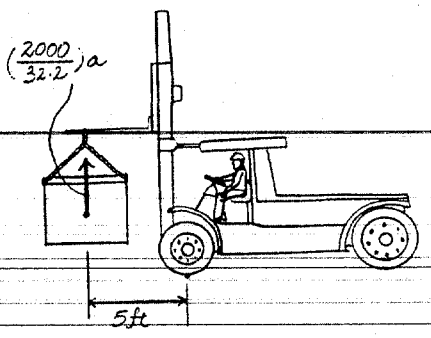


It is required that $N_B = 0$.

$$(+\Sigma M_A = \Sigma (M_k)_A: \quad 2000(5) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)a\right](5)$$

$$a = 96.6 \text{ ft/s}^2 \qquad \textbf{Ans}$$





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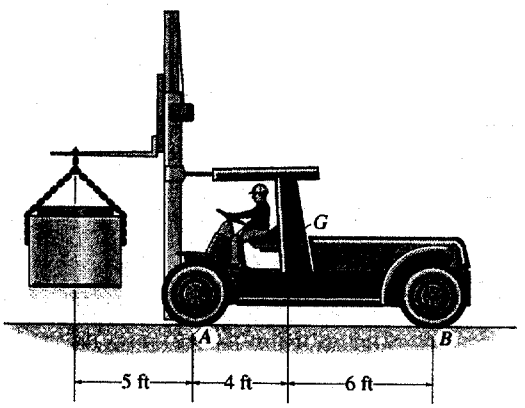
***17-44.** The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the normal reactions on each of its four wheels if the pipe is given an upward acceleration of 4 ft/s^2.

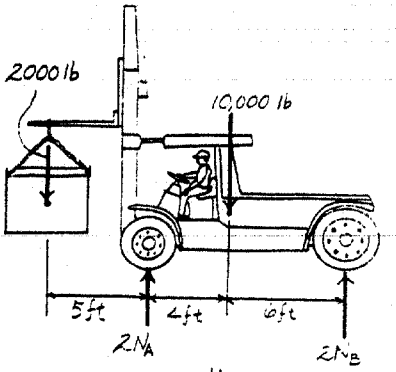


$(+\Sigma M_A = \Sigma(M_k)_A;$ $\quad 2000(5) + 2N_B(10) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)(4)\right](5)$

$N_B = 1437.89 \text{ lb} = 1.44 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 2N_A + 2(1437.89) - 2000 - 10000 = \left(\frac{2000}{32.2}\right)(4)$

$N_A = 4686.34 \text{ lb} = 4.69 \text{ kip}$ **Ans**



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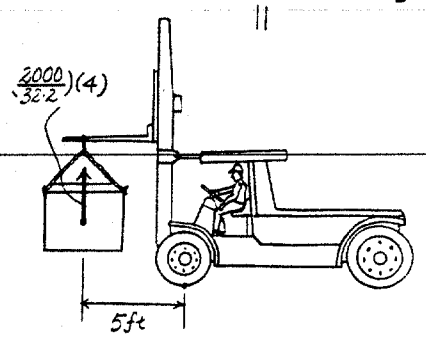

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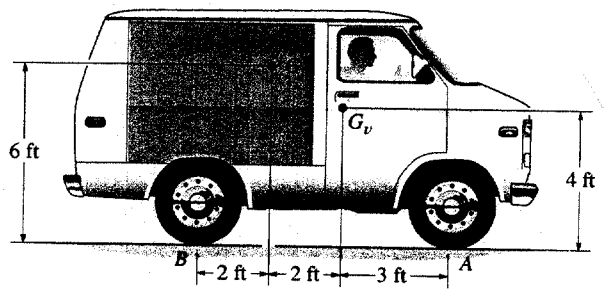
17-45. The van has a weight of 4500 lb and center of gravity at G_v. It carries a fixed 800-lb load which has a center of gravity at G_l. If the van is traveling at 40 ft/s, determine the distance it skids before stopping. The brakes cause *all* the wheels to lock or skid. The coefficient of kinetic friction between the wheels and the pavement is $\mu_k = 0.3$. Compare this distance with that of the van being empty. Neglect the mass of the wheels.



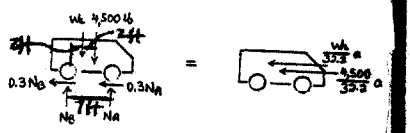
$\overset{+}{\leftarrow} \Sigma F_x = ma_x;$ $\quad 0.3N_B + 0.3N_A = \frac{W_L}{32.2}a + \frac{4500}{32.2}a$ (1)

$+\uparrow \Sigma F_y = ma_y;$ $\quad N_B + N_A - W_L - 4500 = 0$ (2)

Set $W_L = 800$ lb in Eqs. (1) and (2)

$N_A + N_B = 5300$

$a = 9.66 \text{ ft/s}^2$



$(\overset{+}{\rightarrow})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (40)^2 + 2(-9.66)(s - 0)$

$s = 82.8$ ft **Ans**

For empty van $W_L = 0$ in Eqs. (1) and (2)

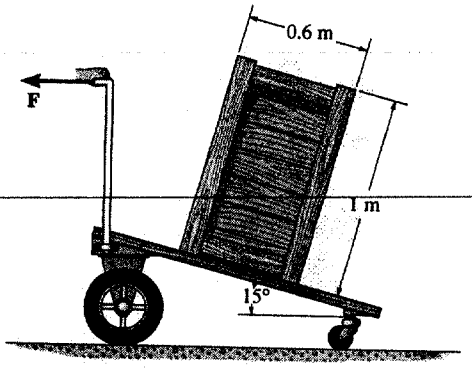
$N_A + N_B = 4500$

$a = 9.66 \text{ ft/s}^2$

Thus,

$s = 82.8$ ft as before **Ans**

17-46. The crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine if the crate will tip over or slide relative to the cart when the cart is subjected to the smallest acceleration necessary to cause one of these relative motions. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



Equation of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A;$ $\quad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5) = 50a\cos 15°(0.5) + 50a\sin 15°(x)$ [1]

$+ \Sigma F_{y'} = m(a_G)_{y'};$ $\quad N - 50(9.81)\cos 15° = -50a\sin 15°$ [2]

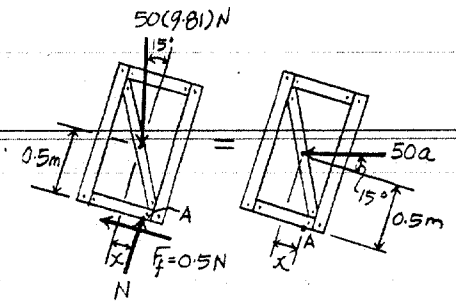
$+ \Sigma F_{x'} = m(a_G)_{x'};$ $\quad 50(9.81)\sin 15° - 0.5N = -50a\cos 15°$ [3]

Solving Eqs. [1], [2] and [3] yields

$N = 447.81$ N $\quad x = 0.250$ m

$a = 2.01 \text{ m/s}^2$ **Ans**

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips.** **Ans**



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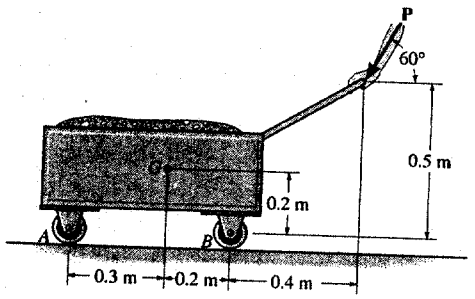
17-47. The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at each of the two wheels at A and the two wheels at B if a force of $P = 50$ N is applied to the handle. Neglect the mass of the wheels.



$\leftarrow \Sigma F_x = m(a_G)_x$; $\quad 50\cos 60° = 200a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A + N_B - 200(9.81) - 50\sin 60° = 0$

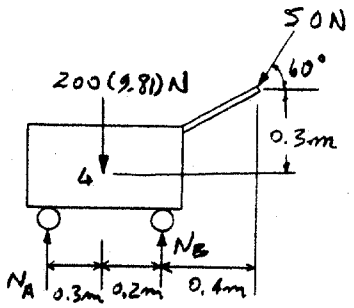
$\circlearrowleft + \Sigma M_G = 0$; $\quad -N_A(0.3) + N_B(0.2) + 50\cos 60°(0.3) - 50\sin 60°(0.6) = 0$

$a_G = 0.125\ \text{m/s}^2$; $\quad N_A = 765.2$ N; $\quad N_B = 1240$ N

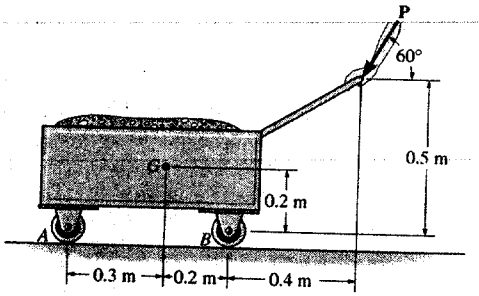
At each wheel,

$N_A' = \frac{N_A}{2} = 383$ N **Ans**

$N_B' = \frac{N_B}{2} = 620$ N **Ans**



***17-48.** The handcart has a mass of 200 kg and center of mass at G. Determine the magnitude of the largest force **P** that can be applied to the handle so that the wheels at A or B continue to maintain contact with the ground. Neglect the mass of the wheels.



$\leftarrow \Sigma F_x = m(a_G)_x$; $\quad P\cos 60° = 200a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A + N_B - 200(9.81) - P\sin 60° = 0$

$\circlearrowleft + \Sigma M_G = 0$; $\quad -N_A(0.3) + N_B(0.2) + P\cos 60°(0.3) - P\sin 60°(0.6) = 0$

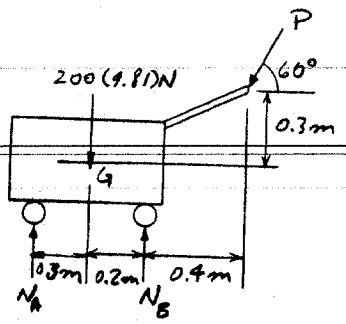
For P_{max}, require

$N_A = 0$

$P = 1998$ N $= 2.00$ kN **Ans**

$N_B = 3692$ N

$a_G = 4.99\ \text{m/s}^2$



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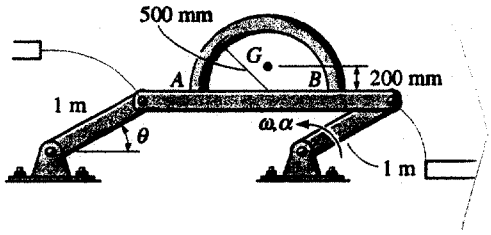
17-49. The arched pipe has a mass of 80 kg and rests on the surface of the platform. As it is hoisted from one level to the next, $\alpha = 0.25\ \text{rad/s}^2$ and $\omega = 0.5\ \text{rad/s}$ at the instant $\theta = 30°$. If it does not slip, determine the normal reactions of the arch on the platform at this instant.



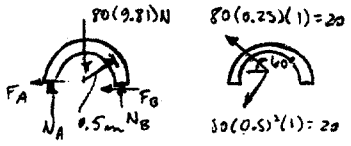
$+\uparrow \Sigma F_y = m(a_G)_y;$ $N_A + N_B - 80(9.81) = 20\sin 60° - 20\cos 60°$

$N_A + N_B = 792.12$

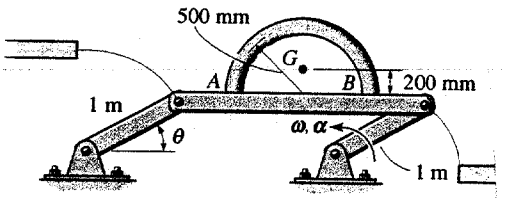
$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;$ $N_B(1) - 80(9.81)(0.5) = 20\cos 60°(0.2) + 20\sin 60°(0.5) - 20\cos 60°(0.5) + 20\sin 60°(0.2)$

$N_B = 402\ \text{N}$ **Ans**

$N_A = 391\ \text{N}$ **Ans**



17-50. The arched pipe has a mass of 80 kg and rests on the surface of the platform for which the coefficient of static friction is $\mu_s = 0.3$. Determine the greatest angular acceleration α of the platform, starting from rest when $\theta = 45°$, without causing the pipe to slip on the platform.



$a_G = (a_G)_t = (1)(\alpha)$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;$ $N_B(1) - 80(9.81)(0.5) = 80(1\alpha)(\sin 45°)(0.2) + 80(1\alpha)(\cos 45°)(0.5)$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $0.3N_A + 0.3N_B = 80(1\alpha)\sin 45°$

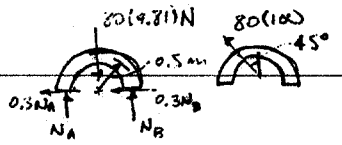
$+\uparrow \Sigma F_y = m(a_G)_y;$ $N_A + N_B - 80(9.81) = 80(1\alpha)\cos 45°$

Solving,

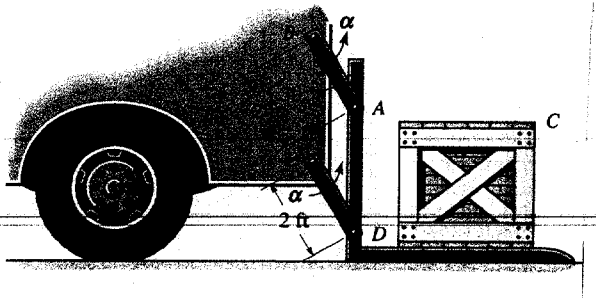
$\alpha = 5.95\ \text{rad/s}^2$ **Ans**

$N_A = 494\ \text{N}$

$N_B = 628\ \text{N}$



17-51. The crate C has a weight of 150 lb and rests on the truck elevator for which the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.



$\overset{+}{\rightarrow} \Sigma F_x = ma_x;$ $0.4N_C = \frac{150}{32.2}(a)\cos 30°$

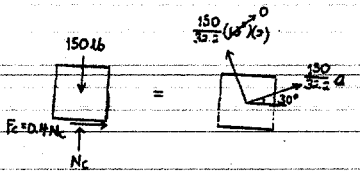
$+\uparrow \Sigma F_y = ma_y;$ $N_C - 150 = \frac{150}{32.2}(a)\sin 30°$

$N_C = 195.0\ \text{lb}$

$a = 19.34\ \text{ft/s}^2$

$19.34 = 2\alpha$

$\alpha = 9.67\ \text{rad/s}^2$ **Ans**



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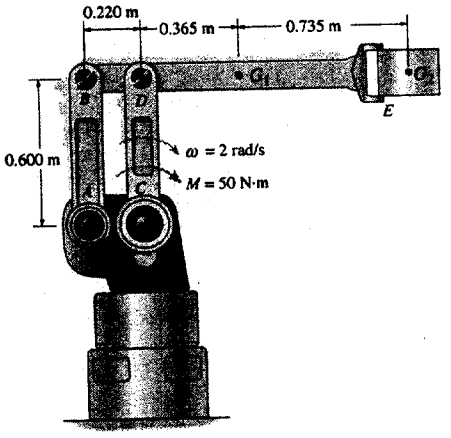
***17-52.** The arm BDE of the industrial robot manufactured by Cincinnati Milacron is activated by applying the torque of $M = 50$ N·m to link CD. Determine the reactions at the pins B and D when the links are in the position shown and have an angular velocity of 2 rad/s. The uniform arm BDE has a mass of 10 kg and a center of mass at G_1. The container held in its grip at E has a mass of 12 kg and center of mass at G_2. Neglect the mass of links AB and CD.



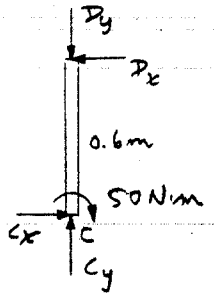
Curvilinear translation :

$(a_D)_n = (a_G)_n = (2)^2(0.6) = 2.4 \text{ m/s}^2$

Member DC :

$\zeta + \Sigma M_C = 0; \quad -D_x(0.6) + 50 = 0$

$D_x = 83.33 \text{ N} = 83.3 \text{ N}$ **Ans**

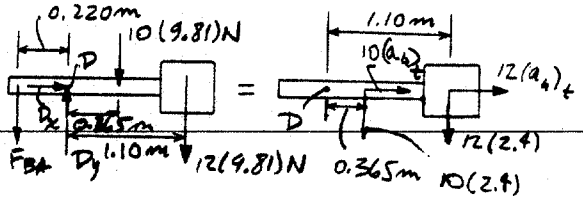


Member BDE :

$\zeta + \Sigma M_D = \Sigma (M_k)_D; \quad -F_{BA}(0.220) + 10(9.81)(0.365) + 12(9.81)(1.10)$

$= 10(2.4)(0.365) + 12(2.4)(1.10)$

$F_{BA} = 567.54 \text{ N} = 568 \text{ N}$ **Ans**



$+\uparrow \Sigma F_y = m(a_G)_y; \quad -567.54 + D_y - 10(9.81) - 12(9.81) = -10(2.4) - 12(2.4)$

$D_y = 731 \text{ N}$ **Ans**

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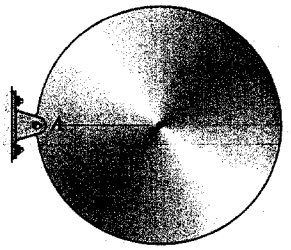
17-53. The 80-kg disk is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.



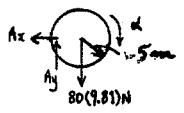
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 80(9.81) = -80(1.5)(\alpha)$

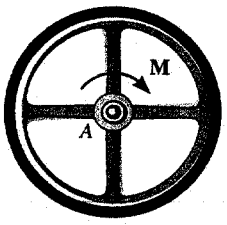
$\curvearrowleft + \Sigma M_A = I_A \alpha;$ $\quad 80(9.81)(1.5) = [\frac{3}{2}(80)(1.5)^2]\alpha$

$\alpha = 4.36 \text{ rad/s}^2$

$A_y = 262 \text{ N}$ **Ans**



17-54. The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 10(9.81) = 0$

$\curvearrowleft + \Sigma M_A = I_A \alpha;$ $\quad 5t = 10(0.2)^2\alpha$

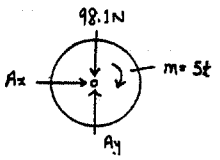
$\alpha = \frac{d\omega}{dt} = 12.5t$

$\omega = \int_0^3 12.5t\, dt = \frac{12.5}{2}(3)^2$

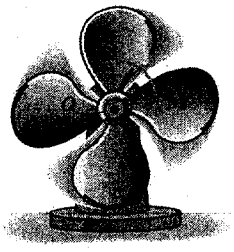
$\omega = 56.2 \text{ rad/s}$ **Ans**

$A_x = 0$ **Ans**

$A_y = 98.1 \text{ N}$ **Ans**



17-55. The fan blade has a mass of 2 kg and a moment of inertia $I_O = 0.18 \text{ kg·m}^2$ about an axis passing through its center O. If it is subjected to a moment of $M = 3(1 - e^{-0.2t})$ N·m, where t is in seconds, determine its angular velocity when $t = 4$ s starting from rest.



$\curvearrowleft + \Sigma M_O = I_O \alpha;$ $\quad 3\left(1 - e^{-0.2t}\right) = 0.18\alpha$

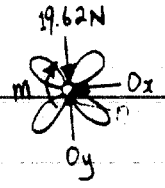
$\alpha = 16.67\left(1 - e^{-0.2t}\right)$

$d\omega = \alpha\, dt$

$\int_0^{\omega} d\omega = \int_0^4 16.67\left(1 - e^{-0.2t}\right) dt$

$\omega = 16.67\left[t + \frac{1}{0.2}e^{-0.2t}\right]_0^4$

$\omega = 20.8 \text{ rad/s}$ **Ans**



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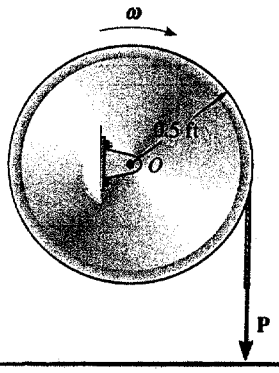
***17-56.** The drum has a weight of 80 lb and a radius of gyration $k_O = 0.4$ ft. If the cable, which is wrapped around the drum, is subjected to a vertical force $P = 15$ lb, determine the time needed to increase the drum's angular velocity from $\omega_1 = 5$ rad/s to $\omega_2 = 25$ rad/s. Neglect the mass of the cable.



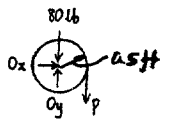
$(\zeta+\Sigma M_O = I_O\alpha;\quad 15(0.5) = [\frac{80}{32.2}(0.4)^2]\alpha \quad (1)$

$\alpha = 18.87\ \text{rad/s}^2$

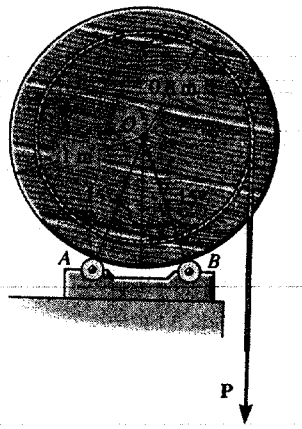
$(\zeta+)\omega = \omega_0 + \alpha t$

$25 = 5 + 18.87t$

$t = 1.06\ \text{s}$ **Ans**



17-57. The spool is supported on small rollers at A and B. Determine the constant force P that must be applied to the cable in order to unwind 8 m of cable in 4 s starting from rest. Also calculate the normal forces at A and B during this time. The spool has a mass of 60 kg and a radius of gyration $k_O = 0.65$ m. For the calculation neglect the mass of the cable and the mass of the rollers at A and B.



$(\downarrow+)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

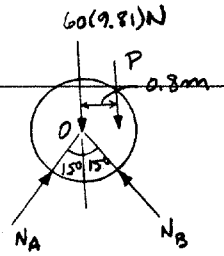
$8 = 0 + 0 + \frac{1}{2}a_c(4)^2$

$a_c = 1\ \text{m/s}^2$

$\alpha = \frac{1}{0.8} = 1.25\ \text{rad/s}^2$

$(+\Sigma M_O = I_O\alpha;\quad P(0.8) = 60(0.65)^2(1.25)$

$P = 39.6\ \text{N}$ **Ans**



$\overset{+}{\rightarrow}\Sigma F_x = ma_x;\quad N_A \sin 15° - N_B \sin 15° = 0$

$+\uparrow \Sigma F_y = ma_y;\quad N_A \cos 15° + N_B \cos 15° - 39.6 - 588.6 = 0$

$N_A = N_B = 325\ \text{N}$ **Ans**

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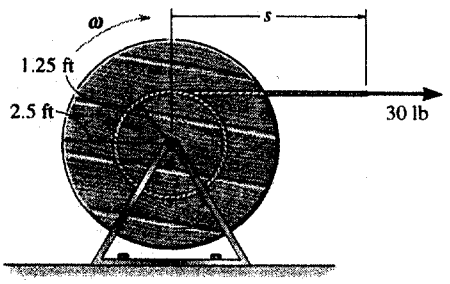
17-58. A cord is wrapped around the inner core of a spool. If the cord is pulled with a constant tension of 30 lb and the spool is originally at rest, determine the spool's angular velocity when $s = 8$ ft of cord has unwound. Neglect the weight of the 8-ft portion of cord. The spool and the entire cord have a total weight of 400 lb, and the radius of gyration about the axle A is $k_A = 1.30$ ft.



$$I_A = mk_A^2 = \left(\frac{400}{32.2}\right)(1.30)^2 = 20.99 \text{ slug} \cdot \text{ft}^2$$

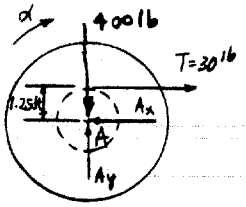
$$\curvearrowleft +\Sigma M_A = I_A\alpha; \quad 30(1.25) = 20.99(\alpha) \quad \alpha = 1.786 \text{ rad/s}^2$$

The angular displacement is $\theta = \frac{s}{r} = \frac{8}{1.25} = 6.4$ rad.

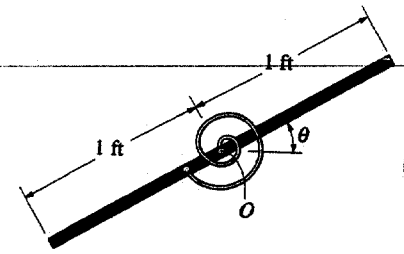
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\omega^2 = 0 + 2(1.786)(6.4 - 0)$$

$\omega = 4.78$ rad/s **Ans**



17-59. The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5$ lb·ft/rad, so that the torque developed is $M = (5\theta)$ lb·ft, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



$$\curvearrowleft +\Sigma M_O = I_O\alpha; \quad -5\theta = [\tfrac{1}{12}(\tfrac{10}{32.2})(2)^2]\alpha$$

$$-48.3\,\theta = \alpha$$

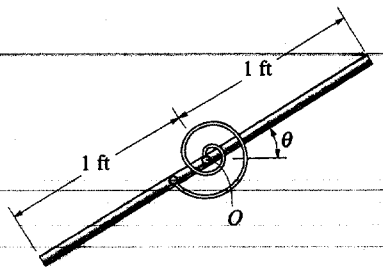
$$\alpha\, d\theta = \omega\, d\omega$$

$$-\int_{\frac{\pi}{2}}^{0} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

$$\frac{48.3}{2}(\tfrac{\pi}{2})^2 = \tfrac{1}{2}\omega^2$$

$\omega = 10.9$ rad/s **Ans**

***17-60.** The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5$ lb·ft/rad, so that the torque developed is $M = (5\theta)$ lb·ft, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



$$\curvearrowleft +\Sigma M_O = I_O\alpha; \quad 5\theta = [\tfrac{1}{12}(\tfrac{10}{32.2})(2)^2]\alpha$$

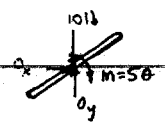
$$\alpha = -48.3\theta$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 48.3\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

$$-24.15\left((\tfrac{\pi}{4})^2 - (\tfrac{\pi}{2})^2\right) = \tfrac{1}{2}\omega^2$$

$\omega = 9.45$ rad/s **Ans**



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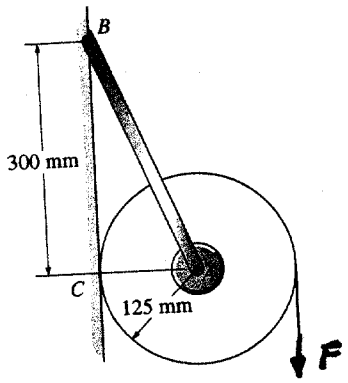
17-61. The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin-supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad N_C - T_{AB}\cos 67.38° = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad T_{AB}\sin 67.38° - 0.2N_C - 20(9.81) - 30 = 0$

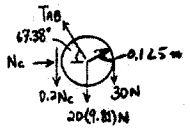
$\zeta + \Sigma M_A = I_A \alpha;$ $\quad -0.2N_C(0.125) + 30(0.125) = 20(0.09)^2\alpha$

Solving:

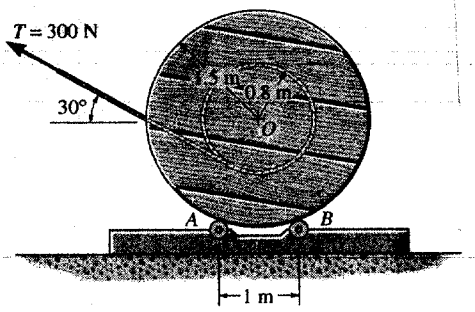
$N_C = 103$ N

$T_{AB} = 267$ N

$\alpha = 7.28$ rad/s^2 Ans



17-62. Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of rollers at A and B. The rollers turn with no friction.



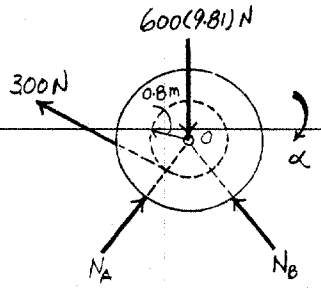
Equation of Motion : The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600(1.2^2) = 864$ kg · m^2. Applying Eq. 17 – 16, we have

$\zeta + \Sigma M_O = I_O \alpha;$ $\quad -300(0.8) = -864\alpha$ $\quad \alpha = 0.2778$ rad/s^2

Kinematic : Here, the angular displacement $\theta = \frac{s}{r} = \frac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$, we have

$(+)$ $\quad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$

$t = 6.71$ s Ans



17-63. The door will close automatically using torsional springs mounted on the hinges. Each spring has a stiffness $k = 50$ N · m/rad so that the torque on each hinge is $M = (50\theta)$ N · m, where θ is measured in radians. If the door is released from rest when it is open at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.

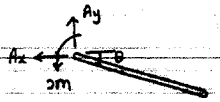
$I_{AB} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(70)(1.2)^2 + 70(0.6)^2 = 33.6$ kg · m^2

$\Sigma M_{AB} = I_{AB}\alpha;$ $\quad 2(50\theta) = -33.6(\alpha)$ $\quad \alpha = -2.976\theta$

$\omega d\omega = \alpha d\theta$

$\int_0^\omega \omega d\omega = -\int_{\frac{\pi}{2}}^0 2.976\theta d\theta$

$\omega = 2.71$ rad/s Ans



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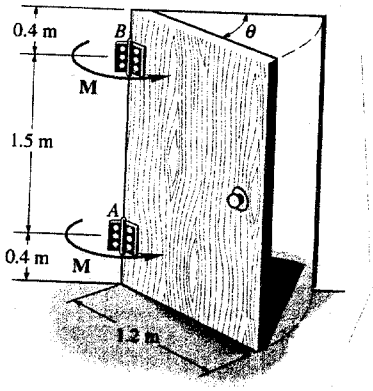
***17-64.** The door will close automatically using torsional springs mounted on the hinges. If the torque on each hinge is $M = k\theta$, where θ is measured in radians, determine the required torsional stiffness k so that the door will close ($\theta = 0°$) with an angular velocity $\omega = 2$ rad/s when it is released from rest at $\theta = 90°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



$$\Sigma M_A = I_A \alpha; \quad 2M = -\left[\frac{1}{12}(70)(1.2)^2 + 70(0.6)^2\right](\alpha)$$

$$M = -16.8\alpha$$

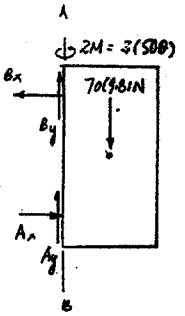
$$k\theta = -16.8\alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

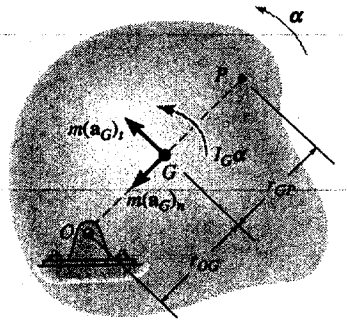
$$-k\int_{\frac{\pi}{2}}^{0} \theta\, d\theta = 16.8\int_0^2 \omega\, d\omega$$

$$\frac{k}{2}\left(\frac{\pi}{2}\right)^2 = \frac{16.8}{2}(2)^2$$

$$k = 27.2 \text{ N·m/rad} \qquad \textbf{Ans}$$



17-65. The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis at O is shown in the figure. Show that $I_G\alpha$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about G. The point P is called the *center of percussion* of the body.

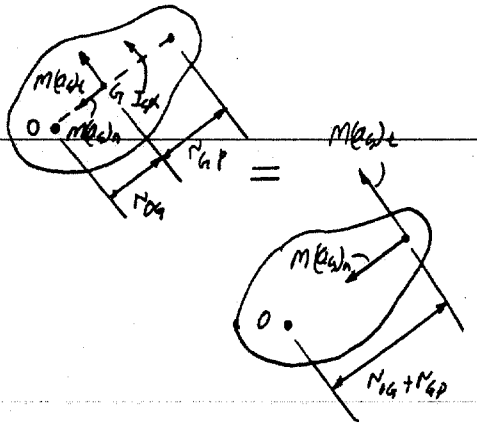


$$m(a_G)_t r_{OG} + I_G\alpha = m(a_G)_t r_{OG} + \left(mk_G^2\right)\alpha$$

However, $k_G^2 = r_{OG}r_{GP}$ and $\alpha = \dfrac{(a_G)_t}{r_{OG}}$

$$m(a_G)_t r_{OG} + I_G\alpha = m(a_G)_t r_{OG} + (mr_{OG}r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t (r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



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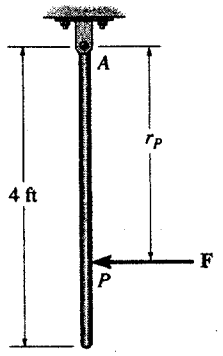
17-66. Determine the position r_P of the center of percussion P of the 10-lb slender bar. (See Prob. 17-65.) What is the horizontal force A_x at the pin when the bar is struck at P with a force of $F = 20$ lb?



Using the result of Prob 17 - 65

$$r_{GP} = \frac{k_G^2}{r_{AG}} = \frac{\left[\sqrt{\frac{1}{12}\left(\frac{ml^2}{m}\right)}\right]^2}{\frac{l}{2}} = \frac{1}{6}l$$

Thus,

$$r_P = \frac{1}{6}l + \frac{1}{2}l = \frac{2}{3}l = \frac{2}{3}(4) = 2.67 \text{ ft} \qquad \textbf{Ans}$$

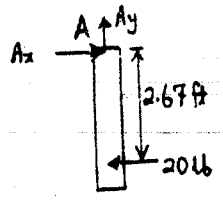
$$\zeta + \Sigma M_A = I_A\alpha; \quad 20(2.667) = \left[\frac{1}{3}\left(\frac{10}{32.2}\right)(4)^2\right]\alpha$$

$$\alpha = 32.2 \text{ rad/s}^2$$

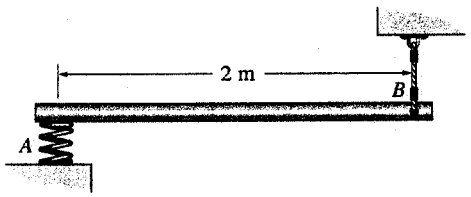
$$(a_G)_t = 2(32.2) = 64.4 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_t; \quad -A_x + 20 = \left(\frac{10}{32.2}\right)(64.4)$$

$$A_x = 0 \qquad \textbf{Ans}$$



17-67. The 4-kg slender rod is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint:* The stiffness of the spring is not needed for the calculation.



Since the deflection of the spring is unchanged at the instant the cord is cut, the reaction at A is

$$F_A = \frac{4}{2}(9.81) = 19.62 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0 = 4(a_G)_x$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 4(9.81) - 19.62 = 4(a_G)_y$$

$$\zeta + \Sigma M_G = I_G\alpha; \qquad (19.62)(1) = \left[\frac{1}{12}(4)(2)^2\right]\alpha$$

Solving :

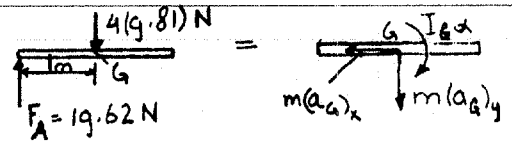
$$(a_G)_x = 0$$

$$(a_G)_y = 4.905 \text{ m/s}^2$$

$$\alpha = 14.7 \text{ rad/s}^2 \qquad \textbf{Ans}$$

Thus,

$$(a_G) = 4.90 \text{ m/s}^2 \qquad \textbf{Ans}$$



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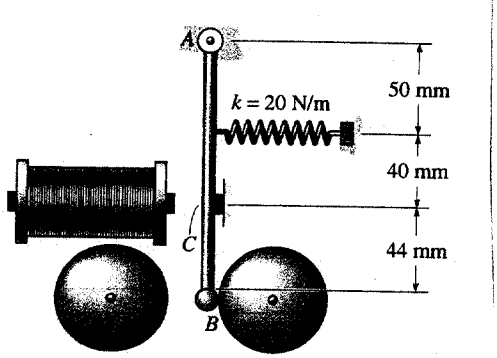
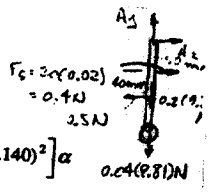
***17-68.** The operation of the doorbell requires the use of the electromagnet, which attracts the iron clapper AB, pinned at end A and consisting of a 0.2-kg slender rod to which is attached the 0.04-kg steel ball having a radius of 6 mm. If the attractive force of the magnet at C is 0.5 N on the center of the ball when the button is pushed, determine the initial angular acceleration of the clapper. The spring is originally stretched 20 mm.



$$\zeta + \Sigma M_A = I_A \alpha; \quad 0.5(0.09) - 0.4(0.05) =$$

$$\left[\frac{1}{3}(0.2)(0.134)^2 + \frac{2}{5}(0.04)(0.006)^2 + 0.04(0.140)^2\right]\alpha$$

$$\alpha = 12.6 \text{ rad/s}^2 \qquad \textbf{Ans}$$



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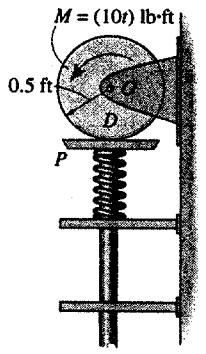
17-69. The 10-lb disk D is subjected to a counterclockwise moment of $M = (10t)$ lb·ft, where t is in seconds. Determine the angular velocity of the disk 2 s after the moment is applied. Due to the spring the plate P exerts a constant force of 100 lb on the disk. The coefficients of static and kinetic friction between the disk and the plate are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. *Hint:* First find the time needed to start the disk rotating.



Determine time required to start disk in motion.

$F = 0.3(100) = 30$ lb

$\zeta + \Sigma M_O = 0; \quad 10t - 30(0.5) = 0$

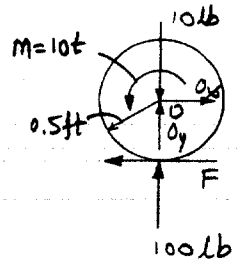
$t = 1.5$ s

Thus,

$F = 0.2(100) = 20$ lb

$\zeta + \Sigma M_O = I_O\alpha; \quad 10t - 20(0.5) = \left[\left(\frac{1}{2}\right)\left(\frac{10}{32.2}\right)(0.5)^2\right]\alpha$

$\alpha = 257.6(t - 1)$



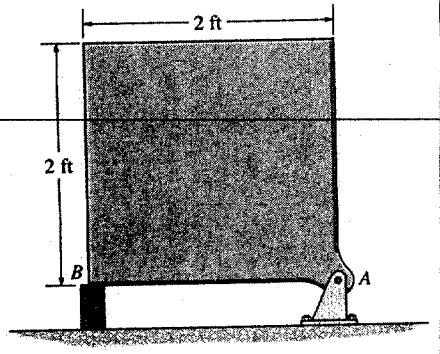
Since $\alpha = \frac{d\omega}{dt}$,

$\int_0^\omega d\omega = \int_{1.5}^2 257.6(t-1)\,dt$

$\omega = 257.6\left(\frac{t^2}{2} - t\right)\Big|_{1.5}^2$

$\omega = 96.6$ rad/s **Ans**

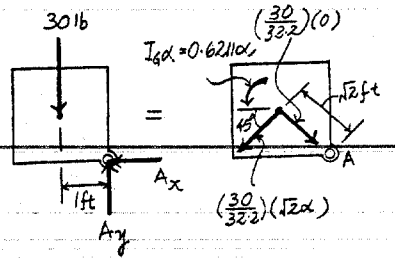
17-70. If the support at B is suddenly removed, determine the initial reactions at the pin A. The plate has a weight of 30 lb.



Equation of Motion : The mass moment inertia of the plate about its mass center is $I_G = \frac{1}{12}m(a^2 + b^2) = \frac{1}{12}\left(\frac{30}{32.2}\right)(2^2 + 2^2) = 0.6211$ slug · ft^2. At the instant shown, the normal component of acceleration of the mass center for the plate $(a_G)_n = 0$ since the angular velocity of the plate $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for the plate $(a_G)_t = \alpha r_G = \sqrt{2}\alpha$.

$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 30(1) = 0.6211\alpha + \left(\frac{30}{32.2}\right)(\sqrt{2}\alpha)(\sqrt{2})$

$\alpha = 12.075$ rad/s^2



$\rightarrow \Sigma F_x = m(a_G)_x; \quad -A_x = -\left(\frac{30}{32.2}\right)[\sqrt{2}(12.075)]\cos 45°$

$A_x = 11.25$ lb

$+\uparrow \Sigma F_y = m(a_G)_y; \quad A_y - 30 = -\left(\frac{30}{32.2}\right)[\sqrt{2}(12.075)]\sin 45°$

$A_y = 18.75$ lb

Thus,

$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{11.25^2 + 18.75^2} = 21.9$ lb **Ans**

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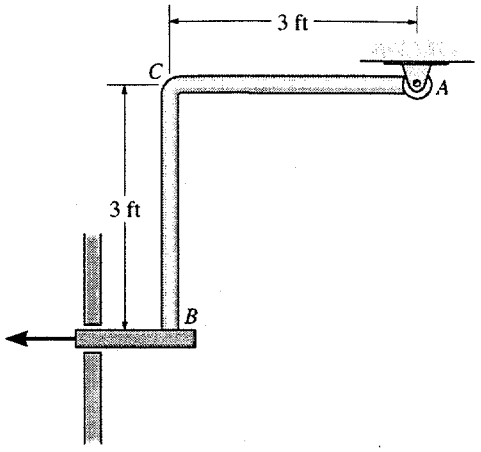
17-71. If the support at B is suddenly removed, determine the initial downward acceleration of point C. Segments AC and CB each have a weight of 10 lb.

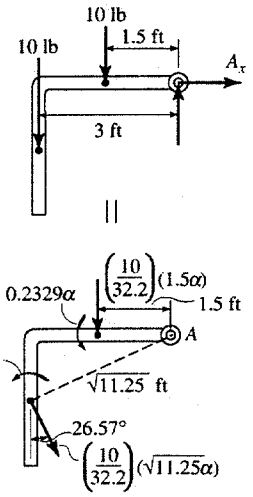


$$I_A = \frac{1}{3}\left(\frac{10}{32.2}\right)(3)^2 + \frac{1}{12}\left(\frac{10}{32.2}\right)(3)^2 + \left(\frac{10}{32.2}\right)(1.5^2 + 3^2)$$

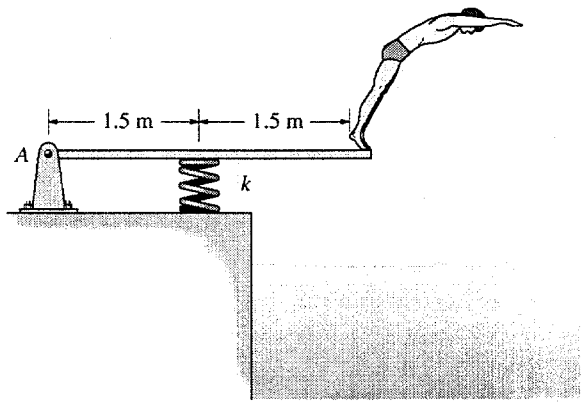
$$= 4.6584 \text{ slng} \cdot \text{ft}^2$$

$$\curvearrowleft + \sum M_A = I_A\alpha; \quad 10(3) + 10(1.5) = 4.6584(\alpha) \quad \alpha = 9.66 \text{ rad/s}^2$$

$$a_C = \alpha r = 9.66(3) = 29.0 \text{ ft/s}^2 \quad \textbf{Ans}$$



***17-72.** Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



$$\curvearrowleft + \sum M_A = I_A\alpha; \quad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \quad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

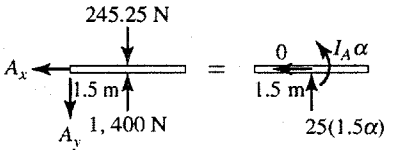
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \quad A_x = 0$$

Solving,

$A_x = 0$ **Ans**

$A_y = 289$ N **Ans**

$\alpha = 23.1$ rad/s^2 **Ans**



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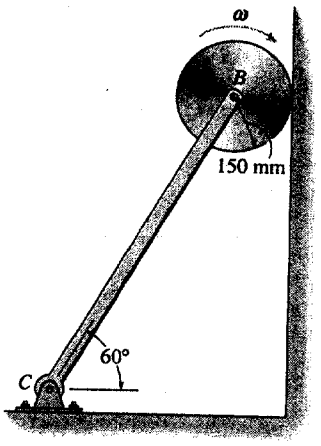
17-73. The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, for which the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad F_{CB}\sin 30° - N_A = 0$

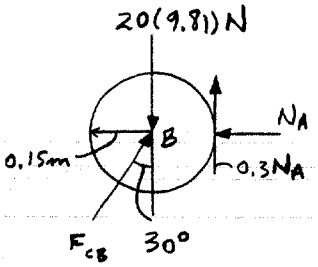
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad F_{CB}\cos 30° - 20(9.81) + 0.3N_A = 0$

$\zeta + \Sigma M_B = I_B\,\alpha;$ $\quad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$N_A = 96.6$ N

$F_{CB} = 193$ N **Ans**

$\alpha = 19.3$ rad/s^2

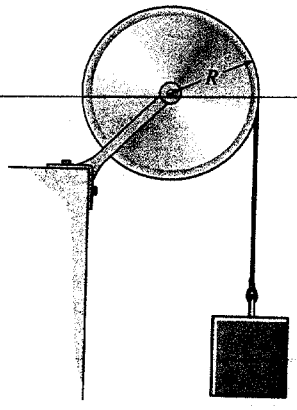


$\zeta + \omega = \omega_0 + \alpha_c t$

$0 = 60 + (-19.3)t$

$t = 3.11$ s **Ans**

17-74. The disk has a mass M and a radius R. If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also, what is the velocity of the block after it falls a distance $2R$ starting from rest?



FBD & KD

$\zeta + \Sigma M_O = \Sigma(M_k)_O;$ $\quad mgR = \frac{1}{2}MR^2(\alpha) + m(\alpha R)R$

$\alpha = \frac{2mg}{R(M+2m)}$ **Ans**

$a = \alpha R$

$v^2 = v_0^2 + 2a(s - s_0)$

$v^2 = 0 + 2\left(\frac{2mgR}{R(M+2m)}\right)(2R - 0)$

$v = \sqrt{\frac{8mgR}{(M+2m)}}$ **Ans**

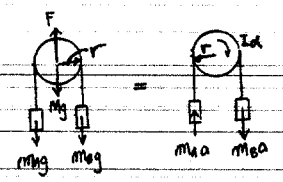
17-75. The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.

$a = \alpha r$

$\zeta + \Sigma M_O = \Sigma(M_O);$ $\quad M_B g(r) - M_A g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + M_B r^2\alpha + M_A r^2\alpha$

$\alpha = \frac{g(M_B - M_A)}{r\left(\frac{1}{2}M + M_B + M_A\right)}$

$a = \frac{g(M_B - M_A)}{\left(\frac{1}{2}M + M_B + M_A\right)}$ **Ans**



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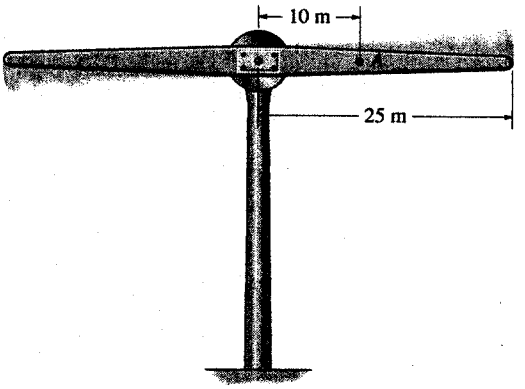
***17-76.** The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and an angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



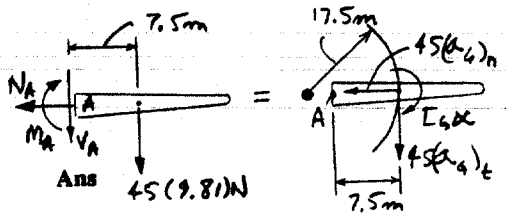
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n$; $N_A = 45(15)^2(17.5) = 177$ kN **Ans**

$+\downarrow \Sigma F_t = m(a_G)_t$; $V_A + 45(9.81) = 45(8)(17.5)$

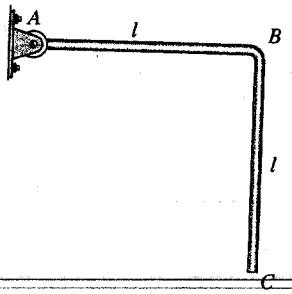
$V_A = 5.86$ kN **Ans**

$\zeta + \Sigma M_A = \Sigma(M_k)_A$; $M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

$M_A = 50.7$ kN · m **Ans**



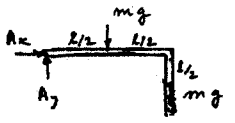
17-77. The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.



Assembly:

$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$

$= 1.667\,ml^2$



$\zeta + \Sigma M_A = I_A\alpha$; $mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$

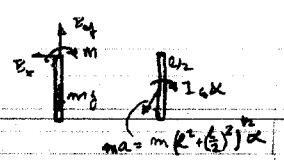
$\alpha = \frac{0.9g}{l}$

Segment BC:

$\zeta + \Sigma M_B = \Sigma(M_k)_B$; $M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\alpha(\frac{l/2}{l^2+(\frac{l}{2})^2})(\frac{l}{2})$

$M = \frac{1}{3}ml^2\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$

$M = 0.3gml$ **Ans**



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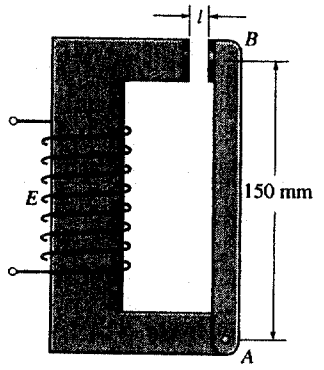
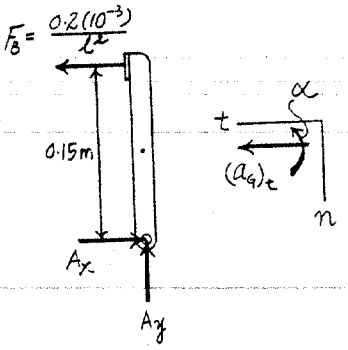
17-78. The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.



Equation of Motion : The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)(0.15^2) + 0.2(0.075^2) = 1.50(10^{-3})$ kg · m^2. Applying Eq. 17 – 16, we have

$$\zeta + \Sigma M_A = I_A \alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50(10^{-3})\alpha$$

$$\alpha = \frac{0.02}{l^2}$$



Kinematic : From the geometry, $l = 0.02 - 0.15\theta$. Then, $dl = -0.15d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

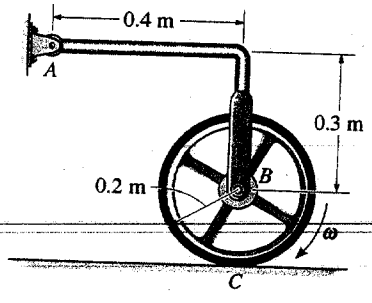
$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

$$v\,dv = -0.15\alpha\,dl$$

$$\int_0^v v\,dv = \int_{0.02\text{m}}^{0.01\text{m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548 \text{ m/s} \qquad \textbf{Ans}$$

17-79. The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega_1 = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.

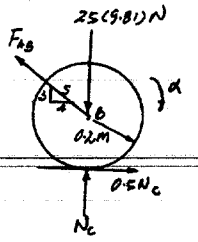


$I_B = mk_B^2 = 25(0.15)^2 = 0.5625 \text{ kg} \cdot \text{m}^2$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad \left(\frac{3}{5}\right)F_{AB} + N_C - 25(9.81) = 0 \qquad [1]$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.5N_C - \left(\frac{4}{5}\right)F_{AB} = 0 \qquad [2]$

$\zeta + \Sigma M_B = I_B\alpha; \quad 0.5N_C(0.2) = 0.5625(-\alpha) \qquad [3]$



Solvings Eqs.[1], [2] and [3] yields :

$F_{AB} = 111.48$ N $\quad N_C = 178.4$ N

$\alpha = -31.71$ rad/s^2

$A_x = \frac{4}{5}F_{AB} = 0.8(111.48) = 89.2$ N **Ans**

$A_y = \frac{3}{5}F_{AB} = 0.6(111.48) = 66.9$ N **Ans**

$\omega = \omega_0 + \alpha_c t$

$0 = 40 + (-31.71)t$

$t = 1.26$ s **Ans**

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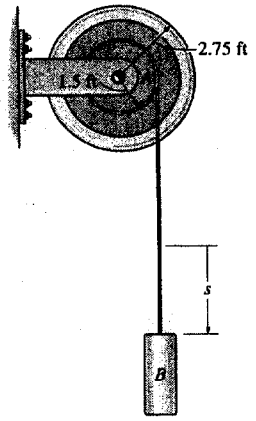
***17-80.** The cord is wrapped around the inner core of the spool. If a 5-lb block B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = 3$ s. Neglect the mass of the cord. The spool has a weight of 180 lb and the radius of gyration about the axle A is $k_A = 1.25$ ft. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.



System :

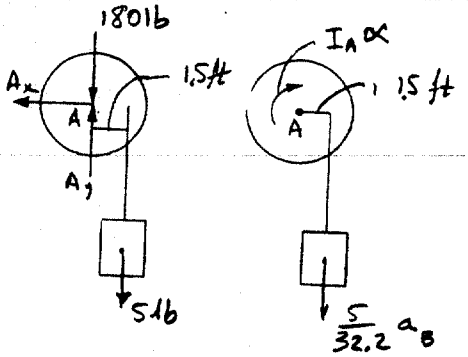
$$(+\circlearrowleft \Sigma M_A = \Sigma (M_k)_A; \quad 5(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha + \left(\frac{5}{32.2}\right)(1.5\alpha)(1.5)$$

$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \quad \omega = \omega_0 + \alpha_c t$$

$$\omega = 0 + (0.8256)(3)$$

$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans}$$



Also,

Spool :

$$(+\circlearrowleft \Sigma M_A = I_A \alpha; \quad T(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha$$

Weight :

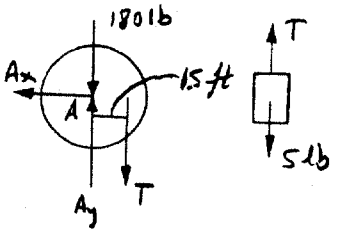
$$+\downarrow \Sigma F_y = m(a_G)_y; \quad 5 - T = \left(\frac{5}{32.2}\right)(1.5\alpha)$$

$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \quad \omega = \omega_0 + \alpha_c t$$

$$\omega = 0 + (0.8256)(3)$$

$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans}$$



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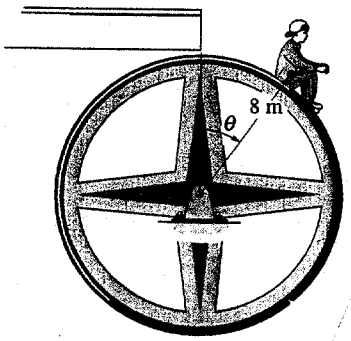
■17-81. A 40-kg boy sits on top of the large wheel which has a mass of 400 kg and a radius of gyration $k_G = 5.5$ m. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is $\mu_s = 0.5$ Neglect the size of the boy in the calculation.



$\zeta + \Sigma M_O = \Sigma (M_k)_O$; $392.4(8\sin\theta) = 400(5.5)^2\alpha + 40(8)(\alpha)(8)$

$0.2141\sin\theta = \alpha$

$\alpha\, d\theta = \omega\, d\omega$

$\int_0^\theta 0.2141 \sin\theta\, d\theta = \int_0^\omega \omega\, d\omega$

$-0.2141\cos\theta\Big|_0^\theta = \frac{1}{2}\omega^2$

$\omega^2 = 0.4283(1-\cos\theta)$

$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}$; $392.4\cos\theta - N = 40(\omega^2)(8)$

$+\nwarrow \Sigma F_{x'} = m(a_G)_{x'}$; $392.4\sin\theta - 0.5N = 40(8)(\alpha)$

$N = 392.4\cos\theta - 137.05(1-\cos\theta) = 529.45\cos\theta - 137.05$

$392.4\sin\theta - 0.5(529.45\cos\theta - 137.05) = 320(0.2141\sin\theta)$

$323.89\sin\theta - 264.73\cos\theta + 68.52 = 0$

$-\sin\theta + 0.8173\cos\theta = 0.2116$

Solve by trial and error

$\theta = 29.8°$ **Ans**

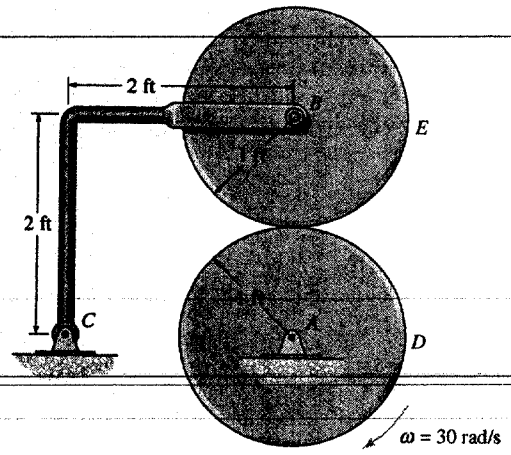
Note: The boy will loose contact with the wheel when $N = 0$, i.e.

$N = 529.45\cos\theta - 137.05 = 0$

$\theta = 75.0° > 29.8°$

Hence slipping occurs first.

17-82. Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.



Equation of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317$ slug·ft^2. Applying Eq. 17 – 16, we have

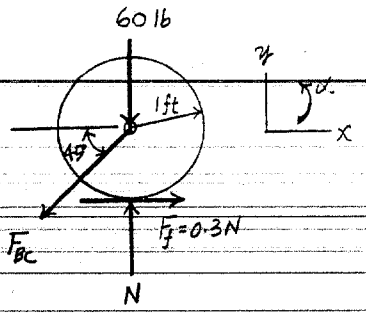
$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x$; $0.3N - F_{BC}\cos 45° = 0$ [1]

$+\uparrow \Sigma F_y = m(a_G)_y$; $N - F_{BC}\sin 45° - 60 = 0$ [2]

$\zeta + \Sigma M_O = I_O\alpha$; $0.3N(1) = 0.9317\alpha$ [3]

Solving Eqs [1], [2] and [3] yields

$F_{BC} = 36.37$ lb $N = 85.71$ lb $\alpha = 27.60$ rad/s^2



Kinematic: Applying equation $\omega = \omega_0 + \alpha t$, we have

$(+)$ $30 = 0 + 27.60t$

$t = 1.09$ s **Ans**

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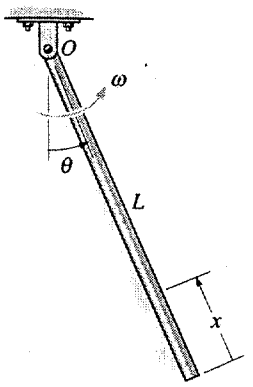
17-83. The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.



$$a = \omega^2\left(L - \frac{x}{2}\right)\measuredangle\theta$$

Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{2}\right)\measuredangle\theta = N\measuredangle\theta + S\angle\theta + wx\downarrow \quad (1)$$

Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

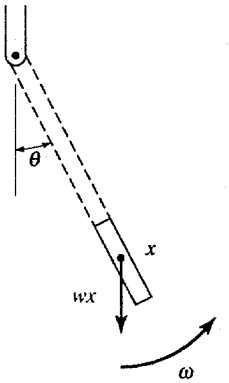
$$O = M - \frac{1}{2}Sx \quad (2)$$

Solving (1) and (2),

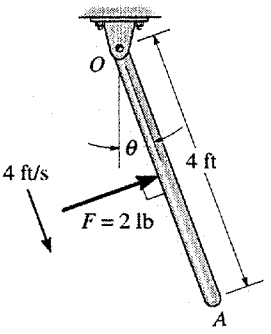
$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right]$$

$$S = wx\sin\theta$$

$$M = \frac{1}{2}wx^2\sin\theta$$



***17-84.** A force $F = 2$ lb is applied perpendicular to the axis of the 5-lb rod and moves from O to A at a constant rate of 4 ft/s. If the rod is at rest when $\theta = 0°$ and **F** is at O when $t = 0$, determine the rod's angular velocity at the instant the force is at A. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane*.



$$I_O = \frac{1}{3}mR^2 = \frac{1}{3}\left(\frac{5}{32.2}\right)(4)^2 = 0.8282 \text{ slug}\cdot\text{ft}^2$$

$$\curvearrowleft + \sum M_O = I_O\alpha; \quad 2(4t) = 0.8282(\alpha)$$

$$\alpha = 9.66t$$

$$d\omega = \alpha\, dt$$

$$\int_0^\omega d\omega = \int_0^t 9.66t\, dt$$

$$\omega = 4.83t^2$$

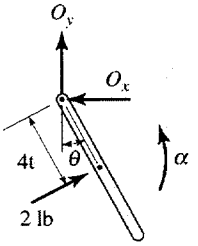
When $t = 1$ s,

$$\omega = 4.83(1)^2 = 4.83 \text{ rad/s} \quad \textbf{Ans}$$

$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t 4.83t^2 dt$$

$$\theta = 1.61 \text{ rad} = 92.2° \quad \textbf{Ans}$$



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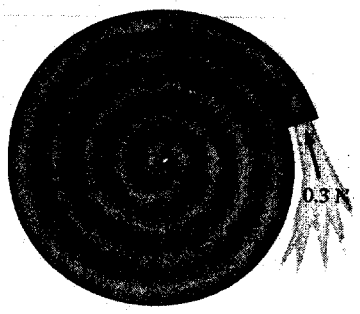
17-85. The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of $r = 75$ mm. For the calculation, consider the wheel to always be a thin disk.



Mass of wheel when 75% of the powder is burned = 0.025 kg

Time to burn off 75% $= \dfrac{0.075\text{ kg}}{0.02\text{ kg/s}} = 3.75\text{ s}$

$m(t) = 0.1 - 0.02t$

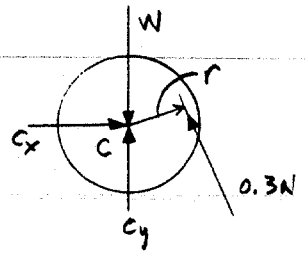
Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\text{ kg}}{\pi(0.075\text{ m})^2} = 5.6588\text{ kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$



$$+\Sigma M_C = I_C\alpha; \quad 0.3r = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$d\omega = \alpha\, dt$

$$\int_0^{\omega} d\omega = 2.530\int_0^t [0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$\omega = 800$ rad/s **Ans**

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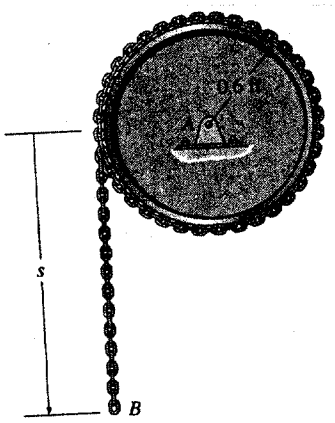
17-86. The drum has a weight of 50 lb and a radius of gyration $k_A = 0.4$ ft. A 35-ft-long chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of $s = 3$ ft is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B has descended $s = 13$ ft. Neglect the thickness of the chain.



$$\left(+\Sigma M_A = \Sigma (M_k)_A\right); \quad 2s(0.6) = \left(\frac{2s}{32.2}\right)[(\alpha)(0.6)](0.6) + \left[\left(\frac{50}{32.2}\right)(0.4)^2 + \frac{2(35-s)}{32.2}(0.6)^2\right]\alpha$$

$$1.2s = 0.02236s\alpha + (0.24845 + 0.7826 - 0.02236s)\alpha$$

$$1.164s = \alpha$$

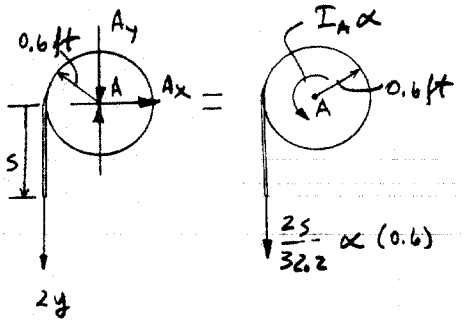
$$\alpha\, d\theta = \alpha\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

$$1.164s\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

$$1.9398\int_3^{13} s\, ds = \int_0^{\omega} \omega\, d\omega$$

$$1.9398\left[\frac{(13)^2}{2} - \frac{(3)^2}{2}\right] = \frac{1}{2}\omega^2$$

$\omega = 17.6$ rad/s **Ans**



17-87. If the disk in Fig. 17-21*a rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, IC, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

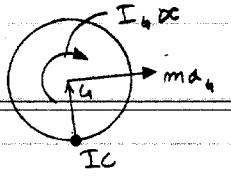
$$\left(+\Sigma M_{IC} = \Sigma (M_k)_{IC}\right); \quad \Sigma M_{IC} = I_G\alpha + (ma_G)r$$

Since there is no slipping, $a_G = \alpha r$

Thus, $\Sigma M_{IC} = (I_G + mr^2)\alpha$

By the parallel-axis thoerem, the term in parenthesis represents I_{IC}. Thus,

$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D.**



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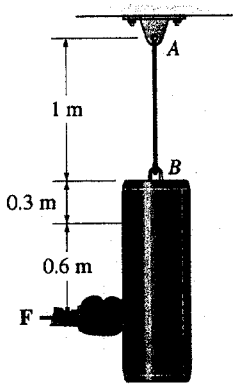
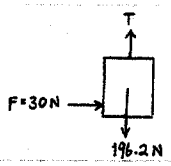
***17-88.** The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha;$ $\quad 30(0.6) = 20(0.4)^2\alpha$

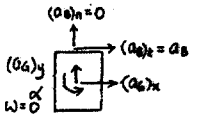


$\alpha = 5.62 \text{ rad/s}^2$ **Ans**

$(a_G)_x = 1.5 \text{ m/s}^2$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B\mathbf{i} = (a_G)_y\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$



$(+\uparrow) \quad (a_G)_y = 0$

Thus,

$T = 196$ N **Ans**

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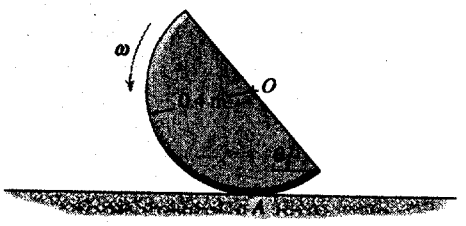
17-89. The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.

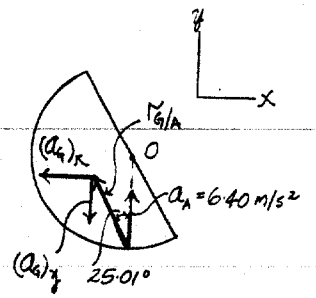


Equation of Motion : The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\left(0.1698^2\right) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m. Also, using law of sines, $\frac{\sin\theta}{0.1698} = \frac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17 – 16, we have

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha + 10(a_G)_x \cos 25.01°(0.3477) + 10(a_G)_y \sin 25.01°(0.3477) \quad [1]$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad F_f = 10(a_G)_x \quad [2]$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 10(9.81) = -10(a_G)_y \quad [3]$$



Kinematics : Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}\ \text{m} = \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16 – 18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$
$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k}\times(-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$
$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (2.3523 - 0.3151\alpha)\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \quad [4]$$
$$(a_G)_y = 0.1470\alpha - 1.3581 \quad [5]$$

Solving Eqs. [1], [2], [3], [4] and [5] yields

$$\alpha = 13.85\ \text{rad/s}^2 \quad (a_G)_x = 2.012\ \text{m/s}^2 \quad (a_G)_y = 0.6779\ \text{m/s}^2$$
$$F_f = 20.12\ \text{N} \quad N = 91.32\ \text{N}$$

Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular disk **does not slip.** **Ans**

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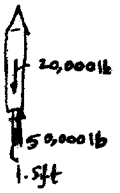
17-90. The rocket has a weight of 20 000 lb, mass center at G, and radius of gyration about the mass center of $k_G = 21$ ft when it is fired. Each of its two engines provides a thrust $T = 50\,000$ lb. At a given instant, engine A suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose B.

$+\Sigma M_G = I_G\alpha;$ $\quad 50\,000(1.5) = \frac{20\,000}{32.2}(21)^2\alpha$

$\alpha = 0.2738 \text{ rad/s}^2 = 0.274 \text{ rad/s}^2$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 50\,000 - 20\,000 = \frac{20\,000}{32.2}a_G$

$a_G = 48.3 \text{ ft/s}^2$



$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

Since $\omega = 0$

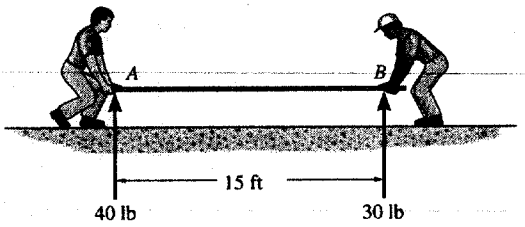
$\mathbf{a}_B = 48.3\mathbf{j} - 0.2738(30)\mathbf{i}$

$= 48.3\mathbf{j} - 8.214\mathbf{i}$

$a_B = \sqrt{(48.3)^2 + (8.214)^2} = 49.0 \text{ ft/s}^2$ **Ans**

$\theta = \tan^{-1}\frac{48.3}{8.214} = 80.3°$ ∠

***17-91.** Two men exert constant vertical forces of 40 lb and 30 lb at the ends A and B of a uniform plank which has a weight of 50 lb. If the plank is originally at rest, determine the acceleration of its center and its angular acceleration. Assume the plank to be a slender rod.



Equation of Motion : The mass moment of inertia of the plank about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{50}{32.2}\right)(15^2) = 29.115 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17 – 14, we have

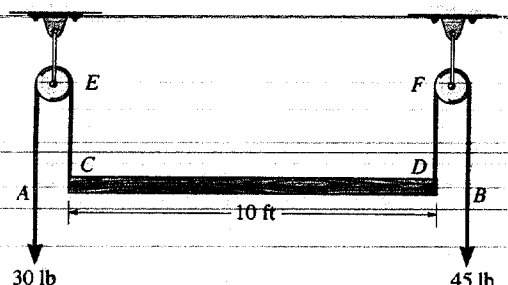
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 40 + 30 - 50 = \left(\frac{50}{32.2}\right)a_G$

$a_G = 12.9 \text{ ft/s}^2$ **Ans**

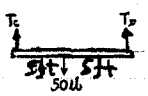
$\circlearrowleft +\Sigma M_G = I_G\alpha;$ $\quad 30(7.5) - 40(7.5) = -29.115\alpha$

$\alpha = 2.58 \text{ rad/s}^2$ **Ans**

***17-92.** The uniform 50-lb board is suspended from cords at C and D. If these cords are subjected to constant forces of 30 lb and 45 lb, respectively, determine the acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.



$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 45 + 30 - 50 = \frac{50}{32.2}a_G$



$a_G = 16.1 \text{ ft/s}^2$ **Ans**

$\circlearrowleft +\Sigma M_G = I_G\alpha;$ $\quad -30(5) + 45(5) = \left[\frac{1}{12}\left(\frac{50}{32.2}\right)(10^2)\right]\alpha$

$\alpha = 5.80 \text{ rad/s}^2$ **Ans**

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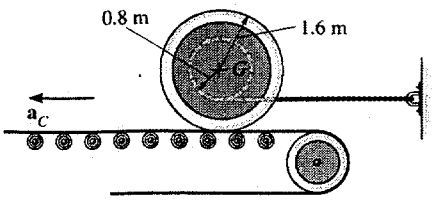
17-93. The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.4$. If the conveyor accelerates at $a_C = 1$ m/s^2, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.



$\overset{+}{\rightarrow}\sum F_x = m(a_G)_x;\quad -F_s + T = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y;\quad N_s - 500(9.81) = 0$

$\curvearrowleft + \sum M_G = I_G\alpha;\quad F_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$\mathbf{a}_P = \mathbf{a}_G + \mathbf{a}_{P/G}$

$(a_P)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$\alpha_G = 0.8\alpha$

$N_s = 4905$ N

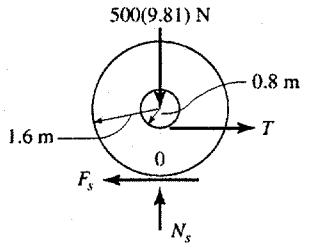
Assume no slipping

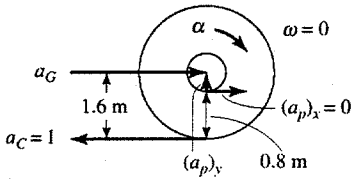
$\alpha = \dfrac{a_C}{0.8} = \dfrac{1}{0.8} = 1.25$ rad/s **Ans**

$a_G = 0.8(1.25) = 1$ m/s^2

$T = 2.32$ kN **Ans**

$F_s = 1.82$ kN



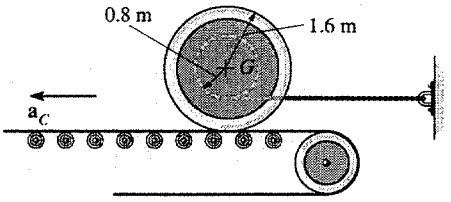


Since

$(F_s)_{max} = 0.5(4.905) = 2.45 > 1.82$

(No slipping occurs)

17-94. The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



$\overset{+}{\rightarrow}\sum F_x = m(a_G)_x;\quad T - 0.5N_s = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y;\quad N_s - 500(9.81) = 0$

$\curvearrowleft + \sum M_G = I_G\alpha,\quad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_P = a_C + a_{P/G}$

$(a_P)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

Solving;

$N_s = 4905$ N

$T = 3.13$ kN **Ans**

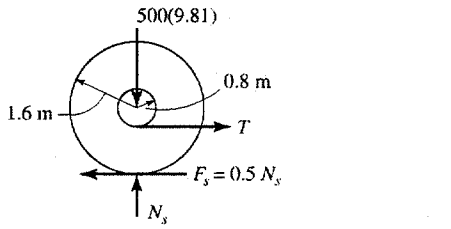
$\alpha = 1.684$ rad/s **Ans**

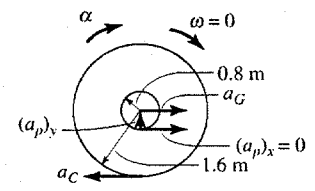
$a_G = 1.347$ m/s^2

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$a_C = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$





$a_C = 1.35$ m/s^2 **Ans**

Also,

$\curvearrowleft + \sum M_{IC} = I_{IC}\alpha;\quad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

Since $N_s = 4905$ N

$\alpha = 1.684$ rad/s

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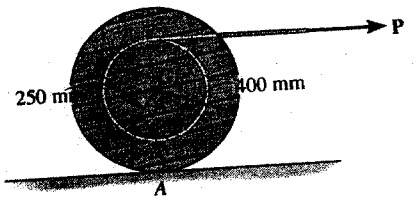
17-95. The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x$; $\quad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A - 100(9.81) = 0$

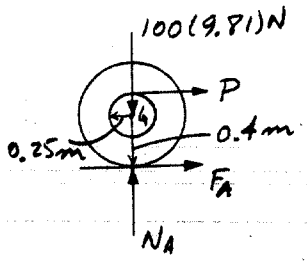
$\zeta + \Sigma M_G = I_G\alpha$; $\quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping : $\quad a_G = 0.4\alpha$

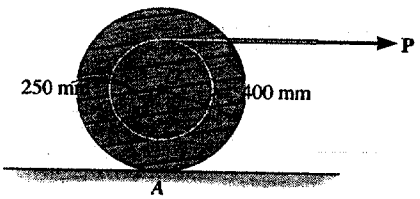
$\alpha = 1.30 \text{ rad/s}^2$ **Ans**

$a_G = 0.520 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**



***17-96.** Solve Prob. 17-95 if the cord and force $P =$ 50 N are directed vertically upwards.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x$; $\quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A + 50 - 100(9.81) = 0$

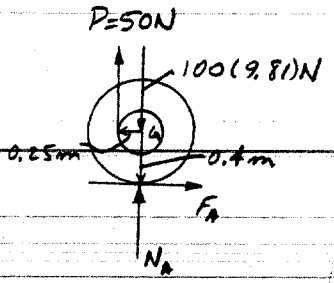
$\zeta + \Sigma M_G = I_G\alpha$; $\quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping : $\quad a_G = 0.4\alpha$

$\alpha = 0.500 \text{ rad/s}^2$ **Ans**

$a_G = 0.2 \text{ m/s}^2 \quad N_A = 931 \text{ N} \quad F_A = 20 \text{ N}$

Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$ **OK**



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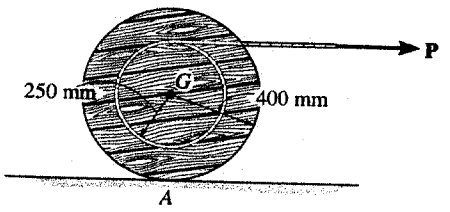
17-97. The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x$; $\quad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A - 100(9.81) = 0$

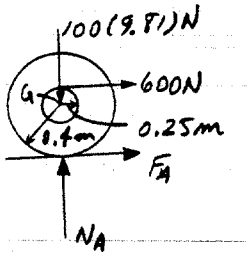
$(+\Sigma M_G = I_G\alpha$; $\quad 600(0.25) - F_A(0.4) = \left[100(0.3)^2\right]\alpha$

Assume no slipping : $\quad a_G = 0.4\alpha$

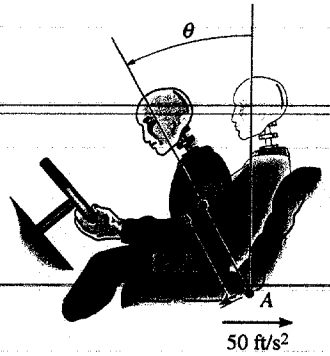
$\alpha = 15.6$ rad/s^2 **Ans**

$a_G = 6.24$ m/s^2 $\quad N_A = 981$ N $\quad F_A = 24.0$ N

Since $(F_A)_{max} = 0.2(981) = 196.2$ N > 24.0 N **OK**



17-98. The upper body of the crash dummy has a mass of 75 lb, a center of gravity at G, and a radius of gyration about G of $k_G = 0.7$ ft. By means of the seat belt this body segment is assumed to be pin-connected to the seat of the car at A. If a crash causes the car to decelerate at 50 ft/s^2, determine the angular velocity of the body when it has rotated to $\theta = 30°$.



$(+\Sigma M_A = \Sigma(M_k)_A$; $\quad 75(1.9\sin\theta) = \left[\left(\frac{75}{32.2}\right)(0.7)^2\right]\alpha + \left[\frac{75}{32.2}(a_G)_t\right](1.9)$

$+\swarrow a_G = a_A + (a_{G/A})_n + (a_{G/A})_t$

$(a_G)_t = -50\cos\theta + 0 + (\alpha)(1.9)$

$142.5\sin\theta = 1.1413\alpha - 221.273\cos\theta + 8.4084\alpha$

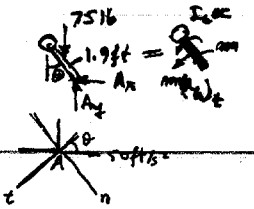
$142.5\sin\theta + 221.273\cos\theta = 9.5497\alpha$

$\omega\, d\omega = \alpha\, d\theta$

$\int_0^{\omega} \omega\, d\omega = \int_0^{30°} (14.922\sin\theta + 23.17\cos\theta)\, d\theta$

$\frac{1}{2}\omega^2 = -14.922(\cos 30° - \cos 0°) + 23.17(\sin 30° - \sin 0°)$

$\omega = 5.21$ rad/s **Ans**



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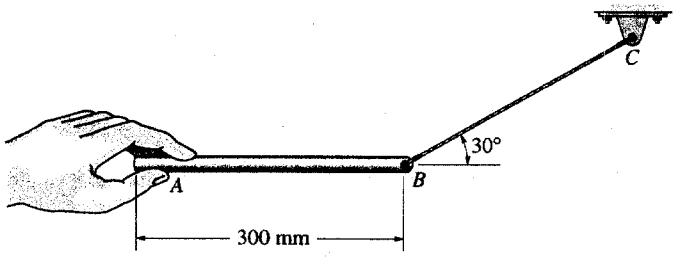
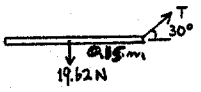
17-99. The 2-kg slender bar is supported by cord BC and then released from rest at A. Determine the initial angular acceleration of the bar and the tension in the cord.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad T\cos 30° = 2(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad T\sin 30° - 19.62 = 2(a_G)_y$



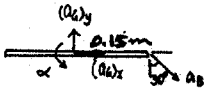
$(+\Sigma M_G = I_G\alpha; \quad T\sin 30°(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B \sin 30°\mathbf{i} - a_B \cos 30°\mathbf{j} = (a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} + \alpha(0.15)\mathbf{j}$

$(\overset{+}{\rightarrow}) \quad (a_B)\sin 30° = (a_G)_x$

$(+\uparrow) \quad (a_B)\cos 30° = -(a_G)_y - \alpha(0.15)$



Thus,

$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$

$T = 5.61$ N Ans

$(a_G)_x = 2.43$ m/s^2

$(a_G)_y = -8.41$ m/s^2

$\alpha = 28.0$ rad/s^2 Ans

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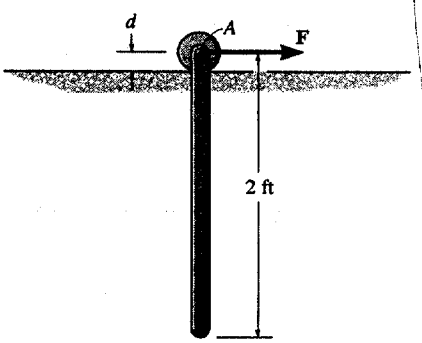
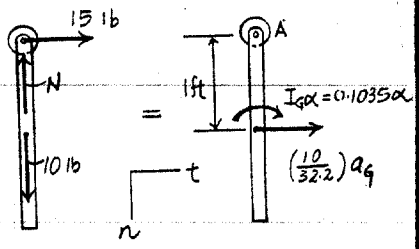
***17-100.** A uniform rod having a weight of 10 lb is pin-supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.



Equation of Motion : The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035\ \text{slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17 – 16, we have

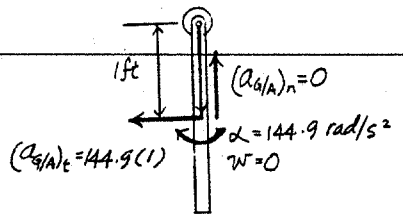


$$\Sigma F_t = m(a_G)_t; \qquad 15 = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 48.3\ \text{ft/s}^2$$

$$\left(+\Sigma M_A = \Sigma (M_k)_A\right); \qquad 0 = \left(\frac{10}{32.2}\right)(48.3)(1) - 0.1035\alpha$$

$$\alpha = 144.9\ \text{rad/s}^2$$

Kinematic : Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16 – 17, we have



$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\left[\underset{\rightarrow}{48.3}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{144.9(1)}\right] + \left[0\right]$$

$$\left(\overset{+}{\rightarrow}\right) \qquad 48.3 = a_A - 144.9$$

$$a_A = 193\ \text{ft/s}^2 \qquad \textbf{Ans}$$

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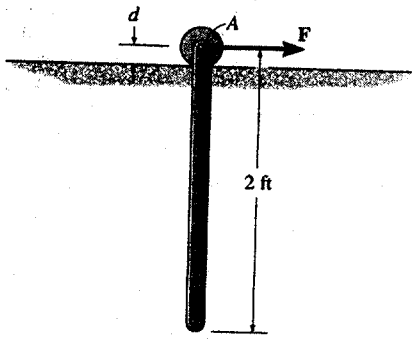
17-101. Solve Prob. 17–100 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



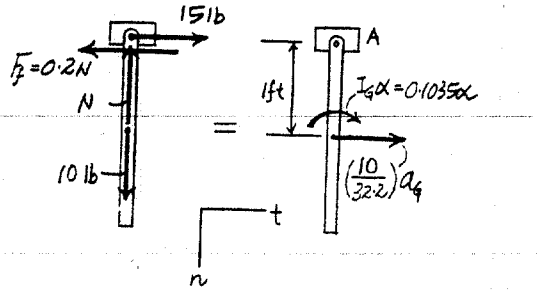
Equation of Motion : The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug} \cdot \text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17 – 16, we have

$$\Sigma F_n = m(a_G)_n; \qquad 10 - N = 0 \qquad N = 10.0 \text{ lb}$$

$$\Sigma F_t = m(a_G)_t; \qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 41.86 \text{ ft/s}^2$$

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$$

$$\alpha = 125.58 \text{ rad/s}^2$$



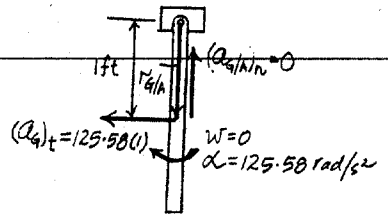
Kinematic : Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16 – 17, we have

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\left[\underset{\rightarrow}{41.86}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{125.58(1)}\right] + \left[0\right]$$

$$(\overset{+}{\rightarrow}) \qquad 41.86 = a_A - 125.58$$

$$a_A = 167 \text{ ft/s}^2 \qquad \textbf{Ans}$$



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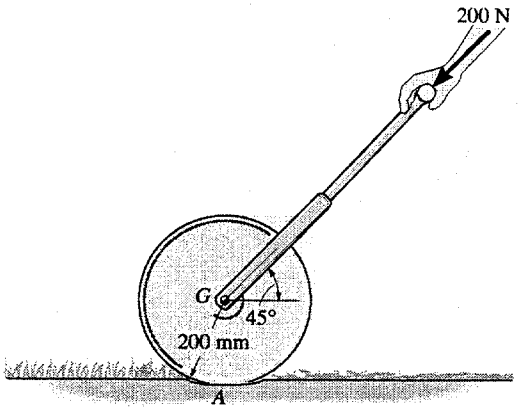
17-102. The lawn roller has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If it is pushed forward with a force of 200 N when the handle is at 45°, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.1$, respectively.



Assume no slipping.

$$I_A = I_G + md^2 = 80(0.175)^2 + 80(0.200)^2 = 5.65 \text{ kg} \cdot \text{m}^2$$

$$M_A = 200\cos 45°(0.200) = 28.284 \text{ N} \cdot \text{m}$$

$$M_A = I_A\alpha$$

$$\alpha = \frac{28.284}{5.65} = 5.01 \text{ rad/s}^2 \quad \textbf{Ans}$$

Check no slippage assumption.

$$(\uparrow +) \sum F_y = 0 \quad -200\sin 45° - 80(9.81) + N = 0$$

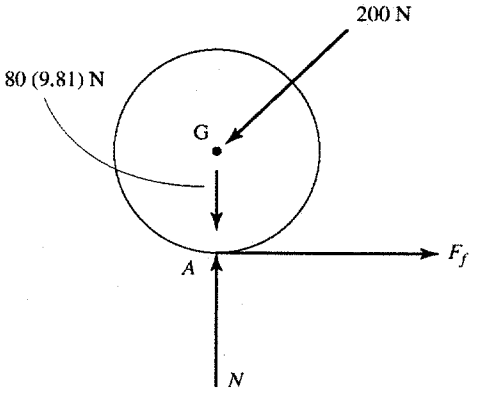
$$N = 926.22 \text{ N}$$

$$(F_f)_{max} = 0.12(926.22) = 111.15 \text{ N}$$

$$I_G = 80(0.175)^2 = 2.45 \text{ kg} \cdot \text{m}^2$$

$$M_G = I_G\alpha = 2.45(5.01) = 12.27 \text{ N} \cdot \text{m}$$

$$F_f = \frac{12.27}{0.200} = 61.4 \text{ N} < 111.15 \text{ N} \quad \textbf{O.K.}$$



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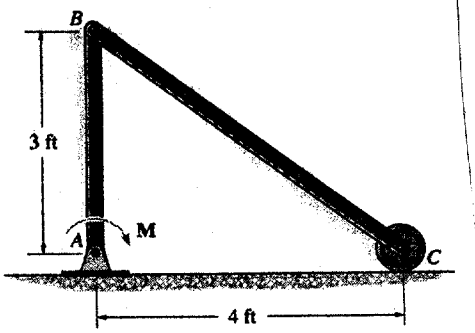
17-103. The two pin-connected bars each have a weight of 10 lb/ft. If a moment of $M = 60$ lb·ft is applied to bar AB, determine the initial vertical reaction at C and the horizontal and vertical components of reaction at B. Neglect the size of the roller at C. The bars are initially at rest.



Equation of Motion : The mass moment of inertia of rod AB and BC about their mass center are given by $(I_G)_{AB} = \frac{1}{12}\left(\frac{30}{32.2}\right)(3^2) = 0.6988 \text{ slug}\cdot\text{ft}^2$ and $(I_G)_{BC} = \frac{1}{12}\left(\frac{50}{32.2}\right)(5^2) = 3.2350 \text{ slug}\cdot\text{ft}^2$. At the instant moment M is applied, the angular velocity of the rod AB, $\omega_{AB} = 0$. Thus, the normal component of acceleration of the mass center for the rod AB $\left[(a_G)_{AB}\right]_n = 0$. Since rod AB is rotating about fixed point A, $(a_G)_{AB} = \left[(a_G)_{AB}\right]_t = \alpha_{AB} r_{AG} = 1.5\alpha_{AB}$. Applying Eq. 17 – 16 to FBD(a), we have

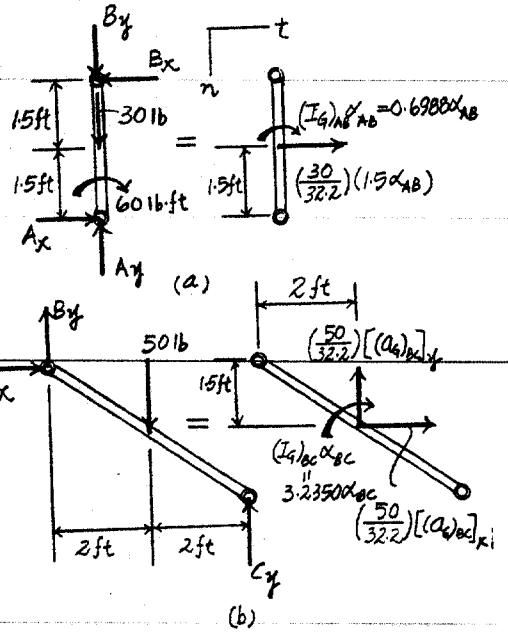
$$+\Sigma M_A = \Sigma (M_k)_A; \quad B_x(3) - 60 = -0.6988\alpha_{AB} - \left(\frac{30}{32.2}\right)(1.5\alpha_{AB})(1.5) \qquad [1]$$

Since rod AB is rotating about fixed point A and $\omega_{AB} = 0$, the normal and tangential components of acceleration of point B are given by $(a_B)_n = \omega_{AB}^2 r_{AB} = 0$ and $(a_B)_t = \alpha_{AB} r_{AB} = 3\alpha_{AB}$. Applying Eq. 17 – 16 to FBD(b), we have

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad B_x = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_x \qquad [2]$$

$$+\uparrow \Sigma F_y = m(a_G)_t; \quad C_y + B_y - 50 = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_y \qquad [3]$$

$$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B; \quad C_y(4) - 50(2) = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_x (1.5) + \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_y (2) - 3.2350\alpha_{BC} \qquad [4]$$



Kinematic : It can be proven that $\omega_{BC} = 0$. Analyzing the motion of points B and C by applying Eq. 16 – 18 with $\mathbf{r}_{C/B} = \{-4\mathbf{i} - 3\mathbf{j}\}$ ft, we have

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$
$$a_C\mathbf{i} = 3\alpha_{AB}\mathbf{i} + (-\alpha_{BC}\mathbf{k}) \times (-4\mathbf{i} - 3\mathbf{j}) - 0$$
$$a_C\mathbf{i} = (3\alpha_{AB} - 3\alpha_{BC})\mathbf{i} + 4\alpha_{BC}\mathbf{j}$$

Equating j components, we have

$$\alpha_{BC} = 0$$

Analyzing the motion of points B and G by applying Eq. 16 – 18 with $\mathbf{r}_{G/B} = \{-2\mathbf{i} - 1.5\mathbf{j}\}$ ft, we have

$$\mathbf{a}_G = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B}$$
$$\left[(a_G)_{BC}\right]_x \mathbf{i} + \left[(a_G)_{BC}\right]_y \mathbf{j} = 3\alpha_{AB}\mathbf{i} + 0\mathbf{k} \times (-2\mathbf{i} - 1.5\mathbf{j}) - 0$$
$$\left[(a_G)_{BC}\right]_x \mathbf{i} + \left[(a_G)_{BC}\right]_y \mathbf{j} = 3\alpha_{AB}\mathbf{i}$$

Equating i and j components, we have

$$\left[(a_G)_{BC}\right]_x = 3\alpha_{AB} \qquad \left[(a_G)_{BC}\right]_y = 0$$

Substitute the results obtained above into Eqs. [1], [2], [3] and [4] and solve yields

$$B_x = 16.7 \text{ lb} \quad B_y = 18.75 \text{ lb} \quad C_y = 31.25 \text{ lb} \quad \textbf{Ans}$$
$$\alpha_{AB} = 3.578 \text{ rad/s}^2$$

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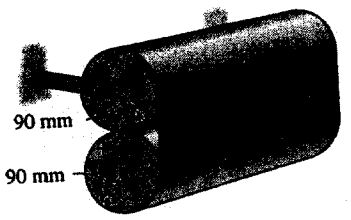
***17-104.** A long strip of paper is wrapped into two rolls, each having a mass of 8 kg. Roll A is pin supported about its center whereas roll B is not centrally supported. If B is brought into contact with A and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.



For roll A.

$$(+\Sigma M_A = I_A\alpha; \quad T(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_A \qquad (1)$$

For roll B

$$(+\Sigma M_O = \Sigma(M_k)_O; \quad 8(9.81)(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_B + 8a_B(0.09) \qquad (2)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad T - 8(9.81) = -8a_B \qquad (3)$$

Kinematics:

$$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_t + (\mathbf{a}_{B/O})_n$$

$$\left[a_B\right] = \left[a_O\right] + \left[\alpha_B(0.09)\right] + [0]$$

$$(+\downarrow) \qquad a_B = a_O + 0.09\alpha_B \qquad (4)$$

also, $$(+\downarrow) \qquad a_O = \alpha_A(0.09) \qquad (5)$$

Solving Eqs. (1) – (5) yields:

$\alpha_A = 43.6$ rad/s^2 **Ans**

$\alpha_B = 43.6$ rad/s^2 **Ans**

$T = 15.7$ N **Ans**

$a_B = 7.85$ m/s^2 $\qquad a_O = 3.92$ m/s^2

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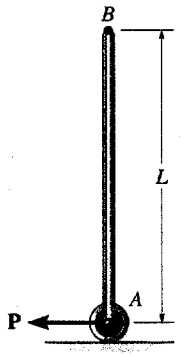
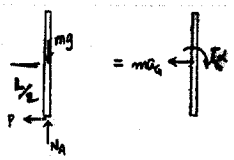
17-105. The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad P = ma_G$

$\zeta + \Sigma M_G = I_G\alpha; \quad P(\frac{L}{2}) = (\frac{1}{12}mL^2)\alpha$



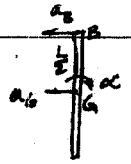
$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$ **Ans**

$a_G = \frac{P}{m}$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\mathbf{i}$



$(\overset{+}{\leftarrow}) \quad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

$= \frac{P}{m} - \frac{L}{2}(\frac{6P}{mL})$

$a_B = -\frac{2P}{m} = \frac{2P}{m} \rightarrow$ **Ans**

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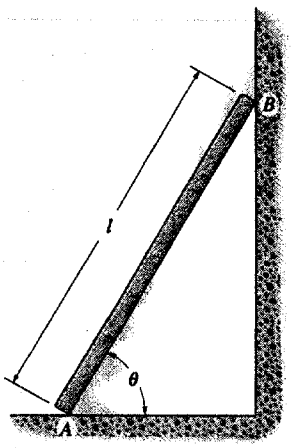
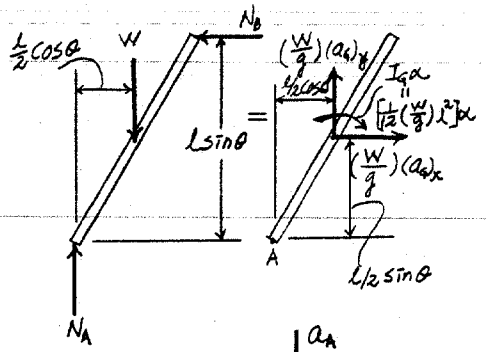
***17-106.** The ladder has a weight W and rests against the smooth wall and ground. Determine its angular acceleration as a function of θ when it is released and allowed to slide downward. For the calculation, treat the ladder as a slender rod.



Equation of Motion : The mass moment of inertia of the ladder about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{W}{g}\right)(l^2)$. Applying Eq. 17 – 16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad N_B(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = -\left[\frac{1}{12}\left(\frac{W}{g}\right)l^2\right]\alpha + \left(\frac{W}{g}\right)(a_G)_y\left(\frac{l}{2}\cos\theta\right) - \left(\frac{W}{g}\right)(a_G)_x\left(\frac{l}{2}\sin\theta\right) \qquad [1]$$

$$\stackrel{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad -N_B = \left(\frac{W}{g}\right)(a_G)_x \qquad [2]$$

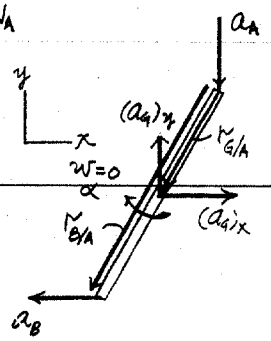


Kinematic : At the instant the ladder being released, the angular velocity of the ladder, $\omega = 0$. Analysing the motion of points A and B by applying Eq. 16 – 18 with $\mathbf{r}_{B/A} = -l\cos\theta\mathbf{i} - l\sin\theta\mathbf{j}$, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha\times\mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$
$$-a_B\mathbf{i} = -a_A\mathbf{j} + (-\alpha\mathbf{k})\times(-l\cos\theta\mathbf{i} - l\sin\theta\mathbf{j}) - \mathbf{0}$$
$$-a_B\mathbf{i} = -(l\sin\theta)\alpha\mathbf{i} + [(l\cos\theta)\alpha - a_A]\mathbf{j}$$

Equating **j** component, we have

$$0 = (l\cos\theta)\alpha - a_A \qquad a_A = (l\cos\theta)\alpha$$



Analysing the motion of points A and G by applying Eq. 16 – 18 with $\mathbf{r}_{G/A} = -\frac{l}{2}\cos\theta\mathbf{i} - \frac{l}{2}\sin\theta\mathbf{j}$, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha\times\mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -(l\cos\theta)\alpha\mathbf{j} + (-\alpha\mathbf{k})\times\left(-\frac{l}{2}\cos\theta\mathbf{i} - \frac{l}{2}\sin\theta\mathbf{j}\right) - \mathbf{0}$$
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -\left(\frac{l}{2}\sin\theta\right)\alpha\mathbf{i} - \left(\frac{l}{2}\cos\theta\right)\alpha\mathbf{j}$$

Equating **i** and **j** component, we have

$$(a_G)_x = -\left(\frac{l}{2}\sin\theta\right)\alpha \qquad (a_G)_y = -\left(\frac{l}{2}\cos\theta\right)\alpha$$

Substitute the results obtained above into Eqs. [1] and [2] and solve yields

$$N_B = \frac{3W}{8}\sin 2\theta$$

$$\alpha = \frac{3g}{2l}\cos\theta \qquad \textbf{Ans}$$

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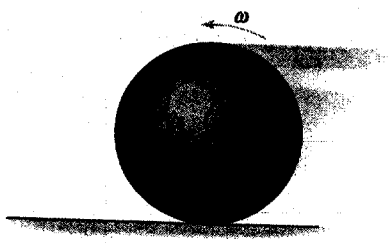
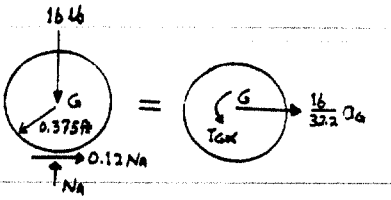
17-107. The 16-lb bowling ball is cast horizontally onto a lane such that initially $\omega = 0$ and its mass center has a velocity $v = 8$ ft/s. If the coefficient of kinetic friction between the lane and the ball is $\mu_k = 0.12$, determine the distance the ball travels before it rolls without slipping. For the calculation, neglect the finger holes in the ball and assume the ball has a uniform density.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.12N_A = \frac{16}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 16 = 0$

$\zeta + \Sigma M_G = I_G\alpha; \quad 0.12N_A(0.375) = \left[\frac{2}{5}\left(\frac{16}{32.2}\right)(0.375)^2\right]\alpha$



Solving,

$N_A = 16$ lb; $\quad a_G = 3.864$ ft/s^2; $\quad \alpha = 25.76$ rad/s^2

When the ball rolls without slipping $v = \omega(0.375)$,

$(\zeta +) \qquad \omega = \omega_0 + \alpha_c t$

$\frac{v}{0.375} = 0 + 25.76t$

$v = 9.660t$

$\left(\overset{+}{\leftarrow}\right) \qquad v = v_0 + a_c t$

$9.660t = 8 - 3.864t$

$t = 0.592$ s

$\left(\overset{+}{\leftarrow}\right) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 8(0.592) - \frac{1}{2}(3.864)(0.592)^2$

$s = 4.06$ ft **Ans**

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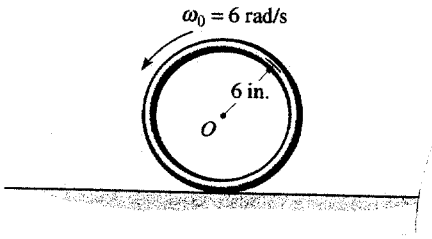
***17-108.** The 10-lb hoop or thin ring is given an initial angular velocity of 6 rad/s when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is $\mu_k = 0.3$, determine the distance the hoop moves before it stops slipping.



$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N - 10 = 0$ $\quad N = 10$ lb

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad 0.3(10) = \left(\frac{10}{32.2}\right) a_G$ $\quad a_G = 9.66$ ft/s^2

$\curvearrowright + \Sigma M_G = I_G \alpha;$ $\quad 0.3(10)\left(\frac{6}{12}\right) = \left(\frac{10}{32.2}\right)\left(\frac{6}{12}\right)^2 \alpha$ $\quad \alpha = 19.32$ rad/s^2

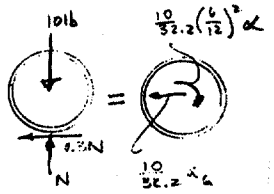
When slipping ceases, $v_G = \omega r = 0.5\omega$ [1]

$(\circlearrowleft +)$ $\quad \omega = \omega_0 + \alpha t$

$\omega = 6 + (-19.32)t$ [2]

$(\overset{+}{\leftarrow})$ $\quad v_G = (v_G)_0 + a_G t$

$v_G = 0 + 9.66t$ [3]



Solving Eqs. [1] to [3] yields:

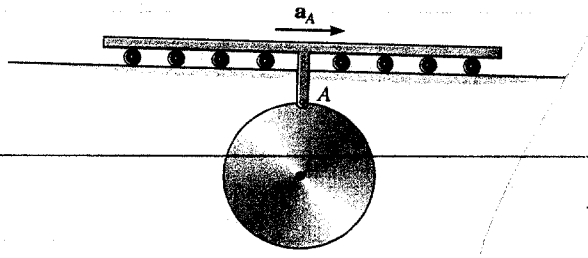
$t = 0.1553$ s $\quad v_G = 1.5$ ft/s $\quad \omega = 3$ rad/s

$(\overset{+}{\leftarrow})$ $\quad s = s_0 + (v_G)_0 t + \frac{1}{2} a_G t^2$

$= 0 + 0 + \frac{1}{2}(9.66)(0.1553)^2$

$= 0.116$ ft $= 1.40$ in. Ans

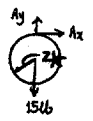
17-109. The 15-lb circular plate is suspended from a pin at A. If the pin is connected to a track which is given an acceleration $a_A = 3$ ft/s^2, determine the horizontal and vertical components of reaction at A and the acceleration of the plate's mass center G. The plate is originally at rest.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad A_x = \frac{15}{32.2}(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 15 = \frac{15}{32.2}(a_G)_y$

$\curvearrowright + \Sigma M_G = I_G \alpha;$ $\quad A_x(2) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(2)^2\right]\alpha$

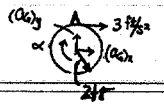


$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$

$\mathbf{a}_G = 3\mathbf{i} - 2\alpha\mathbf{i}$

$(+\uparrow)$ $\quad (a_G)_y = 0$

$(\overset{+}{\rightarrow})$ $\quad (a_G)_x = 3 - 2\alpha$



Thus,

$A_y = 15.0$ lb Ans

$A_x = 0.466$ lb Ans

$\alpha = 1$ rad/s^2

$a_G = (a_G)_x = 1.00$ ft/s^2 Ans

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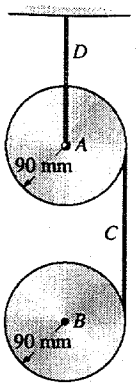
17-110. A cord C is wrapped around each of the two 10-kg disks. If they are released from rest, determine the tension in the fixed cord D. Neglect the mass of the cord.



For A:

$$\curvearrowleft + \sum M_A = I_A\alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \quad (1)$$

For B:

$$\curvearrowright + \sum M_B = I_B\alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \quad (2)$$

$$+\downarrow \sum F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \quad (3)$$

$$a_B = a_P + (a_{B/P})_t + (a_{B/P})_n$$

$$(+\downarrow)a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \quad (4)$$

Solving,

$a_B = 7.85 \text{ m/s}^2$

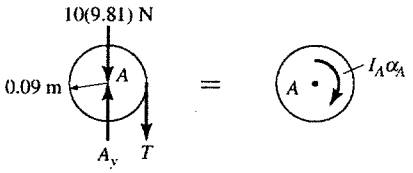
$\alpha_A = 43.6 \text{ rad/s}^2$

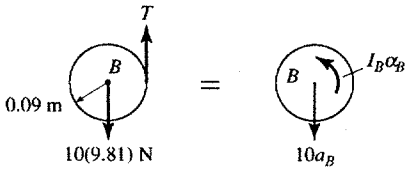
$\alpha_B = 43.6 \text{ rad/s}^2$

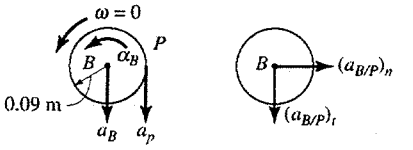
$T = 19.62 \text{ N}$

$A_y = 10(9.81) + 19.62$

$= 118 \text{ N}$ **Ans**







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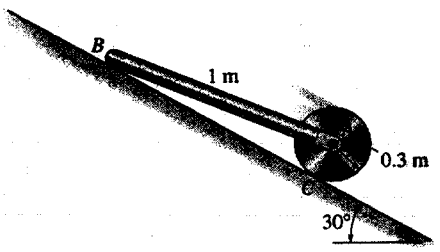
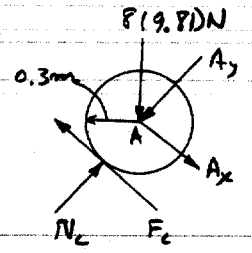
17-111. The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



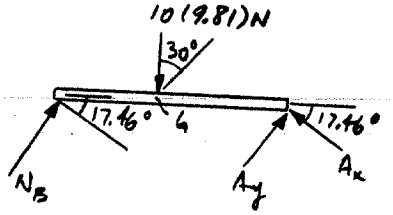


Disk :

$+\searrow \Sigma F_x = m(a_G)_x;$ $\quad A_x - F_C + 8(9.81)\sin 30° = 8a_G \quad (1)$

$+\swarrow \Sigma F_y = m(a_G)_y;$ $\quad N_C - A_y - 8(9.81)\cos 30° = 0 \quad (2)$

$\zeta + \Sigma M_A = I_A \alpha;$ $\quad F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha \quad (3)$



Bar :

$+\searrow \Sigma F_x = m(a_G)_x;$ $\quad 10(9.81)\sin 30° - A_x = 10a_G \quad (4)$

$+\swarrow \Sigma F_y = m(a_G)_y;$ $\quad N_B + A_y - 10(9.81)\cos 30° = 0 \quad (5)$

$\zeta + \Sigma M_G = I_G \alpha;$ $\quad -N_B(0.5\cos 17.46°) + A_x(0.5\sin 17.46°) + A_y(0.5\cos 17.46°) = 0 \quad (6)$

Assume no slipping of the disk,

$a_G = 0.3\alpha \quad (7)$

Solving, Eqs. (1) – (7),

$A_x = 8.92$ N, $\quad A_y = 41.1$ N, $\quad N_B = 43.9$ N

$a_G = 4.01$ m/s^2

$\alpha = 13.4$ rad/s^2 **Ans**

$N_C = 109$ N

$F_C = 16.1$ N

$(F_C)_{max} = 0.6(109) = 65.4$ N > 16.1 N OK

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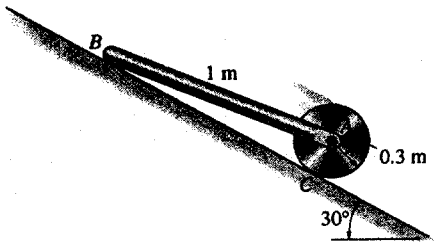
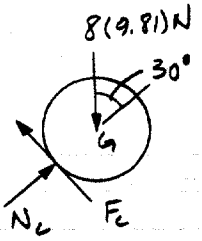
***17-112.** Solve Prob. 17-111 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.





$+\nwarrow \Sigma F_x = m(a_G)_x$; $\quad 8(9.81)\sin 30° - F_C = 8a_G \quad (1)$

$+\nearrow \Sigma F_y = m(a_G)_y$; $\quad -8(9.81)\cos 30° + N_C = 0 \quad (2)$

$\circlearrowleft + \Sigma M_G = I_G\alpha$; $\quad F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha \quad (3)$

Assume no slipping : $\quad a_G = 0.3\alpha$

Solving Eqs. (1) – (3) :

$N_C = 67.97$ N

$a_G = 3.27$ m/s^2

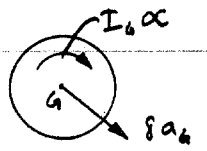
$\alpha = 10.9$ rad/s^2

$F_C = 13.08$ N

$(F_C)_{max} = 0.15(67.97) = 10.2$ N < 13.08 N NG

Slipping occurs

$F_C = 0.1N_C$



Solving Eqs. (1) – (3) :

$N_C = 67.97$ N

$\alpha = 5.66$ rad/s^2 Ans

$a_G = 4.06$ m/s^2

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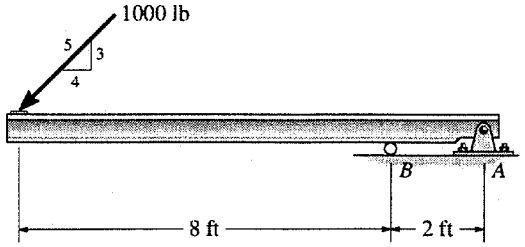
17-113. The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



$$\stackrel{+}{\leftarrow}\sum F_x = m(a_G)_x; \qquad 1000\left(\frac{4}{5}\right) = \frac{500}{32.2}(a_G)_x$$

$$+\downarrow \sum F_y = m(a_G)_y; \qquad 1000\left(\frac{3}{5}\right) + 500 - B_y = \frac{500}{32.2}(a_G)_y$$

$$\curvearrowleft + \sum M_B = \sum (M_k)_B; \quad 500(3) + 1000\left(\frac{3}{5}\right)(8) = \frac{500}{32.2}(a_G)_y(3) + \left[\frac{1}{12}\left(\frac{500}{32.2}\right)(10)^2\right]\alpha$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

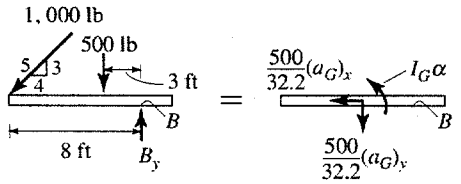
$$-a_B\mathbf{i} = -(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} + \alpha(3)\mathbf{j}$$

$$(+\downarrow) \quad (a_G)_y = \alpha(3)$$

$\alpha = 23.4 \text{ rad/s}^2$ **Ans**

$B_y = 9.62 \text{ lb}$ **Ans**

$B_y > 0$ means that the beam stays in contact with the roller support.



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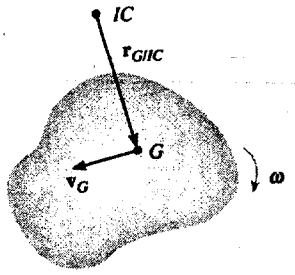
18-1. At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity v_G. Show that its kinetic energy can be represented as $T = \frac{1}{2} I_{IC} \omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



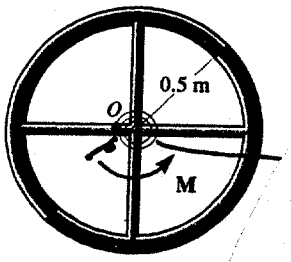
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

$$= \frac{1}{2} m (\omega r_{G/IC})^2 + \frac{1}{2} I_G \omega^2$$

$$= \frac{1}{2} \left(m r_{G/IC}^2 + I_G \right) \omega^2 \qquad \text{However } m r_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2} I_{IC} \omega^2 \qquad \textbf{Q.E.D.}$$

18-2. The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N} \cdot \text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \text{N} \cdot \text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



$$I_O = 2\left[\frac{1}{12}(2)(1)^2\right] + 5(0.5)^2 = 1.583$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{4\pi} 2\theta \, d\theta = \frac{1}{2}(1.583)\omega^2$$

$$(4\pi)^2 = 0.7917\omega^2$$

$$\omega = 14.1 \text{ rad/s} \qquad \textbf{Ans}$$

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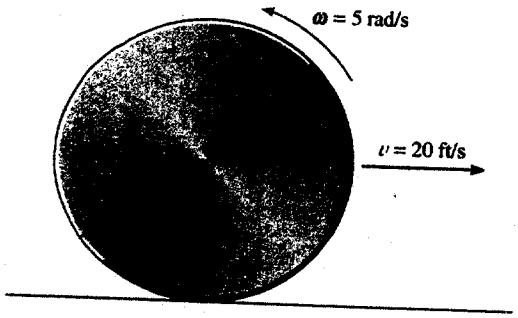
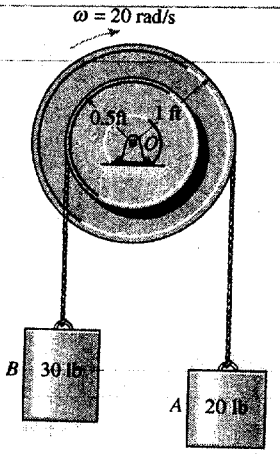
18-3. At the instant shown, the 30-lb disk has a counterclockwise angular velocity of 5 rad/s when its center has a velocity of 20 ft/s. Determine the kinetic energy of the disk at this instant.



$$T = \frac{1}{2}\left[\frac{1}{2}\left(\frac{30}{32.2}\right)(2)^2\right](5)^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)(20)^2 = 210 \text{ ft·lb}$$ **Ans**

***18-4.** The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



$$T = \frac{1}{2} I_O \omega_O^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2 = 283 \text{ ft·lb}$$ **Ans**

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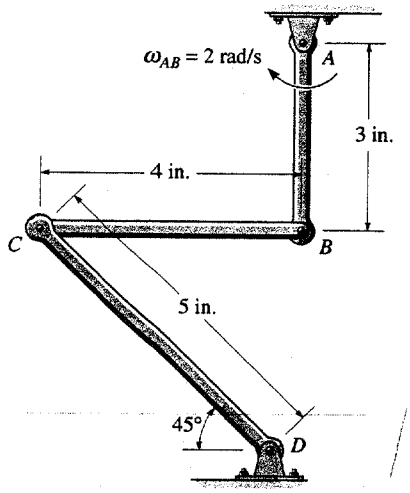
18-5. At the instant shown, link AB has an angular velocity $\omega_{AB} = 2$ rad/s. If each link is considered as a uniform slender bar with a weight of 0.5 lb/in., determine the total kinetic energy of the system.



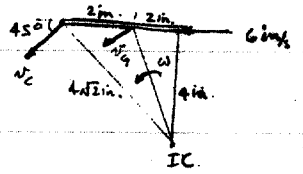
$$\omega_{BC} = \frac{6}{4} = 1.5 \text{ rad/s}$$

$$v_C = 1.5(4\sqrt{2}) = 8.4853 \text{ in./s}$$

$$r_{IC-G} = \sqrt{(2)^2 + (4)^2} = 4.472$$

$$v_G = 1.5(4.472) = 6.7082 \text{ in./s}$$

$$\omega_{BC} = \frac{8.4853}{5} = 1.697 \text{ rad/s}$$



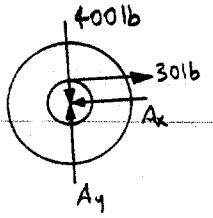
$$T = \frac{1}{2}\left[\frac{1}{3}\left(\frac{3(0.5)}{32.2}\right)\left(\frac{3}{12}\right)^2\right](2)^2 + \frac{1}{2}\left[\frac{4(0.5)}{32.2}\right]\left(\frac{6.7082}{12}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4(0.5)}{32.2}\right)\left(\frac{4}{12}\right)^2\right](1.5)^2$$
$$+ \frac{1}{2}\left[\frac{1}{3}\left(\frac{5(0.5)}{32.2}\right)\left(\frac{5}{12}\right)^2\right](1.697)^2 = 0.0188 \text{ ft·lb} \quad \textbf{Ans}$$

18-6. Solve Prob. 17-58 using the principle of work and energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 30(8) = \frac{1}{2}\left[\left(\frac{400}{32.2}\right)(1.30)^2\right]\omega^2$$

$$\omega = 4.78 \text{ rad/s} \quad \textbf{Ans}$$



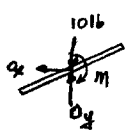
18-7. Solve Prob. 17–59 using the principle of work and energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{\pi/2} 5\theta \, d\theta = \frac{1}{2}\left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2\right]\omega^2$$

$$\frac{5}{2}\left(\frac{\pi}{2}\right)^2 = 0.05176\omega^2$$

$$\omega = 10.9 \text{ rad/s} \quad \textbf{Ans}$$



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***18-8.** Solve Prob. 17–63 using the principle of work and energy.

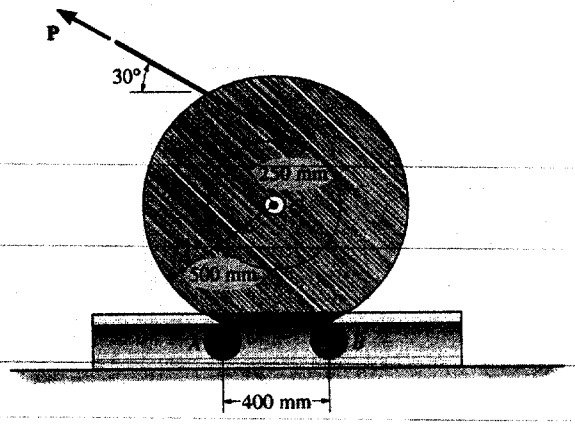
$T_1 + \Sigma U_{1-2} = T_2$

$0 + 2\int_0^{\pi/2} 50\theta \, d\theta = \frac{1}{2}[\frac{1}{3}(70)(1.2)^2]\omega^2$

$100(\frac{\theta^2}{2})\Big|_0^{\pi/2} = 16.8\,\omega^2$

$\omega = 2.71$ rad/s **Ans**

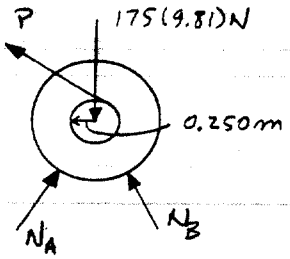
18-9. A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}[175(0.42)^2]\omega^2$

$\omega = 2.02$ rad/s **Ans**



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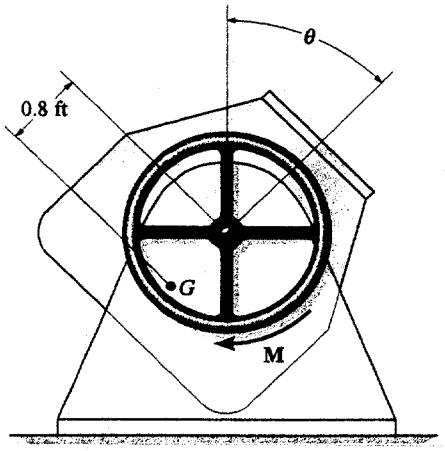
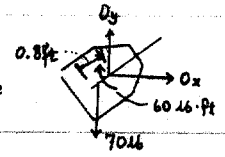
18-10. The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60$ lb·ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$.



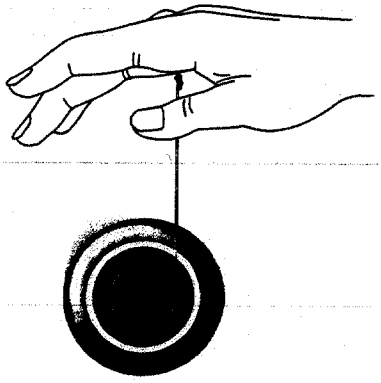
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$



$\omega = 3.89$ rad/s Ans

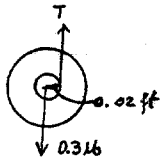
18-11. A yo-yo has a weight of 0.3 lb and a radius of gyration $k_O = 0.06$ ft. If it is released from rest, determine how far it must descend in order to attain an angular velocity $\omega = 70$ rad/s. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is $r = 0.02$ ft.



$v_G = (0.02)70 = 1.40$ ft/s

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (0.3)(s) = \frac{1}{2}\left(\frac{0.3}{32.2}\right)(1.40)^2 + \frac{1}{2}\left[(0.06)^2\left(\frac{0.3}{32.2}\right)\right](70)^2$$



$s = 0.304$ ft Ans

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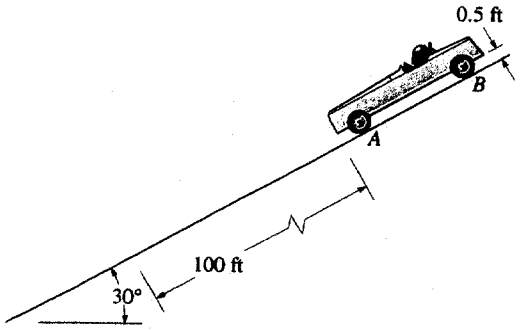
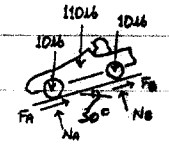
***18-12.** The soap-box car has a weight of 110 lb, including the passenger but *excluding* its four wheels. Each wheel has a weight of 5 lb, radius of 0.5 ft, and a radius of gyration $k = 0.3$ ft, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled 100 ft starting from rest. The wheels roll without slipping. Neglect air resistance.



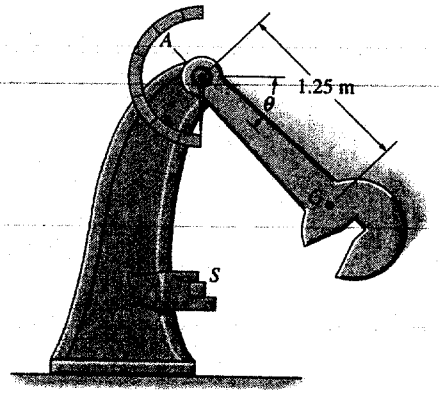
$T_1 + \Sigma U_{1-2} = T_2$

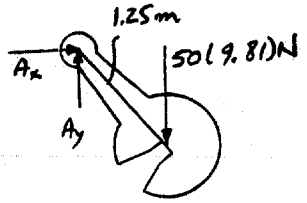
$$0 + 130(100\sin 30°) = 4\{\frac{1}{2}(\frac{5}{32.2})(v_C)^2 + \frac{1}{2}[(\frac{5}{32.2})(0.3)^2](\frac{v_C}{0.5})^2\} + \frac{1}{2}(\frac{110}{32.2})(v_C)^2$$

$v_C = 55.2$ ft/s **Ans**



18-13. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.





$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (50)(9.81)(1.25) = \frac{1}{2}[(50)(1.75)^2]\omega_2^2$$

$\omega_2 = 2.83$ rad/s **Ans**

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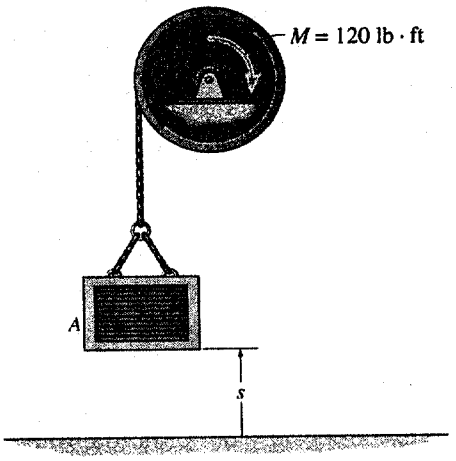
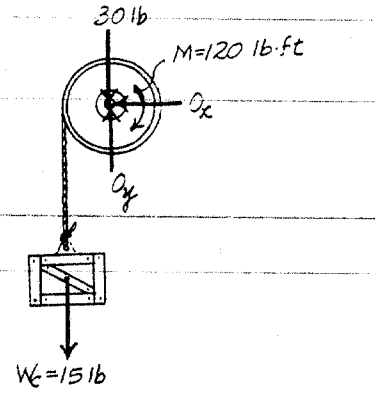
18-14. A motor supplies a constant torque or twist of $M = 120$ lb · ft to the drum. If the drum has a weight of 30 lb and a radius of gyration of $k_O = 0.8$ ft, determine the speed of the 15-lb crate A after it rises $s = 4$ ft starting from rest. Neglect the mass of the cord.



Free Body Diagram : The weight of the crate does *negative* work since it acts in the opposite direction to that of its displacement s_W. Also. the couple moment M does positive work as it acts in the same direction of its angular displacement θ. The reactions O_x, O_y and the weight of the drum do no work since point O does not displace.

Kinematic : Since the drum rotates about point O, the angular velocity of the drum and the speed of the crate can be related by $\omega_D = \frac{v_A}{r_D} = \frac{v_A}{1.5} = 0.6667 v_A$. When the crate rises $s = 4$ ft, the angular displacement of the drum is given by $\theta = \frac{s}{r_D} = \frac{4}{1.5} = 2.667$ rad.



Principle of Work and Energy : The mass moment of inertia of the drum about point O is $I_O = mk_O^2 = \left(\frac{30}{32.2}\right)(0.8^2) = 0.5963$ slug · ft^2. Applying Eq. 18 – 13, we have

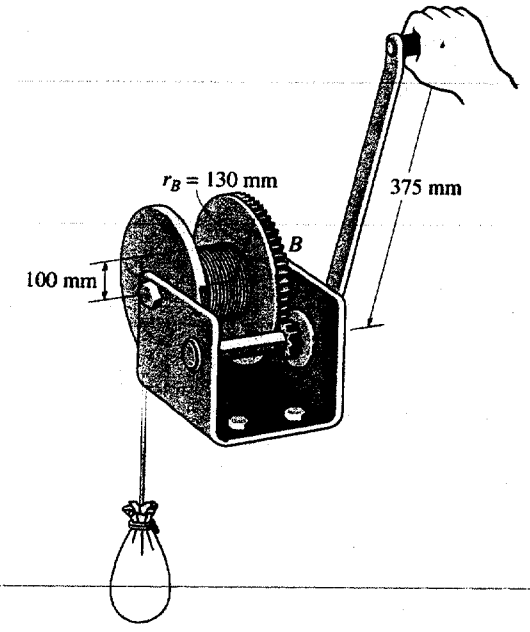
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + M\theta - W_C s_C = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_C v_C^2$$

$$0 + 120(2.667) - 15(4) = \frac{1}{2}(0.5963)(0.6667 v_A)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right) v_A^2$$

$$v_A = 26.7 \text{ ft/s} \qquad \textbf{Ans}$$

18-15. The hand winch is used to lift the 50-kg load. Determine the work required to rotate the handle five revolutions. The gear at A has a radius of 20 mm.



$20(\theta_A) = \theta_B(130)$

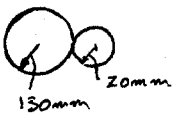
When $\theta_A = 5$ rev. $= 10\pi$

$\theta_B = 4.8332$ rad

Thus load moves up

$s = 4.8332(0.1 \text{ m}) = 0.48332$ m

$U = 50(9.81)(0.48332) = 237$ J **Ans**



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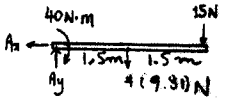
***18-16.** The 4-kg slender rod is subjected to the force and couple moment. When it is in the position shown it has an angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.

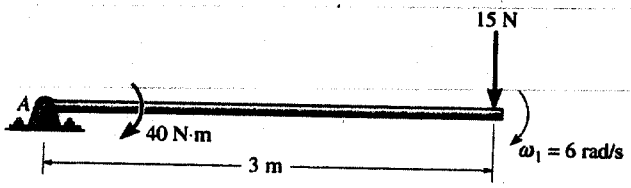


$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(\frac{\pi}{2})(3) + 4(9.81)(1.5) + 40(\frac{\pi}{2}) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

$\omega = 8.25$ rad/s **Ans**

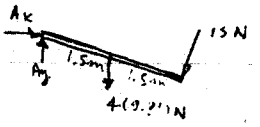
18-17. The 4-kg slender rod is subjected to the force and couple moment. When the rod is in the position shown it has a angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.



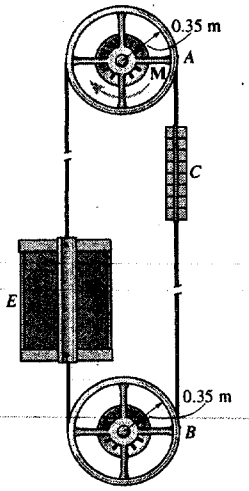
$T_1 + \Sigma U_{1-2} = T_2$

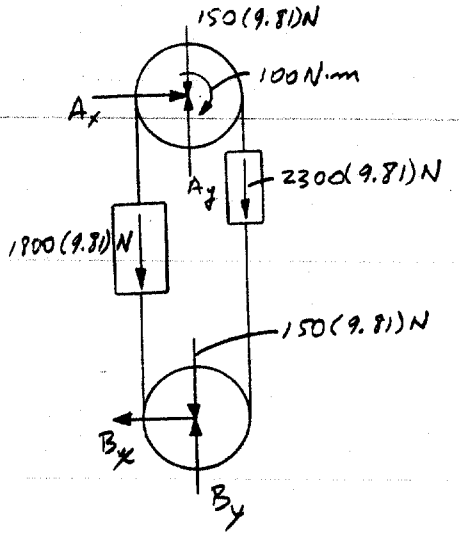
$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(2\pi)(3) + 40(2\pi) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

$\omega = 11.2$ rad/s **Ans**



18-18. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a constant torque of $M = 100$ N·m, determine the speed of the elevator when it has ascended 10 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.





$\theta = \frac{10}{0.35} = 28.57$ rad

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 2300(9.81)(10) - 1800(9.81)(10) + 100(28.57)$

$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}[150(0.2)^2]\left(\frac{v}{0.35}\right)^2$$

$51\,907.1 = 2099v^2$

$v = 4.97$ m/s **Ans**

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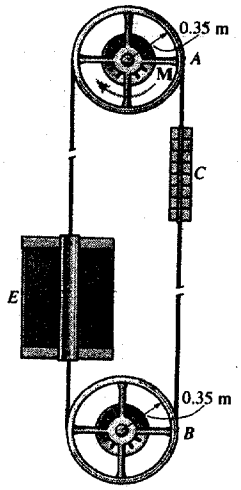
18-19. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a torque of $M = (0.06\theta^2 + 7.5)$ N·m, where θ is in radians, determine the speed of the elevator when it has ascended 12 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



$$\theta = \frac{12}{0.35} = 34.29 \text{ rad}$$

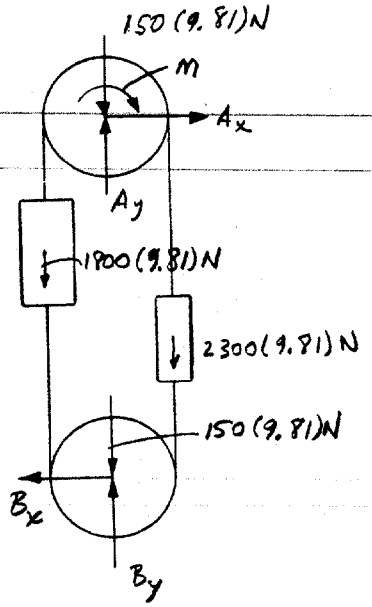
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2300(9.81)(12) - 1800(9.81)(12) + \int_0^{34.29} (0.06\theta^2 + 7.5)\,d\theta$$

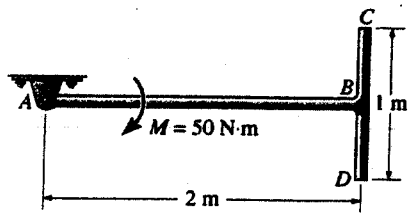
$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}\left[150(0.2)^2\right]\left(\frac{v}{0.35}\right)^2$$

$$58860 + \left(0.02\theta^3 + 7.5\theta\right)\Big|_0^{34.29} = 2098.98v^2$$

$v = 5.34$ m/s **Ans**



***18-20.** The pendulum consists of two slender rods each having a mass of 4 kg/m. If it is acted upon by a moment $M = 50$ N·m and released from the position shown, determine its angular velocity when it has rotated (a) 90° and (b) 180°. Motion occurs in the vertical plane.

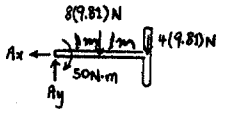


a) $T_1 + \Sigma U_{1-2} = T_2$

$$[0+0] + 8(9.81)(1) + 4(9.81)(2) + 50\left(\frac{\pi}{2}\right) = \frac{1}{2}\left[\frac{1}{3}(8)(2)^2\right]\omega^2 + \frac{1}{2}\left[\frac{1}{12}(4)(1)^2 + 4(2)^2\right]\omega^2$$

$$235.5 = 13.5\,\omega^2$$

$\omega = 4.18$ rad/s **Ans**



b) $T_1 + \Sigma U_{1-2} = T_2$

$$[0+0] + 50(\pi) = 13.5\omega^2$$

$\omega = 3.41$ rad/s **Ans**

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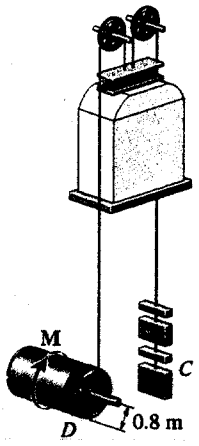
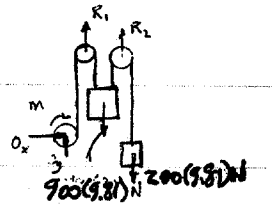
18-21. A motor supplies a constant torque $M = 6\ \text{kN}\cdot\text{m}$ to the winding drum that operates the elevator. If the elevator has a mass of 900 kg, the counterweight C has a mass of 200 kg, and the winding drum has a mass of 600 kg and radius of gyration about its axis of $k = 0.6$ m, determine the speed of the elevator after it rises 5 m starting from rest. Neglect the mass of the pulleys.



$v_E = v_C$

$\theta = \frac{s}{r} = \frac{5}{0.8}$

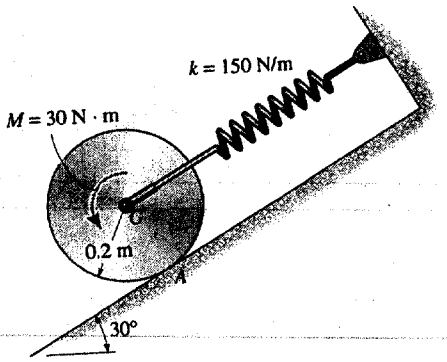


$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 6000\left(\frac{5}{0.8}\right) - 900(9.81)(5) + 200(9.81)(5) = \frac{1}{2}(900)(v)^2 + \frac{1}{2}(200)(v)^2 + \frac{1}{2}[600(0.6)^2]\left(\frac{v}{0.8}\right)^2$$

$v = 2.10$ m/s **Ans**

118-22. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved 0.8 m down along the inclined plane. The disk rolls without slipping.



Initial tension in spring :

$+\Sigma M_A = 0;\quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$

$F_s = 98.1$ N

$s_1 = \frac{98.1}{150} = 0.654$ m

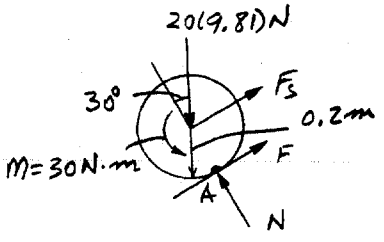
When $s = 0.8$ m the disk rotates $\theta = \frac{0.8}{0.2} = 4$ rad

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 20(9.81)(0.8\sin 30°) + 30(4) - \left[\frac{1}{2}(150)(0.8 + 0.654)^2 - \frac{1}{2}(150)(0.654)^2\right] = \frac{1}{2}\left[\frac{1}{2}(20)(0.2)^2\right]\omega^2 + \frac{1}{2}(20)(0.2\omega)^2$$

$72 = 0.6\omega^2$

$\omega = 11.0$ rad/s **Ans**



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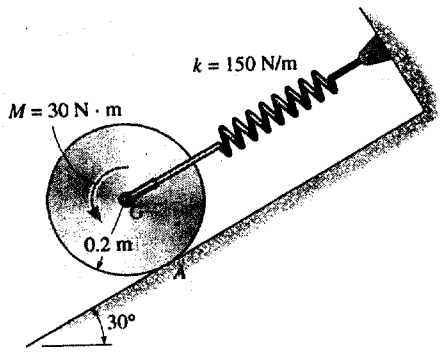
18-23. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.

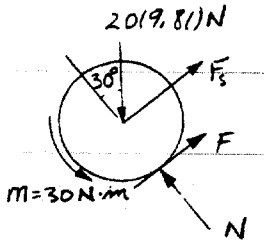


$+\Sigma M_A = 0; \quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$

$F_s = 98.1\ \text{N}$

$s_1 = \dfrac{98.1}{150} = 0.654\ \text{m}$

When G moves s, the disk rotates $\theta = \dfrac{s}{0.2}$



$T_1 + \Sigma U_{1-2} = T_2$

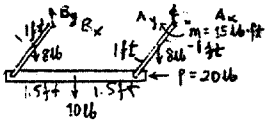
$$0 + 20(9.81)(s)\sin 30° + 30\left(\frac{s}{0.2}\right) - \left[\frac{1}{2}(150)(s+0.654)^2 - \frac{1}{2}(150)(0.654)^2\right] = 0$$

$248.1s = 75\left(s^2 + 1.308s + 0.4277\right) - 32.08$

$248.1 = 75s + 98.1$

$s = 2.00\ \text{m}$ **Ans**

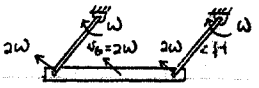
***18-24.** The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2\ \text{rad/s}$. If rod CD is subjected to a couple moment $M = 15\ \text{lb}\cdot\text{ft}$ and bar AD is subjected to a horizontal force $P = 20\ \text{lb}$ as shown, determine ω_{AB} at the instant $\theta = 90°$.



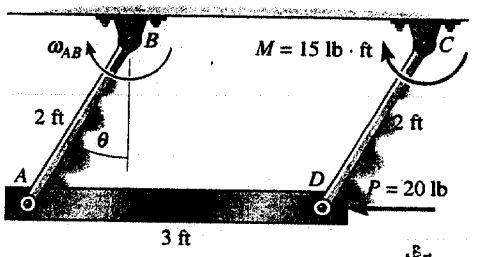
$T_1 + \Sigma U_{1-2} = T_2$

$$2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}(2)^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + \left[20(2) + 15\left(\frac{\pi}{2}\right) - 2(8)(1) - 10(2)\right] =$$
$$2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}\omega^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(2\omega)^2$$

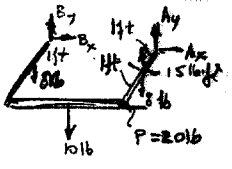
$\omega = 5.74\ \text{rad/s}$ **Ans**



18-25. The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2\ \text{rad/s}$. If rod CD is subjected to a couple moment $M = 15\ \text{lb}\cdot\text{ft}$ and bar AD is subjected to a horizontal force $P = 20\ \text{lb}$ as shown, determine ω_{AB} at the instant $\theta = 45°$.



$T_1 + \Sigma U_{1-2} = T_2$



$$2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}(2)^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2$$
$$+ \left[20(2\sin 45°) + 15\left(\frac{\pi}{4}\right) - 2(8)(1-\cos 45°) - 10(2-2\cos 45°)\right]$$
$$= 2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}\omega^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(2\omega)^2$$

$\omega = 5.92\ \text{rad/s}$ **Ans**

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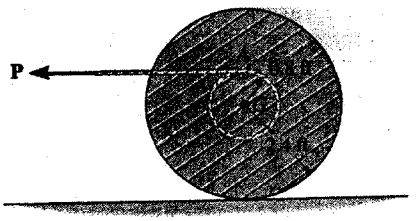
18-26. The spool has a weight of 500 lb and a radius of gyration of $k_G = 1.75$ ft. A horizontal force of $P = 15$ lb is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved 6 ft to the left. The spool rolls without slipping. Neglect the mass of the cable.



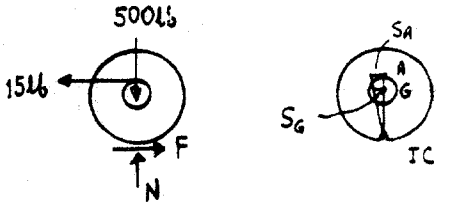
$$\frac{s_G}{2.4} = \frac{s_A}{3.2}$$

For $s_G = 6$ ft, then $s_A = 8$ ft

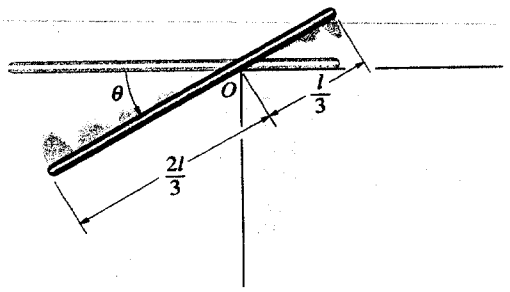
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(8) = \frac{1}{2}\left[\left(\frac{500}{32.2}\right)(1.75)^2\right]\omega^2 + \frac{1}{2}\left(\frac{500}{32.2}\right)(2.4\omega)^2$$

$$\omega = 1.32 \text{ rad/s} \qquad \textbf{Ans}$$



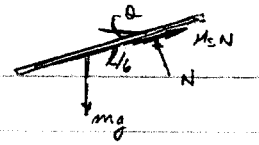
18-27. The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine the angle θ at which it first begins to slip. The coefficient of static friction at O is $\mu_s = 0.3$.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + mg\left(\frac{l}{6}\sin\theta\right) = \frac{1}{2}\left[\frac{1}{12}ml^2 + m\left(\frac{l}{6}\right)^2\right]\omega^2$$

$$\omega = \sqrt{\frac{3g\sin\theta}{l}}$$



$$(+\Sigma M_O = I_O\alpha; \quad mg\cos\theta\left(\frac{l}{6}\right) = \left[\frac{1}{12}ml^2 + m\left(\frac{l}{6}\right)^2\right]\alpha$$

$$\alpha = \frac{3g\cos\theta}{2l}$$

$$+\Sigma F_n = m(a_G)_n; \quad \mu_s N - mg\sin\theta = m\left(\frac{3g\sin\theta}{l}\right)\left(\frac{l}{6}\right)$$

$$\mu_s N = 1.5mg\sin\theta$$

$$+\Sigma F_t = m(a_G)_t; \quad -N + mg\cos\theta = m\left(\frac{3g\cos\theta}{2l}\right)\left(\frac{l}{6}\right)$$

$$N = 0.75mg\cos\theta$$

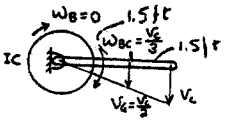
Thus,

$$\mu_s = \frac{1.5}{0.75}\tan\theta$$

$$0.3 = 2\tan\theta$$

$$\theta = 8.53° \qquad \textbf{Ans}$$

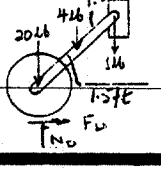
***18-28.** The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2$$

$$v_C = 13.3 \text{ ft/s} \qquad \textbf{Ans}$$



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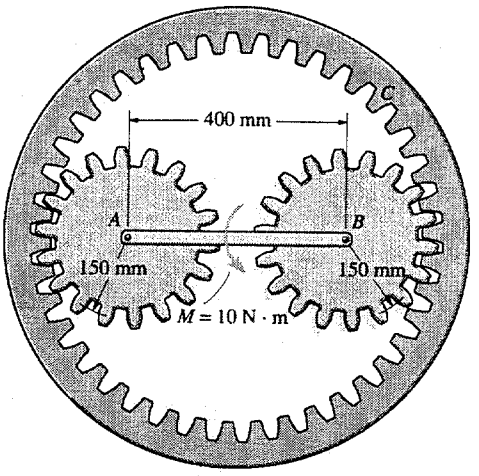
18-29. The two 2-kg gears A and B are attached to the ends of a 3-kg slender bar. The gears roll within the fixed ring gear C, which lies in the horizontal plane. If a 10-N · m torque is applied to the center of the bar as shown, determine the number of revolutions the bar must rotate starting from rest in order for it to have an angular velocity of $\omega_{AB} = 20$ rad/s. For the calculation, assume the gears can be approximated by thin disks. What is the result if the gears lie in the vertical plane?



Energy equation (where G refers to the center of one of the two gears):

$$M\theta = T_2$$

$$10\theta = 2\left(\frac{1}{2}I_G\omega_{\text{gear}}^2\right) + 2\left(\frac{1}{2}m_{\text{gear}}\right)(0.200\omega_{AB})^2 + \frac{1}{2}I_{AB}\omega_{AB}^2$$

Using $m_{\text{gear}} = 2$ kg, $I_G = \frac{1}{2}(2)(0.150)^2 = 0.0225$ kg · m^2,

$I_{AB} = \frac{1}{12}(3)(0.400)^2 = 0.0400$ kg · m^2, and $\omega_{\text{gear}} = \frac{200}{150}\omega_{AB}$,

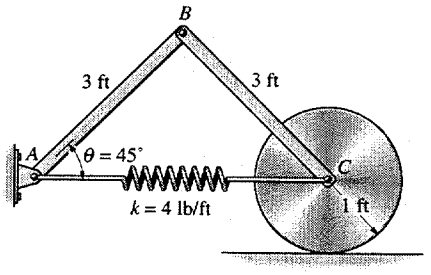
$$10\theta = 0.0225\left(\frac{200}{150}\right)^2\omega_{AB}^2 + 2(0.200)^2\omega_{AB}^2 + 0.0200\omega_{AB}^2$$

When $\omega_{AB} = 20$ rad/s,

$$\theta = 5.60 \text{ rad}$$

$$= 0.891 \text{ rev, regardless of orientation} \quad \textbf{Ans}$$

18-30. The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45°$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0°$. The disk rolls without slipping.

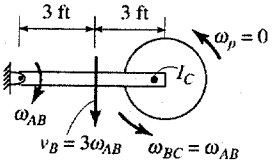


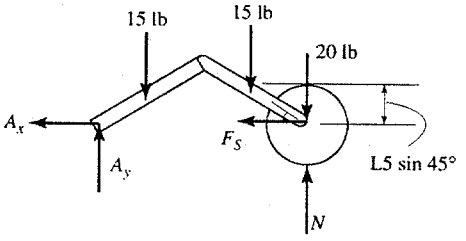
$$T_1 + \sum U_{1-2} = T_2$$

$$[0 + 0] + 2(15)(1.5)\sin 45° - \frac{1}{2}(4)[6 - 2(3)\cos 45°]^2$$

$$= 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$$\omega_{AB} = 4.28 \text{ rad/s} \quad \textbf{Ans}$$





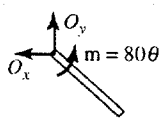
18-31. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80$ N · m/rad, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint*: For a torsional spring $M = k\theta$, when k is the stiffness and θ is the angle of twist.

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \int_{\theta_0}^{\theta_0 + \frac{\pi}{2}} 80\theta \, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$

$$40\left[\left(\theta_0 + \frac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$$\theta_0 = 1.66 \text{ rad} \quad \textbf{Ans}$$



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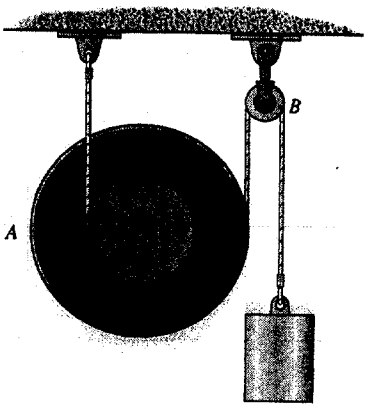
***18-32.** Pulley A weighs 15 lb and has a radius of gyration of $k_O = 0.8$ ft. If the system is released from rest, determine the velocity of the center O of the pulley after the 10-lb block moves downward 4 ft. Neglect the mass of the pulley at B.



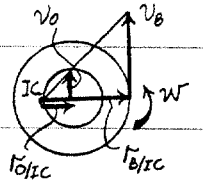
Kinematic : Since the cylinder rolls without slipping at point C, the instantaneous center of zero velocity is located at point C. Thus,

$$\frac{s_O}{0.5} = \frac{4}{1.5} \qquad s_O = 1.333 \text{ ft}$$

Also,

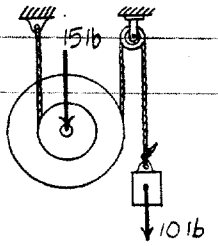
$$\omega = \frac{v_O}{r_{O/IC}} = \frac{v_B}{r_{B/IC}}$$

Thus, $\omega = \frac{v_O}{0.5} = 2v_O$ and $\frac{v_O}{0.5} = \frac{v_B}{1.5}$, $v_B = 3v_O$.



Free Body Diagram : The weight of the block does *positive* work since it acts in the same direction of its displacement, whereas the weight of the pulley acts in the opposite direction to that of its displacement and hence does *negative* work.

Principle of Work and Energy : The mass moment of inertia of the pulley about point O is $I_O = mk_O^2 = \left(\frac{15}{32.2}\right)(0.8^2) = 0.2981 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18 – 13, we have



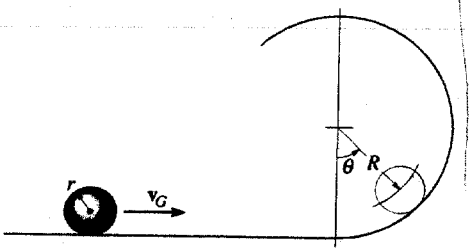
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_B s_B - W_P s_O = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_P v_O^2 + \frac{1}{2} I_O \omega^2$$

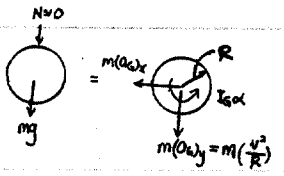
$$0 + 10(4) - 15(1.333) = \frac{1}{2}\left(\frac{10}{32.2}\right)(3v_O)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right)v_O^2 + \frac{1}{2}(0.2981)(2v_O)^2$$

$$v_O = 3.00 \text{ ft/s} \qquad \textbf{Ans}$$

18-33. A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.



$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad mg = m\left(\frac{v^2}{R}\right)$$



$$v^2 = gR$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_G^2}{r^2}\right) + \frac{1}{2}mv_G^2 - mg(2R) = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{gR}{r^2}\right) + \frac{1}{2}m(gR)$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2gR + \frac{1}{5}gR + \frac{1}{2}gR$$

$$v_G = 3\sqrt{\frac{3}{7}gR} \qquad \textbf{Ans}$$

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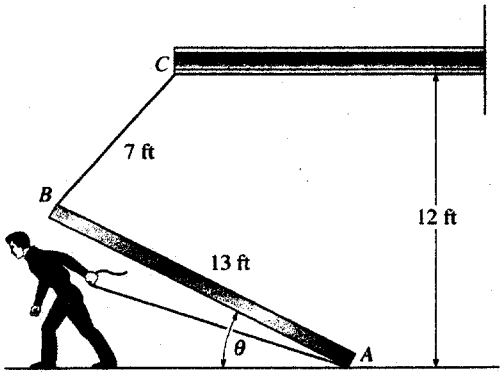
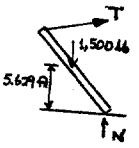
18-34. The beam has a weight of 1500 lb and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = 60°$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

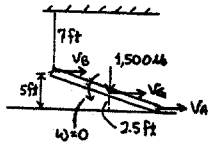


$T_1 + \Sigma U_{1-2} = T_2$

$0 + 1500(5.629) - 1500(2.5) = \frac{1}{2}\left(\frac{1500}{32.2}\right)(v_G)^2$

$v_G = v_A = 14.2$ ft/s **Ans**





18-35. Solve Prob. 18-13 using the conservation of energy equation.

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$0 + 50(9.81)(1.25) = \frac{1}{2}\left[50(1.75)^2\right]\omega^2 + 0$

$\omega = 2.83$ rad/s **Ans**

***18-36.** Solve Prob. 18–12 using the conservation of energy equation.

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 4\left[\frac{1}{2}\left(\frac{5}{32.2}\right)v_C^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(0.3)^2\left(\frac{v_C}{0.5}\right)^2\right] + \frac{1}{2}\left(\frac{110}{32.2}\right)v_C^2 - 130(100\sin 30°)$

$v_C = 55.2$ ft/s **Ans**

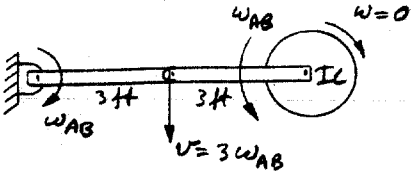
18-37. Solve Prob. 18-30 using the conservation of energy equation.

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$0 + 2\left[15(1.5\sin 45°)\right] = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right] + \frac{1}{2}(4)\left[6 - 2(3\cos 45°)\right]^2 + 0$

$\omega_{AB} = 4.28$ rad/s **Ans**



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18-38. Solve Prob. 18–11 using the conservation of energy equation.

$v_G = (0.02)\omega = 0.02(70) = 1.4$ ft/s

$T_1 + V_1 = T_2 + V_2$

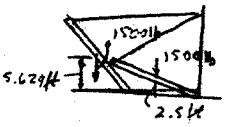
$0 + 0 = \frac{1}{2}(\frac{0.3}{32.2})(1.4)^2 + \frac{1}{2}(0.06)^2(\frac{0.3}{32.2})](70)^2 - (0.3)\,s$

$s = 0.304$ ft **Ans**

18-39. Solve Prob. 18–34 using the conservation of energy equation.

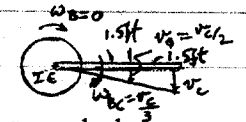
$T_1 + V_1 = T_2 + V_2$

$0 + 1500(5.629) = \frac{1}{2}(\frac{1500}{32.2})(v_G)^2 + (1500)(2.5)$



$v_G = v_A = 14.2$ ft/s **Ans**

***18-40.** Solve Prob. 18–28 using the conservation of energy equation.

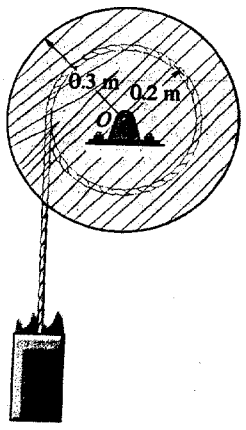


$T_1 + V_1 = T_2 + V_2$

$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}[\frac{1}{3}(\frac{4}{32.2})(3)^2](\frac{v_C}{3})^2 + \frac{1}{2}(\frac{1}{32.2})(v_C)^2 + 0$

$v_C = 13.3$ ft/s **Ans**

18-41. The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



$v_A = 0.2\,\omega = 0.2(5) = 1$ m/s

System:

$T_1 + V_1 = T_2 + V_2$

$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$

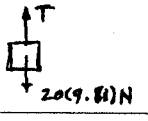
$s = 0.30071$ m $= 0.301$ m **Ans**

Block:

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$

$T = 163$ N **Ans**



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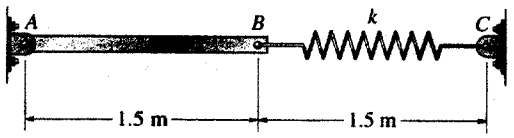
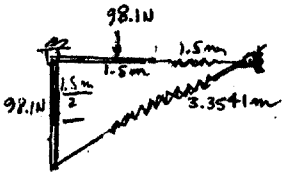
18-42. When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.



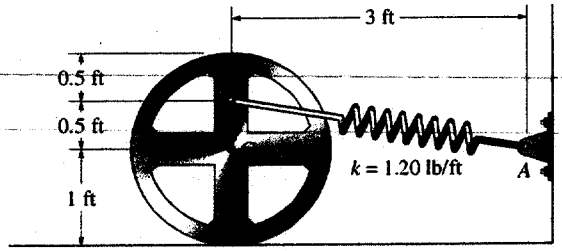
$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1(\frac{1.5}{2})$

$k = 42.8$ N/m **Ans**



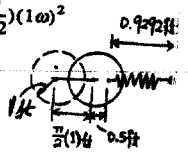
18-43. The 50-lb wheel has a radius of gyration about its center of gravity G of $k_G = 0.7$ ft. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 1.20$ lb/ft and an unstretched length of 0.5 ft. The wheel is released from rest.



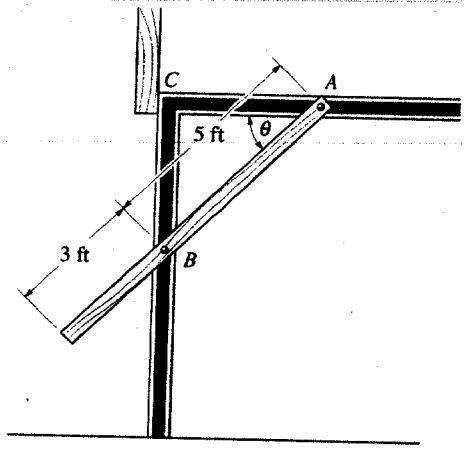
$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(1.20)[\sqrt{(3)^2 + (0.5)^2} - 0.5]^2 = \frac{1}{2}[\frac{50}{32.2}(0.7)^2]\omega^2 + \frac{1}{2}(\frac{50}{32.2})(1\omega)^2 + \frac{1}{2}(1.20)(0.9292 - 0.5)^2$

$\omega = 1.80$ rad/s **Ans**



***18-44.** The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a 180-lb thin plate having a width of 10 ft.

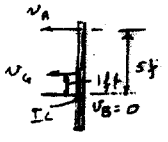


$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}[\frac{1}{12}(\frac{180}{32.2})(8)^2]\omega^2 + \frac{1}{2}(\frac{180}{32.2})(1\omega)^2 - 180(4)$

$\omega = 6.3776$ rad/s

$v_C = \omega(5) = 6.3776(5) = 31.9$ m/s **Ans**



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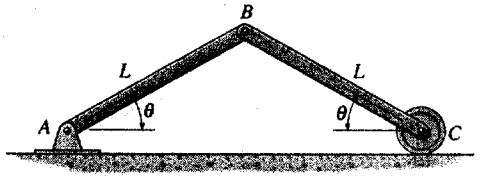
18-45. The two bars are released from rest at the position θ. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at C. Each bar has a mass m and length L.

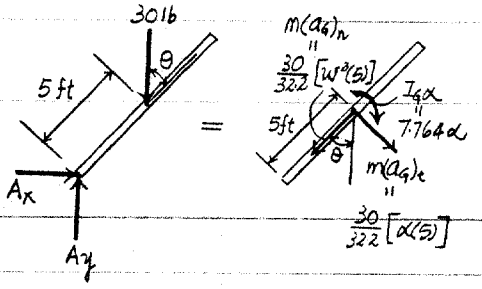


Potential Energy : Datum is set at point A . When links AB and BC is at their initial position, their center of gravity is located $\frac{L}{2}\sin\theta$ *above* the datum. Their gravitational potential energy at this position is $mg\left(\frac{L}{2}\sin\theta\right)$. Thus, the initial and final potential energies are

$$V_1 = 2\left(\frac{mgL}{2}\sin\theta\right) = mgL\sin\theta \qquad V_2 = 0$$

Kinetic Energy : When links AB and BC are in the horizontal position, then $v_B = \omega_{AB}L$ which is directed vertically downward since link AB is rotating about fixed point A. Link BC is subjected to general plane motion and its instantaneous center of zero velocity is located at point C. Thus, $v_B = \omega_{BC}r_{B/IC}$ or $\omega_{AB}L = \omega_{BC}L$, hence $\omega_{AB} = \omega_{BC} = \omega$. The mass moment inertia for link AB and BC about point A and C is $(I_{AB})_A = (I_{BC})_C = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$ $= \frac{1}{3}mL^2$. Since links AB and CD are at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

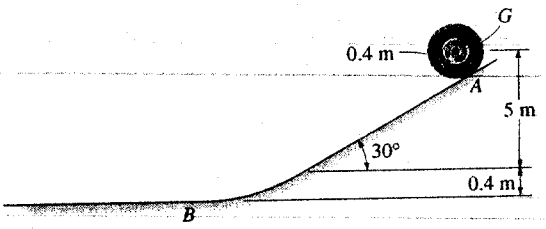
$$\begin{aligned} T_2 &= \frac{1}{2}(I_{AB})_A\,\omega_{AB}^2 + \frac{1}{2}(I_{BC})_C\,\omega_{BC}^2 \\ &= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 \\ &= \frac{1}{3}mL^2\omega^2 \end{aligned}$$



Conservation of Energy : Applying Eq. 18 – 18, we have

$$\begin{aligned} T_1 + V_1 &= T_2 + V_2 \\ 0 + mgL\sin\theta &= \frac{1}{3}mL^2\omega^2 + 0 \\ \omega_{AB} = \omega_{BC} = \omega &= \sqrt{\frac{3g}{L}\sin\theta} \end{aligned}$$ **Ans**

18-46. An automobile tire has a mass of 7 kg and radius of gyration $k_G = 0.3$ m. If it is released from rest at A on the incline, determine its angular velocity when it reaches the horizontal plane. The tire rolls without slipping.



$v_G = 0.4\omega$

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$$0 + 7(9.81)(5) = \frac{1}{2}(7)(0.4\omega)^2 + \frac{1}{2}\left[7(0.3)^2\right]\omega^2 + 0$$

$\omega = 19.8$ rad/s **Ans**

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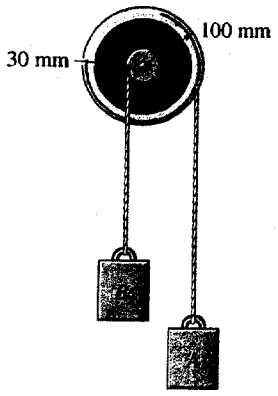
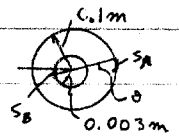
18-47. The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.



$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0+0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$$\theta = \frac{s_B}{0.03} = \frac{s_A}{0.1}$$



$s_B = 0.3\, s_A$

Set $s_A = 0.2$ m, $s_B = 0.06$ m

Substituting and solving yields,

$\omega = 14.04$ rad/s

$v_A = 0.1(14.04) = 1.40$ m/s **Ans**

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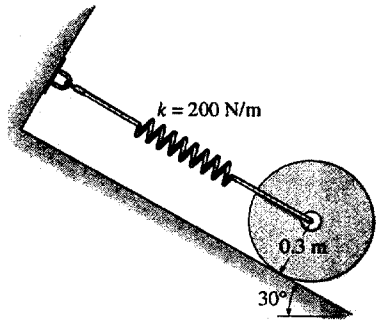
***18-48.** At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.



Datum at lowest point.

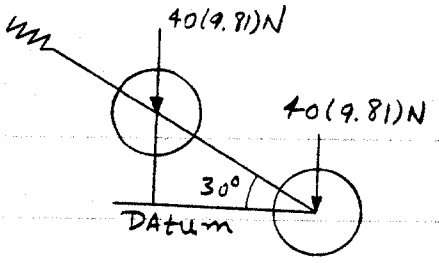
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{2}(40)(0.3)^2\right]\left(\frac{4}{0.3}\right)^2 + \frac{1}{2}(40)(4)^2 + 40(9.81)d\sin 30° = 0 + \frac{1}{2}(200)d^2$$

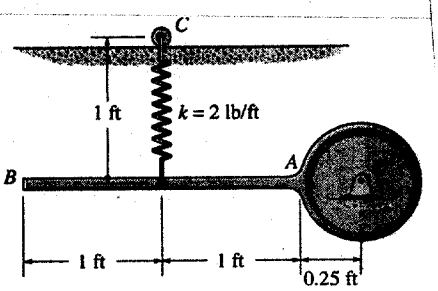
$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root

$d = 3.38$ m **Ans**



18-49. The pendulum consists of a 2-lb rod BA and a 6-lb disk. The spring is stretched 0.3 ft when the rod is horizontal as shown. If the pendulum is released from rest and rotates about point D, determine its angular velocity at the instant the rod becomes vertical. The roller at C allows the spring to remain vertical as the rod falls.



Potential Energy : Datum is set at point D. When rod AB is at vertical position, its center of gravity is located 1.25 ft *below* the datum. Its gravitational potential energy at this position is $-2(1.25)$ ft · lb. The initial and final stretch of the spring are 0.3 ft and $(1.25 + 0.3)$ ft $= 1.55$ ft, respectively. Hence, the initial and final elastic potential energy are $\frac{1}{2}(2)\left(0.3^2\right) = 0.09$ lb · ft and $\frac{1}{2}(2)\left(1.55^2\right)$ $= 2.4025$ lb · ft. Thus,

$$V_1 = 0.09 \text{ lb} \cdot \text{ft} \qquad V_2 = 2.4025 + [-2(1.25)] = -0.0975 \text{ lb} \cdot \text{ft}$$

Kinetic Energy : The mass moment inertia for rod AB and the disk about point D are $(I_{AB})_O = \frac{1}{12}\left(\frac{2}{32.2}\right)\left(2^2\right) + \left(\frac{2}{32.2}\right)\left(1.25^2\right) = 0.1178$ slug · ft^2 and $(I_D)_O$ $= \frac{1}{2}\left(\frac{6}{32.2}\right)\left(0.25^2\right) = 0.005823$ slug · ft^2. Since rod AB and the disk are initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$\begin{aligned} T_2 &= \frac{1}{2}(I_{AB})_O\,\omega^2 + \frac{1}{2}(I_D)_O\,\omega^2 \\ &= \frac{1}{2}(0.1178)\,\omega^2 + \frac{1}{2}(0.005823)\,\omega^2 \\ &= 0.06179\omega^2 \end{aligned}$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0.09 = 0.06179\omega^2 + (-0.0975)$$
$$\omega = 1.74 \text{ rad/s}$$ **Ans**

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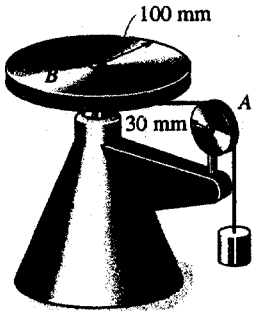
18-50. The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



$T_1 + V_1 = T_2 + V_2$

$[0 + 0 + 0] + [0] = \frac{1}{2}[\frac{1}{2}(3)(0.03)^2]\omega_A^2 + \frac{1}{2}[\frac{1}{2}(10)(0.1)^2]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$

$v_C = \omega_B(0.1) = 0.03\omega_A$

Thus,

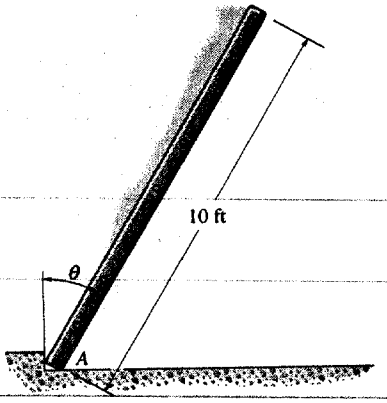
$\omega_B = 10\,v_C$

$\omega_A = 33.33 v_C$

Substituting and solving yeilds,

$v_C = 1.52$ m/s **Ans**

18-51. A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle θ at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



Potential Energy : Datum is set at point A. When the ladder is at its initial and final position, its center of gravity is located 5 ft and (5cos θ) ft *above* the datum. Its initial and final gravitational potential energy are $30(5) = 150$ ft · lb and $30(5\cos\theta) = 150\cos\theta$ ft · lb respectively. Thus, the initial and final potential energy are

$$V_1 = 150 \text{ ft} \cdot \text{lb} \qquad V_2 = 150\cos\theta \text{ ft} \cdot \text{lb}$$

Kinetic Energy : The mass moment inertia of the ladder about point A is $I_A = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) + \left(\frac{30}{32.2}\right)(5^2) = 31.06 \text{ slug} \cdot \text{ft}^2$. Since the ladder is initially at rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(31.06)\omega^2 = 15.53\omega^2$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 150 = 15.53\omega^2 + 150\cos\theta$$
$$\omega^2 = 9.66(1 - \cos\theta)$$

Equation of Motion : The mass moment inertia of the ladder about its mass center is $I_G = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) = 7.764 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17 – 16, we have

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -30\sin\theta(5) = -7.764\alpha - \left(\frac{30}{32.2}\right)[\alpha(5)](5)$$
$$\alpha = 4.83\sin\theta$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 30 = -\frac{30}{32.2}[9.66(1 - \cos\theta)(5)]\cos\theta - \frac{30}{32.2}[4.83\sin\theta(5)]\sin\theta$$

$$A_y = 30 - \frac{30}{32.2}(48.3\cos\theta - 48.3\cos^2\theta + 24.15\sin^2\theta)$$
$$= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5\sin^2\theta$$
$$= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5(1 - \cos^2\theta)$$
$$= 7.50(9\cos^2\theta - 6\cos\theta + 1)$$
$$= 7.50(3\cos\theta - 1)^2$$

If the ladder lifts off the ground, then $A_y = 0$. Thus,

$$7.50(3\cos\theta - 1)^2 = 0$$
$$\theta = 70.5° \qquad \textbf{Ans}$$

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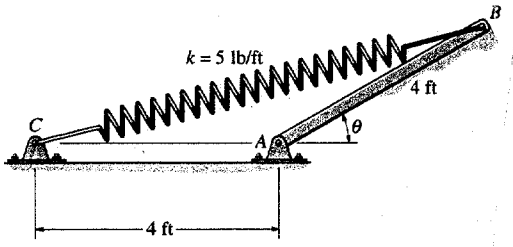
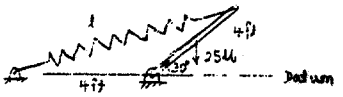
***18-52.** The 25-lb slender rod AB is attached to a spring BC which, has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$.



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

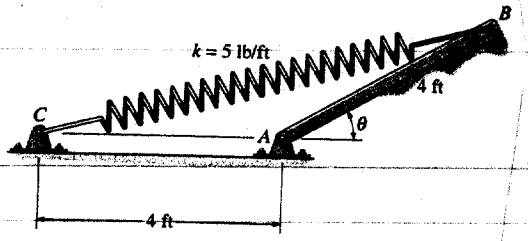


$$T_1 + V_1 = T_2 + V_2$$

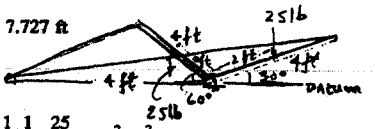
$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727-4)^2 = \frac{1}{2}[\frac{1}{3}(\frac{25}{32.2})(4)^2]\omega^2 + 25(2) + \frac{1}{2}(5)(4\sqrt{2}-4)^2$$

$$\omega = 1.18 \text{ rad/s} \qquad \textbf{Ans}$$

18-53. The 25-lb slender rod AB is attached to a spring BC which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

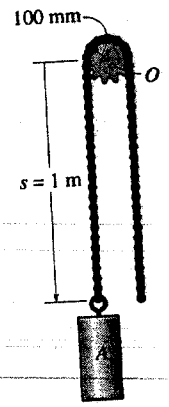


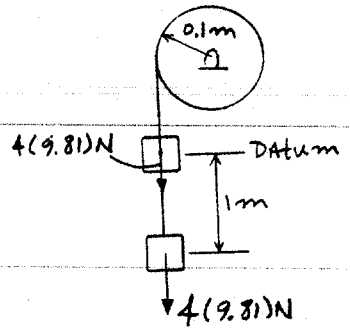
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727-4)^2 = \frac{1}{2}[\frac{1}{3}(\frac{25}{32.2})(4)^2]\omega^2 + 25(2)(\sin 60°) + 0$$

$$\omega = 2.82 \text{ rad/s} \qquad \textbf{Ans}$$

18-54. A chain that has a negligible mass is draped over the sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block A is released from rest in the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.





$T_1 + V_1 = T_2 + V_2$

$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}[2(0.05)^2]\omega^2 - 4(9.81)(1)$$

$$\omega = 41.8 \text{ rad/s} \qquad \textbf{Ans}$$

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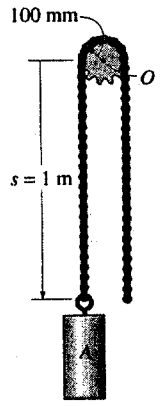
18-55. Solve Prob. 18-54 if the chain has a mass of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.

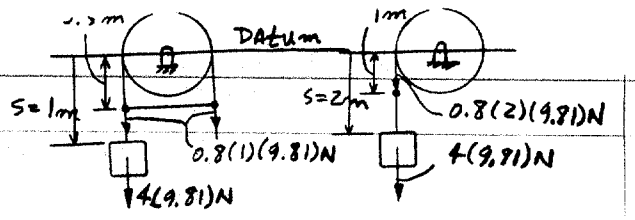


$T_1 + V_1 = T_2 + V_2$

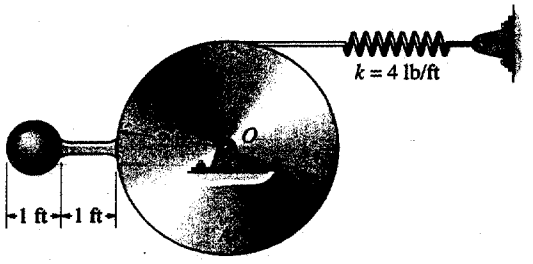
$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 + \frac{1}{2}(0.8)(2)(0.1\omega)^2$$

$$-4(9.81)(2) - 0.8(2)(9.81)(1)$$

$\omega = 39.3$ rad/s **Ans**



18-56. The disk A is pinned at O and weighs 15 lb. A 1-ft rod weighing 2 lb and a 1-ft-diameter sphere weighing 10 lb are welded to the disk, as shown. If the spring is orginally stretched 1 ft and the sphere is released from the position shown, determine the angular velocity of the disk when it has rotated 90°.



$T_1 + V_1 = T_2 + V_2$

$$[0+0+0] + \frac{1}{2}(4)(1)^2 = \frac{1}{2}\left[\frac{1}{2}\left(\frac{15}{32.2}\right)(2)^2\right]\omega^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{2}{32.2}\right)(1)^2\right]\omega^2 + \frac{1}{2}\left(\frac{2}{32.2}\right)(v_G)_R^2$$
$$+\frac{1}{2}\left[\frac{2}{5}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(v_G)_S^2 - 2(2.5) - 10(3.5) + \frac{1}{2}(4)\left(1+2\left(\frac{\pi}{2}\right)\right)^2$$

Since

$(v_G)_S = 3.5\omega$

$(v_G)_R = 2.5\omega$

Substituting and solving, yields

$\omega = 1.73$ rad/s **Ans**

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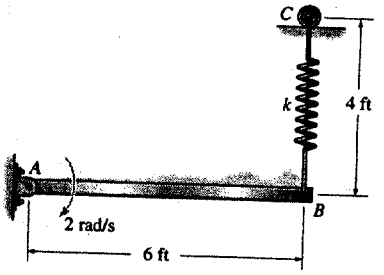
18-57. At the instant shown, the 50-lb bar is rotating downwards at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at *C*. If the spring has an unstretched length of 2 ft and a stiffness of $k = 6$ lb/ft, determine the angular velocity of the bar the instant it has rotated downward 30° below the horizontal.

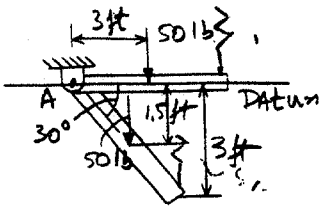


Datum through *A*.

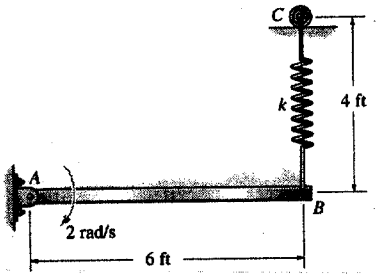
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(6)(4-2)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right]\omega^2 + \frac{1}{2}(6)(7-2)^2 - 50(1.5)$$

$\omega = 2.30$ rad/s **Ans**



18-58. At the instant shown, the 50-lb bar is rotating downwards at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at *C*. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle θ, measured below the horizontal, to which the bar rotates before it momentarily stops.



$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4+6\sin\theta-2)^2 - 50(3\sin\theta)$$

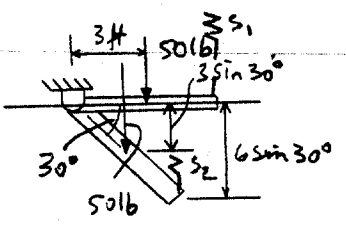
$$61.2671 = 24(1+3\sin\theta)^2 - 150\sin\theta$$

$$37.2671 = -6\sin\theta + 216\sin^2\theta$$

Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$\sin\theta = 0.4295$

$\theta = 25.4°$ **Ans**



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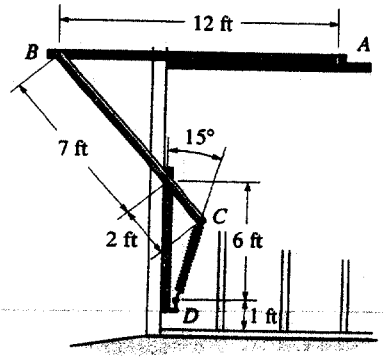
18-59. The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is similar connection and spring on the other side of the door.

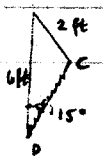


$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$

$CD^2 - 11.591CD + 32 = 0$

Selecting the smaller root:

$CD = 4.5352$ ft



$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 + 2[\frac{1}{2}(k)(8-4.5352)^2] - 200(6)$

$k = 100$ lb/ft **Ans**

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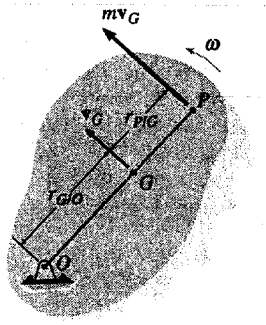
19-1. The rigid body (slab) has a mass m and is rotating with an angular velocity ω about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion,* which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.

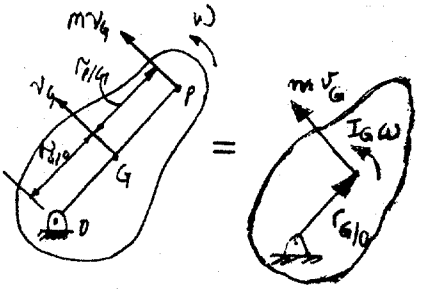


$$H_O = (r_{G/O} + r_{P/G}) m v_G = r_{G/O}(m v_G) + I_G \omega, \quad \text{where } I_G = m k_G^2$$

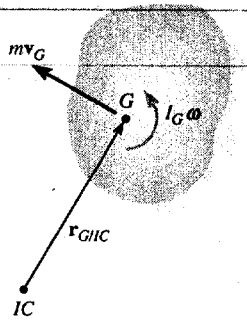
$$r_{G/O}(m v_G) + r_{P/G}(m v_G) = r_{G/O}(m v_G) + (m k_G^2)\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega} \quad \text{However, } v_G = \omega r_{G/O} \text{ or } r_{G/O} = \frac{v_G}{\omega}$$

$$r_{P/G} = \frac{k_G^2}{r_{G/O}} \quad \textbf{Q.E.D.}$$



19-2. At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



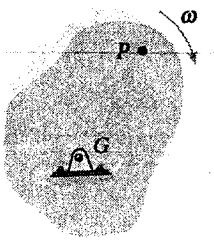
$$H_{IC} = r_{G/IC}(m v_G) + I_G \omega, \quad \text{where } v_G = \omega r_{G/IC}$$

$$= r_{G/IC}(m\omega r_{G/IC}) + I_G\omega$$

$$= (I_G + m r_{G/IC}^2)\omega$$

$$= I_{IC}\omega \quad \textbf{Q.E.D.}$$

19-3. Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P on the slab.



Since $v_G = 0$, the linear momentum $L = m v_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$. **Q.E.D.**

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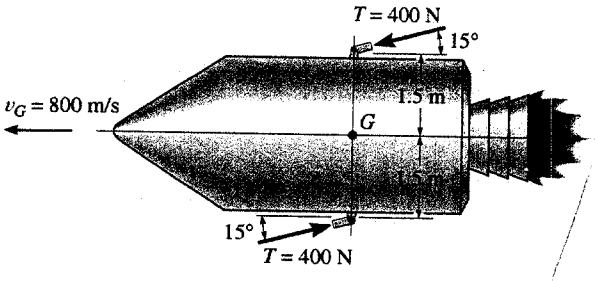
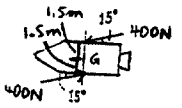
***19-4.** The space capsule has a mass of 1200 kg and a moment of inertia $I_G = 900\ \text{kg}\cdot\text{m}^2$ about an axis passing through G and directed perpendicular to the page. If it is traveling forward with a speed $v_G = 800$ m/s and executes a turn by means of two jets, which provide a constant thrust of 400 N for 0.3 s, determine the capsule's angular velocity just after the jets are turned off.





$(+)\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$0 + 2[400\cos 15°(0.3)(1.5)] = 900\,\omega_2$

$\omega_2 = 0.386$ rad/s **Ans**

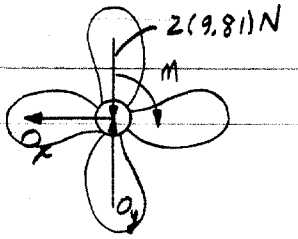
19-5. Solve Prob. 17-55 using the principle of impulse and momentum.

$(\overset{+}{\curvearrowleft})\quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$0 + \int_0^4 3(1 - e^{-0.2t})\,dt = (0.18)\omega$

$3(t + 5e^{-0.2t})\Big|_0^4 = 0.18\omega$

$\omega = 20.8$ rad/s **Ans**



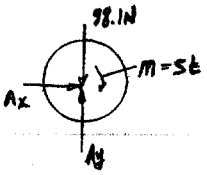
19-6. Solve Prob. 17–54 using the principle of impulse and momentum.

$(H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

$0 + \int_0^3 5t\,dt = [10(0.2)^2]\omega$

$\frac{5}{2}(3)^2 = 0.4\omega$

$\omega = 56.2$ rad/s **Ans**



$m(v_G)_1 + \Sigma\int F\,dt = m(v_G)_2$

$(\overset{+}{\rightarrow})\quad 0 + A_x(3) = 0$

$A_x = 0$ **Ans**

$(+\uparrow)\quad 0 + A_y(3) - 98.1(3) = 0$

$A_y = 98.1$ N **Ans**

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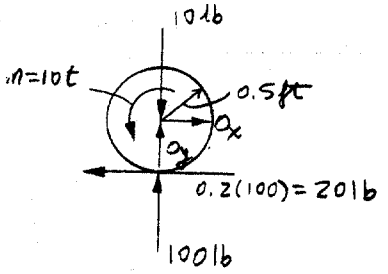
19-7. Solve Prob. 17-69 using the principle of impulse and momentum.



Time to start motion :

$F = 100(0.3) = 30$ lb

$+\Sigma M_O = 0; \quad 10t - 30(0.5) = 0$

$t = 1.5$ s

$(\circlearrowleft +) \quad (H_O)_1 + \Sigma \int M_O \, dt = (H_O)_2$

$$0 + \int_{1.5}^{2} 10t \, dt - 20(2 - 1.5)(0.5) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega$$

$$5t^2\Big|_{1.5}^{2} - 5 = 0.038820\omega$$

$\omega = 96.6$ rad/s **Ans**

***19-8.** Solve Prob. 17-80 using the principle of impulse and momentum.

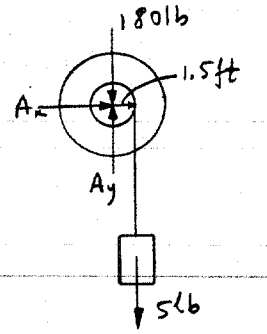
System :

$v_B = \omega(1.5)$

$(\circlearrowleft +) \quad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$

$$0 + 5(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega + \left(\frac{5}{32.2}\right)[\omega(1.5)](1.5)$$

$\omega = 2.48$ rad/s **Ans**

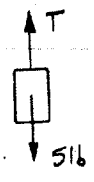


Block :

$v_B = \omega(1.5)$

$(+\downarrow) \quad m(v_G)_1 + \Sigma \int F_y \, dt = m(v_G)_2$

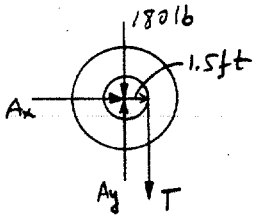
$$0 + 5(3) - T(3) = \frac{5}{32.2}[\omega(1.5)] \qquad (1)$$



Spool :

$(\circlearrowleft +) \quad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$

$$0 + T(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega \qquad (2)$$



Solving Eqs. (1) and (2) :

$T = 4.81$ lb

$\omega = 2.48$ rad/s **Ans**

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19-9. Solve Prob. 17-73 using the principle of impulse and momentum.

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_{Gx})_1 + \Sigma \int F_x\, dt = m(v_{Gx})_2$$

$$0 + T_{BC}\sin 30^\circ (t) - N_A t = 0$$

$$0.5T_{BC} = N_A$$

$$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y\, dt = m(v_{Gy})_2$$

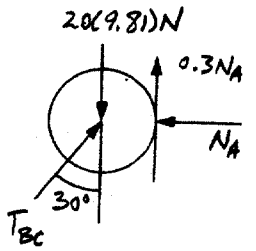
$$0 + T_{BC}\cos 30^\circ t - 20(9.81)t + 0.3N_A t = 0$$

$$0.86603T_{BC} + 0.3N_A = 196.2$$

Solving:

$T_{BC} = 193$ N **Ans**

$N_A = 96.553$ N



$$(\circlearrowleft +) \qquad (H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

$$\left[\frac{1}{2}(20)(0.15)^2\right](60) - 0.3(96.553)t(0.15) = 0$$

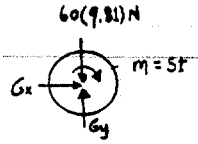
$t = 3.11$ s **Ans**

19-10. A flywheel has a mass of 60 kg and a radius of gyration of $k_G = 150$ mm about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude of $M = (5t)$ N·m, where t is in seconds, determine the flywheel's angular velocity in $t = 3$ s. Initially the flywheel is rotating clockwise at $\omega_1 = 2$ rad/s.

$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M\, dt = (H_G)_2$$

$$60(0.15)^2(2) + \int_0^3 5t\, dt = 60(0.15)^2\omega$$

$\omega = 18.7$ rad/s **Ans**



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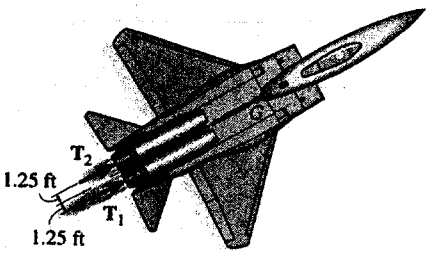
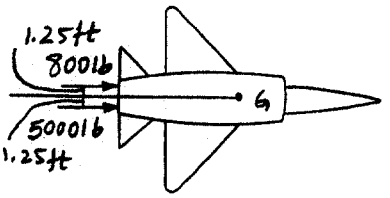
19-11. The pilot of a crippled F-15 fighter was able to control his plane by throttling the two engines. If the plane has a weight of 17 000 lb and a radius of gyration of $k_G = 4.7$ ft about the mass center G, determine the angular velocity of the plane and the velocity of its mass center G in $t = 5$ s if the thrust in each engine is altered to $T_1 = 5000$ lb and $T_2 = 800$ lb as shown. Originally the plane is flying straight at 1200 ft/s. Neglect the effects of drag and the loss of fuel.



$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

$$0 + 5000(5)(1.25) - 800(5)(1.25) = \left[\left(\frac{17\,000}{32.2}\right)(4.7)^2\right]\omega$$

$$\omega = 2.25 \text{ rad/s} \qquad \textbf{Ans}$$

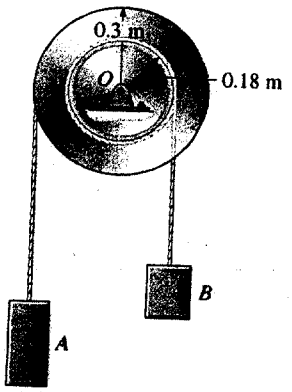


$$\left(\stackrel{+}{\rightarrow}\right) \qquad m(v_{Gx})_1 + \Sigma \int F_x\, dt = m(v_{Gx})_2$$

$$\left(\frac{17\,000}{32.2}\right)(1200) + 5800(5) = \left(\frac{17\,000}{32.2}\right)(v_G)_2$$

$$(v_G)_2 = 1.25(10^3) \text{ ft/s} \qquad \textbf{Ans}$$

***19-12.** The spool has a mass of 30 kg and a radius of gyration $k_O = 0.25$ m. Block A has a mass of 25 kg, and block B has a mass of 10 kg. If they are released from rest, determine the time required for block A to attain a speed of 2 m/s. Neglect the mass of the ropes.



$$v_A = 2 \text{ m/s}$$

$$\omega = \frac{2}{0.3} = 6.667 \text{ rad/s}$$

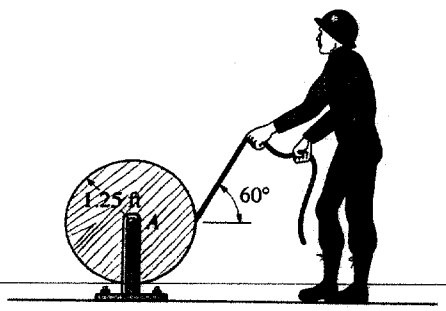
$$v_B = 6.667(0.18) = 1.20 \text{ m/s}$$

$$(\circlearrowleft +) \quad (H_O)_1 + \Sigma \int M\, dt = (H_O)_2$$

$$0 + 25(9.81)(0.3)\,t - 10(9.81)(0.18)(t) = 25(2)(0.3) + 30(0.25)^2(6.667) + 10(1.20)(0.$$

$$t = 0.530 \text{ s} \qquad \textbf{Ans}$$

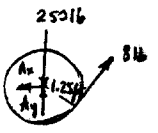
19-13. The man pulls the rope off the reel with a constant force of 8 lb in the direction shown. If the reel has a weight of 250 lb and radius of gyration $k_G = 0.8$ ft about the trunnion (pin) at A, determine the angular velocity of the reel in 3 s starting from rest. Neglect friction and the weight of rope that is removed.



$$(\circlearrowleft +) \quad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$$

$$0 + 8(1.25)(3) = \left[\frac{250}{32.2}(0.8)^2\right]\omega$$

$$\omega = 6.04 \text{ rad/s} \qquad \textbf{Ans}$$



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19-14. Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at a constant rate of 16 rad/s, and the coefficient of kinetic friction between the wheels is $\mu_k = 0.2$, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force of 50 N. What is the final angular velocity of wheel B? Wheel B has a mass of 90 kg and a radius of gyration about its axis of rotation of $k_G = 120$ mm.

$F = 0.2(50) = 10\ \text{N}$

$(\zeta+) \quad (H_O)_1 + \Sigma \int M_O dt = (H_O)_2$

$0 + (10)(0.09)\, t = [90(0.120)^2]\omega$

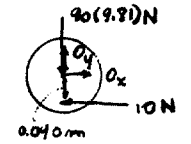
$\omega_A r_A = \omega_B r_B$

$16(20) = \omega_B(90)$

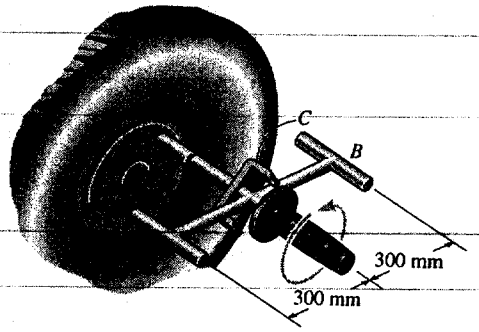
$\omega_B = 3.556\ \text{rad/s} = 3.56\ \text{rad/s}$ **Ans**

Thus,

$t = 5.12\ \text{s}$ **Ans**



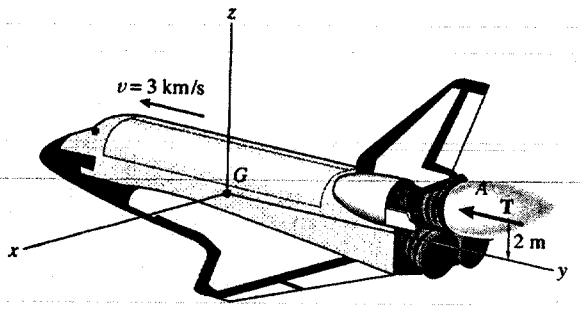
19-15. The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to turn about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



$$I_{axle} = \frac{1}{12}(1)(0.6-0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081\ \text{kg}\cdot\text{m}^2$$

$$\int M dt = I_{axle}\omega = 0.2081(4) = 0.833\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans**

***19-16.** The space shuttle is located in "deep space," where the effects of gravity can be neglected. It has a mass of 120 Mg, a center of mass at G, and a radius of gyration $(k_G)_x = 14$ m about the x axis. It is originally traveling forward at $v = 3$ km/s when the pilot turns on the engine at A, creating a thrust $T = 600(1 - e^{-0.3t})$ kN, where t is in seconds. Determine the shuttle's angular velocity 2 s later.

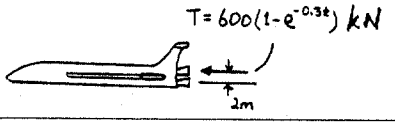


$(\zeta+) \quad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$

$$0 + \int_0^2 600(10^3)(1 - e^{-0.3t})(2)\, dt = \left[120(10^3)(14)^2\right]\omega$$

$$1200(10^3)\left[t + \frac{1}{0.3}e^{-0.3t}\right]_0^2 = 120(10^3)(14)^2\omega$$

$\omega = 0.0253\ \text{rad/s}$ **Ans**



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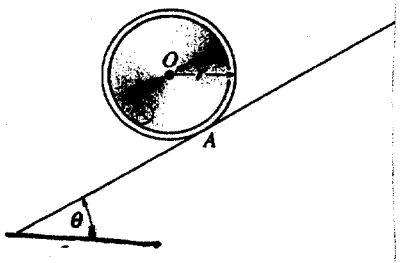
19-17. The drum has a mass of 70 kg, a radius of 300 mm, and radius of gyration $k_O = 125$ mm. If the coefficients of static and kinetic friction at A are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the drum's angular velocity 2 s after it is released from rest. Take $\theta = 30°$.



$(\zeta +)$ $(H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$

$0 + 70(9.81)(\sin 30°)(0.3)(2) = (70(0.125)^2 + 70(0.3)^2)\omega$

$\omega = 27.863$ rad/s $= 27.9$ rad/s **Ans**

$m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$

$+\nearrow$ $0 + 70(9.81)\sin 30°(2) - F(2) = 70(27.863)(0.3)$

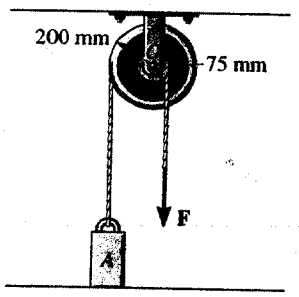
$F = 50.79$ N

$+\nwarrow$ $0 + N_D(2) - 70(9.81)\cos 30°(2) = 0$

$N_D = 594.7$ N

$F_{max} = 0.4[(594.7] = 237.87$ N > 50.79 N **OK**

19-18. The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration $k_O = 110$ mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force $F = 2$ kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.

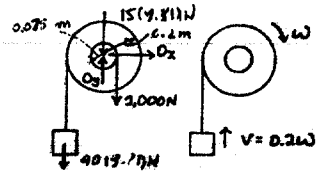


$(\zeta +)$ $(H_O)_1 + \Sigma \int M_O dt = (H_O)_2$

$0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2\omega + 40(0.2\omega)(0.2)$

$\omega = 120.4$ rad/s

$v_A = 0.2(120.4) = 24.1$ m/s **Ans**



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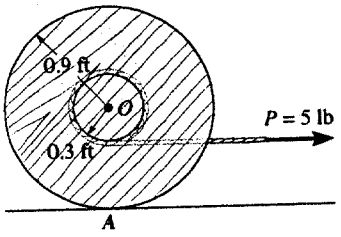
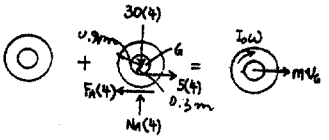
19-19. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around its inner hub and the end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.



$(+\uparrow)$ $\quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$0 + N_A(4) - 30(4) = 0$

$N_A = 30$ lb



$(\overset{+}{\rightarrow})$ $\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$0 + 5(4) - F_A(4) = \dfrac{30}{32.2}v_G$

$(\overset{+}{\circlearrowleft})$ $\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$0 + F_A(4)(0.9) - 5(4)(0.3) = \dfrac{30}{32.2}(0.45)^2\,\omega$

Since no slipping occurs

Set $v_G = 0.9\omega$

$F_A = 2.33$ lb

$\omega = 12.7$ rad/s **Ans**

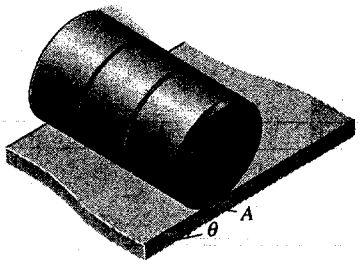
Also,

$(\overset{+}{\circlearrowleft})$ $\quad (H_A)_1 + \Sigma M_A\,dt = (H_A)_2$

$0 + 5(4)(0.6) = \left[\dfrac{30}{32.2}(0.45)^2 + \dfrac{30}{32.2}(0.9)^2\right]\omega$

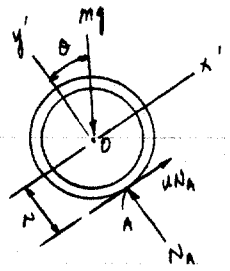
$\omega = 12.7$ rad/s **Ans**

***19-20.** The drum of mass m, radius r, and radius of gyration k_O rolls along an inclined plane for which the coefficient of static friction is μ_s. If the drum is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping.



$(+\nwarrow)$ $\quad m(v_{Oy'})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Oy'})_2$

$0 + N_A(t) - mg\cos\theta(t) = 0 \qquad N_A = mg\cos\theta$



$(+\swarrow)$ $\quad m(v_{Ox'})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ox'})_2$

$0 + mg\sin\theta(t) - \mu_s mg\cos\theta(t) = mv_O \qquad (1)$

Since no slipping occurs, $v_O = \omega r$. Hence Eq.(1) becomes

$mg\sin\theta(t) - \mu_s mg\cos\theta(t) = m\omega r$

$t = \dfrac{m\omega r}{mg(\sin\theta - \mu_s\cos\theta)} \qquad (2)$

$(\overset{+}{\circlearrowleft})$ $\quad I_A\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$

$0 + mg\sin\theta(r)(t) = \left[mk_O^2 + mr^2\right]\omega$

$t = \dfrac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta} \qquad (3)$

Equating Eqs.(2) and (3)

$\dfrac{m\omega r}{mg(\sin\theta - \mu_s\cos\theta)} = \dfrac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta}$

$\theta = \tan^{-1}\left[\dfrac{\mu_s(k_O^2 + r^2)}{k_O^2}\right]$ **Ans**

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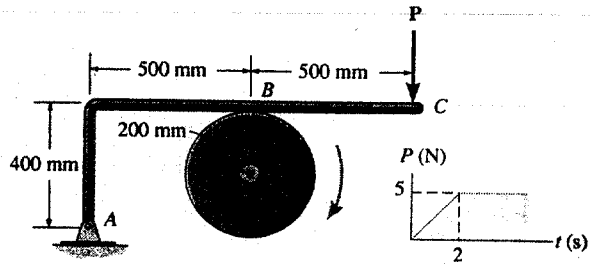
19-21. The 12-kg disk has an angular velocity of $\omega = 20$ rad/s. If the brake ABC is applied such that the magnitude of force **P** varies with time as shown, determine the time needed to stop the disk. The coefficient of kinetic friction at B is $\mu_k = 0.4$.



Equations of Equilibrium : Since slipping occurs at B, the friction $F_f = \mu_k N_B = 0.4N_B$. From FBD(a), the normal reaction N_B can be obtained directly by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \qquad N_B(0.5) - 0.4N_B(0.4) - P(1) = 0$$
$$N_B = 2.941P$$

Thus, the friction $F_f = 0.4N_B = 0.4(2.941P) = 1.176P$.

Principle of Impulse and Momentum : The mass moment inertia of the cylinder about its mass center is $I_O = \frac{1}{2}(12)(0.2^2) = 0.240\ \text{kg}\cdot\text{m}^2$. Applying Eq. 19 – 14, we have

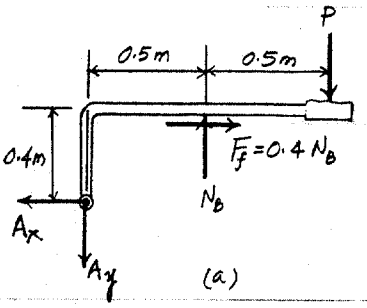
$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

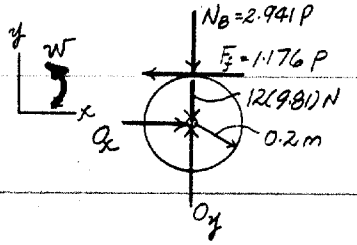
$$(\zeta+) \qquad -0.240(20) + \left(1.176\int_0^t P\,dt\right)(0.2) = 0 \qquad [1]$$

However, $\int_0^t P\,dt$ is the area under the $P-t$ graph. Assuming $t > 2$ s, then $\int_0^t P\,dt = \frac{1}{2}(5)(2) + 5(t-2) = (5t-5)$ N · s. Substitute into Eq. [1] yields

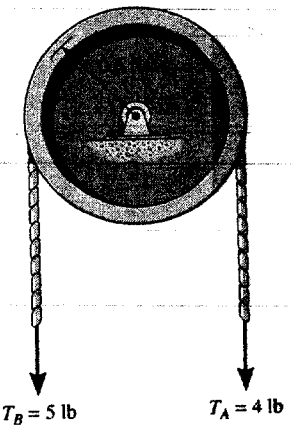
$$-0.240(20) + 1.176(5t-5)(0.2) = 0$$
$$t = 5.08\ \text{s} \qquad \textbf{Ans}$$

Since $t = 5.08$ s > 2 s, the above assumption is correct.





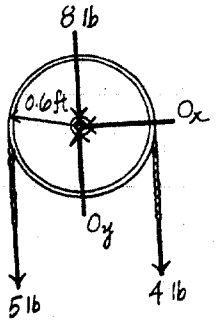
19-22. The pulley has a weight of 8 lb and may be treated as a thin disk. A cord wrapped over its surface is subjected to forces $T_A = 4$ lb and $T_B = 5$ lb. Determine the angular velocity of the pulley when $t = 4$ s if it starts from rest when $t = 0$. Neglect the mass of the cord.



Principle of Impulse and Momentum : The mass moment inertia of the pulley about its mass center is $I_O = \frac{1}{2}\left(\frac{8}{32.2}\right)(0.6^2) = 0.04472\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19 – 14, we have

$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$(\zeta+) \qquad 0 + [5(4)](0.6) - [4(4)](0.6) = 0.04472\omega_2$$
$$\omega_2 = 53.7\ \text{rad/s} \qquad \textbf{Ans}$$



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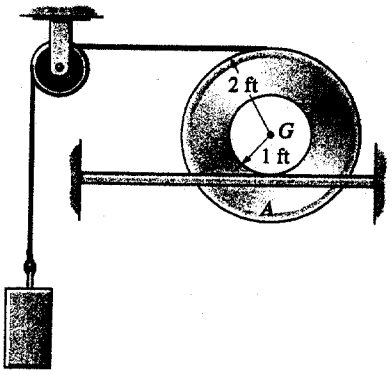
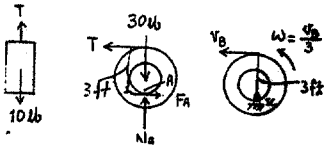
19-23. The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.



Spool,



$$(\zeta+) \quad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$$

$$0 + T(3)(2) = \left[\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2\right]\left(\frac{v_B}{3}\right)$$

Block,

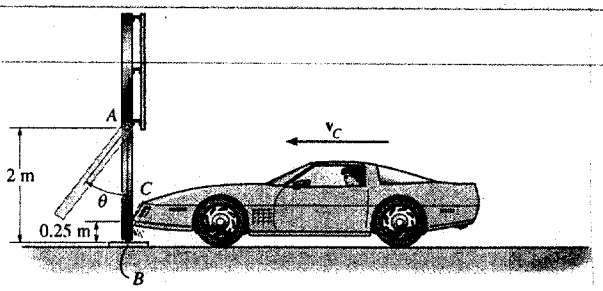
$$(+\downarrow) \quad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$$

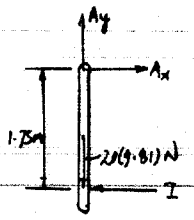
$$0 + 10(2) - T(2) = \frac{10}{32.2} v_B$$

$$v_B = 34.0 \text{ ft/s} \qquad \textbf{Ans}$$

$$T = 4.73 \text{ lb}$$

***19-24.** For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at B when the leg is subjected to the impact of a car. Assuming that the leg is pin supported at A and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate upward to an angle of $\theta_{max} = 150°$.





$$(\zeta+) \quad I_A \omega_1 + \Sigma \int_{t_1}^{t_2} M_A\, dt = I_A \omega_2$$

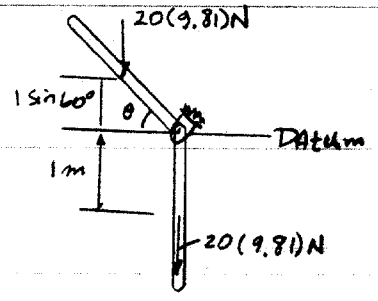
$$0 + I(1.75) = \left[\frac{1}{3}(20)(2)^2\right]\omega_2$$

$$\omega_2 = 0.065625I$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{1}{3}(20)(2)^2\right](0.065625I)^2 + 20(9.81)(-1) = 0 + 20(9.81)(1\sin 60°)$$

$$I = 79.8 \text{ N} \cdot \text{s} \qquad \textbf{Ans}$$



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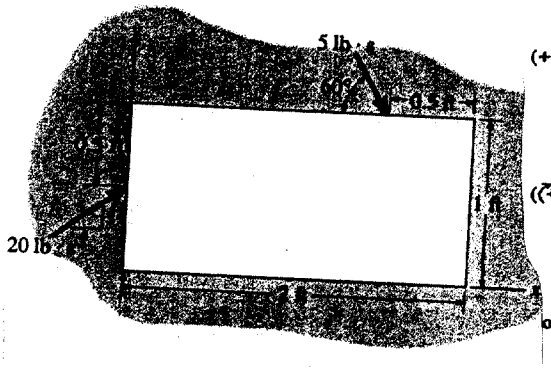
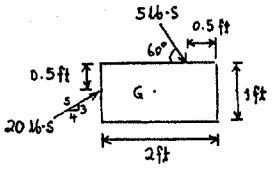
19-25. The 10-lb rectangular plate is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.





$$(\overset{+}{\rightarrow}) \quad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$$

$$0 + 20\left(\frac{4}{5}\right) + 5\cos 60° = \frac{10}{32.2}(v_G)_x$$

$$(+\uparrow) \quad m(v_{Gy})_1 + \Sigma\int F_y\,dt = m(v_{Gy})_2$$

$$0 + 20\left(\frac{3}{5}\right) - 5\sin 60° = \frac{10}{32.2}(v_G)_y$$

$$(\overset{+}{\circlearrowleft}) \quad I_G\omega_1 + \Sigma\int M\,dt = I_G\omega_2$$

$$0 + 20\left(\frac{3}{5}\right)(1) + 5\sin 60°(0.5) + 5\cos 60°(0.5) = \frac{1}{12}\left(\frac{10}{32.2}\right)[(1)^2 + (2)^2]\omega$$

Solving,

$$\omega = 119 \text{ rad/s} \qquad \text{Ans}$$

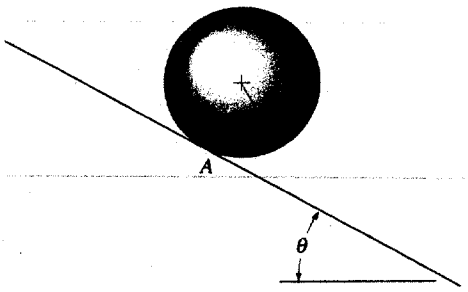
$$(v_G)_x = 59.6 \text{ ft/s}$$

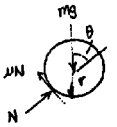
$$(v_G)_y = 24.7 \text{ ft/s}$$

$$v_G = \sqrt{(59.6)^2 + (24.7)^2} = 64.5 \text{ ft/s} \qquad \text{Ans}$$

$$\theta = \tan^{-1}\frac{24.7}{59.6} = 22.5° \quad \measuredangle\theta$$

19-26. The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.





$$+\nearrow \quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$$

$$0 + N(t) - (mg\cos\theta)t = 0$$

$$\searrow+ \quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$0 + (mg\sin\theta)t - \mu Nt = m(r\omega)$$

$$(\overset{+}{\circlearrowleft}) \quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$$

$$0 + \mu Nrt = \left(\frac{2}{5}mr^2\right)\omega$$

$$N = mg\cos\theta$$

$$mg\sin\theta\, t - \mu mg\cos\theta t = mr\omega$$

$$t = \frac{r\omega}{g(\sin\theta - \mu\cos\theta)}$$

$$\mu(mg\cos\theta)r\left(\frac{r\omega}{g(\sin\theta - \mu\cos\theta)}\right) = \frac{2}{5}mr^2\omega$$

$$\mu\cos\theta = \frac{2}{5}(\sin\theta - \mu\cos\theta)$$

$$3.5\mu\cos\theta = \sin\theta$$

$$\tan\theta = 3.5\mu$$

$$\theta = \tan^{-1}(3.5\mu) \qquad \text{Ans}$$

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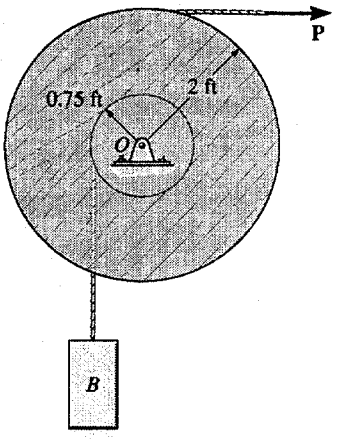
19-27. The spool has a weight of 75 lb and a radius of gyration $k_O = 1.20$ ft. If the block B weighs 60 lb, and a force $P = 25$ lb is applied to the cord, determine the speed of the block in 5 s starting from rest. Neglect the mass of the cord.

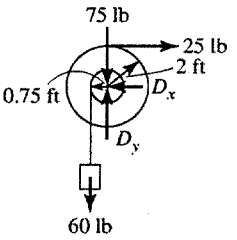


$$\circlearrowleft + \quad (H_O)_1 + \sum \int M_O dt = (H_O)_2$$

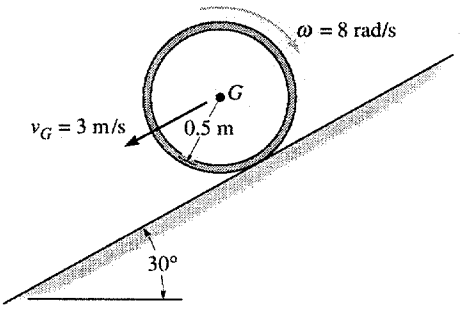
$$0 - 60(0.75)(5) + 25(2)(5) = \frac{75}{32.2}(1.20)^2\omega + \left[\frac{60}{32.2}(0.75\omega)\right](0.75)$$

$$\omega = 5.679 \text{ rad/s}$$

$$v_B = \omega r = (5.679)(0.75) = 4.26 \text{ ft/s} \qquad \textbf{Ans}$$



***19-28.** The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine how long the hoop rolls before it stops slipping.



$$\swarrow + \quad mv_{x1} + \sum \int F_x dt = mv_{x2}$$

$$5(3) + 49.05 \sin 30°(t) - 25.487t = 5v_G$$

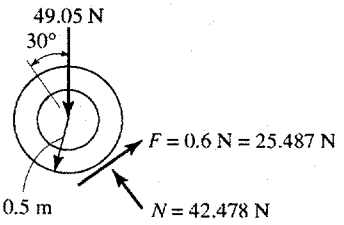
$$\circlearrowleft + \quad (H_G)_1 + \sum \int M_G dt = (H_G)_2$$

$$-5(0.5)^2(8) + 25.487(0.5)(t) = 5(0.5)^2\left(\frac{v_G}{0.5}\right)$$

Solving,

$$v_G = 2.75 \text{ m/s}$$

$$t = 1.32 \text{ s} \qquad \textbf{Ans}$$



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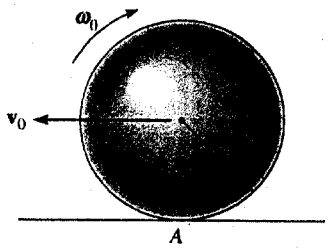
19-29. If the ball has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_0$ parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of friction at A for the calculation.

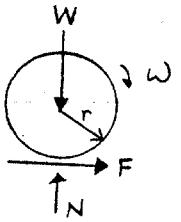


$$\left(\overset{+}{\leftarrow}\right) \quad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$$

$$\frac{W}{g} v_0 - Ft = 0 \qquad (1)$$

$$(\curvearrowleft +) \quad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

$$-\frac{2}{5}\left(\frac{W}{g} r^2\right)\omega_0 + Ft(r) = 0 \qquad (2)$$



Eliminate Ft between Eqs. (1) and (2):

$$\frac{2}{5}\left(\frac{W}{g} r^2\right)\omega_0 = \left[\frac{W}{g}\left(\frac{v_0}{t}\right)\right] t(r)$$

$$\omega_0 = 2.5\left(\frac{v_0}{r}\right) \qquad \textbf{Ans}$$

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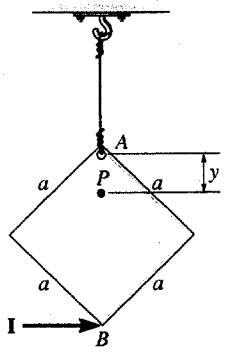
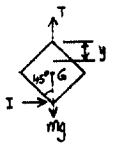
19-30. The square plate has a mass m and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y of the point P about which the plate appears to rotate during the impact.



$$(\circlearrowleft+) \quad (H_G)_1 + \Sigma \int M_G dt = (H_G)_2$$

$$0 + I\left(\frac{a}{\sqrt{2}}\right) = \frac{m}{12}(a^2 + a^2)\omega$$

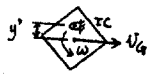


$$(\overset{+}{\rightarrow}) \quad m(v_{Gx})_1 + \Sigma \int F_x dt = m(v_{Gx})_2$$

$$0 + I = mv_G$$

$$\omega = \frac{6I}{\sqrt{2}am}$$

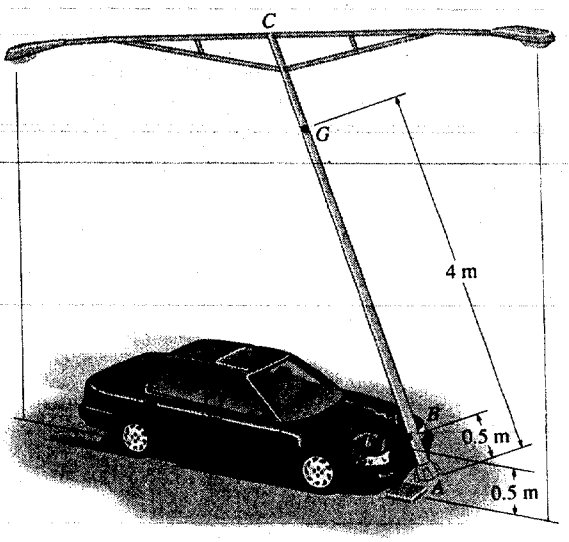
$$v_G = \frac{I}{m}$$

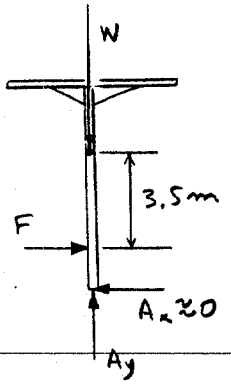


$$y' = \frac{v_G}{\omega} = \frac{\frac{I}{m}}{\frac{6I}{\sqrt{2}am}} = \frac{\sqrt{2}a}{6}$$

$$y = \frac{3\sqrt{2}}{6}a - \frac{\sqrt{2}}{6}a = \frac{\sqrt{2}}{3}a \qquad \textbf{Ans}$$

19-31. The car strikes the side of a light standard, which is designed to break away from its base with negligible resistance. From a video taken of the collision it is observed that the pole was given an angular velocity of 60 rad/s when AC was vertical. The pole has a mass of 175 kg, a center of mass at G, and a radius of gyration about an axis perpendicular to the plane of the pole assembly and passing through G of $k_G = 2.25$ m. Determine the horizontal impulse which the car exerts on the pole while AC is essentially vertical.





$$(\circlearrowleft+) \quad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

$$0 + \left[\int F\, dt\right](3.5) = 175(2.25)^2(60)$$

$$\int F\, dt = 15.2\ \text{kN} \cdot \text{s} \qquad \textbf{Ans}$$

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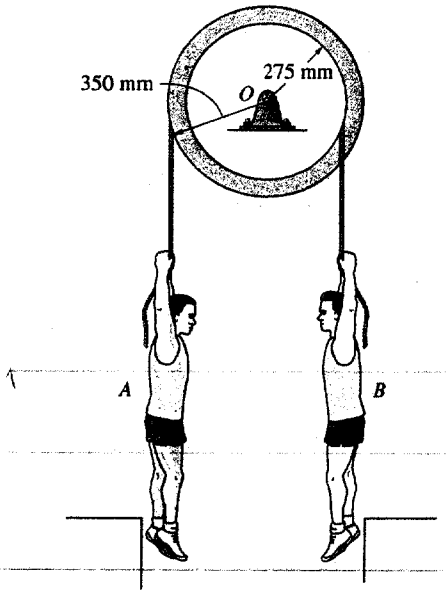
***19-32.** The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 30 kg and a radius of gyration $k_O = 250$ mm. If two men A and B grab the suspended ropes and step off the ledges at the same time, determine their speeds in 4 s starting from rest. The men A and B have a mass of 60 kg and 70 kg, respectively. Assume they do not move relative to the rope during the motion. Neglect the mass of the rope.



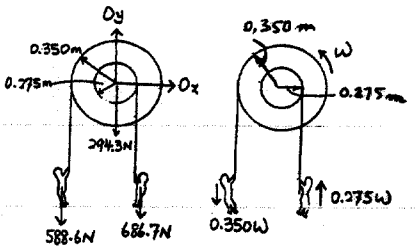
$(+ \quad (H_O)_1 + \Sigma \int M_O dt = (H_O)_2$

$0 + 588.6(0.350)(4) - 686.7(0.275)(4) = 30(0.25)^2\omega + 60(0.35\omega)(0.35) + 70(0.275\omega)(0.275$

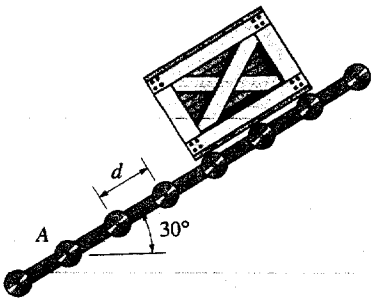
$\omega = 4.73$ rad/s

$v_A = 0.35(4.73) = 1.66$ m/s Ans

$v_B = 0.275(4.73) = 1.30$ m/s Ans



19-33. The crate has a mass m_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have a radius of r, mass m, and are spaced d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

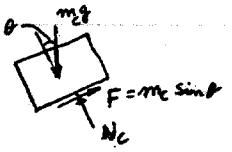


The number of rollers per unit length is $1/d$.

Thus in one second, $\frac{v_0}{d}$ rollers are contacted.

If a roller is brought to full angular speed of $\omega = \frac{v_0}{r}$ in t_0 seconds,

then the moment of inertia that is effected is



$I' = I(\frac{v_0}{d})(t_0) = (\frac{1}{2}mr^2)(\frac{v_0}{d})t_0$

Since the frictional impluse is

$F = m_c \sin\theta$ then

$(+ \quad (H_G)_1 + \Sigma \int M_G dt = (H_G)_2$

$0 + (m_c \sin\theta)\, r t_0 = \left[\left(\frac{1}{2}mr^2\right)\left(\frac{v_0}{d}\right)t_0\right]\left(\frac{v_0}{r}\right)$

$v_0 = \sqrt{(2g \sin\theta\, d)(\frac{m_c}{m})}$ Ans

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19-34. Two wheels A and B have masses m_A and m_B, and radii of gyration about their central vertical axes of k_A and k_B, respectively. If they are freely rotating in the same direction at ω_A and ω_B about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

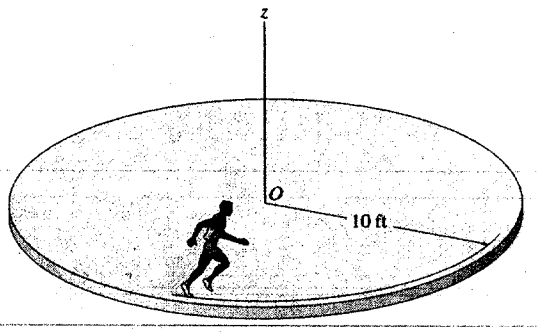
$(\Sigma \text{Syst. Ang. Mom.})_1 = (\Sigma \text{Syst. Ang. Mom.})_2$

$$(m_A k_A^2)\omega_A + (m_B k_B^2)\omega_B = (m_A k_A^2)\omega'_A + (m_B k_B^2)\omega'_B$$

Set $\omega'_A = \omega'_B = \omega$, then

$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2} \qquad \textbf{Ans}$$

19-35. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he has a speed of 4 ft/s and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



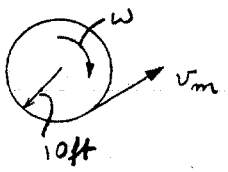
$\mathbf{v}_m = \mathbf{v}_p + \mathbf{v}_{m/p}$

$\left(\overset{+}{\rightarrow}\right)$ $v_m = -10\omega + 4$

$(\circlearrowleft +)$ $(H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{300}{32.2}\right)(8)^2\omega + \left(\frac{150}{32.2}\right)(-10\omega + 4)(10)$$

$\omega = 0.175$ rad/s **Ans**



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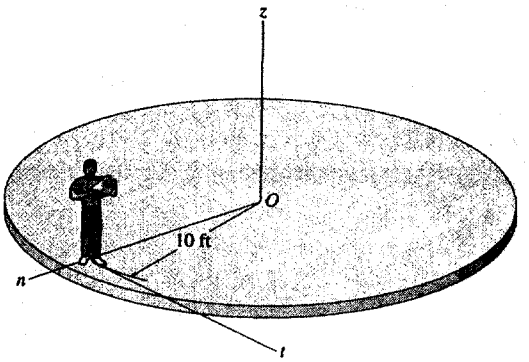
***19-36.** A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb throws a 15-lb block off the edge of the platform with a horizontal velocity of 5 ft/s, *measured relative to the platform*. Determine the angular velocity of the platform if the block is thrown (a) tangent to the platform, along the $+t$ axis, and (b) outward along a radial line, or $+n$ axis. Neglect the size of the man.



$$0+0=\left(\frac{15}{32.2}\right)(v_b)(10)-\left(\frac{300}{32.2}\right)(8)^2\omega-\left(\frac{150}{32.2}\right)(10\omega)(10)$$

$$v_b = 228\omega$$

$$\mathbf{v}_b = \mathbf{v}_m + \mathbf{v}_{b/m}$$

$$\left(\overset{+}{\rightarrow}\right) \quad v_b = -10\omega + 5$$

$$228\omega = -10\omega + 5$$

$$\omega = 0.0210 \text{ rad/s} \qquad \textbf{Ans}$$

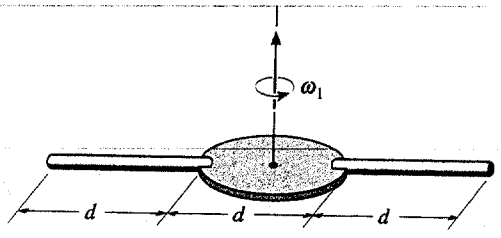
b)

$$(H_z)_1 = (H_z)_2$$

$$0+0=0-\left(\frac{300}{32.2}\right)(8)^2\omega-\left(\frac{150}{32.2}\right)(10\omega)(10)$$

$$\omega = 0 \qquad \textbf{Ans}$$

19-37. Each of the two slender rods and the disk have the same mass m. Also, the length of each rod is equal to the diameter d of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.



$$H_1 = H_2$$

$$[\tfrac{1}{2}m(\tfrac{d}{2})^2]\omega_1 + 2(\tfrac{1}{12}md^2)\omega_1 + 2(md^2\omega_1) = [\tfrac{1}{2}m(\tfrac{d}{2})^2]\omega' + 2m(\tfrac{d}{2})^2\omega'$$

$$\omega' = \frac{11}{3}\omega_1 \qquad \textbf{Ans}$$

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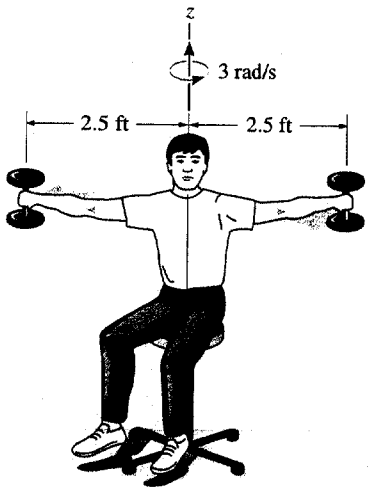
19-38. The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is turning at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.



Mass Moment of Inertia : The mass moment inertia of the man and the weights about z axis when the man arms is fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug} \cdot \text{ft}^2$$

The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug} \cdot \text{ft}^2$$

Conservation of Angular Momentum : Applying Eq. 19 – 17, we have

$$(H_z)_1 = (H_z)_2$$
$$3.444(3) = 1.531(\omega_z)_2$$
$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans}$$

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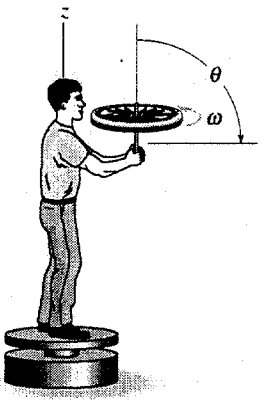
19-39. A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel when it is at *rest* and he starts it spinning with an angular velocity ω, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out, $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$.



(a)

$$\sum (H_z)_1 = \sum (H_z)_2; \quad 0 + I\omega = I_z\omega_M + I\omega \quad \omega_M = 0 \qquad \textbf{Ans}$$

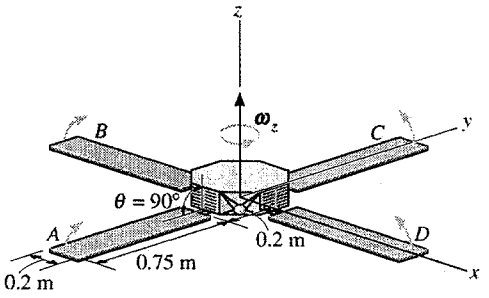
(b)

$$\sum (H_z)_1 = \sum (H_z)_2; \quad 0 + I\omega = I_z\omega_M + 0 \quad \omega_M = \frac{I}{I_z}\omega \qquad \textbf{Ans}$$

(c)

$$\sum (H_z)_1 = \sum (H_z)_2; \quad 0 + I\omega = I_z\omega_M - I\omega \quad \omega_M = \frac{2I}{I_z}\omega \qquad \textbf{Ans}$$

***19-40.** The space satellite has a mass of 125 kg and a moment of inertia $I_z = 0.940\ \text{kg} \cdot \text{m}^2$, excluding the four solar panels *A*, *B*, *C*, and *D*. Each solar panel has a mass of 20 kg and can be approximated as a thin plate. If the satellite is originally spinning about the z axis at a constant rate $\omega_z = 0.5$ rad/s when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instant



$$\curvearrowleft + \quad H_1 = H_2$$

$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega$$

$$+ 2[4(0.75\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$$\omega = 0.0906 \text{ rad/s} \qquad \textbf{Ans}$$

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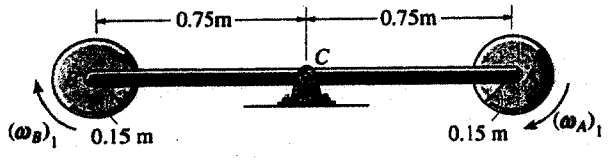
19-41. The 2-kg rod *ACB* supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins *A* and *B*. Motion is in the *horizontal plane*. Neglect friction at pin *C*.

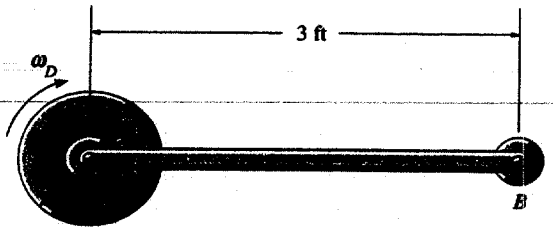


$$\zeta + \quad H_1 = H_2$$

$$2[\frac{1}{2}(4)(0.15)^2](5) = 2[\frac{1}{2}(4)(0.15)^2]\omega + 2[4(0.75\omega)(0.75)] + [\frac{1}{12}(2)(1.50)^2]\omega$$

$$\omega = 0.0906 \text{ rad/s} \qquad \textbf{Ans}$$

19-42. The 5-lb rod *AB* supports the 3-lb disk at its end. If the disk is given an angular velocity $\omega_D = 8$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing *A*. Motion is in the *horizontal plane*. Neglect friction at the fixed bearing *B*.



$$\Sigma(H_B)_1 = \Sigma(H_B)_2$$

$$\left[\tfrac{1}{2}\left(\tfrac{3}{32.2}\right)(0.5)^2\right](8) + 0 = \left[\tfrac{1}{3}\left(\tfrac{5}{32.2}\right)(3)^2\right]\omega + \left[\tfrac{1}{2}\left(\tfrac{3}{32.2}\right)(0.5)^2\right]\omega + \left(\tfrac{3}{32.2}\right)(3\omega)(3)$$

$$\omega = 0.0708 \text{ rad/s} \qquad \textbf{Ans}$$

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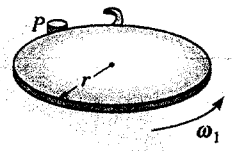
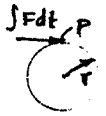
19-43. A thin disk of mass m has an angular velocity ω_1 while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.



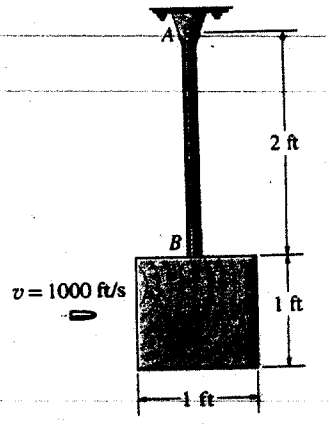
$$H_1 = H_2$$

$$\left(\frac{1}{2}mr^2\right)\omega_1 = \left[\frac{1}{2}mr^2 + mr^2\right]\omega_2$$

$$\omega_2 = \frac{1}{3}\omega_1 \qquad \textbf{Ans}$$



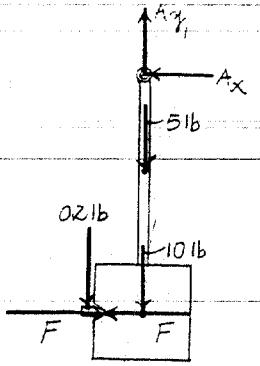
***19-44.** The pendulum consists of a 5-lb slender rod AB and a 10-lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.



Mass Moment of Inertia : The mass moment inertia of the pendulum and the embeded bullet about point A is

$$(I_A)_2 = \frac{1}{12}\left(\frac{5}{32.2}\right)(2^2) + \frac{5}{32.2}(1^2) + \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2 + 1^2) + \frac{10}{32.2}(2.5^2) + \frac{0.2}{32.2}(2.5^2)$$

$$= 2.239 \text{ slug} \cdot \text{ft}^2$$



Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the pendulum and the bullet. it will cancel out. Thus, angular momentum is conserved about point A. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$

$$(m_b v_b)(r_b) = (I_A)_2 \omega_2$$

$$\left(\frac{0.2}{32.2}\right)(1000)(2.5) = 2.239\omega_2$$

$$\omega_2 = 6.94 \text{ rad/s} \qquad \textbf{Ans}$$

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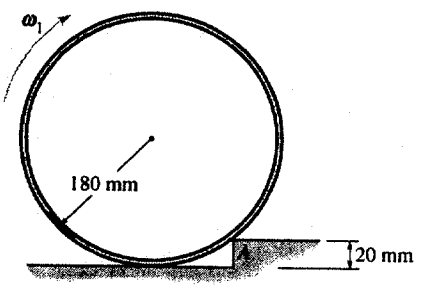
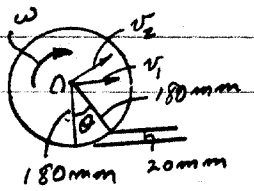
19-45. A thin ring having a mass of 15 kg strikes the 20-mm-high step. Determine the largest angular velocity ω_1 the ring can have so that it will not rebound off the step at A when it strikes it.



The weight is non – impulsive.

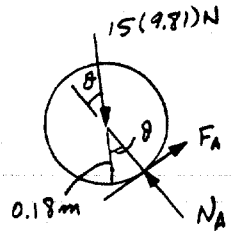


$(H_A)_1 = (H_A)_2$

$$15(\omega_1)(0.18)(0.18-0.02)+\left[15(0.18)^2\right](\omega_1)=\left[15(0.18)^2+15(0.18)^2\right]\omega_2$$

$$\omega_2 = 0.9444\omega_1$$

$+\nwarrow \Sigma F_n = m(a_G)_n;\quad (15)(9.81)\cos\theta - N_A = 15\omega_2^2(0.18)$



When hoop is about to rebound, $N_A = 0$. Also, $\cos\theta = \frac{160}{180}$, and so

$$\omega_2 = 6.9602 \text{ rad/s}$$

$$\omega_1 = \frac{6.9602}{0.9444} = 7.37 \text{ rad/s}$$ **Ans**

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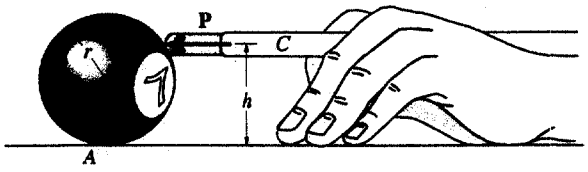
19-46. Determine the height h at which a billiard ball of mass m must be struck so that no frictional force develops between it and the table at A. Assume that the cue C only exerts a horizontal force **P** on the ball.



For the ball

$$(\overset{+}{\leftarrow})\ mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + P(\Delta t) = mv_2 \qquad (1)$$

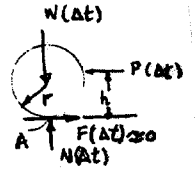
$$\circlearrowleft +\ (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + (P)\Delta t(h) = \left[\frac{2}{5}mr^2 + mr^2\right]\omega_2 \qquad (2)$$

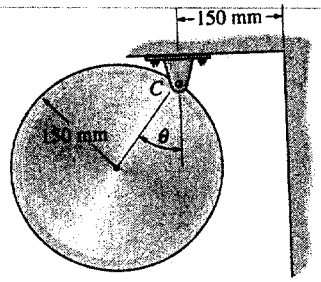
Require $v_2 = \omega_2 r$ (3)

Solving Eqs. (1) – (3) for h yields

$$h = \frac{7}{5}r \qquad \textbf{Ans}$$



19-47. The disk has a mass of 15 kg. If it is released from rest when $\theta = 30°$, determine the maximum angle θ of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is $e = 0.6$. When $\theta = 0°$, the disk hangs such that it just touches the wall. Neglect friction at the pin C.



Datum at lower position of G.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (15)(9.81)(0.15)(1 - \cos 30°) = \frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right]\omega^2 + 0$$

$$\omega = 3.418 \text{ rad/s}$$

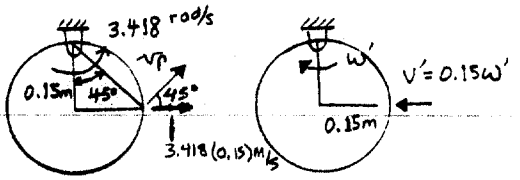
$$\left(\overset{+}{\rightarrow}\right)\quad e = 0.6 = \frac{0 - (-0.15\omega')}{3.418(0.15) - 0}$$

$$\omega' = 2.0508 \text{ rad/s}$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right](2.0508)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 17.9° \qquad \textbf{Ans}$$



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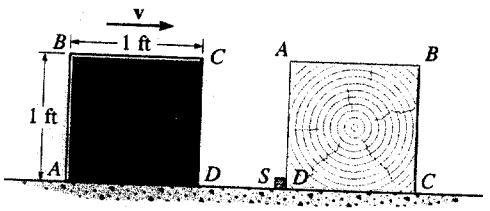
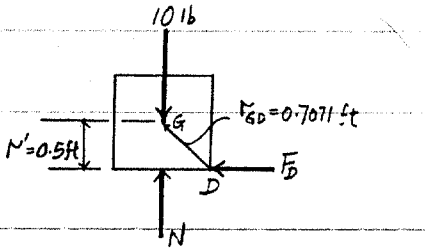
***19-48.** The 10-lb block is sliding on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity $\mathbf{v}$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.



Conservation of Energy : If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00$ ft · lb and $10(0.7071) = 7.071$ ft · lb. The mass moment of inertia of the block about point D is $I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2+1^2) + \left(\frac{10}{32.2}\right)\left(\sqrt{0.5^2+0.5^2}\right)^2 = 0.2070 \text{ slug} \cdot \text{ft}^2$. The initial kinetic energy of the block (after the impact) is $\frac{1}{2} I_D \omega_2^2 = \frac{1}{2}(0.2070)\,\omega_2^2$. Applying Eq. 18 – 18, we have

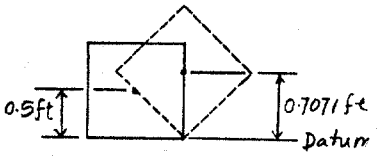


$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\,\omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472 \text{ rad/s}$$

Conservation of Angular Momentum : Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserved about point D. Applying Eq. 19 – 17, we have



$$(H_D)_1 = (H_D)_2$$

$$(mv_G)(r') = I_D \omega_2$$

$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96 \text{ ft/s} \qquad \textbf{Ans}$$

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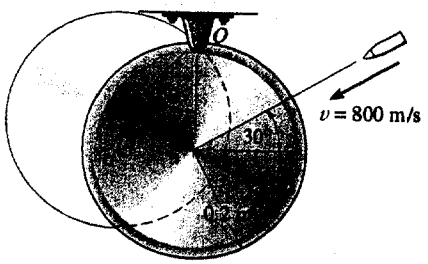
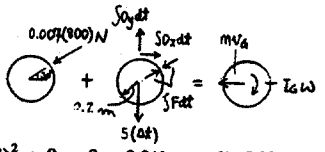
19-49. A 7-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.



$\zeta+ \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$$0.007(800)\cos 30°(0.2) + 0 = \frac{1}{2}(5.007)(0.2)^2\omega + 5.007(0.2\omega)(0.2)$$

$\omega = 3.23$ rad/s **Ans**

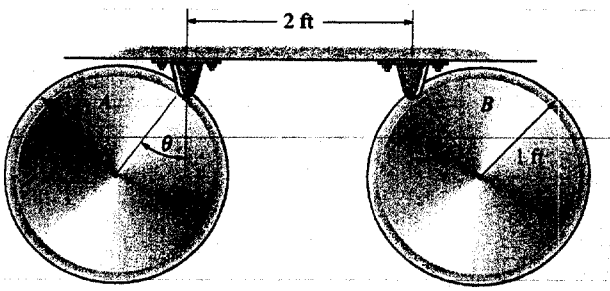


$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(5.007)[3.23(0.2)]^2 + \frac{1}{2}[\frac{1}{2}(5.007)(0.2)^2](3.23)^2 + 0 = 0 + 0.2(1-\cos\theta)(5.007)(9.81)$$

$\theta = 32.8°$ **Ans**

19-50. The two disks each weigh 10 lb. If they are released from rest when $\theta = 30°$, determine θ after they collide and rebound from each other. The coefficient of restitution is $e = 0.75$. When $\theta = 0°$, the disks hang so that they just touch one another.



$$I_C = \frac{1}{2}(\frac{10}{32.2})(1)^2 + \frac{10}{32.2}(1)^2 = 0.46584 \text{ slug}\cdot\text{ft}^2$$

$T_1 + V_1 = T_2 + V_2$

$$0 + 10(1-\cos 30°) = \frac{1}{2}(0.46584)\omega_1^2 + 0$$

$\omega_1 = 2.398$ rad/s

Coefficient of restitution:

$$e = \frac{(v_D)_{B_2} - (v_D)_{A_2}}{(v_D)_{A_1} - (v_D)_{B_1}} = 0.75 = \frac{\omega_2 - (-\omega_2)}{2.398 - (-2.398)} \qquad (1)$$

Where, v_D is the speed of point D on disk A or B. Note that $(v_D)_B = -(v_D)_A$ and $(v_D)_A = r\omega = (v_D)_B$.

Solving Eq.(1); $\omega_2 = 1.799$ rad/s

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(0.46584)(1.799)^2 + 0 = 0 + 10(1-\cos\theta)$$

$\theta = 22.4°$ **Ans**

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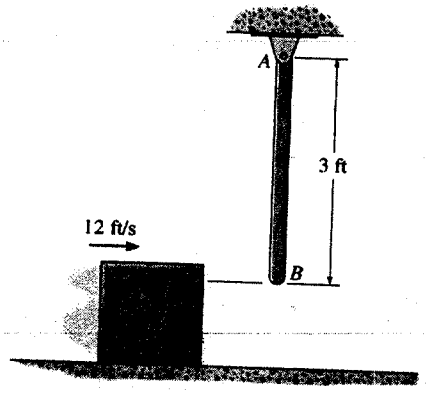
***19-51.** The 4-lb rod AB is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.

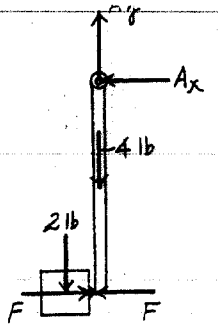


Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727 \text{ slug} \cdot \text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r_b) = I_A\omega_2 + [m_b(v_b)_2](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad [1]$$



Coefficient of Restitution : Applying Eq. 19 – 20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

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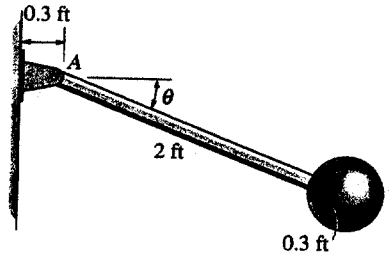
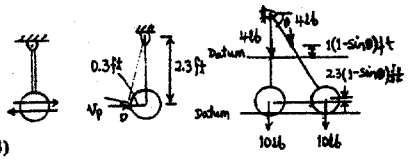
***19-52.** The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \frac{10}{32.2}(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$

Just before impact:



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}[1.8197]\omega^2 - 4(1) - 10(2.3)$$

$$\omega = 5.4475 \text{ rad/s}$$

$$v_P = 2.3(5.4475) = 12.53 \text{ ft/s}$$

Since wall does not move

$$e = 0.6 = \frac{v_P'}{12.529}$$

$$v_P' = 7.518 \text{ ft/s}$$

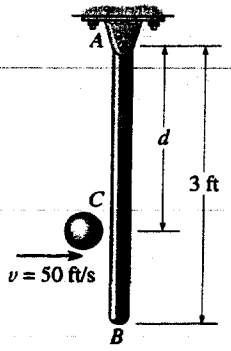
$$\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(1.8197)(3.2685)^2 = 4(1)(1-\sin\theta) + 10(2.3)(1-\sin\theta)$$

$$\theta = 39.8° \qquad \text{Ans}$$

19-53. The 6-lb slender rod AB is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at C. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



$$(+) (H_A)_1 = (H_A)_2$$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

Thus,

$$\omega_2 = 7.73 \text{ rad/s} \qquad \text{Ans}$$

$$v_{BL} = -19.5 \text{ ft/s}$$

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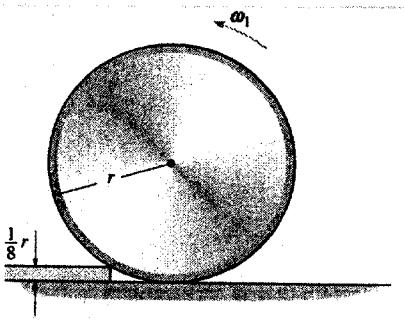
19-54. The disk has a mass m and radius r. If it strikes the rough step having a height $\frac{1}{8}r$ as shown, determine the largest angular velocity ω_1 the disk can have and not rebound off the step when it strikes it.



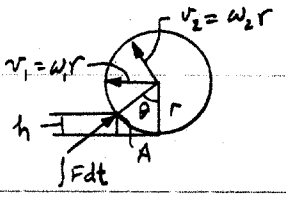
$$(H_A)_1 = \frac{1}{2}m r^2(\omega_1) + m(\omega_1 r)(r-h)$$

$$(H_A)_2 = \frac{1}{2}m r^2(\omega_2) + m(\omega_2 r)(r)$$

$$(H_A)_1 = (H_A)_2$$

$$\left[\frac{1}{2}m r^2 + m r(r-h)\right]\omega_1 = \frac{3}{2}m r^2 \omega_2$$

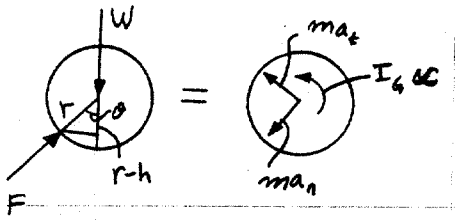
$$\left(\frac{3}{2}r - h\right)\omega_1 = \frac{3}{2}r\,\omega_2$$



$+\nearrow \Sigma F_n = m a_n$: $\quad W\cos\theta - F = m(\omega_2^2 r)$

$$F = mg\left(\frac{r-h}{r}\right) - m(\omega_2^2 r)$$

$$F = mg\left(\frac{r-h}{r}\right) - m r\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r - h}{r}\right)\omega_1^2$$



Set $h = \frac{1}{8}r$; also note that for maximum ω_1 F will approach zero. Thus

$$mg\left(\frac{r-\frac{1}{8}r}{r}\right) - mr\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r-\frac{r}{8}}{r}\right)\omega_1^2$$

$$\omega_1 = 1.02\sqrt{\frac{g}{r}} \qquad \textbf{Ans}$$

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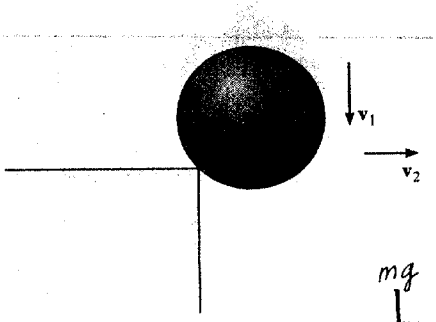
19-55. The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



Conservation of Angular Momentum : Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A.

The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$.

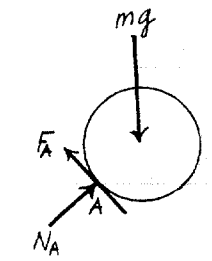
Here, $\omega_2 = \frac{v_2 \cos\theta}{r}$. Applying Eq. 19 – 17, we have

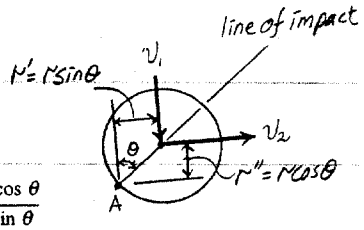
$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r') = I_G\omega_2 + [m_b(v_b)_2](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad [1]$$





Coefficient of Restitution : Applying Eq. 19 – 20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad [2]$$

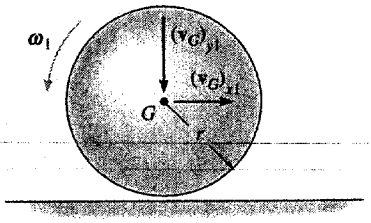
Equating Eqs. [1] and [2] yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans}$$

***19-56.** A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components $(\mathbf{v}_G)_{x1}$ and $(\mathbf{v}_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



Coefficient of Restitution (y direction) :

$$(+\downarrow) \qquad e = \frac{0 - (v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1}\uparrow \qquad \textbf{Ans}$$

Conservation of angular momentum about point on the ground :

$$(\curvearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}r$$

Since no slipping, $(v_G)_{x2} = \omega_2 r$ then,

$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans}$$

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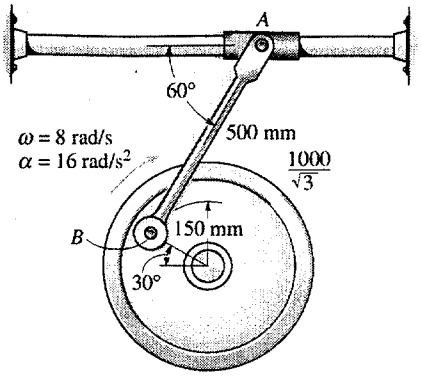
R2-1. At a given instant, the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.



$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$$

$$v_B = 8(0.150) = 1.20 \text{ m/s}$$

$$\omega_{AB} = \frac{1.20}{\left(\frac{0.500}{\sqrt{3}}\right)} = 4.1569 \text{ rad/s}$$

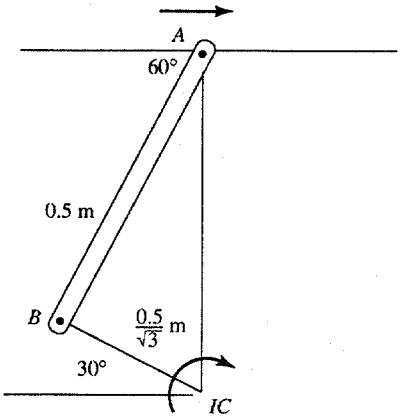
$$\underset{\rightarrow}{a_A} = \underset{\angle 60°}{16(0.150)} + \underset{\searrow 30°}{(8)^2(0.150)} + \underset{\searrow 30°}{\alpha_{AB}(0.500)} + \underset{60°\swarrow}{(4.1569)^2(0.500)}$$

$$0 = 1.2\sqrt{3} - 4.8 - 0.250\alpha_{AB} - 4.32\sqrt{3}$$

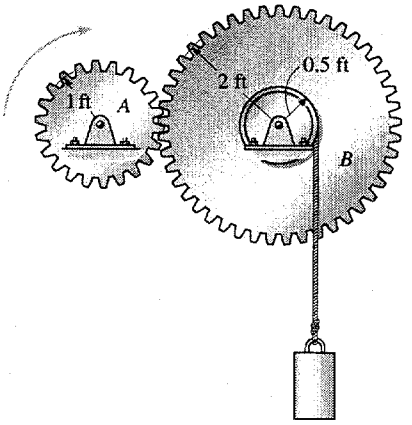
$$a_A = 1.2 + 4.8\sqrt{3} + (0.250\sqrt{3})\alpha_{AB} - 4.32$$

$$\alpha_{AB} = -40.8 \text{ rad/s}^2$$

$$a_A = -12.5 \text{ m/s}^2 = 12.5 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans}$$



R2-2. The hoisting gear A has an initial angular velocity of 60 rad/s and a constant deceleration of 1 rad/s^2. Determine the velocity and deceleration of the block which is being hoisted by the hub on gear B when $t = 3$ s.



$$(\omega_A)_O = 60 \text{ rad/s}$$

$$\alpha_A = -1 \text{ rad/s}^2$$

$$\omega_A = (\omega_A)_O + \alpha_A t$$

$$\omega_A = 60 + (-1)(3) = 57 \text{ rad/s}$$

$$v_A = r\omega_A = (1)(57) = 57 \text{ ft/s} = v_B$$

$$\omega_B = \frac{v_B}{r} = 57/2 = 28.5 \text{ rad/s}$$

$$v_W = r_C\omega_C = (0.5)(28.5) = 14.2 \text{ ft/s} \quad \textbf{Ans}$$

$$\alpha_A = 1$$

$$a_{A_t} = l(1) = 1 \text{ ft/s}^2$$

$$\alpha_B = \frac{1}{2} = 0.5 \text{ rad/s}^2$$

$$a_W = r\alpha_B = (0.5)(0.5) = 0.25 \text{ ft/s}^2 \quad \textbf{Ans}$$

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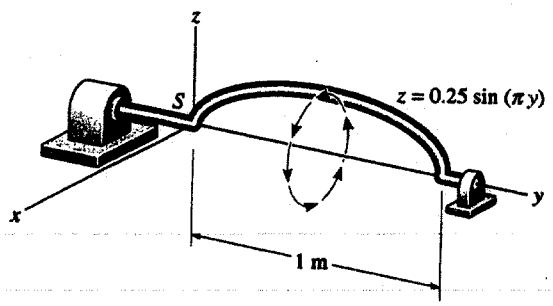
R2-3. The rod is bent into the shape of a sine curve and is forced to rotate about the y axis by connecting the spindle S to a motor. If the rod starts from rest in the position shown and a motor drives it for a short time with an angular acceleration $\alpha = (1.5e^t)$ rad/s^2, where t is in seconds, determine the magnitudes of the angular velocity and angular displacement of the rod when $t = 3$ s. Locate the point on the rod which has the greatest velocity and acceleration, and compute the magnitudes of the velocity and acceleration of this point when $t = 3$ s. The curve defining the rod is $z = 0.25 \sin(\pi y)$, where the argument for the sine is given in radians when y is in meters.



$d\omega = \alpha\, dt$

$\int_0^\omega d\omega = \int_0^t 1.5\, e^t\, dt$

$\omega = 1.5\, e^t|_0^t = 1.5(e^t - 1)$

$d\theta = \omega\, dt$

$\int_0^\theta d\theta = 1.5\int_0^t (e^t - 1)\, dt$

$\theta = 1.5[e^t - t]_0^t = 1.5(e^t - t - 1)$

When $t = 3$ s,

$\omega = 1.5(e^3 - 1) = 28.6$ rad/s **Ans**

$\theta = 1.5(e^3 - 3 - 1) = 24.1$ rad **Ans**

The point having the greatest velocity and acceleration is located furthest from the axis of rotation. This is at $y = 0.5$ m, where

$z = 0.25 \sin(\pi(0.5)) = 0.25$ m

Hence,

$v_P = \omega(z) = 28.6(0.25)$

$v_P = 7.16$ m/s **Ans**

and

$(a_t)_P = \alpha(z) = (1.5e^3)(0.25) = 7.53$

$(a_n)_P = \omega^2(z) = (28.6)^2(0.25) = 204.90$

So that

$a_P = \sqrt{(7.53)^2 + (204.90)^2} = 205$ m/s^2 **Ans**

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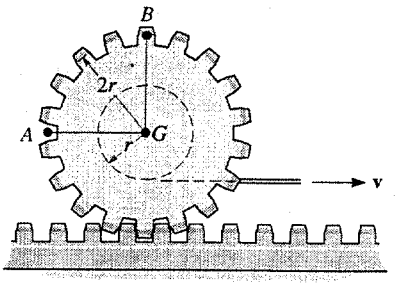
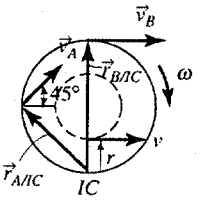
***R2-4.** A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity v, determine the velocity and acceleration of points A and B. The gear rolls on the fixed gear rack.



$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \qquad \textbf{Ans}$$

$$v_A = \omega r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \qquad \textbf{Ans}$$



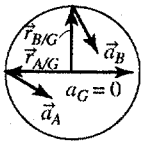
From Example 16-3, Since $\alpha_G = 0, a = 0$

$\mathbf{r}_{B/G} = 2r\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\mathbf{i}$

$\mathbf{a}_B = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$

$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (2r\mathbf{j})$

$= \frac{2v^2}{r}$



$$a_B = \frac{2v^2}{r} \qquad \textbf{Ans}$$

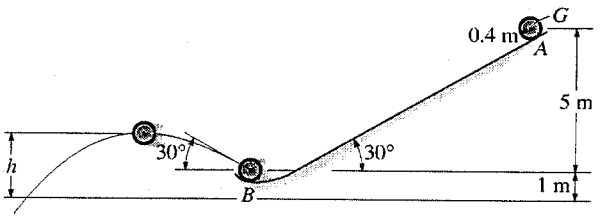
$\mathbf{a}_A = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$

$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i})$

$= \frac{2v^2}{r}\mathbf{i}$

$$a_A = \frac{2v^2}{r} \qquad \textbf{Ans}$$

R2-5. A 7-kg automobile tire is released from rest at A on the incline and rolls without slipping to point B, where it then travels in free flight. Determine the maximum height h the tire attains. The radius of gyration of the tire about its mass center is $k_G = 0.3$ m.



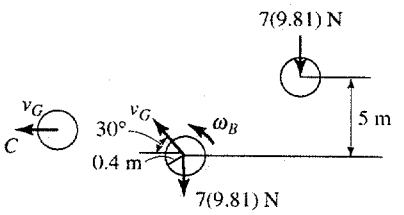
Establish the datum at B.

Note that $v_G = 0.4\omega_B$

$$0 + 7(9.81)(5) = \frac{1}{2}[7(0.3)^2]\omega_B^2 + \frac{1}{2}(7)(0.4\omega_B)^2$$

$\omega_B = 19.81$ rad/s

$v_G = 0.4(19.81) = 7.92$ m/s



Let C be the high point of the tire's flight.

At C, $(v_C)_y = 0$. Thus,

$(+\uparrow) \qquad (v_C)_y^2 = (v_B)_y^2 + 2g((s_C)_y - (s_B)_y)$

$0 = (7.92 \sin 30°)^2 + 2(-9.81)(h - 1)$

$h = 1.80$ m $\qquad$ **Ans**

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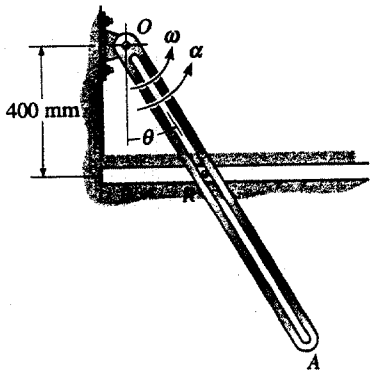
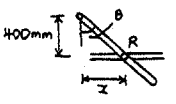
R2-6. The link OA is pinned at O and rotates because of the sliding action of rod R along the horizontal groove. If R starts from rest. when $\theta = 0°$ and has a constant acceleration $a_R = 60\ \text{mm/s}^2$ to the right, determine the angular velocity and angular acceleration of OA when $t = 2$ s.





$$x = 400\tan\theta \qquad (1)$$

$$\dot{x} = 400\sec^2\theta\,\dot{\theta} \qquad (2)$$

$$\ddot{x} = 400(2\sec^2\theta\tan\theta\,\dot{\theta})\dot{\theta} + 400\sec^2\theta\,\ddot{\theta}$$

$$= 400\sec^2\theta(2\tan\theta(\dot{\theta})^2 + \ddot{\theta}) \qquad (3)$$

Since $\ddot{x} = 60\ \text{mm/s}^2$ and $\dot{x} = x = 0$ at $t = 0$;

Then in $t = 2$ s,

$$v = v_0 + a_c t$$

$$\dot{x} = 0 + 60(2) = 120\ \text{mm/s}$$

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$x = 0 + 0 + 30(2)^2 = 120\ \text{mm}$$

From Eqs. (1)–(3),

$$\theta = \tan^{-1}\left(\frac{120}{400}\right) = 16.7°$$

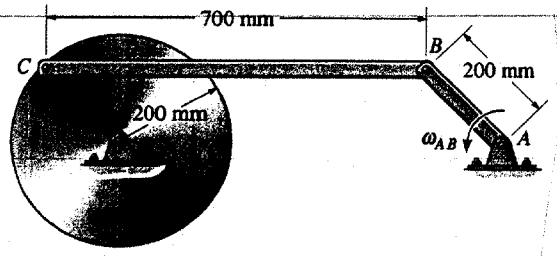
$$120 = 400\sec^2 16.7°(\dot{\theta})$$

$$\omega = \dot{\theta} = 0.275\ \text{rad/s} \qquad \textbf{Ans}$$

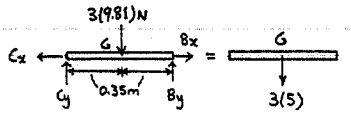
$$60 = 400\sec^2 16.7°[2\tan 16.7°(0.275)^2 + \ddot{\theta}]$$

$$\alpha = \ddot{\theta} = 0.0922\ \text{rad/s}^2 \qquad \textbf{Ans}$$

R2-7. The uniform connecting rod BC has a mass of 3 kg and is pin-connected at its end points. Determine the vertical forces which the pins exert on the ends B and C of the rod at the instant (a) $\theta = 0°$, and (b) $\theta = 90°$. The crank AB is turning with a constant angular velocity $\omega_{AB} = 5$ rad/s.



Rod has curvilinear motion



$$\mathbf{a}_B = \mathbf{a}_G = (\mathbf{a}_B)_n = (5)^2(0.2) = 5$$

a) when $\theta = 0°$;

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad C_y + B_y - 3(9.81) = -3(5)$$

$$\circlearrowleft + \Sigma M_G = 0; \quad -C_y(0.35) + B_y(0.35) = 0$$

Solving,

$$C_y = 7.22\ \text{N} \qquad \textbf{Ans}$$

$$B_y = 7.22\ \text{N} \qquad \textbf{Ans}$$

b) When $\theta = 90°$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad C_y + B_y - 3(9.81) = 0$$

$$\circlearrowleft + \Sigma M_G = 0; \quad -C_y(0.35) + B_y(0.35) = 0$$

Solving,

$$C_y = 14.7\ \text{N} \qquad \textbf{Ans}$$

$$B_y = 14.7\ \text{N} \qquad \textbf{Ans}$$

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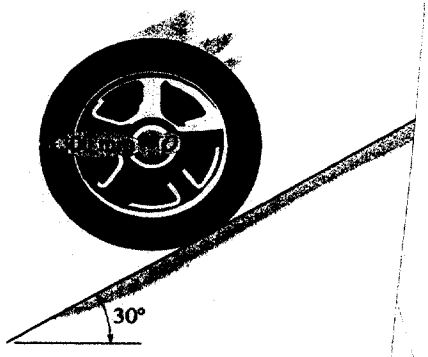
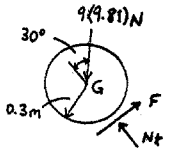
***R2-8.** The tire has a mass of 9 kg and a radius of gyration $k_O = 225$ mm. If it is released from rest and rolls down the plane without slipping, determine the speed of its center O when $t = 3$ s.





$+\nearrow m(v_G)_1 + \Sigma\int F\,dt = m(v_G)_2$

$0 + 9(9.81)(\sin 30°)(3) - \int_0^3 F\,dt = 9(v_G)_2$ (1)

$(+ (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$0 + (\int_0^3 F\,dt)(0.3) = [9(0.225)^2]\omega_2$ (2)

Since $(v_G)_2 = 0.3\omega_2$,

Eliminating $\int_0^3 F\,dt$ from Eqs. (1) and (2) and solving for $(v_G)_2$ yields,

$(v_G)_2 = 9.42$ m/s **Ans**

Also,

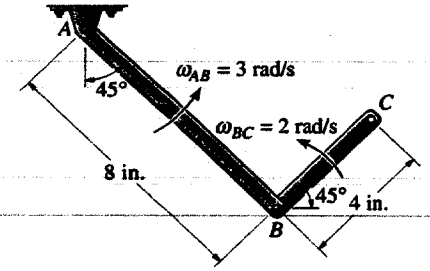
$(+ (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

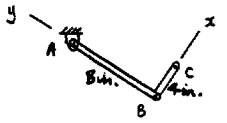
$0 + 9(9.81)\sin 30°(3)(0.3) = [9(0.225)^2 + 9(0.3)^2]\omega$

$\omega = 31.39$ rad/s

$v = 0.3(31.39) = 9.42$ m/s **Ans**

R2-9. The double pendulum consists of two rods. Rod AB has a constant angular velocity of 3 rad/s, and rod BC has a constant angular velocity of 2 rad/s. Both of these absolute motions are measured counterclockwise. Determine the velocity and acceleration of point C at the instant shown.





$\mathbf{v}_B = 3(8)\mathbf{i} = \{24\mathbf{i}\}$ in./s

$\mathbf{a}_B = (3)^2(8)\mathbf{j} = \{72\mathbf{j}\}$ in./s^2

$\Omega = 3\mathbf{k}$

$\alpha = 0$

$\mathbf{r}_{C/B} = \{4\mathbf{i}\}$ in.

Note: The x,y axes rotate at 3 rad/s while BC rotates at 2 rad/s

Relative rotation of BC is therefore,

$(\omega_{BC})_{xyz} = 2\mathbf{k} - 3\mathbf{k} = -1\mathbf{k}$

$\mathbf{v}_{C/B} = (\omega_{BC})_{xyz} \times \mathbf{r}_{C/B} = -\mathbf{k} \times 4\mathbf{i} = -4\mathbf{j}$

$\mathbf{a}_{C/B} = (\omega_{BC})_{xyz} \times [(\omega_{BC}) \times \mathbf{r}_{C/B}] = -4\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$

$= 24\mathbf{i} + (3\mathbf{k} \times 4\mathbf{i}) + (-4\mathbf{j})$

$= \{24\mathbf{i} + 8\mathbf{j}\}$ in./s

$v_C = 25.3$ in./s, $\theta = 63.4°$ ∠ **Ans**

$\mathbf{a}_C = \mathbf{a}_B + \alpha \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times \mathbf{v}_{C/B} + \mathbf{a}_{C/B}$

$= 72\mathbf{j} + 0 + 3\mathbf{k} \times (3\mathbf{k} \times 4\mathbf{i}) + 2(3\mathbf{k}) \times (-4\mathbf{j}) + (-4\mathbf{i})$

$= \{-16\mathbf{i} + 72\mathbf{j}\}$ in./s^2

$a_C = 73.8$ in./s^2, $\theta = 32.5°$ ◁ **Ans**

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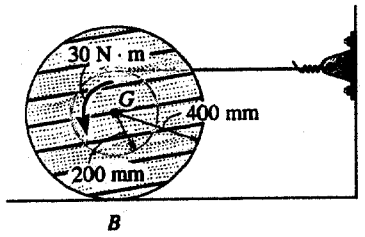
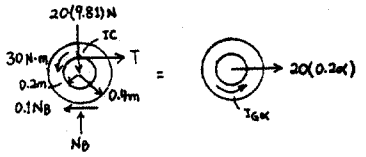
R2-10. The spool and wire wrapped around its core have a mass of 20 kg and a centroidal radius of gyration $k_G = 250$ mm. If the coefficient of kinetic friction at the ground is $\mu_B = 0.1$, determine the angular acceleration of the spool when the 30-N·m couple moment is applied.



$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B - 20(9.81) = 0$



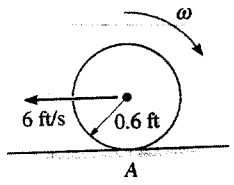
$N_B = 196.2$ N

$F_B = 0.1(196.2) = 19.62$ N

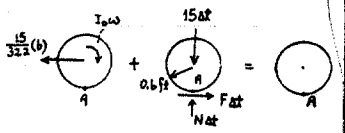
$\circlearrowleft +\Sigma M_{IC} = \Sigma(M_k)_{IC}; \quad 30 - 19.62(0.6) = 20(0.2\alpha)(0.2) + [20(0.25)^2]\alpha$

$\alpha = 8.89$ rad/s² Ans

R2-11. If the ball has a weight of 15 lb and is thrown onto a *rough surface* so that its center has a velocity of 6 ft/s parallel to the surface, determine the amount of backspin, ω, the ball must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.



$\circlearrowleft + \quad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$



$\frac{15}{32.2}(6)(0.6) - [\frac{2}{5}(\frac{15}{32.2})(0.6)^2]\omega + 0 = 0$

$\omega = 25.0$ rad/s Ans

Also,

$\overset{+}{\leftarrow}\ m(v_G)_1 + \Sigma\int F\, dt = m(v_G)_2$

$\frac{15}{32.2}(6) - \int F\, dt = 0$

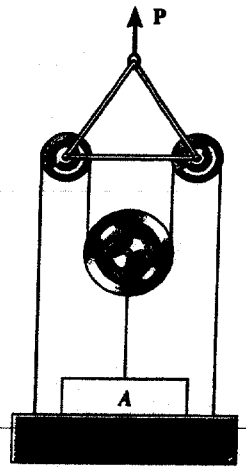
$\circlearrowleft + (H_G)_1 + \Sigma\int M_G\, dt = (H_G)_2$

$\left[\frac{2}{5}(\frac{15}{32.2})(0.6)^2\right]\omega - (0.6)\int F\, dt = 0$

Eliminating $\int F\, dt$ and solving for ω yields

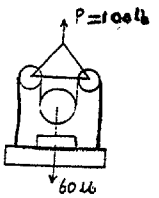
$\omega = 25.0$ rad/s Ans

***R2-12.** Blocks A and B weigh 50 and 10 lb, respectively. If $P = 100$ lb, determine the normal force exerted by block A on block B. Neglect friction and the weights of the pulleys, cord, and bars of the triangular frame.



System:

$+\uparrow \Sigma F_y = m a_y; \quad 100 - 60 = \frac{60}{32.2}a$



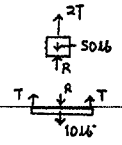
$a = 21.5$ ft/s²

Block A:

$+\uparrow \Sigma F_y = m a_y; \quad 2T + R - 50 = \frac{50}{32.2}(21.5) \qquad (1)$

Block B:

$+\uparrow \Sigma F_y = m a_y; \quad 2T - R - 10 = \frac{10}{32.2}(21.5) \qquad (2)$



Solving Eqs. (1) and (2),

$R = 33.3$ lb Ans

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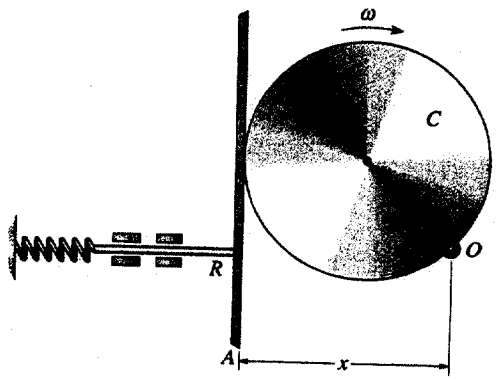
R2-13. Determine the velocity and acceleration of rod R for any angle θ of cam C if the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of A on C.



$$x = r + r\cos\theta$$

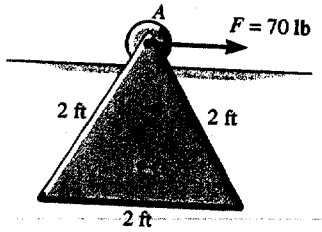
$$\dot{x} = -r\sin\theta\,\dot{\theta}$$

$$v = -r\omega\sin\theta \qquad \text{Ans}$$

$$\ddot{x} = -r\sin\theta\,\ddot{\theta} - r\cos\theta(\dot{\theta})^2$$

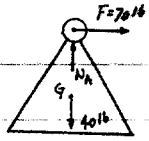
$$a = -r\omega^2\cos\theta \qquad \text{Ans}$$

R2-14. The uniform plate weighs 40 lb and is supported by a roller at A. If a horizontal force $F = 70$ lb is suddenly applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. The plate has a moment of inertia about its center of mass of $I_G = 0.414$ slug $\cdot$ ft^2. Neglect the weight of the roller.



At the instant force F is applied, the normal component of acceleration of G is zero since the angular velocity is zero at this instant. As a result $a_G = (a_G)_t$.

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad 70 = \left(\tfrac{40}{32.2}\right)a_G \quad a_G = 56.35 \text{ ft/s}^2$$



$$\Sigma M_A = \Sigma(M_k)_A; \quad 0 = \left(\tfrac{40}{32.2}\right)(56.35)\left(\tfrac{2}{3}\right)(2\sin 60°) - 0.414\alpha$$

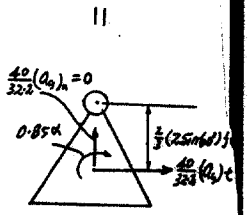
$$\alpha = 195.2 \text{ rad/s}^2$$

Kinematics:

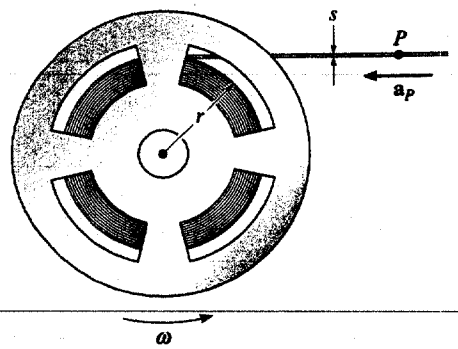
$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\left[a_A \rightarrow\right] = \left[56.35 \rightarrow\right] + \left[195.2\left(\tfrac{2}{3}\right)(2\sin 60°) \rightarrow\right] + [0]$$

$$(\overset{+}{\rightarrow}) \qquad a_A = 56.35 + 225.4 = 282 \text{ ft/s}^2 \qquad \text{Ans}$$



R2-15. A tape having a thickness s wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P on the tape when the radius is r. *Hint:* Since $v_p = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



$$v = \omega r$$

$$a = \frac{dv}{dt} = \frac{d\omega}{dt}r + \omega\frac{dr}{dt}$$

Since $\dfrac{d\omega}{dt} = 0$,

$$a = \omega\left(\frac{dr}{dt}\right)$$

In one revolution $(2\pi \text{ rad})r$ increases by s, so that

$$\frac{2\pi}{\theta} = \frac{s}{\Delta r}$$

$$\Delta r = \frac{s}{2\pi}\theta$$

$$\frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$a = \frac{s}{2\pi}\omega^2 \qquad \text{Ans}$$

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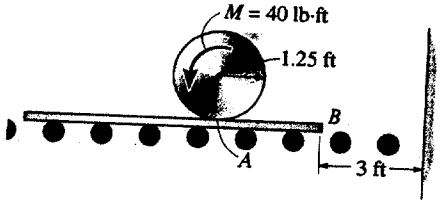
***R2-16.** The 15-lb cylinder is initially at rest on a 5-lb plate. If a couple moment $M = 40\ \text{lb}\cdot\text{ft}$ is applied to the cylinder, determine the angular acceleration of the cylinder and the time needed for the end B of the plate to travel 3 ft and strike the wall. Assume the cylinder does not slip on the plate, and neglect the mass of the rollers under the plate.



Cylinder:

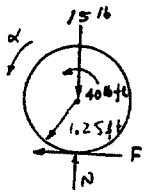
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad F = \left(\frac{15}{32.2}\right) a_G$ [1]

$\zeta + \Sigma M_G = I_G \alpha; \quad 40 - F(1.25) = \frac{1}{2}\left(\frac{15}{32.2}\right)(1.25)^2 \alpha$ [2]

$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$

$\left[(a_A)_t\right] + \left[(a_A)_n\right] = \left[a_G\right] + \left[1.25\alpha\right] + \left[(a_{A/G})_n\right]$

$(\overset{+}{\rightarrow})\ (a_A)_t = -a_G + 1.25\alpha$ [3]



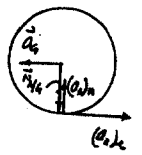
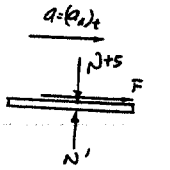


Plate: Since no slipping occurs at the point of contact, the plate will move with an acceleration of $a = (a_A)_t$.

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad F = \left(\frac{5}{32.2}\right)(a_A)_t$ [4]



Solving Eqs.[1] to [4] yields:

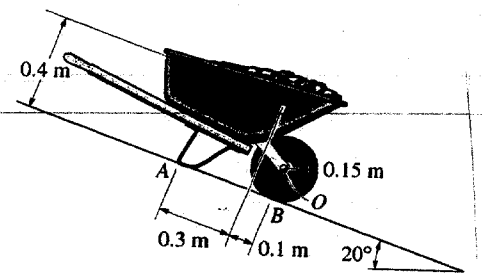
$a_G = 22.89\ \text{ft/s}^2 \quad F = 10.67\ \text{lb} \quad (a_A)_t = 68.69\ \text{ft/s}^2$

$\alpha = 73.3\ \text{rad/s}^2$ Ans

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$3 = 0 + 0 + \frac{1}{2}(68.69)t^2 \qquad t = 0.296\ \text{s}$ Ans

R2-17. The wheel barrow and its contents have a mass of 40 kg and a mass center at G, excluding the wheel. The wheel has a mass of 2 kg and a radius of gyration $k_O = 0.120$ m. If the wheelbarrow is released from rest from the position shown, determine its speed after it travels 4 m down the incline. The coefficient of kinetic friction between the incline and A is $\mu_A = 0.3$. The wheels roll without slipping at B.



$\nearrow + \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 44(9.81)\cos 20° = 0$ (1)

$+\searrow \Sigma F_x = ma_x; \quad 44(9.81)\sin 20° - 0.3N_A = 40a_G + 4a_O$ (2)

$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 40(9.81)\sin 20°(0.4) + 40(9.81)\cos 20°(0.3) + 4(9.81)\sin 20°(0.15) + 4(9.81)\cos 20°(0.4) - N_B(0.4) = 40a_G(0.4) + 4a_O(0.15) + [4(0.12)^2]\alpha$ (3)

Kinematics

$a_O = a_G$ (since the cart translates)

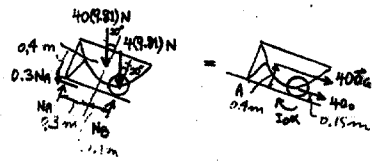
$a_O = a_G = 0.15\alpha$ (since the wheels roll without slipping)

Eqs.(1)–(3) reduce to

$N_A + N_B = 405.61$

$147.63 - 0.3N_A = 6.60\alpha$

$181.067 - 0.4N_B = 2.548\alpha$



Solving,

$\alpha = 19.0\ \text{rad/s}^2$

$a_G = 2.85\ \text{m/s}^2$

$N_A = 74.0\ \text{N}$

$N_B = 331.6\ \text{N}$

$v^2 = (v_0)^2 + 2a_G(s - s_0)$

$v^2 = 0 + 2(2.85)(4 - 0)$

$v = 4.78\ \text{m/s}$ Ans

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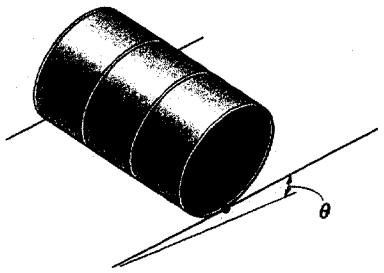
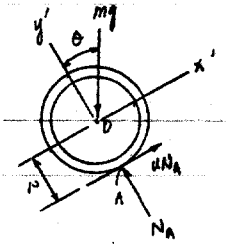
R2-18. The drum of mass m, radius r, and radius of gyration k_O rolls along an inclined plane for which the coefficient of static friction is μ. If the drum is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping.



$$m(v_{Oy'})_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_{Oy'})_2$$



$$0 + N_A(t) - mg\cos\theta(t) = 0 \qquad N_A = mg\cos\theta$$

$$m(v_{Ox'})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{Ox'})_2$$

$$0 + mg\sin\theta(t) - \mu mg\cos\theta(t) = mv_O \qquad [1]$$

Since no slipping occurs, $v_O = \omega r$. Hence Eq.[1] becomes

$$mg\sin\theta(t) - \mu mg\cos\theta(t) = m\omega r$$

$$t = \frac{m\omega r}{mg(\sin\theta - \mu\cos\theta)} \qquad [2]$$

$$I_A \omega_1 + \Sigma \int_{t_1}^{t_2} M_A \, dt = I_A \omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[mk_O^2 + mr^2\right]\omega$$

$$t = \frac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta} \qquad [3]$$

Equating Eqs.[2] and [3]

$$\frac{m\omega r}{mg(\sin\theta - \mu\cos\theta)} = \frac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta}$$

$$\theta = \tan^{-1}\left[\frac{\mu(k_O^2 + r^2)}{k_O^2}\right] \qquad \text{Ans}$$

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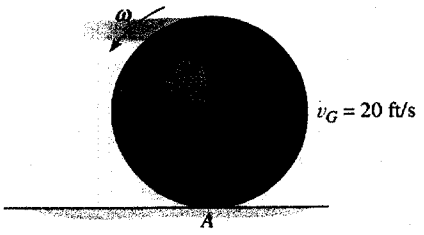
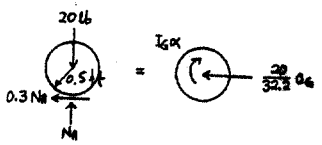
R2-19. The 20-lb solid ball is cast on the floor such that it has a backspin $\omega = 15$ rad/s and its center has an initial horizontal velocity $v_G = 20$ ft/s. If the coefficient of kinetic friction between the floor and the ball is $\mu_A = 0.3$, determine the distance it travels before it stops spinning.





$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \quad 0.3N_A = \frac{20}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 20 = 0$

$\zeta + \Sigma M_G = I_G\alpha; \quad 0.3N_A(0.5) = [\frac{2}{5}(\frac{20}{32.2})(0.5)^2]\alpha$

Solving,

$N_A = 20$ lb

$a_G = 9.66$ ft/s^2

$\alpha = 48.3$ rad/s^2

$(\circlearrowleft +) \quad \omega = \omega_0 + \alpha_c t$

$0 = 15 - 48.3t$

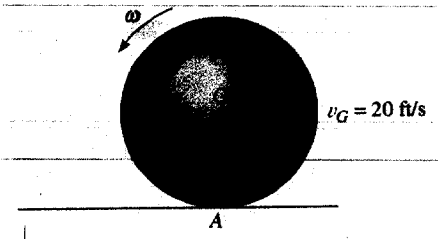
$t = 0.311$ s

$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 20(0.311) - \frac{1}{2}(9.66)(0.311)^2$

$s = 5.75$ ft **Ans**

***R2-20.** Determine the backspin ω which should be given to the 20-lb ball so that when its center is given an initial horizontal velocity $v_G = 20$ ft/s it stops spinning and translating at the same instant. The coefficient of kinetic friction is $\mu_A = 0.3$.



$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \quad 0.3N_A = \frac{20}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 20 = 0$

$\zeta + \Sigma M_G = I_G\alpha; \quad 0.3N_A(0.5) = [\frac{2}{5}(\frac{20}{32.2})(0.5)^2]\alpha$

Solving,

$N_A = 20$ lb

$a_G = 9.66$ ft/s^2

$\alpha = 48.3$ rad/s^2

$(\circlearrowleft +) \quad \omega = \omega_0 + \alpha_c t$

$0 = \omega_1 - 48.3t$

$\omega_1 = 48.3t$

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$0 = 20 - 9.66(\frac{\omega}{48.3})$

$\omega = 100$ rad/s **Ans**

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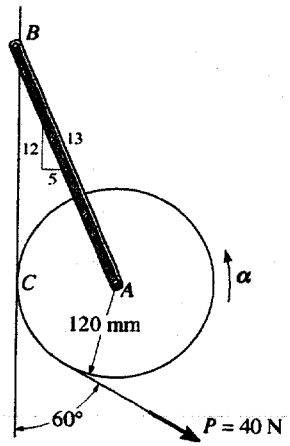
R2-21. A 20-kg roll of paper, originally at rest, is pin-supported at its ends to bracket AB. The roll rests against a wall for which the coefficient of kinetic friction at C is $\mu_C = 0.3$. If a force of 40 N is applied uniformly to the end of the sheet, determine the initial angular acceleration of the roll and the tension in the bracket as the paper unwraps. For the calculation, treat the roll as a cylinder.



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad 40\sin 60° + N_C - \left(\frac{5}{13}\right)T = 0$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad -40\cos 60° + 0.3N_C - 20(9.81) + \frac{12}{13}T = 0$$

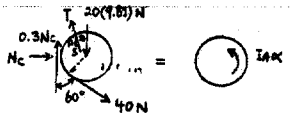
$$\zeta + \Sigma M_A = I_A\alpha; \quad 40(0.120) - 0.3N_C(0.120) = \left[\frac{1}{2}(20)(0.120)^2\right]\alpha$$

Solving,

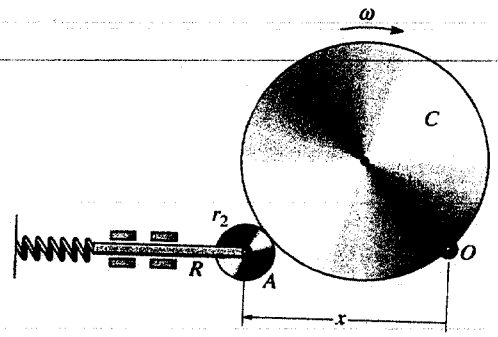
$T = 218$ N Ans

$N_C = 49.28$ N

$\alpha = 21.0$ rad/s^2 Ans



R2-22. Compute the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of A on C.



$$x = r_1\cos\theta + \sqrt{(r_1 + r_2)^2 - (r_1\sin\theta)^2}$$

$$\dot{x} = -r_1\sin\theta\dot{\theta} - \frac{2(r_1\sin\theta)r_1\cos\theta\dot{\theta}}{2\sqrt{(r_1 + r_2)^2 - (r_1\sin\theta)^2}}$$

$$v = -r_1\omega\sin\theta - \frac{r_1^2\omega\sin 2\theta}{2\sqrt{(r_1 + r_2)^2 - (r_1\sin\theta)^2}}$$ Ans

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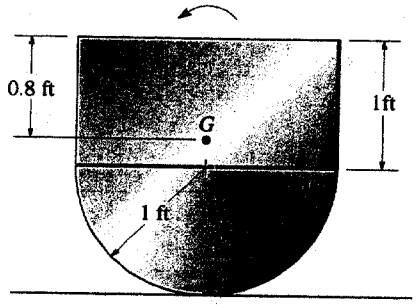
R2-23. The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft-lb when it is in the position shown. If it is rolling counterclockwise on the surface without slipping, determine its linear momentum at this instant.

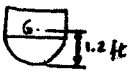


$$I_G = (0.6)^2\left(\frac{10}{32.2}\right) = 0.1118 \text{ slug}\cdot\text{ft}^2$$

$$T = \frac{1}{2}\left(\frac{10}{32.2}\right)v_G^2 + \frac{1}{2}(0.1118)\omega^2 = 31 \qquad (1)$$

$$v_G = 1.2\,\omega$$

Substitute into Eq. (1),

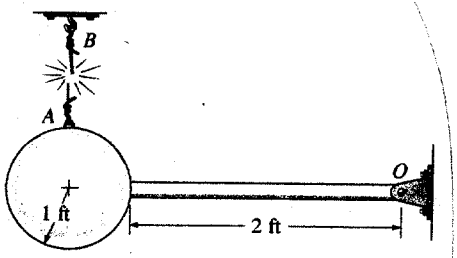


$$\omega = 10.53 \text{ rad/s}$$

$$v_G = 10.53(1.2) = 12.64 \text{ ft/s}$$

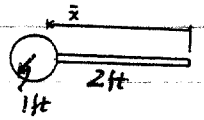
$$L = mv_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug}\cdot\text{ft/s} \quad \text{Ans}$$

***R2-24.** The pendulum consists of a 30-lb sphere and a 10-lb slender rod. Compute the reaction at the pin O just after the cord AB is cut.



$$I_O = \frac{2}{5}\left(\frac{30}{32.2}\right)(1)^2 + \left(\frac{30}{32.2}\right)(3)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2 = 9.17 \text{ slug}\cdot\text{ft}^2$$

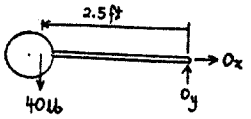
$$\bar{x} = \frac{30(3) + 10(1)}{30 + 10} = 2.5 \text{ ft}$$



$$\stackrel{+}{\rightarrow}\Sigma F_n = ma_n; \quad O_x = 0$$

$$+\downarrow \Sigma F_t = ma_t; \quad 40 - O_y = \frac{40}{32.2}a_G$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad 40(2.5) = 9.17\alpha$$



Kinematics,

$$a_G = 2.5\alpha$$

Solving,

$$\alpha = 10.90 \text{ rad/s}^2$$

$$a_G = 27.3 \text{ ft/s}^2$$

$$O_x = 0$$

$$O_y = 6.14 \text{ lb}$$

Thus;

$$F_O = 6.14 \text{ lb} \qquad \text{Ans}$$

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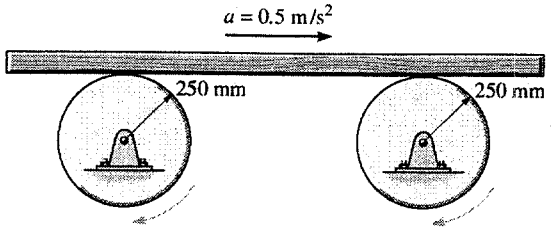
R2-25. The board rests on the surface of two drums. At the instant shown, it has an acceleration of 0.5 m/s^2 to the right, while at the same instant points on the outer rim of each drum have an acceleration with a magnitude of 3 m/s^2. If the board does not slip on the drums, determine its speed due to the motion.



A point on the drum which is in contact with the board has a tangential acceleration of

$a_t = 0.5 \text{ m/s}^2$

$a^2 = a_t^2 + a_n^2$

$(3)^2 = (0.5)^2 + a_n^2$

$a_n = 2.96 \text{ m/s}^2$

$a_n = \omega^2 r, \qquad \omega = \sqrt{\frac{2.96}{0.25}} = 3.44 \text{ rad/s}$

$v_B = \omega r = 3.44(0.25) = 0.860 \text{ m/s}$ **Ans**

R2-26. The center of the pulley is being lifted vertically with an acceleration of 4 m/s^2 at the instant it has a velocity of 2 m/s. If the cable does not slip on the pulley's surface, determine the accelerations of the cylinder B and point C on the pulley.

$\omega = \frac{2}{0.08} = 25 \text{ rad/s}$

$\alpha = \frac{4}{0.08} = 50 \text{ rad/s}^2$

$\mathbf{a}_C = \mathbf{a}_A + (\mathbf{a}_{C/A})_n + (\mathbf{a}_{C/A})_t$

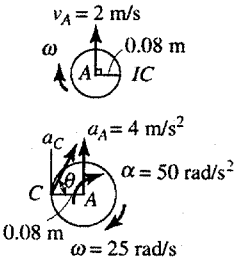
$\mathbf{a}_C = 4\mathbf{j} + (25)^2(0.08)\mathbf{i} + 50(0.08)\mathbf{j}$

$\stackrel{+}{\rightarrow} \quad a_C \cos\theta = 0 + 50$

$+\uparrow \quad a_C \sin\theta = 4 + 0 + 4$

Solving, $a_C = 50.6 \text{ m/s}^2$ **Ans**

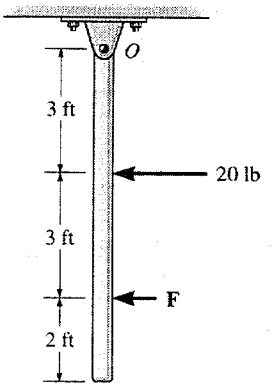
$\theta = 9.09°$ **Ans**



The block moves up with an acceleration

$a_B = (a_C)_t = 50.6 \sin 9.09° = 8.00 \text{ m/s}^2 \uparrow$ **Ans**

R2-27. At the instant shown, two forces act on the 30-lb slender rod which is pinned at O. Determine the magnitude of force **F** and the initial angular acceleration of the rod so that the horizontal reaction which the *pin exerts on the rod* is 5 lb directed to the right.xsa



$(a_G)_t = 4\alpha$

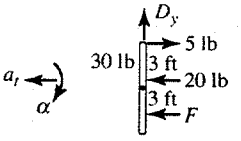
$\stackrel{+}{\leftarrow} \sum F_t = m(a_G)_t; \qquad F + 20 - 5 = \frac{30}{32.2}(4\alpha)$

$\curvearrowleft + \sum M_O = I_O\alpha; \qquad 20(3) + F(6) = \frac{1}{3}\left(\frac{30}{32.2}\right)(8)^2\alpha$

Solving,

$\alpha = 12.1 \text{ rad/s}^2$ **Ans**

$F = 30.0 \text{ lb}$ **Ans**



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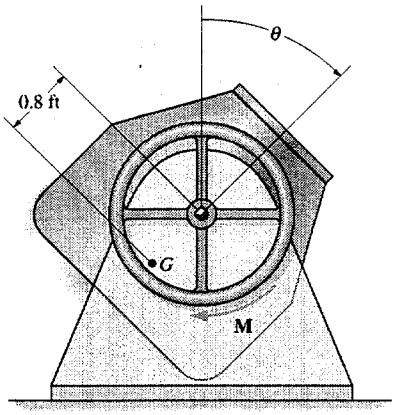
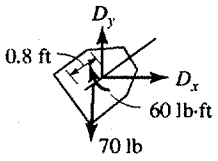
***R-28.** The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60$ lb · ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$.



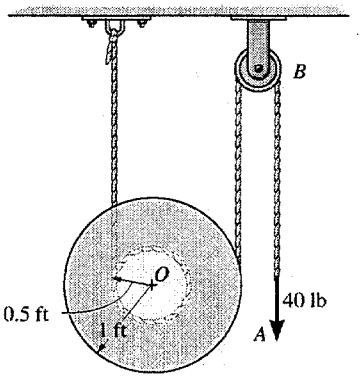
$T_1 + \sum U_{1-2} = T_2$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$\omega = 3.89$ rad/s **Ans**



R2-29. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.65$ ft. If a force of 40 lb is applied to the cord at A, determine the angular velocity of the spool in $t = 3$ s starting from rest. Neglect the mass of the pulley and cord.

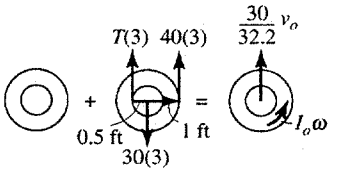


$(+\uparrow)$ $\quad mv_1 + \sum \int F\,dt = mv_2$

$$0 + T(3) - 30(3) + 40(3) = \frac{30}{32.2}v_O$$

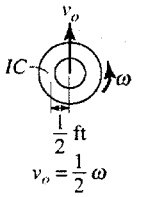
$(\curvearrowleft +)$ $\quad (H_O)_1 + \sum \int M_O\,dt = (H_O)_2$

$$-T(0.5)3 + 40(1)3 = \left[\frac{30}{32.2}(0.65)^2\right]\omega$$



Kinematics,

$v_O = 0.5\omega$



Solving,

$T = 23.5$ lb

$\omega = 215$ rad/s **Ans**

$v_O = 108$ ft/s

Also,

$(\curvearrowleft +)$ $\quad (H_{IC})_1 + \sum \int M_{IC}\,dt = (H_{IC})_2$

$$0 - 30(0.5)(3) + 40(1.5)(3) = \left[\frac{30}{32.2}(0.65)^2 + \frac{30}{32.2}(0.5)^2\right]\omega$$

$\omega = 215$ rad/s **Ans**

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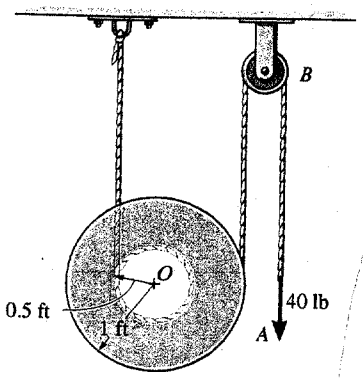
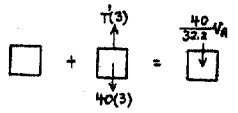
R2-30. Solve Prob. R2–29 if a 40-lb block is suspended from the cord at A, rather than applying the 40-lb force.



Block :

$(+\downarrow)\quad mv_1 + \Sigma\int F\,dt = mv_2$

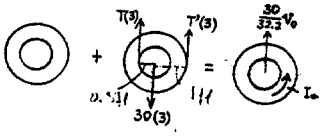
$0 + 40(3) - T'(3) = \frac{40}{32.2}v_A \qquad (1)$



Spool :

$(+\uparrow)\quad mv_1 + \Sigma\int F\,dt = mv_2$

$0 + T(3) - 30(3) + T'(3) = \frac{30}{32.2}v_O$



$(\circlearrowleft +)\quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$0 - T(0.5)3 + T'(1)3 = [\frac{30}{32.2}(0.65)^2]\omega$

Kinematics,

$v_O = 0.5\omega$

$v_A = 1.5\omega$

Solving,

$T' = 15.5$ lb

$T = 20.6$ lb

$\omega = 39.5$ rad/s **Ans**

$v_O = 19.7$ ft/s

$v_A = 59.2$ ft/s

Also,

$(\circlearrowleft +)\quad (H_{IC})_1 + \Sigma\int M_{IC}\,dt = (H_{IC})_2$

$0 + T'(1.5)(3) - 30(0.5)(3) = \left(\frac{30}{32.2}(0.65)^2 + \frac{30}{32.2}(0.5)^2\right)\omega$

Since $v_A = 1.5\omega$, then with Eq. (1) we get

$\omega = 39.5$ rad/s **Ans**

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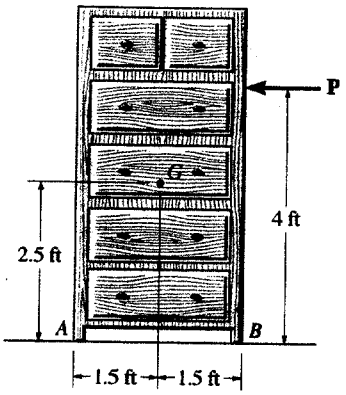
R2-31. The dresser has a weight of 80 lb and is pushed along the floor. If the coefficient of static friction at A and B is $\mu_s = 0.3$ and the coefficient of kinetic friction is $\mu_k = 0.2$, determine the smallest horizontal force P needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at A and B when it begins to move?

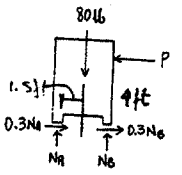


For slipping:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -P + 0.3(N_A + N_B) = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 80 = 0$

$P = 24$ lb **Ans**



For tipping $N_B = 0$, $N_A = 80$ lb

$(+\Sigma M_A = 0; \quad P(4) - 80(1.5) = 0$

$P = 30$ lb < 24 lb

Therefore dresser slips.

$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 80 = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad 24 - 0.2N_A - 0.2N_B = \frac{80}{32.2} a_G$

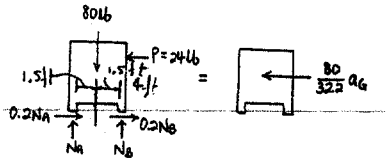
$(+\Sigma M_A = \Sigma (M_k)_A; \quad 24(4) + N_B(3) - 80(1.5) = \frac{80}{32.2} a_G (2.5)$

Solving,

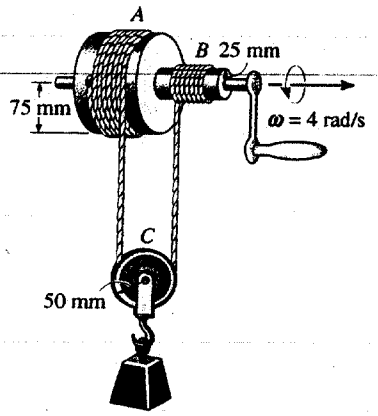
$a_G = 3.22$ ft/s^2 **Ans**

$N_B = 14.8$ lb **Ans**

$N_A = 65.3$ lb **Ans**



***R2-32.** When the crank on the Chinese windlass is turning, the rope on shaft A unwinds while that on shaft B winds up. Determine the speed at which the block lowers if the crank is turning with an angular velocity $\omega = 4$ rad/s. What is the angular velocity of the pulley at C? The rope segments on each side of the pulley are both parallel and vertical, and the rope does not slip on the pulley.



$v_P = \omega r_A = 4(75) = 300$ mm/s $\downarrow$

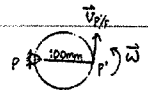
$v_{P'} = \omega r_B = 4(25) = 100$ mm/s $\uparrow$

$\mathbf{v}_{P'} = \mathbf{v}_P + \mathbf{v}_{P'/P}$

$100\mathbf{j} = -300\mathbf{j} + \omega(100)\mathbf{j}$

$100 = -300 + \omega(100)$

$\omega = 4$ rad/s **Ans**

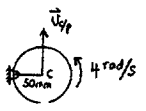


$\mathbf{v}_C = \mathbf{v}_P + \mathbf{v}_{C/P}$

$-v_C\mathbf{j} = -300\mathbf{j} + 4(50)\mathbf{j}$

$-v_C = -300 + 200$

$v_C = 100$ mm/s $\downarrow$ **Ans**



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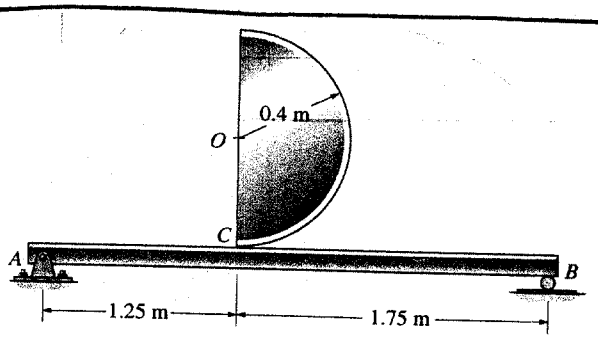
R2-33. The semicircular disk has a mass of 50 kg and is released from rest from the position shown. The coefficients of static and kinetic friction between the disk and the beam are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Determine the initial reactions at the pin A and roller B, used to support the beam. Neglect the mass of the beam for the calculation.



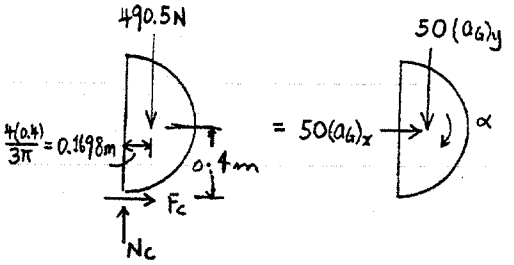
Semicircular disk :

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad F_C = 50(a_G)_x \qquad (1)$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \quad 490.5 - N_C = 50(a_G)_y \qquad (2)$$

$$\curvearrowright + \Sigma M_G = I_G\alpha; \quad N_C(0.1698) - F_C(0.4) = [\frac{1}{2}(50)(0.4)^2 - 50(0.1698)^2]\alpha \qquad (3)$$

Assume no slipping at C,



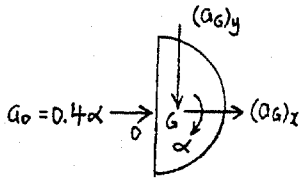
$$\mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 0.4\alpha\,\mathbf{i} - \alpha(0.1698)\mathbf{j}$$

$$(a_G)_x = 0.4\alpha$$

$$(a_G)_y = 0.1698\alpha$$

Solving,



$$\alpha = 6.94 \text{ rad/s}^2$$

$$(a_G)_x = 2.78 \text{ m/s}^2$$

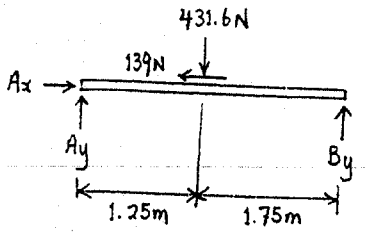
$$(a_G)_y = 1.18 \text{ m/s}^2$$

$$N_C = 431.6 \text{ N}$$

$$F_C = 139 \text{ N}$$

$$(F_C)_{max} = 0.5(431.6) = 216 \text{ N} > 139 \text{ N} \qquad \textbf{(O.K!)}$$

Beam :



$$\curvearrowright + \Sigma M_A = 0; \quad 431.6(1.25) - B_y(3) = 0$$

$$B_y = 179.8 \text{ N} = 180 \text{ N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 179.8 - 431.6 = 0$$

$$A_y = 252 \text{ N} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 139 = 0$$

$$A_x = 139 \text{ N} \qquad \textbf{Ans}$$

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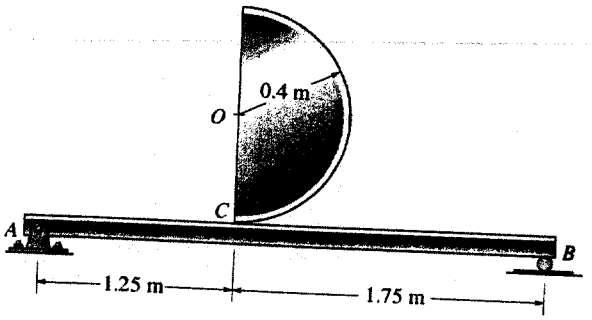
R2-34. The semicircular disk has a mass of 50 kg and is released from rest from the position shown. The coefficients of static and kinetic friction between the disk and the beam are $\mu_s = 0.2$ and $\mu_k = 0.1$, respectively. Determine the initial reactions at the pin A and roller B used to support the beam. Neglect the mass of the beam for the calculation.

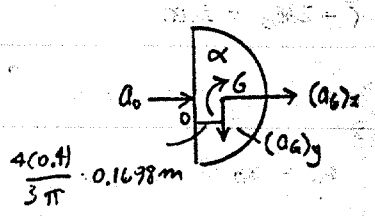


See solution to Prob. R2–33.

Here $F_C = 139\ \text{N} > 0.2(432) = 86.4\ \text{N}$

Therefore disk will slip.

Thus, $F_C = 0.1N_C$ (4)



$\mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$

$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = a_O\mathbf{i} - \alpha(0.1698)\mathbf{j}$

$(+\downarrow)$ $(a_G)_y = \alpha(0.1698)$ (5)

Solving Eqs.(1)–(3) of Prob. R2–33, and Eqs.(4)&(5).

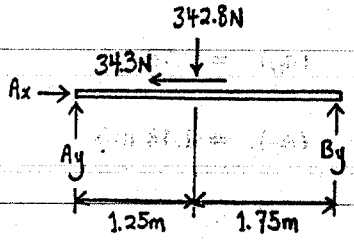
$\alpha = 17.4\ \text{rad/s}^2$

$(a_G)_x = 0.686\ \text{m/s}^2$

$(a_G)_y = 2.95\ \text{m/s}^2$

$N_C = 342.8\ \text{N}$

$F_C = 34.3\ \text{N}$



$\circlearrowleft + \Sigma M_A = 0;\quad 342.8(1.25) - B_y(3) = 0$

$B_y = 143\ \text{N}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + 143 - 342.8 = 0$

$A_y = 200\ \text{N}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 34.3 = 0$

$A_x = 34.3\ \text{N}$ **Ans**

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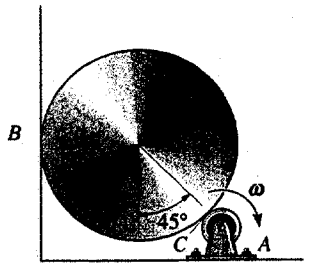
R2-35. The cylinder having a mass of 5 kg is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A \cos 45° + 0.2N_A \sin 45° + 0.2N_B - 5(9.81) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad N_B + 0.2N_A \cos 45° - N_A \sin 45° = 0$$

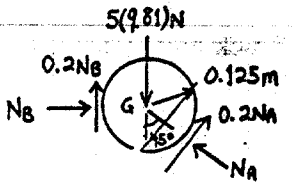
$$\circlearrowleft + \Sigma M_G = I_G \alpha; \quad 0.2N_A(0.125) - 0.2N_B(0.125) = [\tfrac{1}{2}(5)(0.125)^2]\alpha$$

Solving,

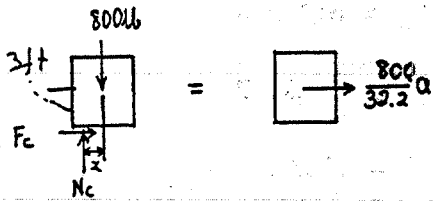
$N_A = 51.0$ N

$N_B = 28.9$ N

$\alpha = 14.2$ rad/s^2 **Ans**



***R2-36.** The truck carries the 800-lb crate which has a center of gravity at G_c. Determine the largest acceleration of the truck so that the crate will not slip or tip on the truck bed. The coefficient of static friction between the crate and the truck is $\mu_s = 0.6$.



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad F_C = \frac{800}{32.2}a$$

$$+\uparrow \Sigma F_y = 0; \quad N_C - 800 = 0$$

$$\circlearrowleft + \Sigma M_{G_c} = 0; \quad -N_C(x) + F_C(3) = 0$$

Assume impending slip; $F_C = 0.6N_C$

$a = 19.32$ ft/s^2

$x = 1.8$ ft > 1.5 ft crate tips!

Thus, $x = 1.5$ ft

$F_C = 400$ lb

$a = 16.1$ ft/s^2 **Ans**

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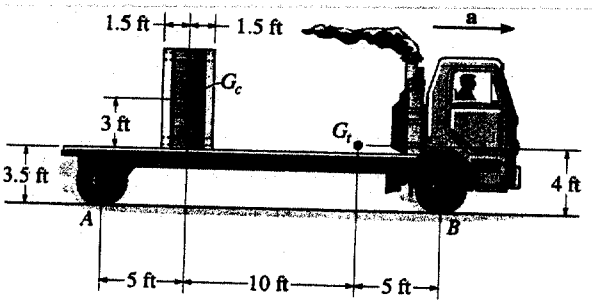
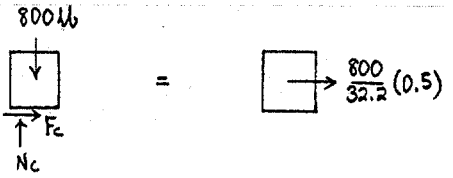
R2-37. The truck has a weight of 8000 lb and center of gravity at G_t. It carries the 800-lb crate, which has a center of gravity at G_c. Determine the normal reaction at *each* of its four tires if it accelerates at $a = 0.5\ \text{ft/s}^2$. Also, what is the frictional force acting between the crate and the truck, and between *each* of the rear tires and the road? Assume that power is delivered only to the rear tires. The front tires are free to roll. Neglect the mass of the tires. The crate does not slip or tip on the truck.



Crate:



$$+\uparrow \Sigma F_y = 0; \quad N_C = 800\ \text{lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad F_C = \frac{800}{32.2}(0.5) = 12.4\ \text{lb} \qquad \textbf{Ans}$$

Truck and crate:

$$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 800 - 8000 = 0$$

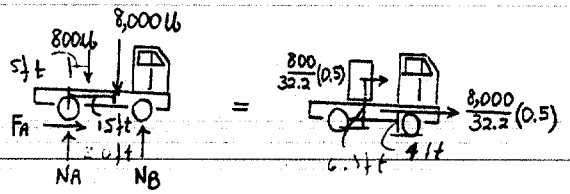
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad F_A = \frac{800}{32.2}(0.5) + \frac{8000}{32.2}(0.5)$$

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad N_B(20) - 800(5) - 8000(15) = -\frac{800}{32.2}(0.5)(6.5) - \frac{8000}{32.2}(0.5)(4)$$

$$N_B = 6171.1\ \text{lb}$$

$$N_A = 2628.9\ \text{lb}$$

$$F_A = 136.6\ \text{lb}$$



Thus,

$$N_{B'} = \frac{N_B}{2} = 3.09\ \text{kip} \qquad \textbf{Ans}$$

$$N_{A'} = \frac{N_A}{2} = 1.31\ \text{kip} \qquad \textbf{Ans}$$

$$F_{A'} = \frac{F_A}{2} = 68.3\ \text{lb} \qquad \textbf{Ans}$$

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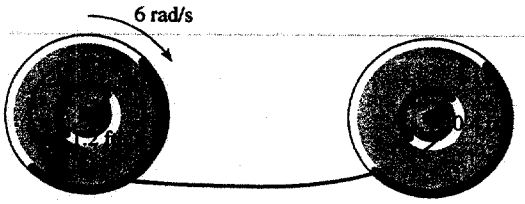
R2-38. Spool B is at rest and spool A is rotating at 6 rad/s when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight. The spools A and B have weights and radii of gyration $W_A = 30$ lb, $k_A = 0.8$ ft and $W_B = 15$ lb, $k_B = 0.6$ ft, respectively.

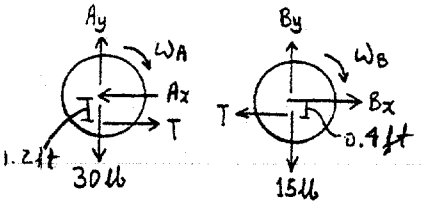


$$\curvearrowright + \quad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$$

$$[\frac{30}{32.2}(0.8)^2](6) - \int T\, dt\,(1.2) = [\frac{30}{32.2}(0.8)^2]\omega_A$$

$$\curvearrowright + \quad (H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

$$0 + \int T\, dt\,(0.4) = [\frac{15}{32.2}(0.6)^2]\omega_B$$



Kinematics:

$$1.2\omega_A = 0.4\omega_B$$

$$\omega_B = 3\omega_A$$

Thus,

$\omega_A = 1.70$ rad/s **Ans**

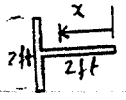
$\omega_B = 5.10$ rad/s **Ans**

R2-39. The two 3-lb rods EF and HI are fixed (welded) to the link AC at E. Determine the internal axial force E_x, shear force E_y, and moment M_E, which the bar AC exerts on FE at E if at the instant $\theta = 30°$ link AB has an angular velocity $\omega = 5$ rad/s and an angular acceleration $\alpha = 8$ rad/s^2 as shown.

Curvilinear translation.

$$(a_G)_t = 8(3) = 24 \text{ ft/s}^2$$

$$(a_G)_n = (5)^2(3) = 75 \text{ ft/s}^2$$

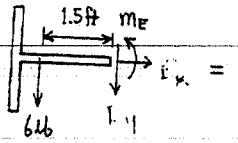


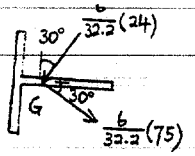
$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{1(3) + 2(3)}{6} = 1.5 \text{ ft}$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \quad E_y + 6 = \frac{6}{32.2}(24)\cos 30° + \frac{6}{32.2}(75)\sin 30°$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad E_x = \frac{6}{32.2}(75)\cos 30° - \frac{6}{32.2}(24)\sin 30°$$

$$\circlearrowleft + \Sigma M_G = 0; \quad M_E - E_y(1.5) = 0$$





$E_x = 9.87$ lb **Ans**

$E_y = 4.86$ lb **Ans**

$M_E = 7.29$ lb·ft **Ans**

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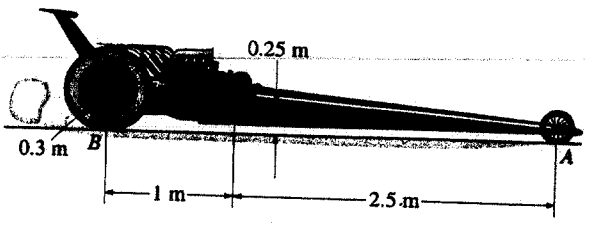
***R2-40.** The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear wheels are slipping. If so, what acceleration is necessary to do this? Neglect the mass of the wheels and assume that the front wheels are free to roll.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.6N_B = 1500a_G \qquad (1)$

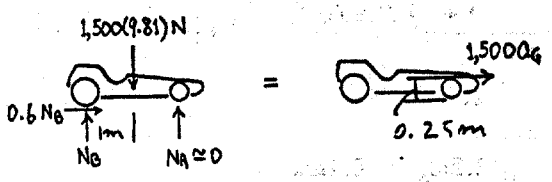
$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B - 1500(9.81) = 0 \qquad (2)$

$(+\Sigma M_B = \Sigma(M_k)_B; \quad -1500(9.81)(1) = -1500(a_G)(0.25) \qquad (3)$

Solving Eqs.(1)&(2)

$N_B = 14715$ N

$a_G = 5.886$ m/s^2



From Eq.(3);

$a_G = 39.24$ m/s$^2 \rightarrow$

Acceleration necessary to lift front wheels is

$a_G = 39.2$ m/s^2 **Ans**

It cannot be done since 39.2 m/s^2 > 5.89 m/s^2 Ans

R2-41. The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the friction force F_B which must be applied to *each* of the rear wheels B in order to develop an acceleration $a = 6$ m/s^2. What are the normal reactions of *each* wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad F_B = 1500(6)$

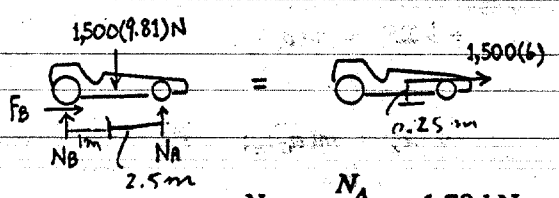
$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B + N_A - 1500(9.81) = 0$

$(+\Sigma M_B = \Sigma(M_k)_B; \quad -1500(9.81)(1) + N_A(3.5) = -1500(6)(0.25)$

$F_B = 9.00$ kN

$N_A = 3.56$ kN

$N_B = 11.15$ kN



$F_{B'} = \frac{F_B}{2} = 4.50$ kN **Ans**

$N_{A'} = \frac{N_A}{2} = 1.78$ kN **Ans**

$N_{B'} = \frac{N_B}{2} = 5.58$ kN **Ans**

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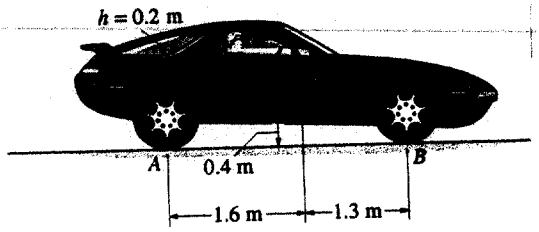
R2-42. The 1.6-Mg car shown has been "raked" by increasing the height $h = 0.2$ m of its center of mass. This was done by raising the springs on the rear axle. If the coefficient of kinetic friction between the rear wheels and the ground is $\mu_k = 0.3$, show that the car can accelerate slightly faster than its counterpart for which $h = 0$. Neglect the mass of the wheels and driver and assume the front wheels at B are free to roll while the rear wheels slip.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.3N_A = 1600a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B + N_A - 1600(9.81) = 0$

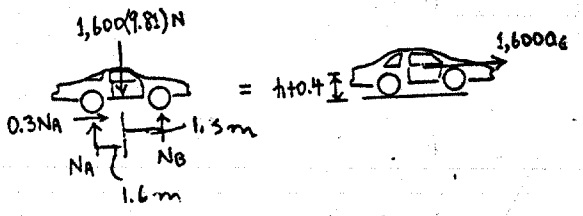
$\curvearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 1600(9.81)(1.6) - N_B(2.9) = 1600a_G(h + 0.4)$

Set $h = 0.2$ m and solving,

$a_G = 1.41 \text{ m/s}^2$ **Ans**

$N_A = 7.50 \text{ kN}$

$N_B = 8.19 \text{ kN}$



Set $h = 0$

$a_G = 1.38 \text{ m/s}^2$ **Ans**

$N_A = 7.34 \text{ kN}$

$N_B = 8.36 \text{ kN}$

R2-43. The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at *each* of the wheels at A and B if a force $P = 50$ N is applied to the handle. Neglect the mass and rolling resistance of the wheels.

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad 50\cos 60° = 200a_G \qquad (1)$

$+\uparrow \Sigma F_y = ma_y; \quad N_A + N_B - 200(9.81) - 50\sin 60° = 0 \qquad (2)$

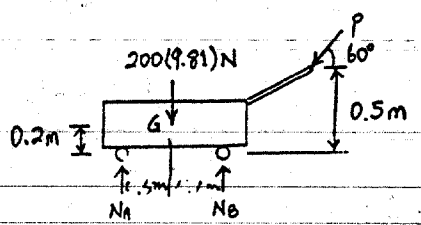
$\curvearrowleft + \Sigma M_G = 0; \quad -N_A(0.3) + N_B(0.2) + 50\cos 60°(0.3) - 50\sin 60°(0.6) = 0 \qquad (3)$

Solving,

$a_G = 0.125 \text{ m/s}^2$

$N_A = 765.2 \text{ N}$

$N_B = 1240 \text{ N}$



At each wheel

$N'_A = \frac{N_A}{2} = 383 \text{ N}$ **Ans** $\quad N'_B = \frac{N_B}{2} = 620 \text{ N}$ **Ans**

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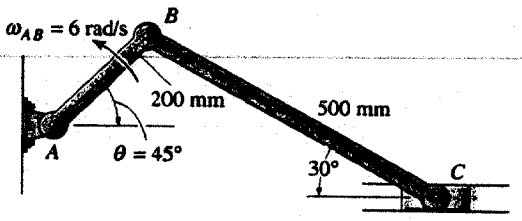
***R2-44.** If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



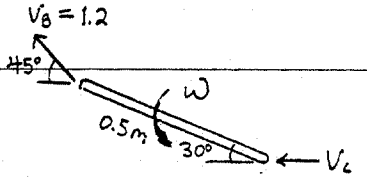
$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$v_C\mathbf{i} = (6\mathbf{k}) \times (0.2\cos 45°\mathbf{i} + 0.2\sin 45°\mathbf{j}) + (\omega\mathbf{k}) \times (0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j})$$

$$v_C = -0.8485 + \omega(0.25)$$

$$0 = 0.8485 + 0.433\,\omega$$



Solving

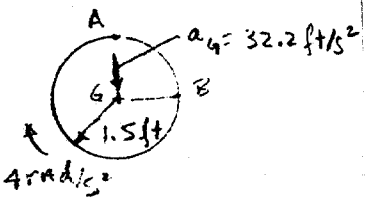
$\omega = 1.96$ rad/s ↻ **Ans**

$v_C = 1.34$ m/s **Ans**

R2-45. The disk is rotating at a constant rate $\omega = 4$ rad/s, and as it falls freely, its center has an acceleration of 32.2 ft/s^2. Determine the acceleration of points A and B on the rim of the disk at the instant shown.

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

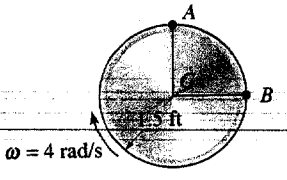
$$\left[(a_A)_{\rightarrow}\right] + \left[(a_A)_{\uparrow}\right] = \left[32.2_{\downarrow}\right] + 0 + \left[(4)^2(1.5)_{\downarrow}\right]$$



$$(\overset{+}{\rightarrow})\,(a_A)_x = 0$$

$$(+\uparrow)(a_A)_y = -32.2 - (4)^2(1.5) = -56.2 \text{ ft/s}^2 = 56.2 \text{ ft/s}^2 \downarrow$$

$$a_A = (a_A)_y = 56.2 \text{ ft/s}^2 \downarrow \quad \textbf{Ans}$$



$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$\left[(a_B)_{\rightarrow}\right] + \left[(a_B)_{\uparrow}\right] = \left[32.2_{\downarrow}\right] + 0 + \left[(4)^2(1.5)_{\leftarrow}\right]$$

$$(\overset{+}{\rightarrow})\,(a_B)_x = -(4)^2(1.5) = -24 \text{ ft/s}^2 = 24 \text{ ft/s}^2 \leftarrow$$

$$(+\uparrow)(a_B)_y = -32.2 \text{ ft/s}^2 = 32.2 \text{ ft/s}^2 \downarrow$$

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{24^2 + 32.2^2} = 40.2 \text{ ft/s}^2 \quad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{(a_B)_y}{(a_B)_x} = \tan^{-1}\frac{32.2}{24} = 53.3° \quad \textbf{Ans}$$

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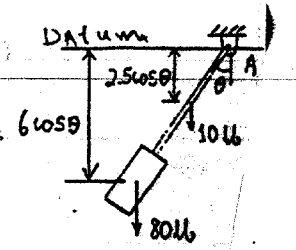
R2-46. The 80-lb cylinder is attached to the 10-lb slender rod which is pinned from point A. At the instant $\theta = 30°$ it has an angular velocity $\omega_1 = 1$ rad/s as shown. Determine the largest angle θ to which the rod swings before it momentarily stops.

$$I_A = \frac{1}{12}\left(\frac{80}{32.2}\right)[3(0.5)^2 + (2)^2] + \frac{80}{32.2}(6)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(5)^2 = 93.01 \text{ slug}\cdot\text{ft}^2$$

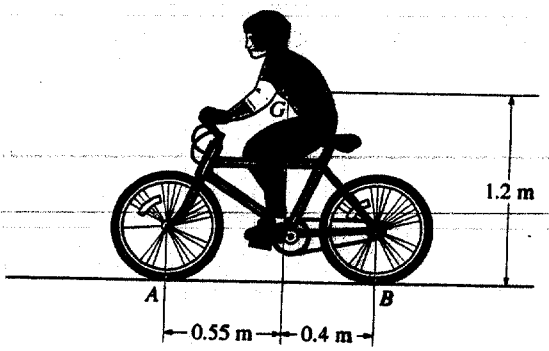
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(93.01)(1)^2 - 10(2.5\cos 30°) - 80(6\cos 30°) = [0] - 10(2.5\cos\theta) - 80(6\cos\theta)$$

$\theta = 39.3°$ **Ans**



R2-47. The bicycle and rider have a mass of 80 kg with center of mass located at G. If the coefficient of kinetic friction at the rear tire is $\mu_B = 0.8$, determine the normal reactions at the tires A and B, and the deceleration of the rider, when the rear wheel locks for braking. What is the normal reaction at the rear wheel when the bicycle is traveling at constant velocity and the brakes are not applied? Neglect the mass of the wheels.



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad 0.8N_B = 80a_B$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 80(9.81) = 0$$

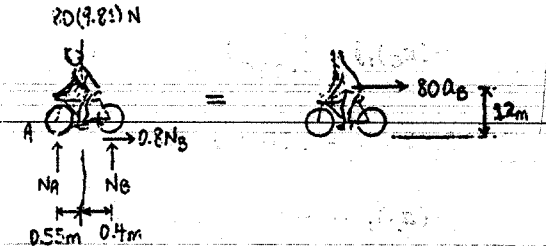
$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad -N_B(0.95) + 80(9.81)(0.55) = 80a_B(1.2)$$

Solving,

$a_B = 2.26 \text{ m/s}^2$ **Ans**

$N_B = 226$ N **Ans**

$N_A = 559$ N **Ans**



For equilibrium,

$$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 80(9.81) = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -N_B(0.95) + 80(9.81)(0.55) = 0$$

$N_A = 330$ N

$N_B = 454$ N **Ans**

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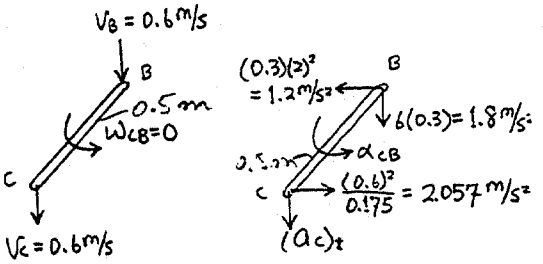
***R2-48.** At the instant shown, link AB has an angular velocity $\omega_{AB} = 2$ rad/s and an angular acceleration $\alpha_{AB} = 6$ rad/s^2. Determine the acceleration of the pin at C and the angular acceleration of link CB at this instant, when $\theta = 60°$.

$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$



$2.057 + (a_C)_t = 1.8 + 1.2 + \alpha_{CB}(0.5)$

$\rightarrow \quad \downarrow \quad \downarrow \quad \leftarrow \quad \measuredangle\theta\ 30°$

$(\overset{+}{\rightarrow}) \quad 2.057 = -1.2 + \alpha_{CB}(0.5)\cos 30°$

$(+\downarrow) \quad (a_C)_t = 1.8 + \alpha_{CB}(0.5)\sin 30°$

$\alpha_{CB} = 7.52$ rad/s^2 **Ans**

$(a_C)_t = 3.68$ m/s^2

$a_C = \sqrt{(3.68)^2 + (2.057)^2} = 4.22$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{3.68}{2.057}\right) = 60.8° \measuredangle\theta$ **Ans**

Also,

$\mathbf{a}_C = \mathbf{a}_B + \alpha_{CB} \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$

$-(a_C)_t\mathbf{j} + \frac{(0.6)^2}{0.175}\mathbf{i} = -(2)^2(0.3)\mathbf{i} - 6(0.3)\mathbf{j} + (\alpha_{CB}\mathbf{k}) \times (-0.5\cos 60°\mathbf{i} - 0.5\sin 60°\mathbf{j}) - 0$

$2.057 = -1.20 + \alpha_{CB}(0.433)$

$-(a_C)_t = -1.8 - \alpha_{CB}(0.250)$

$\alpha_{CB} = 7.52$ rad/s^2 **Ans**

$a_t = 3.68$ m/s^2

$a_C = \sqrt{(3.68)^2 + (2.057)^2} = 4.22$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{3.68}{2.057}\right) = 60.8° \measuredangle\theta$ **Ans**

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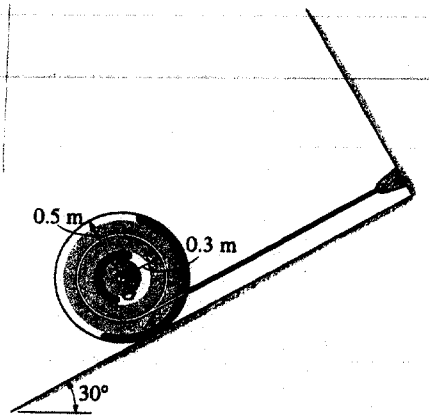
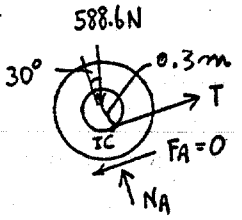
R2-49. The spool has a mass of 60 kg and a radius of gyration $k_G = 0.3$ m. If it is released from rest, determine how far it descends down the smooth plane before it attains an angular velocity $\omega = 6$ rad/s. Neglect friction and the mass of the cord which is wound around the central core.



$v_G = 0.3\omega$

$T_1 + \Sigma U_{1-2} = T_2$



$$0 + 588.6\sin 30°(s) = \frac{1}{2}[60(0.3)^2](6)^2 + \frac{1}{2}(60)[0.3(6)]^2$$

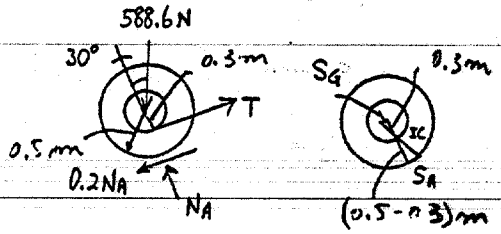
$s = 0.661$ m **Ans**

R2-50. Solve Prob. R2–49 if the plane is rough, such that the coefficient of kinetic friction at A is $\mu_A = 0.2$.

$$\frac{s_G}{0.3} = \frac{s_A}{(0.5-0.3)}$$

$s_A = 0.6667 s_G$

$\nwarrow +\Sigma F_y = 0; \quad N_A - 588.6\cos 30° = 0$



$N_A = 509.7$ N

$v_G = 0.3\omega$

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 588.6\sin 30°(s_G) - 0.2(509.7)(0.6667 s_G) = \frac{1}{2}[60(0.3)^2](6)^2 + \frac{1}{2}(60)[(0.3)(6)]^2$$

$s_G = 0.859$ m **Ans**

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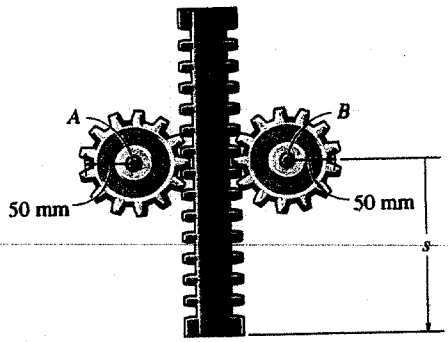
R2-51. The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration $k = 30$ mm at their centers. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to turn about their centers, A and B.



Conservation of energy : Originally, both gears are rotating with an angular velocity of $\omega_1 = \frac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \frac{v_2}{0.05}$, where v_2 is the speed of the rack at that moment.

$$T_1 + V_1 = T_2 + V_2$$

$$\tfrac{1}{2}(6)(2)^2 + 2\{\tfrac{1}{2}[4(0.03)^2](40)^2\} + 0 = \tfrac{1}{2}(6)v_2^2 + 2\left\{\tfrac{1}{2}[4(0.03)^2]\left(\tfrac{v_2}{0.05}\right)^2\right\} - 6(9.81)(0.6)$$

$$v_2 = 3.46 \text{ m/s} \qquad \textbf{Ans}$$

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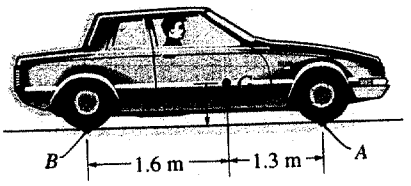
***R2-52.** The car has a mass of 1.50 Mg and a mass center at G. Determine the maximum acceleration it can have if (a) power is supplied only to the rear wheels, (b) power is supplied only to the front wheels. Neglect the mass of the wheels in the calculation, and assume that the wheels that do not receive power are free to roll. Also, assume that slipping of the powered wheels occurs, where the coefficient of kinetic friction is $\mu_k = 0.3$.



(a) Rear wheel drive

Equations of motion :

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.3N_B = 1.5(10)^3 a_G \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 1.5(10)^3(9.81)(1.3) - N_B(2.9) = -1.5(10)^3 a_G(0.4) \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$N_B = 6881\ \text{N} = 6.88\ \text{kN}$

$a_G = 1.38\ \text{m/s}^2$ **Ans**

(b) Front wheel drive

Equations of motion :

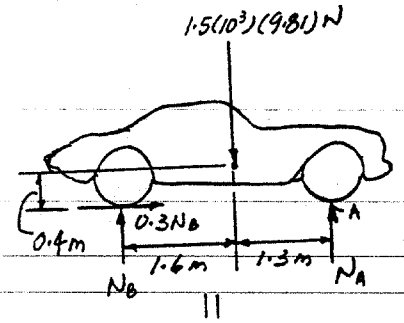
$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.3N_A = 1.5(10)^3 a_G \qquad [1]$$

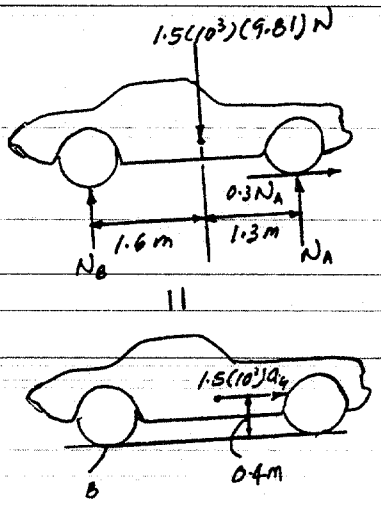
$$\Sigma M_B = \Sigma (M_k)_B; \quad -1.5(10)^3(9.81)(1.6) + N_A(2.9) = -1.5(10)^3 a_G(0.4) \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$N_A = 7796\ \text{N} = 7.80\ \text{kN}$

$a_G = 1.56\ \text{m/s}^2$ **Ans**





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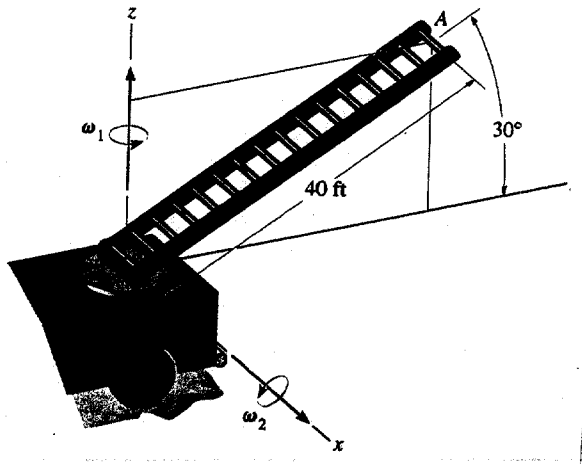
20-1. The ladder of the fire truck rotates around the z axis with an angular velocity $\omega_1 = 0.15$ rad/s, which is increasing at 0.8 rad/s^2. At the same instant it is rotating upward at a constant rate $\omega_2 = 0.6$ rad/s. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



$\omega = \omega_1 + \omega_2 = 0.15\mathbf{k} + 0.6\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

Angular accelerations: For ω_1, $\omega = \omega_1 = \{0.15\mathbf{k}\}$ rad/s.

$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \omega \times \omega_2$

$= 0 + (0.15\mathbf{k}) \times (0.6\mathbf{i}) = \{0.09\mathbf{j}\}$ rad/s^2

For ω_1, $\Omega = 0$.

$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \omega \times \omega_1 = (0.8\mathbf{k}) + 0 = \{0.8\mathbf{k}\}$ rad/s^2

$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$

$\alpha = 0.8\mathbf{k} + 0.09\mathbf{j} = \{0.09\mathbf{j} + 0.8\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\mathbf{v}_A = \omega \times \mathbf{r}_A$

$= (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

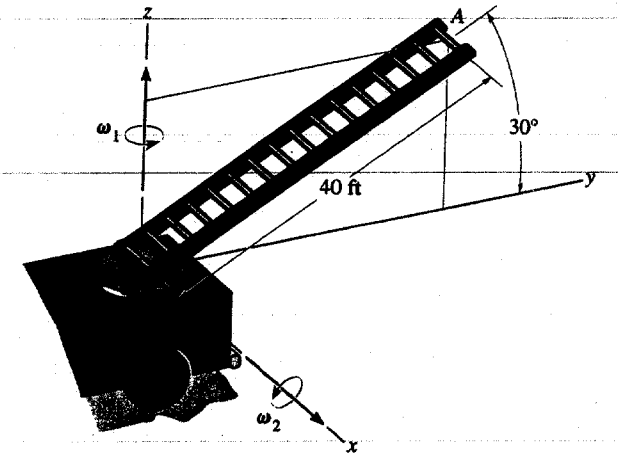
$= \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \alpha \times \mathbf{r} + \omega \times \mathbf{v}_A$

$= (0.09\mathbf{j} + 0.8\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k}) + (0.6\mathbf{i} + 0.15\mathbf{k}) \times (-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k})$

$= \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\}$ ft/s^2 **Ans**

***20-2.** The ladder of the fire truck rotates around the z axis with an angular velocity of $\omega_1 = 0.15$ rad/s, which is increasing at 0.2 rad/s^2. At the same instant it is rotating upwards at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s^2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



$\mathbf{r}_{A/O} = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$

$\mathbf{r}_{A/O} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\Omega = \omega_1\mathbf{k} + \omega_2\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

$\dot{\Omega} = \dot{\omega}_1\mathbf{k} + \dot{\omega}_2\mathbf{i} + \omega_1\mathbf{k} \times \omega_2\mathbf{i}$

$\dot{\Omega} = 0.2\mathbf{k} + 0.4\mathbf{i} + 0.15\mathbf{k} \times 0.6\mathbf{i} = \{0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_A = \Omega \times \mathbf{r}_{A/O} = (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \Omega \times (\Omega \times \mathbf{r}_{A/O}) + \dot{\Omega} \times \mathbf{r}_{A/O}$

$\mathbf{a}_A = (0.6\mathbf{i} + 0.15\mathbf{k}) \times [(0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})]$
$+ (0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2 **Ans**

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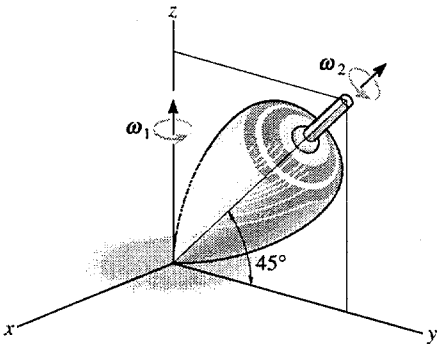
20-3. The motion of the top is such that at the instant shown it is rotating about the z axis at $\omega_1 = 0.6$ rad/s, while it is spinning at $\omega_2 = 8$ rad/s. Determine the angular velocity and angular acceleration of the top at this instant. Express the result as a Cartesian vector.



$\omega = \omega_1 + \omega_2$

$\omega = 0.6\mathbf{k} + 8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}$

$\omega = \{5.66\mathbf{j} + 6.26\mathbf{k}\}$ rad/s **Ans**

$\omega = \omega_1 + \omega_2$

Let x, y, z axes have angular velocity of $\Omega = \omega_1$, thus

$\omega_1 = \mathbf{0}$

$\omega_2 = (\omega_2)_{xyz} + (\omega_1 \times \omega_2) = \mathbf{0} + (0.6\mathbf{k}) \times (8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}) = -3.394\mathbf{i}$

$\alpha = \dot{\omega} = \{-3.39\mathbf{i}\}$ rad/s^2 **Ans**

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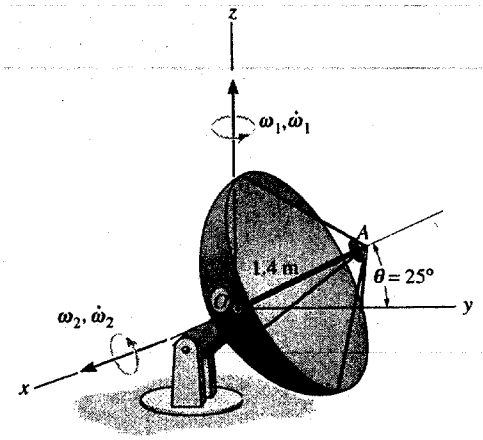
***20-4.** At a given instant, the satellite dish has an angular motion $\omega_1 = 6$ rad/s and $\dot{\omega}_1 = 3$ rad/s^2 about the z axis. At this same insant $\theta = 25°$, the angular motion about the x axis is $\omega_2 = 2$ rad/s, and $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant.



Angular Velocity : The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus. the angular velocity of the satellite at this instant(with reference to X, Y, Z) can be expressed in terms of **i**, **j**, **k** components.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = \{2\mathbf{i} + 6\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration : The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_2 is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s. Applying Eq. 20 – 6 with $(\dot{\omega}_2)_{xyz} = \{1.5\mathbf{i}\}$ rad/s^2, we have

$$\dot{\boldsymbol{\omega}}_2 = (\dot{\boldsymbol{\omega}}_2)_{xyz} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_2 = 1.5\mathbf{i} + 6\mathbf{k} \times 2\mathbf{i} = \{1.5\mathbf{i} + 12\mathbf{j}\} \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis $(\boldsymbol{\Omega} = \mathbf{0})$, then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\boldsymbol{\omega}}_1)_{xyz} + \mathbf{0} \times \boldsymbol{\omega}_1 = \{3\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the satellite is

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 = \{1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration : Applying Eqs. 20 – 3 and 20 – 4 with the ω and α obtained above and $\mathbf{r}_A = \{1.4\cos 25°\mathbf{j} + 1.4\sin 25°\mathbf{k}\}$ m $= \{1.2688\mathbf{j} + 0.5917\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A = (2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$
$$= \{-7.61\mathbf{i} - 1.18\mathbf{j} + 2.54\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_A)$$
$$= (1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k}) + (2\mathbf{i} + 6\mathbf{k}) \times [(2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})]$$
$$= \{10.4\mathbf{i} - 51.6\mathbf{j} - 0.463\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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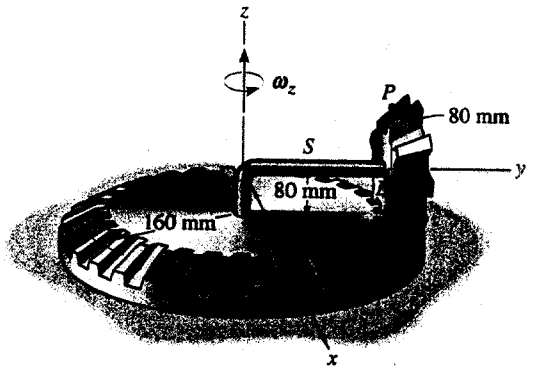
20-5. Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 5$ rad/s, while increasing at 2 rad/s^2, determine the velocity and acceleration of point C at the instant shown. The face of gear B lies in a vertical plane.



From Prob. 20-4.

$\Omega = \{5\mathbf{k} - 10\mathbf{j}\}$ rad/s

$\dot{\Omega} = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_C = \Omega \times \mathbf{r}_C$

$\mathbf{v}_C = (5\mathbf{k} - 10\mathbf{j}) \times (80\mathbf{i} + 160\mathbf{j})$

$\mathbf{v}_C = \{-800\mathbf{i} + 400\mathbf{j} + 800\mathbf{k}\}$ mm/s

$= \{-0.800\mathbf{i} + 0.400\mathbf{j} + 0.800\mathbf{k}\}$ m/s **Ans**

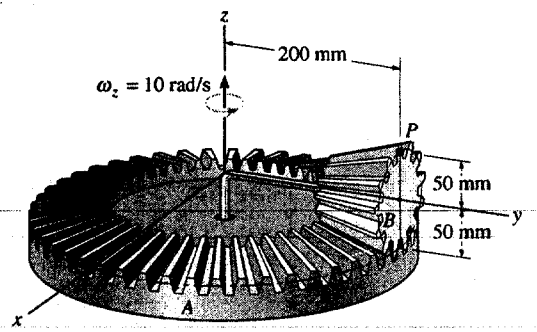
$\mathbf{a}_C = \Omega \times \mathbf{v}_C + \dot{\Omega} \times \mathbf{r}_C$

$\mathbf{a}_C = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \times (80\mathbf{i} + 160\mathbf{j}) + (-10\mathbf{j} + 5\mathbf{k}) \times (-800\mathbf{i} + 400\mathbf{j} + 800\mathbf{k})$

$\mathbf{a}_C = \{-10\,320\mathbf{i} - 3840\mathbf{j} + 320\mathbf{k}\}$ mm/s^2

$\mathbf{a}_C = \{-10.3\mathbf{i} - 3.84\mathbf{j} + 0.320\mathbf{k}\}$ m/s^2 **Ans**

20-6. Gear B is connected to the rotating shaft, while the plate gear A is fixed. If the shaft is turning at a constant rate of $\omega_z = 10$ rad/s about the z axis, determine the magnitudes of the angular velocity and the angular acceleration of gear B. Also, determine the magnitudes of the velocity and acceleration of point P.



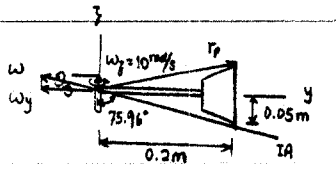
$\omega_z = 10$ rad/s

$\omega_y = -10\tan 75.96° = -40$ rad/s

$\omega_x = 0$

$\omega = \{-40\mathbf{j} + 10\mathbf{k}\}$ rad/s

$\omega = \sqrt{(-40)^2 + (10)^2} = 41.2$ rad/s **Ans**



$\mathbf{r}_P = \{0.2\mathbf{j} + 0.05\mathbf{k}\}$ m

$\mathbf{v}_P = \omega \times \mathbf{r}_P = (-40\mathbf{j} + 10\mathbf{k}) \times (0.2\mathbf{j} + 0.05\mathbf{k})$

$\mathbf{v}_P = \{-4\mathbf{i}\}$ m/s

$v_P = 4.00$ m/s **Ans**

Let $\Omega = \omega_z$,

$\dot{\omega} = \left(\dot{\omega}\right)_{xyz} + \Omega \times \omega$

$= \mathbf{0} + (10\mathbf{k}) \times (-40\mathbf{j} + 10\mathbf{k}) = \{400\mathbf{i}\}$ rad/s^2

$\alpha = \dot{\omega} = 400$ rad/s^2 **Ans**

$\mathbf{a}_P = \alpha \times \mathbf{r}_P + \omega \times \mathbf{v}_P = (400\mathbf{i}) \times (0.2\mathbf{j} + 0.05\mathbf{k}) + (-40\mathbf{j} + 10\mathbf{k}) \times (-4\mathbf{i})$

$= \{-60\mathbf{j} - 80\mathbf{k}\}$ m/s^2

$a_P = \sqrt{(-60)^2 + (-80)^2} = 100$ m/s^2 **Ans**

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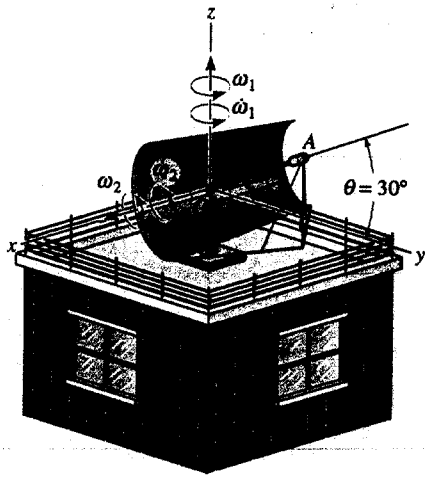
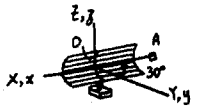
20-7. At a given instant, the antenna has an angular motion $\omega_1 = 3$ rad/s and $\dot{\omega}_1 = 2$ rad/s^2 about the z axis. At this same instant $\theta = 30°$, the angular motion about the x axis is $\omega_2 = 1.5$ rad/s, and $\dot{\omega}_2 = 4$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is $d = 3$ ft.





$\mathbf{r}_A = 3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k} = \{2.598\mathbf{j} + 1.5\mathbf{k}\}$ ft

$\Omega = \omega_1 + \omega_2 = 3\mathbf{k} + 1.5\mathbf{i}$

$\mathbf{v}_A = \Omega \times \mathbf{r}_A$

$\mathbf{v}_A = (3\mathbf{k} + 1.5\mathbf{i}) \times (2.598\mathbf{j} + 1.5\mathbf{k})$

$= -7.794\mathbf{i} + 3.897\mathbf{k} - 2.25\mathbf{j}$

$= \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}$ ft/s **Ans**

$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2$

$= (2\mathbf{k} + 0) + (4\mathbf{i} + 3\mathbf{k} \times 1.5\mathbf{i})$

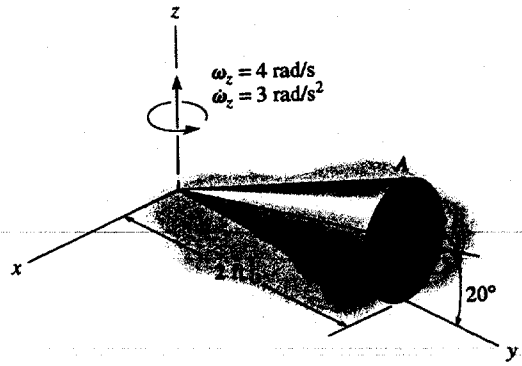
$= 4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_A + \Omega \times \mathbf{v}_A$

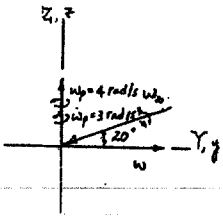
$\mathbf{a}_A = (4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}) \times (2.598\mathbf{j} + 1.5\mathbf{k}) + (3\mathbf{k} + 1.5\mathbf{i}) \times (-7.794\mathbf{i} - 2.25\mathbf{j} + 3.879\mathbf{k})$

$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}$ ft/s^2 **Ans**

***20-8.** The cone rolls without slipping such that at the instant shown $\omega_z = 4$ rad/s and $\dot{\omega}_z = 3$ rad/s^2. Determine the velocity and acceleration of point A at this instant.



Angular velocity : The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity(y axis).



$\omega = \omega_1 + \omega_2$

$\omega\mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20°\mathbf{j} + \omega_2 \sin 20°\mathbf{k})$

$\omega\mathbf{j} = \omega_2 \cos 20°\mathbf{j} + (4 + \omega_2 \sin 20°)\mathbf{k}$

Equating **j** and **k** components

$4 + \omega_2 \sin 20° = 0 \quad \omega_2 = -11.70$ rad/s

$\omega = -11.70 \cos 20° = -10.99$ rad/s

Hence $\omega = \{-10.99\mathbf{j}\}$ rad/s

$\omega_2 = -11.70\cos 20°\mathbf{j} + (-11.70\sin 20°)\mathbf{k} = \{-10.99\mathbf{j} + 4\mathbf{k}\}$ rad/s

Angular acceleration:

$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\}$ rad/s^2

$(\dot{\omega}_2)_{xyz} = \left(\frac{3}{\sin 20°}\right)\cos 20°\mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\}$ rad/s^2

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$

$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$

$\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s then

$\dot{\omega} = [3\mathbf{k} + 0] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} + 4\mathbf{k})]$

$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\}$ rad/s

$\mathbf{r}_A = 2\cos 40°\mathbf{j} + 2\sin 40°\mathbf{k} = \{1.5321\mathbf{j} + 1.2856\mathbf{k}\}$ ft

$\mathbf{v}_A = \omega \times \mathbf{r}_A$

$= (-10.99\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k})$

$= \{-14.1\mathbf{i}\}$ ft/s **Ans**

$\mathbf{a}_A = \mathbf{a} \times \mathbf{r}_A + \omega \times \mathbf{v}_A$

$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k}) + (-10.99\mathbf{j}) \times (-14.1\mathbf{i})$

$= \{-10.6\mathbf{i} - 56.5\mathbf{j} - 87.9\mathbf{k}\}$ ft/s^2 **Ans**

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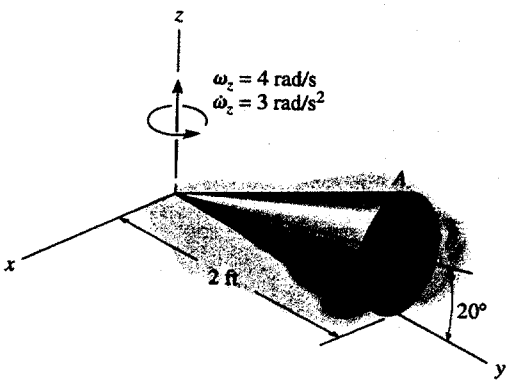
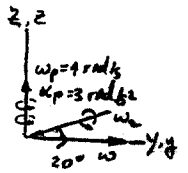
20-9. The cone rolls without slipping such that at the instant shown $\omega_z = 4$ rad/s and $\dot{\omega}_z = 3$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



Angular velocity : The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity(y axis).

$$\omega = \omega_1 + \omega_2$$
$$\omega\,\mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20°\,\mathbf{j} + \omega_2 \sin 20°\,\mathbf{k})$$
$$\omega\,\mathbf{j} = \omega_2 \cos 20°\,\mathbf{j} + (4 + \omega_2 \sin 20°)\,\mathbf{k}$$



Equating j and k components

$$4 + \omega_2 \sin 20° = 0 \qquad \omega_2 = -11.70 \text{ rad/s}$$
$$\omega = -11.70 \cos 20° = -10.99 \text{ rad/s}$$

Hence $\omega = \{-10.99\mathbf{j}\}$ rad/s

$$\omega_2 = -11.70 \cos 20°\,\mathbf{j} + (-11.70 \sin 20°)\,\mathbf{k} = \{-10.99\mathbf{j} + 4\mathbf{k}\} \text{ rad/s}$$

Angular acceleration:

$$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\} \text{ rad/s}^2$$
$$(\dot{\omega}_2)_{xyz} = \left(-\tfrac{3}{\sin 20°}\right) \cos 20°\,\mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\} \text{ rad/s}^2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$
$$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$$

$\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s then

$$\dot{\omega} = [3\mathbf{k} + 0] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} + 4\mathbf{k})]$$
$$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\} \text{ rad/s}$$

$$\mathbf{r}_B = 2\sin 20°\mathbf{i} + 2\cos 20°\mathbf{j} + 2\sin 20°\cos 20°\mathbf{k}$$
$$= -0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k}$$
$$\mathbf{v}_B = \omega \times \mathbf{r}_B$$
$$= (-10.99\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$= -7.0642\mathbf{i} - 7.5176\mathbf{k}$$
$$= \{-7.06\mathbf{i} - 7.52\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B$$
$$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$+ (-10.99\mathbf{j}) \times (-7.0642\mathbf{i} - 7.5176\mathbf{k})$$
$$= \{77.3\mathbf{i} - 28.3\mathbf{j} - 0.657\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

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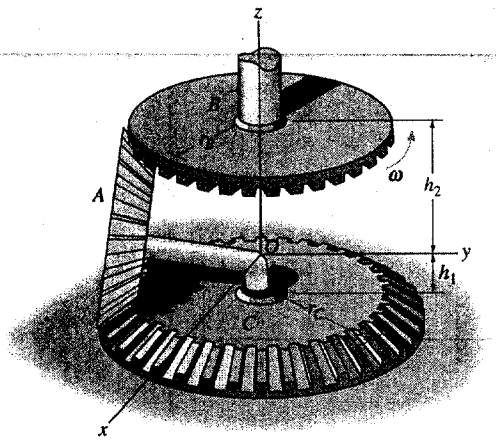
20-10. If the top gear B is rotating at a constant rate of ω, determine the angular velocity of gear A, which is free to turn about the shaft and rolls on the bottom fixed gear C.



$$\mathbf{v}_P = \omega\mathbf{k} \times (-r_B\mathbf{j}) = \omega r_B\mathbf{i}$$

Also,

$$\mathbf{v}_P = \omega_A \times (-r_B\mathbf{j} + h_2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{Ax} & \omega_{Ay} & \omega_{Az} \\ 0 & -r_B & h_2 \end{vmatrix}$$

$$= (\omega_{Ay}h_2 + \omega_{Az}r_B)\mathbf{i} - (\omega_{Ax}h_2)\mathbf{j} - \omega_{Ax}r_B\mathbf{k}$$

Thus,

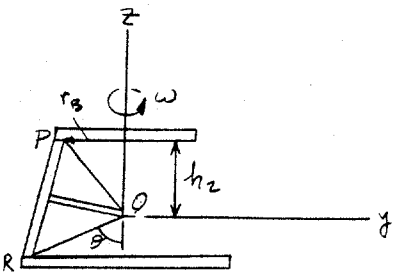
$$\omega r_B = \omega_{Ay}h_2 + \omega_{Az}r_B \qquad (1)$$

$$0 = \omega_{Ax}h_2$$

$$0 = \omega_{Ax}r_B$$

$$\omega_{Ax} = 0$$

$$\mathbf{v}_R = 0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_{Ay} & \omega_{Az} \\ 0 & -r_C & -h_1 \end{vmatrix} = (-\omega_{Ay}h_1 + \omega_{Az}r_C)\mathbf{i}$$



$$\omega_{Ay} = \omega_{Az}\left(\frac{r_C}{h_1}\right)$$

From Eq. (1)

$$\omega r_B = \omega_{Az}\left[\left(\frac{r_C h_2}{h_1}\right) + r_B\right]$$

$$\omega_{Az} = \frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}; \qquad \omega_{Ay} = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)$$

$$\omega_A = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)\mathbf{j} + \left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)\mathbf{k} \qquad \textbf{Ans}$$

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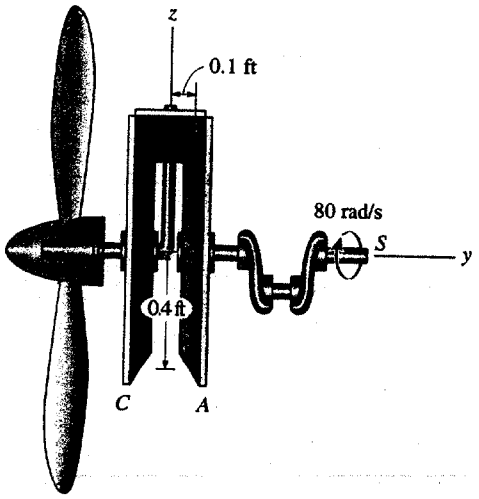
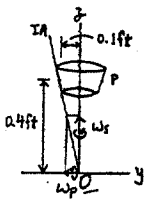
20-11. Gear A is fixed to the crankshaft S, while gear C is fixed and B are free to rotate. The crankshaft is turning at 80 rad/s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.



Point P on gear B has a speed of

$v_P = 80(0.4) = 32$ ft/s

The IA is located along the points of contant of B and C



$$\frac{\omega_P}{0.1} = \frac{\omega_s}{0.4}$$

$$\omega_s = 4\omega_P$$

$$\omega = -\omega_P \mathbf{j} + \omega_s \mathbf{k}$$

$$= -\omega_P \mathbf{j} + 4\omega_P \mathbf{k}$$

$$\mathbf{r}_{PO} = 0.1\mathbf{j} + 0.4\mathbf{k}$$

$$\mathbf{v}_P = -32\mathbf{i}$$

$$\mathbf{v}_P = \omega \times \mathbf{r}_{PO}$$

$$-32\mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\omega_P & 4\omega_P \\ 0 & 0.1 & 0.4 \end{vmatrix}$$

$$-32\mathbf{i} = -0.8\omega_P \mathbf{i}$$

$$\omega_P = 40 \text{ rad/s}$$

$\omega_P = \{-40\mathbf{j}\}$ rad/s **Ans**

$\omega_s = 4(40)\mathbf{k} = \{160\mathbf{k}\}$ rad/s

Thus,

$\omega = \omega_P + \omega_s$

Let the x,y,z axes have an angular velocity of $\Omega \times \omega_P$, then

$\alpha = \dot{\omega} = \dot{\omega}_P + \dot{\omega}_s = 0 + \omega_P \times (\omega_s + \omega_P)$

$\alpha = (-40\mathbf{j}) \times (160\mathbf{k} - 40\mathbf{j})$

$\alpha = \{-6400\mathbf{i}\}$ rad/s^2 **Ans**

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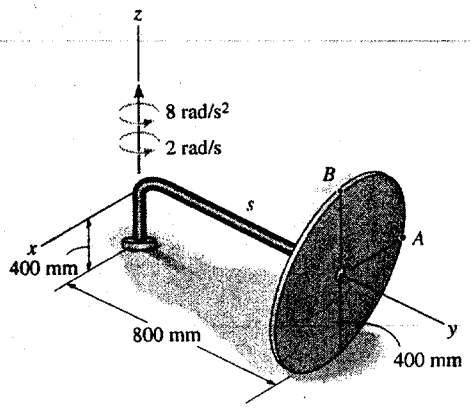
***20-12.** The disk B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 2$ rad/s, while increasing at 8 rad/s^2, determine the veloicty and acceleration of point A at the instant shown.



Angular Velocity : The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant(with reference to X, Y, Z) can be expressed in terms of i, j, k components. The velocity of the center of the disk is $v = \omega_z(0.8) = 2(0.8) = 1.60$ m/s. Since the disk rolls without slipping, its spinning angular velocity is given by $\omega_s = \frac{v}{r} = \frac{1.60}{0.4} = 4$ rad/s and is directed towards $-\mathbf{j}$. Thus, $\omega_s = \{-4\mathbf{j}\}$ rad/s.

$$\omega = \omega_z + \omega_s = \{-4\mathbf{j} + 2\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration : The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{2\mathbf{k}\}$ rad/s. The tangential acceleration of the center of the disk is $a = \dot{\omega}_z(0.8) = 8(0.8) = 6.40$ m/s^2. Since the disk rolls without slipping, its spinning anngular acceleration is given by $\dot{\omega}_s = \frac{a}{r} = \frac{6.40}{0.4} = 16$ rad/s^2 and directed towards $-\mathbf{j}$. Thus, $(\dot{\omega}_s)_{xyz} = \{-16\mathbf{j}\}$ rad/s^2. Applying Eq. 20–6, we have

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_z \times \omega_s = -16\mathbf{j} + 2\mathbf{k} \times (-4\mathbf{j}) = \{8\mathbf{i} - 16\mathbf{j}\} \text{ rad/s}^2$$

Since ω_z is always directed along Z axis $(\Omega = 0)$, then

$$\dot{\omega}_z = (\dot{\omega}_z)_{xyz} + 0 \times \omega_z = \{8\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the disk is

$$\alpha = \dot{\omega}_z + \dot{\omega}_s = \{8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration : Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{-0.4\mathbf{i} + 0.8\mathbf{j}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})$$
$$= \{-1.60\mathbf{i} - 0.800\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$
$$= (8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})$$
$$+ (-4\mathbf{j} + 2\mathbf{k}) \times [(-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})]$$
$$= \{1.6\mathbf{i} - 6.40\mathbf{j} - 6.40\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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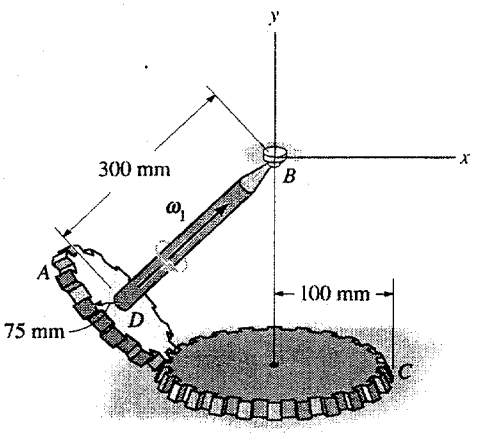
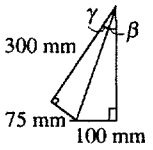
20-13. Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \quad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$

The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.



$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \quad \omega = 13.44 \text{ rad/s}$$

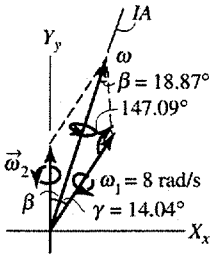
$$\omega = 13.44\sin 18.87^\circ\mathbf{i} + 13.44\cos 18.87^\circ\mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans}$$

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \quad \omega_2 = 6.00 \text{ rad/s}$$

$$\omega_2 = \{6\mathbf{j}\} \text{ rad/s}$$

$$\omega_1 = 8\sin 32.91^\circ\mathbf{i} + 8\cos 32.91^\circ\mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\} \text{ rad/s}$$



For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}$ rad/s.

$$(\omega_1)_{xyz} = (\omega_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\} \text{ rad/s}^2$$

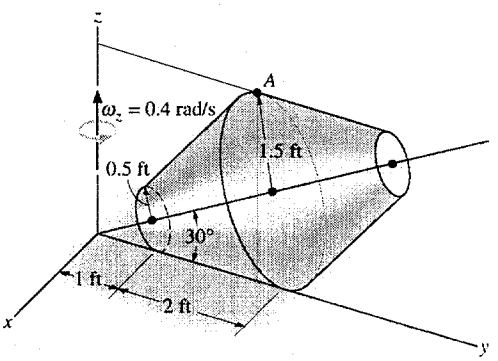
For ω_2, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\boldsymbol{\alpha} = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans}$$

20-14. The truncated cone rotates about the z axis at a constant rate $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8\cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\boldsymbol{\omega} = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\boldsymbol{\Omega} = 0.4\mathbf{k}$$

$$\dot{\boldsymbol{\omega}} = (\dot{\boldsymbol{\omega}})_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega} \qquad (1)$$

$$= 0 + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

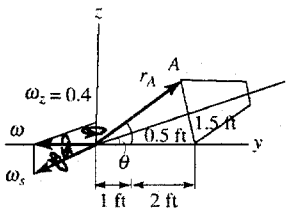
$$\dot{\boldsymbol{\omega}} = \{0.2771\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = (3 - 3\sin 30^\circ)\mathbf{j} + 3\cos 30^\circ\mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$



$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

$$= (0.2771\mathbf{i}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = (-0.720\mathbf{j} - 0.831\mathbf{k}) \text{ ft/s}^2 \qquad \textbf{Ans}$$

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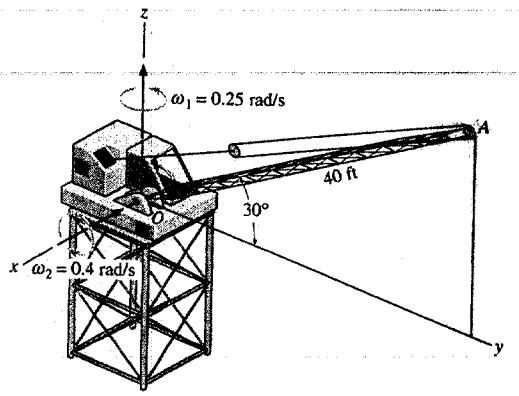
20-15. At the instant shown, the tower crane is rotating about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA is rotating downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the top of the boom at this instant.



$\omega = \omega_1 + \omega_2 = \{-0.4\mathbf{i} + 0.25\mathbf{k}\}$ rad/s

$\Omega = \{0.25\mathbf{k}\}$ rad/s

$\dot{\omega} = \left(\omega\right)_{xyz} + \Omega \times \omega = (-0.8\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\mathbf{i} + 0.25\mathbf{k})$

$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}$ ft

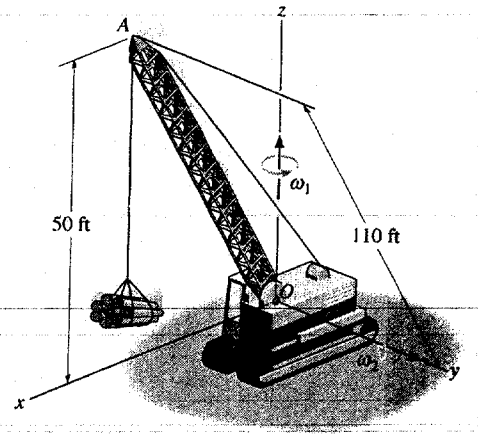
$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$

$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2 **Ans**

***20-16.** The construction boom OA is rotating about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the tip of the boom at the instant shown.



$\omega = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\}$ rad/s

$\omega = \omega_1 + \omega_2$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$\alpha = \dot{\omega} = \left(\omega\right)_{xyz} + \omega_1 \times \omega_2$

$\alpha = 0 + 0.15\mathbf{k} \times 0.2\mathbf{j} = \{-0.03\mathbf{i}\}$ rad/s^2

$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\}$ ft

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}$ ft/s **Ans**

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2 **Ans**

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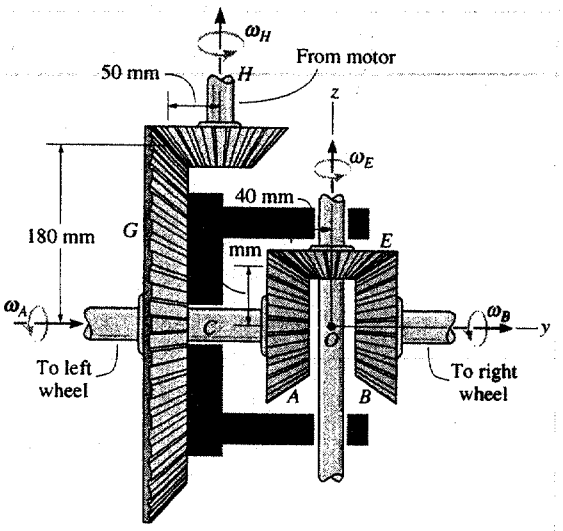
20-17. The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears A and B on their other ends. The differential case D is placed over the left axle but can rotate about C independent of the axle. The case supports a pinion gear E on a shaft, which meshes with gears A and B. Finally, a ring gear G is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning at $\omega_H = 100$ rad/s and the pinion gear E is spinning about its shaft at $\omega_E = 30$ rad/s, determine the angular velocity, ω_A and ω_B, of each axle.



$v_P = \omega_H r_H = 100(50) = 5000$ mm/s

$$\omega_G = \frac{5000}{180} = 27.78 \text{ rad/s}$$

Point O is a fixed point of rotation for gears A, E, and B.

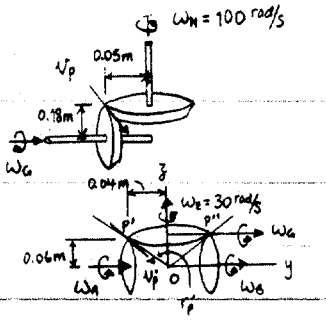
$\Omega = \omega_G + \omega_E = \{27.78\mathbf{j} + 30\mathbf{k}\}$ rad/s

$\mathbf{v}_{P'} = \Omega \times \mathbf{r}_{P'} = (27.78\mathbf{j} + 30\mathbf{k}) \times (-40\mathbf{j} + 60\mathbf{k}) = \{2866.7\mathbf{i}\}$ mm/s

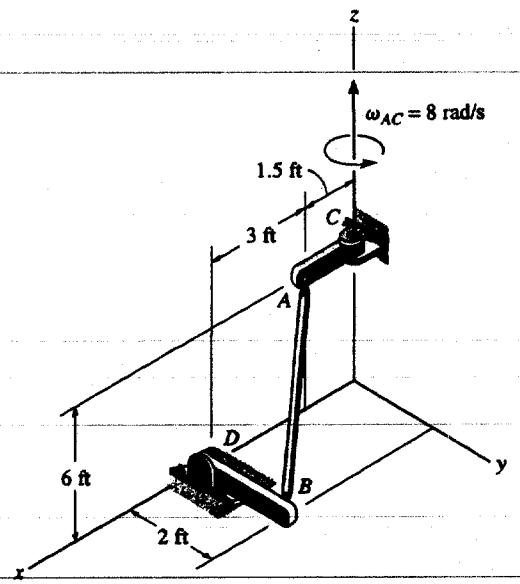
$$\omega_A = \frac{2866.7}{60} = 47.8 \text{ rad/s} \qquad \textbf{Ans}$$

$\mathbf{v}_{P''} = \Omega \times \mathbf{r}_{P''} = (27.78\mathbf{j} + 30\mathbf{k}) \times (40\mathbf{j} + 60\mathbf{k}) = \{466.7\mathbf{i}\}$ mm/s

$$\omega_B = \frac{466.2}{60} = 7.78 \text{ rad/s} \qquad \textbf{Ans}$$



20-18. Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating with a constant angular velocity of 8 rad/s about the pin at C, determine the angular velocity of link BD at the instant shown.



$\mathbf{v}_A = 8(1.5)\mathbf{j} = \{12\mathbf{j}\}$ ft/s

$\mathbf{v}_B = -v_B\mathbf{k}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$-v_B\mathbf{k} = 12\mathbf{j} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$

$0 = -6\omega_y - 2\omega_z$

$0 = 12 + 6\omega_x + 3\omega_z$

$-v_B = 2\omega_x - 3\omega_y$

Thus,

$$-v_B = 2\left(\frac{12}{2} - \frac{3}{6}\omega_z\right) - 3\left(\frac{-\omega_z}{3}\right)$$

$-v_B = -4 - \omega_z + \omega_z$

$v_B = 4$ ft/s

$$\omega_{BD} = -\left(\frac{4}{2}\right)\mathbf{i}$$

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s **Ans**

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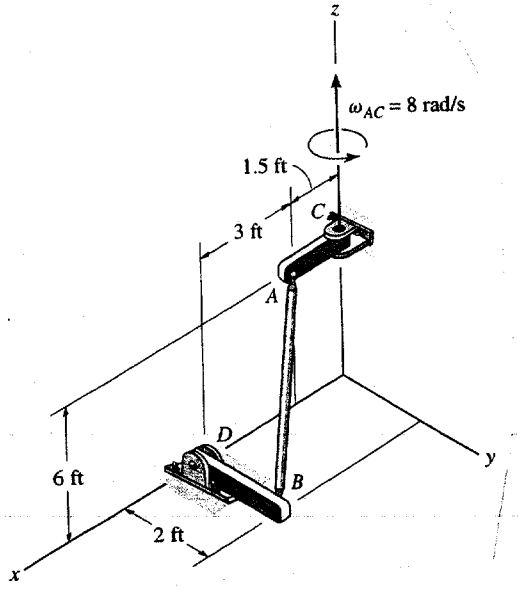
20-19. Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating about point C with an angular velocity of 8 rad/s and has an angular acceleration of 6 rad/s^2 at the instant shown, determine the angular velocity and angular acceleration of link BD at this instant.



See Prob. 20–18.

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s **Ans**

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$-4\mathbf{k} = 12\mathbf{j} + \mathbf{v}_{B/A}$

$\mathbf{v}_{B/A} = \{-12\mathbf{j} - 4\mathbf{k}\}$ ft/s

$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\mathbf{v}_{B/A})$

$$-(2)^2(2)\mathbf{j} + (a_B)_z\mathbf{k} = -96\mathbf{i} + 9\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 2 & -6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.633 & 0.2449 & -0.7347 \\ 0 & -12 & -4 \end{vmatrix}$$

$0 = -96 + \alpha_y(-6) - 2\alpha_z - 9.796$

$-8 = 9 + 6\alpha_x + 3\alpha_z - 6.5308$

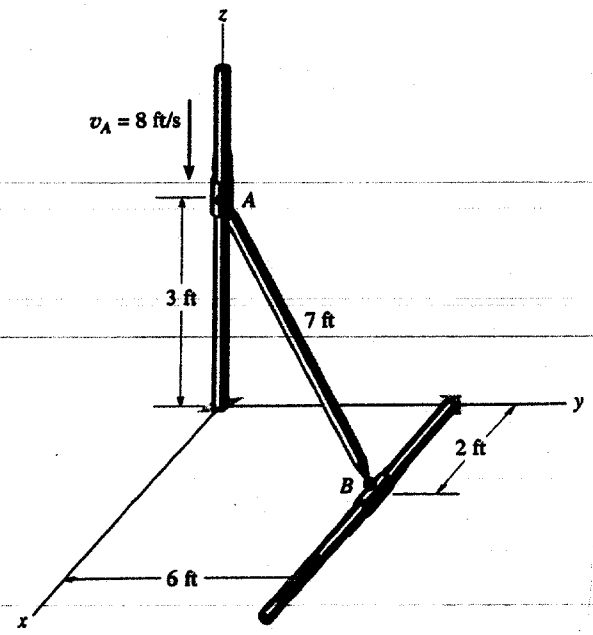
$(a_B)_z = \alpha_x(2) - \alpha_y(3) + 19.592$

$(a_B)_z = 69.00$ ft/s^2

$\alpha_{BD} = \dfrac{69.00}{2} = 34.5$ rad/s^2

$\alpha_{BD} = \{34.5\mathbf{i}\}$ rad/s^2 **Ans**

***20-20.** If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the speed of B at the instant shown if A is moving downward at a constant speed of $v_A = 8$ ft/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.

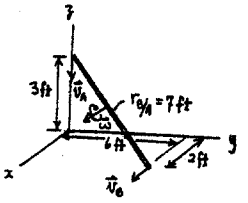


$\mathbf{v}_A = \{-8\mathbf{k}\}$ ft/s

$\mathbf{v}_B = v_B\mathbf{i}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ ft

$\omega = \{\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\}$ rad/s



$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

Expanding and equating components yields :

$v_B = -3\omega_y - 6\omega_z$ (1)

$0 = 3\omega_x + 2\omega_z$ (2)

$0 = -8 + 6\omega_x - 2\omega_y$ (3)

Also, $\omega \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$

$2\omega_x + 6\omega_y - 3\omega_z = 0$ (4)

Solving Eqs. (1)–(4) yields

$\omega_x = 0.9796$ rad/s

$\omega_y = -1.061$ rad/s

$\omega_z = -1.469$ rad/s

$\omega = \{0.980\mathbf{i} - 1.06\mathbf{j} - 1.47\mathbf{k}\}$ rad/s **Ans**

$v_B = \{12.0\mathbf{i}\}$ ft/s **Ans**

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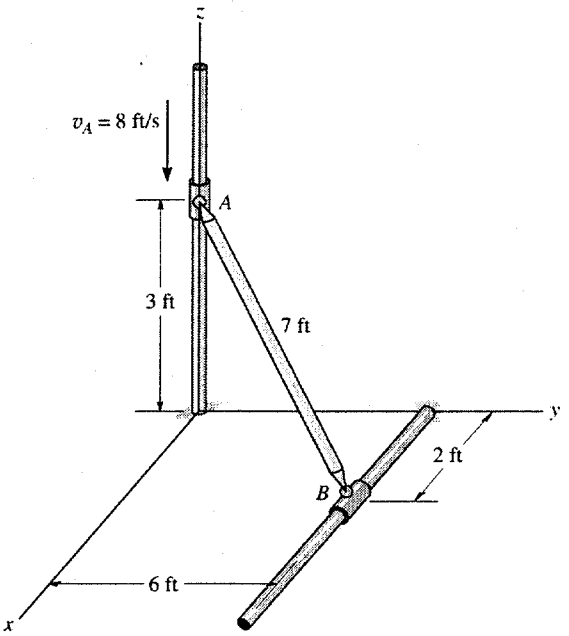
20-21. If the collar at A is moving downward with an acceleration $\mathbf{a}_A = \{-5\mathbf{k}\}$ ft/s^2, at the instant its speed is $v_A = 8$ ft/s, determine the acceleration of the collar at B at this instant.



$\mathbf{a}_B = a_B i, \quad \mathbf{a}_A = -5\mathbf{k}$

From Prob. 20-20.

$\omega_{AB} = 0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$\mathbf{a}_{B/A} = \omega_{AB} \times \mathbf{v}_{B/A} + \alpha_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = 12\mathbf{i} + 8\mathbf{k}$

$\mathbf{a}_{B/A} = \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k}) + \{\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}\} \times (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$

$a_B\mathbf{i} = -5\mathbf{k} + \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k}) + \{(-3\alpha_y - 6\alpha_z)\mathbf{i} + (2\alpha_z + 3\alpha_x)\mathbf{j} + (6\alpha_x - 2\alpha_y)\mathbf{k}\}$

$3\alpha_y + 6\alpha_z + a_B = -8.4897$

$-3\alpha_x - 2\alpha_z = -25.4696$

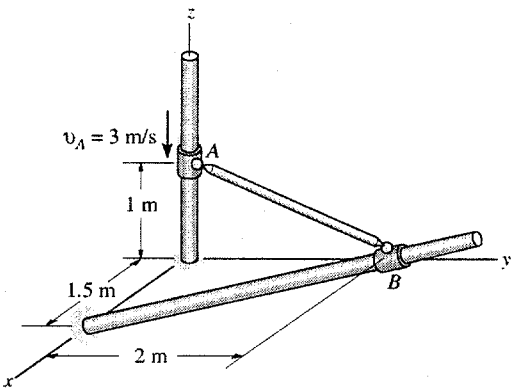
$-6\alpha_x + 2\alpha_y = 7.7344$

Solving these equations

$\mathbf{a}_B = -96.5$

$\mathbf{a}_B = \{-96.5\mathbf{i}\}$ ft/s^2 **Ans**

20-22. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 3$ m/s, determine the speed of collar B at the instant shown.



Velocity Equation: Here,

$\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\mathbf{k}\}$ m $= \{2\mathbf{j} - 1\mathbf{k}\}$ m,

$\mathbf{v}_A = \{-3\mathbf{k}\}$ m/s. $\mathbf{v}_B = v_B\left[\dfrac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right]$

$= -0.6v_B\mathbf{i} + 0.8v_B\mathbf{j}$

and $\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying Eq. 20-7, we have

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$

$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = (-3 - \omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x - 3)\mathbf{k}$

Equating **i**, **j** and **k** components, we have

$-0.6v_B = -\omega_y - 2\omega_z$ [1]

$0.8v_B = \omega_x$ [2]

$0 = 2\omega_x - 3$ [3]

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$\omega \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$

$2\omega_y - \omega_z = 0$ [4]

Solving Eqs. [1], [2], [3] and [4] yields

$\omega_x = 1.50$ rad/s

$\omega_y = 0.225$ rad/s

$\omega_z = 0.450$ rad/s

$v_B = 1.875$ m/s **Ans**

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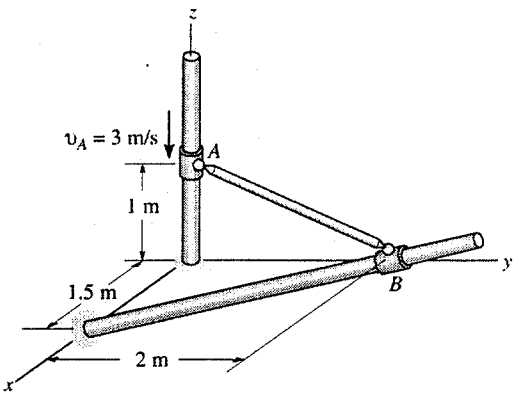
20-23. If the collar at A in Prob 20-22 has an acceleration of $\mathbf{a}_A = [-2\mathbf{k}]\ \text{m/s}^2$ at the instant its speed is $v_A = 3$ m/s, determine the magnitude of the acceleration of the collar at B at this instant.



Velocity Equation: Here,

$$\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\mathbf{k}\}\ \text{m} = \{2\mathbf{j} - 1\mathbf{k}\}\ \text{m}.$$

$$\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s},\ \mathbf{v}_B = v_B\left[\frac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right]$$

$$= -0.6v_B\mathbf{i} + 0.8v_B\mathbf{j}$$

and $\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying Eq. 20-7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = (-3 - \omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x - 3)\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$-0.6v_B = -\omega_y - 2\omega_z \qquad [1]$$

$$0.8v_B = \omega_x \qquad [2]$$

$$0 = 2\omega_x - 3 \qquad [3]$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad [4]$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875\ \text{m/s} \quad \omega_x = 1.50\ \text{rad/s}$$

$$\omega_y = 0.225\ \text{rad/s} \quad \omega_z = 0.450\ \text{rad/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}\}\ \text{rad/s}$$

Acceleration Equation: With $\alpha = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$ and the result obtained above, applying Eq. 20-8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$-0.6a_B\mathbf{i} + 0.8a_B\mathbf{j} = -2\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$+ (1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times [(1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-0.6a_B\mathbf{i} + 0.8a_B\mathbf{j} = (-\alpha_y - 2\alpha_z)\mathbf{i} + (\alpha_x - 5.00625)\mathbf{j}$$

$$+ (2\alpha_x + 0.503125)\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$-0.6a_B = -\alpha_x - 2\alpha_z \qquad [5]$$

$$0.8a_B = \alpha_x - 5.00625 \qquad [6]$$

$$0 = 2\alpha_x + 0.503125 \qquad [7]$$

Solving Eqs. [6] and [7] yields

$$\alpha_x = -0.2515\ \text{rad/s}^2$$

$$a_B = -6.57\ \text{m/s}^2 \qquad \textbf{Ans}$$

Negative sign indicates that $\mathbf{a}_B$ is directed in the opposite direction to that of the above assumed direction.

Note: In order to determine α_y and α_z, one should obtain another equation by specifying the direction of α which acts *perpendicular* to the axis of rod AB.

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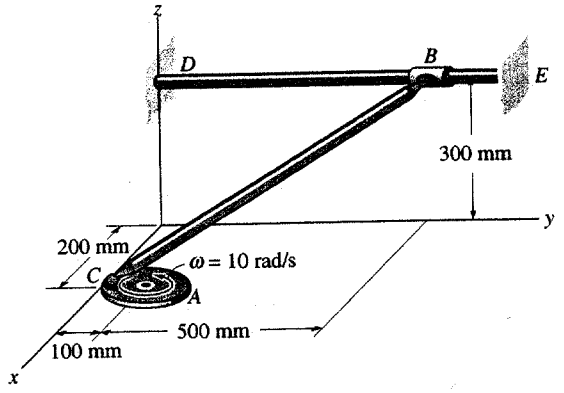
20-24. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity of $v_A = 5$ ft/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.



$\mathbf{v}_A = \{5\mathbf{i}\}$ ft/s

$\mathbf{v}_B = -v_B\mathbf{j}$

$\mathbf{r}_{B/A} = \{-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}\}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$$-v_B\mathbf{j} = 5\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -3 & 4 & -5 \end{vmatrix}$$

$5 - 5\omega_y - 4\omega_z = 0$

$5\omega_x - 3\omega_z = -v_B$

$4\omega_x + 3\omega_y = 0$

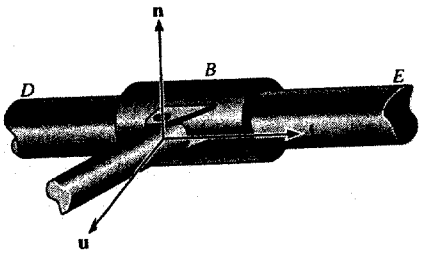
$\boldsymbol{\omega} \cdot \mathbf{r}_{B/C} = 0$

$-3\omega_x + 4\omega_y - 5\omega_z = 0$

$\boldsymbol{\omega} = \{-0.375\mathbf{i} + 0.5\mathbf{j} + 0.625\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_B = \{-3.75\mathbf{j}\}$ ft/s **Ans**

20-25. Solve Prob. 20-24 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both along bar DE ($\mathbf{j}$ direction) and along the axis of the pin ($\mathbf{n}$ direction). Since there is no rotational component in the $\mathbf{u}$ direction, i.e., perpendicular to $\mathbf{n}$ and $\mathbf{j}$ where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector $\mathbf{n}$ is in the same direction as $\mathbf{r}_{B/C} \times \mathbf{r}_{D/C}$.



$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \boldsymbol{\omega}_{BC} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}$ m

$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$

$$-v_B\mathbf{j} = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j** and **k** components

$1 + 0.3\omega_y - 0.6\omega_z = 0$ (1)

$0.3\omega_x + 0.2\omega_z = v_B$ (2)

$0.6\omega_x + 0.2\omega_y = 0$ (3)

Aiso,

$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}$ m

$\mathbf{r}_{D/C} = \{-0.2\mathbf{i} + 0.3\mathbf{k}\}$ m

$$\mathbf{r}_{B/C} \times \mathbf{r}_{D/C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2 & 0.6 & 0.3 \\ -0.2 & 0 & 0.3 \end{vmatrix} = \{0.18\mathbf{i} + 0.12\mathbf{k}\} \text{ m}^2$$

$$\mathbf{n} = \frac{0.18\mathbf{i} + 0.12\mathbf{k}}{\sqrt{0.18^2 + 0.12^2}} = 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times (0.8321\mathbf{i} + 0.5547\mathbf{k}) = 0.5547\mathbf{i} - 0.8321\mathbf{k}$

$\boldsymbol{\omega}_{BC} \cdot \mathbf{u} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (0.5547\mathbf{i} - 0.8321\mathbf{k})$

$0.5547\omega_x - 0.8321\omega_z = 0$ (4)

Solving Eqs.(1) to (4) yields :

$\omega_x = 0.769$ rad/s $\quad \omega_y = -2.31$ rad/s $\quad \omega_z = 0.513$ rad/s $\quad v_B = 0.333$ m/s

Then

$\boldsymbol{\omega}_{BC} = \{0.769\mathbf{i} - 2.31\mathbf{j} + 0.513\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_B = \{-0.333\mathbf{j}\}$ m/s **Ans**

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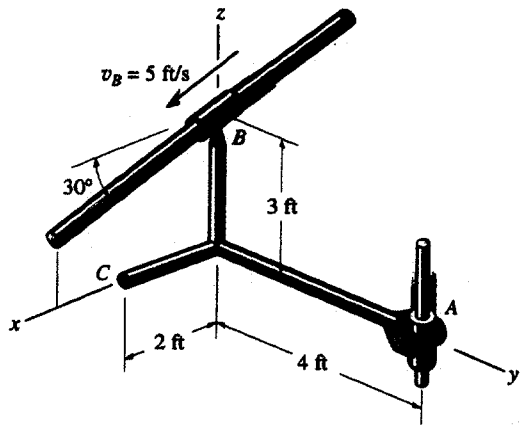
20-26. The rod assembly is supported at B by a ball-and-socket joint and at A by a clevis. If the collar at B moves in the x–z plane with a speed $v_B = 5$ ft/s, determine the velocity of points A and C on the rod assembly at the instant shown. *Hint:* See Prob. 20–25.



$\mathbf{v}_B = \{5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k}\}$ ft/s $\mathbf{v}_A = -v_A\mathbf{k}$ $\boldsymbol{\omega}_{ABC} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{A/B} = \{4\mathbf{j} - 3\mathbf{k}\}$ ft $\mathbf{r}_{C/B} = \{2\mathbf{i} - 3\mathbf{k}\}$ ft

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{A/B}$$

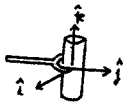
$$-v_A\mathbf{k} = 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 4 & -3 \end{vmatrix}$$

Equating i, j and k components

$$5\cos 30° - 3\omega_y - 4\omega_z = 0 \qquad [1]$$

$$3\omega_x = 0 \qquad [2]$$

$$4\omega_x - 5\sin 30° = -v_A \qquad [3]$$



Also, since there is no rotation abouthe y axis

$$\boldsymbol{\omega}_{ABC} \cdot \mathbf{j} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (\mathbf{j}) = 0$$

$$\omega_y = 0 \qquad [4]$$

Solving Eqs.[1] to [4] yields :

$\omega_x = \omega_y = 0$ $\omega_z = 1.083$ rad/s $v_A = 2.5$ ft/s ↓

Then $\boldsymbol{\omega}_{ABC} = \{1.083\mathbf{k}\}$ rad/s

$\mathbf{v}_A = \{-2.50\mathbf{k}\}$ ft/s **Ans**

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{C/B}$$

$$= 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1.083 \\ 2 & 0 & -3 \end{vmatrix}$$

$= \{4.33\mathbf{i} + 2.17\mathbf{j} - 2.50\mathbf{k}\}$ ft/s **Ans**

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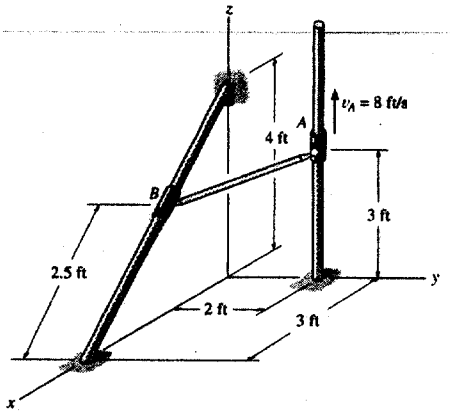
20-27. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with a velocity of 8 ft/s, determine the angular velocity of the rod and the speed of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the rod.



$$\mathbf{v}_A = \{8\mathbf{k}\}\ \text{ft/s} \qquad \mathbf{v}_B = -\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\mathbf{k} \qquad \boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\ \text{ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\mathbf{k} = 8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.5 & -2 & -1 \end{vmatrix}$$

Equating **i**, **j** and **k**

$$-\omega_y + 2\omega_z = -\frac{3}{5}v_B \qquad (1)$$

$$\omega_x + 1.5\omega_z = 0 \qquad (2)$$

$$8 - 2\omega_x - 1.5\omega_y = \frac{4}{5}v_B \qquad (3)$$

Since $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of the rod,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\omega_x - 2\omega_y - \omega_z = 0 \qquad (4)$$

Solving Eqs.(1) to(4) yields :

$\omega_x = 1.1684$ rad/s $\quad \omega_y = 1.2657$ rad/s $\quad \omega_z = -0.7789$ rad/s

$v_B = 4.71$ ft/s **Ans**

Then $\boldsymbol{\omega}_{AB} = \{1.17\mathbf{i} + 1.27\mathbf{j} - 0.779\mathbf{k}\}$ rad/s **Ans**

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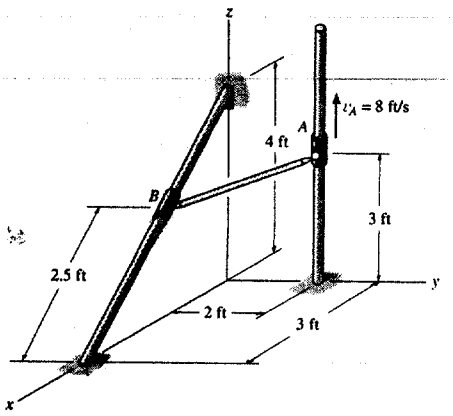
***20-28.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with an acceleration of $a_A = 4$ ft/s^2, determine the angular acceleration of rod AB and the magnitude of acceleration of collar B. Assume that the rod's angular acceleration is directed perpendicular to the rod.



From Prob. 20 – 27

$\omega_{AB} = \{1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}\}$ rad/s

$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}$ ft

$\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$

$\mathbf{a}_A = \{4\mathbf{k}\}$ ft/s^2 $\qquad \mathbf{a}_B = -\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{k} = 4\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})$$
$$+ (1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k})$$
$$\times [(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})]$$

Equating i, j and k components

$$-\alpha_y + 2\alpha_z - 5.3607 = -\frac{3}{5}a_B \qquad (1)$$

$$\alpha_x + 1.5\alpha_z + 7.1479 = 0 \qquad (2)$$

$$7.5737 - 2\alpha_x - 1.5\alpha_y = \frac{4}{5}a_B \qquad (3)$$

Since α_{AB} is perpendicular to the axis of the rod,

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\alpha_x - 2\alpha_y - \alpha_z = 0 \qquad (4)$$

Solving Eqs.(1) to (4) yields :

$\alpha_x = -2.7794$ rad/s^2 $\quad \alpha_y = -0.6285$ rad/s^2 $\quad \alpha_z = -2.91213$ rad/s^2

$a_B = 17.6$ ft/s^2 **Ans**

Then $\alpha_{AB} = \{-2.78\mathbf{i} - 0.628\mathbf{j} - 2.91\mathbf{k}\}$ rad/s^2 **Ans**

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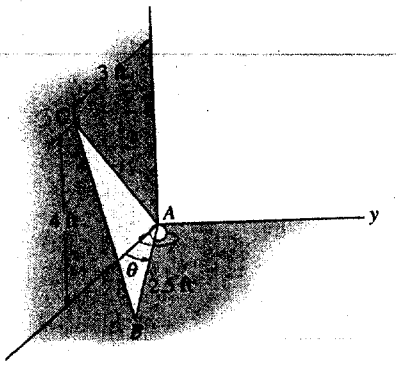
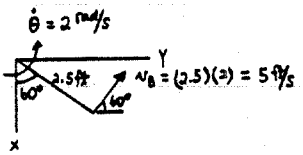
20-29. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the x–z plane. The side AB lies in the x–y plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s and point C has the coordinates shown. Determine the angular velocity of the plate and the velocity of point C at this instant.



$\mathbf{v}_B = -5\sin 60°\mathbf{i} + 5\cos 60°\mathbf{j}$

$= \{-4.33\mathbf{i} + 2.5\mathbf{j}\}$ ft/s



$\mathbf{v}_C = (v_C)_x\mathbf{i} + (v_C)_z\mathbf{k}$

$\mathbf{r}_{C/A} = \{3\mathbf{i} + 4\mathbf{k}\}$ ft

$\mathbf{r}_{B/A} = \{1.25\mathbf{i} + 2.165\mathbf{j}\}$ ft

$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$$-4.33\mathbf{i} + 2.5\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.25 & 2.165 & 0 \end{vmatrix}$$

$-2.165\omega_z = -4.33$; $\quad \omega_z = 2$ rad/s

$2.165\omega_x - 1.25\omega_y = 0$; $\quad \omega_y = 1.732\omega_x$

$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/A}$

$$(v_C)_x\mathbf{i} + (v_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 2 \\ 3 & 0 & 4 \end{vmatrix}$$

$(v_C)_x = 4\omega_y$

$0 = 4\omega_x - 6$; $\quad \omega_x = 1.5$ rad/s

$(v_C)_z = -3\omega_y$

Solving,

$\omega_y = 2.5981$ rad/s

$(v_C)_x = 10.392$ ft/s

$(v_C)_z = -7.7942$ ft/s

Thus,

$\boldsymbol{\omega} = \{1.50\mathbf{i} + 2.60\mathbf{j} + 2.00\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_C = \{10.4\mathbf{i} - 7.79\mathbf{k}\}$ ft/s **Ans**

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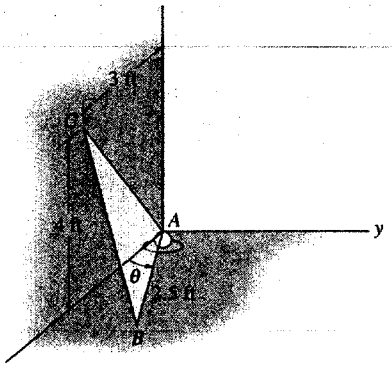
20-30. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the x–z plane. The side AB lies in the x–y plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s, $\ddot{\theta} = 3$ rad/s^2, and point C has the coordinates shown. Determine the angular acceleration of the plate and the acceleration of point C at this instant.



From Prob. 20–29,

$\omega = 1.5\mathbf{i} + 2.5981\mathbf{j} + 2\mathbf{k}$

$\mathbf{r}_{B/A} = 1.25\mathbf{i} + 2.165\mathbf{j}$

$\mathbf{v}_B = -4.33\mathbf{i} + 2.5\mathbf{j}$

$(a_B)_t = 3(2.5) = 7.5$ ft/s^2

$(a_B)_n = (2)^2(2.5) = 10$ ft/s^2

$\mathbf{a}_B = -7.5\sin 60°\mathbf{i} + 7.5\cos 60°\mathbf{j} - 10\cos 60°\mathbf{i} - 10\sin 60°\mathbf{j}$

$\mathbf{a}_B = -11.4952\mathbf{i} - 4.91025\mathbf{j}$

$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} + \omega \times \mathbf{v}_{B/A}$

$$-11.4952\mathbf{i} - 4.91025\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 1.25 & 2.165 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ -4.33 & 2.5 & 0 \end{vmatrix}$$

$-11.4952 = -2.165\alpha_z - 5$

$-4.91025 = 1.25\alpha_z - 8.66$

$\alpha_z = 3$ rad/s^2

$0 = 2.165\alpha_x - 1.25\alpha_y + 15$ (1)

$\mathbf{a}_C = \alpha \times \mathbf{r}_{C/A} + \omega \times \mathbf{v}_{C/A}$

$\mathbf{v}_{C/A} = 10.39\mathbf{i} - 7.794\mathbf{k}$

$$\mathbf{a}_C = (a_C)_x\mathbf{i} + (a_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 0 & 4 \end{vmatrix} + \begin{vmatrix} \\ \\ \\ \end{vmatrix}$$

$(a_C)_x = 4\alpha_y - 20.25$ (2)

$0 = 3\alpha_z - 4\alpha_x + 32.4760$ (3)

$(a_C)_z = -3\alpha_y - 27$ (4)

Solving Eqs. (1)–(4),

$\alpha_x = 10.369$ rad/s^2

$\alpha_y = 29.96$ rad/s^2

$(a_C)_x = 99.6$ ft/s^2

$(a_C)_z = -117$ ft/s^2

$\mathbf{a}_C = \{99.6\mathbf{i} - 117\mathbf{k}\}$ ft/s^2 **Ans**

$\alpha = \{10.4\mathbf{i} + 30.0\mathbf{j} + 3\mathbf{k}\}$ rad/s^2 **Ans**

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20-31. Solve Example 20–5 such that the x, y, z axes move with curvilinear translation, $\boldsymbol{\Omega} = \mathbf{0}$, in which case the collar appears to have both an angular velocity $\boldsymbol{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

Relative to XYZ, let xyz have

$\Omega = 0 \quad \dot{\Omega} = 0$

$\mathbf{r}_B = \{-0.5\mathbf{k}\}$ m

$\mathbf{v}_B = \{2\mathbf{j}\}$ m/s

$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}$ m/s^2

Relative to xyz, let $x'y'z'$ be coincident with xyz and be fixed to BD. Then

$\boldsymbol{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}$ rad/s $\quad \dot{\boldsymbol{\Omega}}_{xyz} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}$ rad/s^2

$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}$ m

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+ \left[(\dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \mathbf{0} + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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***20-32.** Solve Example 20–5 by fixing x, y, z axes to rod BD so that $\Omega = \omega_1 + \omega_2$. In this case the collar appears only to move radially outward along BD; hence $\Omega_{xyz} = 0$.

Relative to XYZ, let $x'y'z'$ be coincident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$.

$$\Omega = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\dot{\omega}_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + 0) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\} \text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_B\right)_{x'y'z'}\right] + \dot{\omega}_1 \times \mathbf{r}_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\} \text{ m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \quad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\} \text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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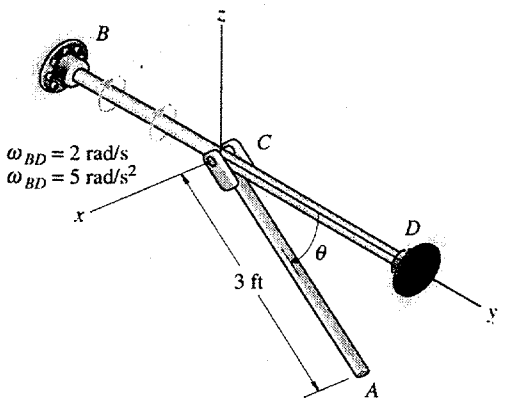
20-33. At a given instant, rod BD is rotating about the y axis with an angular velocity $\omega_{BD} = 2$ rad/s and an angular acceleration $\dot{\omega}_{BD} = 5$ rad/s^2. Also, when $\theta = 60°$ link AC is rotating downward such that $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 8$ rad/s^2. Determine the velocity and acceleration of point A on the link at this instant.



$\Omega = -2\mathbf{i} - 2\mathbf{j}$

$\mathbf{r}_{A/C} = 3\cos 60°\mathbf{j} - 3\sin 60°\mathbf{k} = 1.5\mathbf{j} - 2.5980762\mathbf{k}$

$\dot{\Omega} = -5\mathbf{j} - 8\mathbf{i} + (-2\mathbf{j}) \times (-2\mathbf{i}) = \{-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}\}$ rad/s^2

$\mathbf{v}_A = \mathbf{v}_C + \Omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{A/C})_{xyz}$

$\mathbf{v}_A = (-2\mathbf{i} - 2\mathbf{j}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k}) + 0$

$\mathbf{v}_A = 5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k} + 0$

$\mathbf{v}_A = \{5.20\mathbf{i} - 5.20\mathbf{j} - 3.00\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{A/C} + \Omega \times (\Omega \times \mathbf{r}_{A/C})$

$+ 2(\Omega \times (\mathbf{v}_{A/C})_{xyz}) + (a_{A/C})_{xyz}$

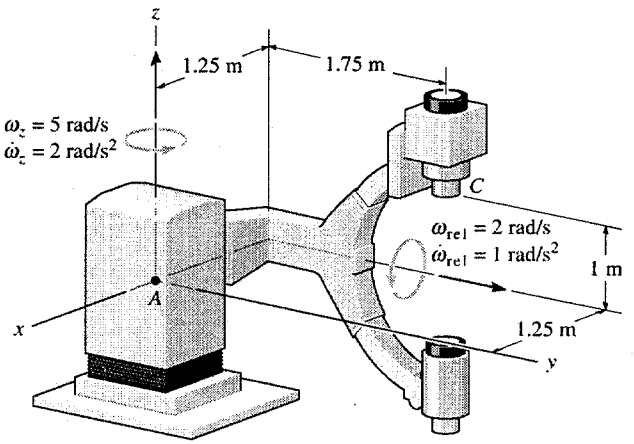
$= \mathbf{0} + (-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k})$

$+ (-2\mathbf{i} - 2\mathbf{j}) \times (5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k}) + \mathbf{0} + \mathbf{0}$

$\mathbf{a}_A = 24.9904\mathbf{i} - 26.7846\mathbf{j} + 8.7846\mathbf{k}$

$\mathbf{a}_A = \{25\mathbf{i} - 26.8\mathbf{j} + 8.78\mathbf{k}\}$ ft/s^2 **Ans**

20-34. During the instant shown the frame of the X-ray camera is rotating about the vertical axis at $\omega_z = 5$ rad/s and $\dot{\omega}_z = 2$ rad/s^2. Relative to the frame the arm is rotating at $\omega_{rel} = 2$ rad/s and $\dot{\omega}_{rel} = 1$ rad/s^2. Determine the velocity and acceleration of the center of the camera C at this instant.



$\Omega = \{5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2

$\mathbf{r}_B = \{-1.25\mathbf{i}\}$ m

$\omega_B = \mathbf{0} + 5\mathbf{k} \times (-1.25\mathbf{i}) = -6.25\mathbf{j}$

$\mathbf{a}_B = \mathbf{0} + 2\mathbf{k} \times (-1.25\mathbf{i}) + \mathbf{0} + 5\mathbf{k} \times (-6.25\mathbf{j})$

$= 31.25\mathbf{i} - 2.5\mathbf{j}$

$\Omega_{xyz} = \{2\mathbf{j}\}$ rad/s

$\dot{\Omega}_{xyz} = \{1\mathbf{j}\}$ rad/s^2

$\mathbf{r}_{C/B} = \{1.75\mathbf{j} + 1\mathbf{k}\}$ m

$(\mathbf{v}_{C/B})_{xyz} = \dot{\mathbf{r}}_{C/B} = \mathbf{0} + (2\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) = 2\mathbf{i}$

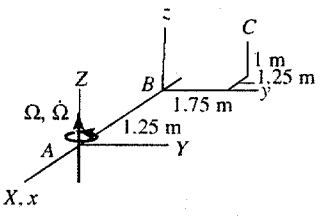
$(\mathbf{a}_{C/B})_{xyz} = \ddot{\mathbf{r}}_{C/B} = \mathbf{0} + (1\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) + \mathbf{0} + (2\mathbf{j}) \times (2\mathbf{i})$

$= 1\mathbf{i} - 4\mathbf{k}$

$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/A})_{xyz}$

$\mathbf{v}_C = -6.25\mathbf{j} + 5\mathbf{k} \times (1.75\mathbf{j} + 1\mathbf{k}) + 2\mathbf{i}$

$\mathbf{v}_C = \{-6.75\mathbf{i} - 6.25\mathbf{j}\}$ m/s **Ans**



$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2(\Omega \times (\mathbf{v}_{C/B})_{xyz}) + (a_{C/B})_{xyz}$

$= (31.25\mathbf{i} - 2.5\mathbf{j}) + (2\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k}) + 5\mathbf{k}|(5\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k})|$

$+ 2(5\mathbf{k}) \times (2\mathbf{i}) + (1\mathbf{i} - 4\mathbf{k})$

$\mathbf{a}_C = \{28.75\mathbf{i} - 26.25\mathbf{j} - 4\mathbf{k}\}$ m/s^2 **Ans**

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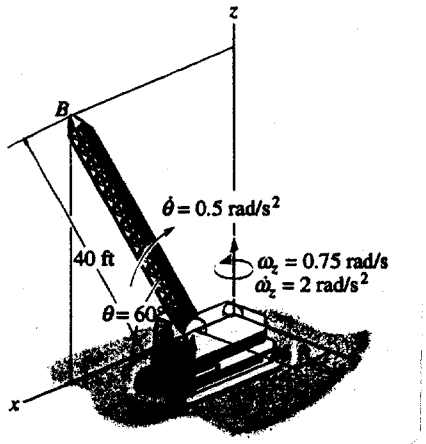
20-35. The boom AB of the crane is rotating about the z axis with an angular velocity $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at a constant rate $\dot{\theta} = 0.5$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



Motion of moving reference:

$\Omega = \omega_z = \{0.75\mathbf{k}\}$ rad/s

$\dot{\Omega} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\}$ rad/s^2

$\mathbf{r}_O = \{5\mathbf{i}\}$ ft

$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \Omega \times \mathbf{r}_O$

$= 0 + (0.75\mathbf{k}) \times (5\mathbf{i})$

$= \{3.75\mathbf{j}\}$ ft/s

$\mathbf{a}_O = \ddot{\mathbf{r}}_O = [(\ddot{\mathbf{r}}_O)_{xyz} + \Omega \times (\dot{\mathbf{r}}_O)_{xyz}] + \dot{\Omega} \times \mathbf{r}_O + \Omega \times \dot{\mathbf{r}}_O$

$= 0 + 0 + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$

$= \{-2.8125\mathbf{i} + 10\mathbf{j}\}$ ft/s^2

Motion of B with respect to moving reference:

$\Omega_{B/O} = \{-0.5\mathbf{j}\}$ rad/s

$\dot{\Omega}_{B/O} = 0$

$\mathbf{r}_{B/O} = 40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\}$ ft

$(\mathbf{v}_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times \mathbf{r}_{B/O}$

$= 0 + (-0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$

$= \{-17.32\mathbf{i} + 10\mathbf{k}\}$ ft/s

$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\Omega}_{B/O} \times \mathbf{r}_{B/O} + \Omega_{B/O} \times \dot{\mathbf{r}}_{B/O}$

$= 0 + 0 + 0 + (-0.5\mathbf{j}) \times (-17.32\mathbf{i} + 10\mathbf{k})$

$= \{-5\mathbf{i} - 8.66\mathbf{k}\}$ ft/s^2

$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$

$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$

$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$

$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$
$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-5\mathbf{i} - 8.66\mathbf{j})$

$\mathbf{a}_B = \{-19.1\mathbf{i} + 24.0\mathbf{j} - 8.66\mathbf{k}\}$ ft/s^2 **Ans**

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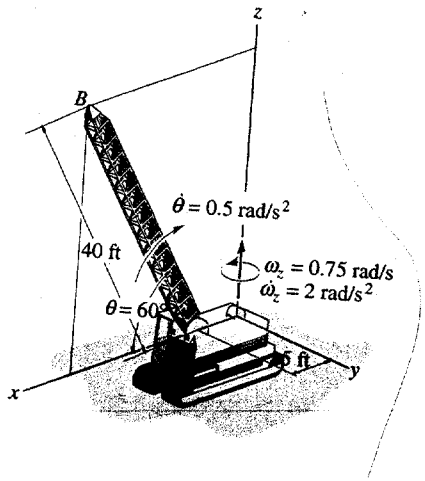
***20-36.** The boom AB of the crane is rotating about the z axis with an angular velocity of $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at $\dot{\theta} = 0.5$ rad/s^2, which is increasing at $\ddot{\theta} = 0.75$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



Motion of moving reference:

$\Omega = \omega_z = \{0.75\mathbf{k}\}$ rad/s

$\dot{\Omega} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\}$ rad/s^2

$\mathbf{r}_O = \{5\mathbf{i}\}$ ft

$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \Omega \times \mathbf{r}_O$

$= \mathbf{0} + (0.75\mathbf{k}) \times (5\mathbf{i})$

$= \{3.75\mathbf{j}\}$ ft/s

$\mathbf{a}_O = \ddot{\mathbf{r}}_O = [(\ddot{\mathbf{r}}_O)_{xyz} + \Omega \times (\dot{\mathbf{r}}_O)_{xyz}] + \dot{\Omega} \times \mathbf{r}_O + \Omega \times \dot{\mathbf{r}}_O$

$= \mathbf{0} + \mathbf{0} + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$

$= \{-2.8125\mathbf{i} + 10\mathbf{j}\}$ ft/s^2

Motion of B with respect to moving reference:

$\Omega_{B/O} = \{-0.5\mathbf{j}\}$ rad/s

$\dot{\Omega}_{B/O} = \{-0.75\mathbf{j}\}$ rad/s^2

$\mathbf{r}_{B/O} = 40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\}$ ft

$(\mathbf{v}_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times \mathbf{r}_{B/O}$

$= \mathbf{0} + (-0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$

$= \{-17.32\mathbf{i} + 10\mathbf{k}\}$ ft/s

$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\Omega}_{B/O} \times \mathbf{r}_{B/O} + \Omega_{B/O} \times \dot{\mathbf{r}}_{B/O}$

$= \mathbf{0} + \mathbf{0} + (-0.75\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-0.5\mathbf{j}) \times (-17.32\mathbf{i} + 10\mathbf{k})$

$= \{-30.98\mathbf{i} + 6.34\mathbf{k}\}$ ft/s^2

$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$

$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$

$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$

$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$
$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-30.98\mathbf{i} + 6.34\mathbf{j})$

$\mathbf{a}_B = \{-45.0\mathbf{i} + 24.0\mathbf{j} + 6.34\mathbf{k}\}$ ft/s^2 **Ans**

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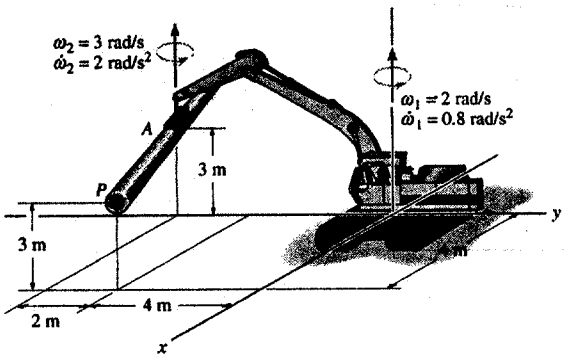
20-37. At the instant shown, the boom is rotating about the z axis with an angular velocity $\omega_1 = 2$ rad/s and angular acceleration $\dot{\omega}_1 = 0.8$ rad/s^2. At this same instant the swivel is rotating at $\omega_2 = 3$ rad/s when $\dot{\omega}_2 = 2$ rad/s^2, both measured relative to the boom. Determine the velocity and acceleration of point P on the pipe at this instant.



Relative to XYZ, let xyz have

$\Omega = \{2\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \{0.8\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_A = \{-6\mathbf{j} + 3\mathbf{k}\}$ m ($\mathbf{r}_A$ changes direction relative to XYZ.)

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = 0 + (2\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}\mathbf{j}) = \{12\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = \left[(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= 0 + 0 + (0.8\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}) + (2\mathbf{k}) \times (12\mathbf{i})$$

$$= \{4.8\mathbf{i} + 24\mathbf{j}\} \text{ m/s}^2$$

Relative to xyz, let $x'y'z'$ have the origin at A and

$\Omega_{xyz} = \{3\mathbf{k}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \{2\mathbf{k}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/A})_{xyz} = \{4\mathbf{i} + 2\mathbf{j}\}$ m ($(\mathbf{r}_{P/A})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/A})_{xyz}$$

$$= 0 + (3\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})$$

$$= \{-6\mathbf{i} + 12\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = (\ddot{\mathbf{r}}_{P/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{x'y'z'}\right] + \left[\dot{\Omega}_{xyz} \times (\mathbf{r}_{P/A})_{xyz}\right] + \left[\Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right]$$

$$= 0 + 0 + \left[(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + \left[(3\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})\right]$$

$$= \{-40\mathbf{i} - 10\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$= 12\mathbf{i} + \left[(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + (-6\mathbf{i} + 12\mathbf{j})$$

$$= \{2\mathbf{i} + 20\mathbf{j}\} \text{ m/s}$$ **Ans**

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$= (4.8\mathbf{i} + 24\mathbf{j}) + \left[(0.8\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + 2\mathbf{k} \times \left[2\mathbf{k} \times (4\mathbf{i} + 2\mathbf{j})\right]$$

$$+ \left[2(2\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})\right] + (-40\mathbf{i} - 10\mathbf{j})$$

$$= \{-101\mathbf{i} - 14.8\mathbf{j}\} \text{ m/s}^2$$ **Ans**

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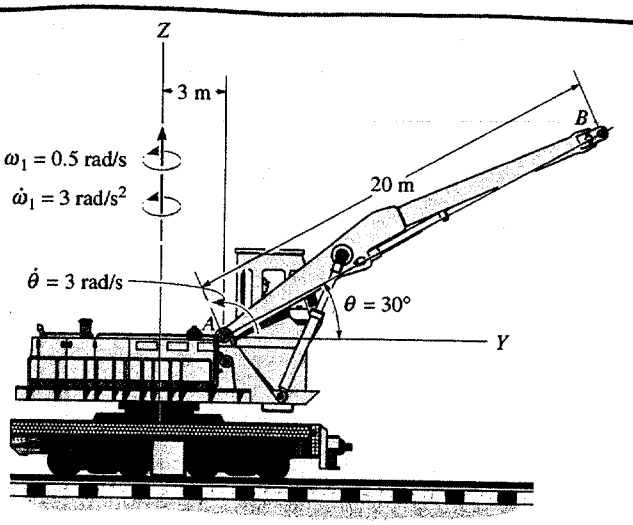
20-38. The boom AB of the locomotive crane is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, $\theta = 30°$ and the boom is rotating upward at a constant rate of $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



$\Omega = \{0.5\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 $\quad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = 0 + (0.5\mathbf{k}) \times (3\mathbf{j}) = \{-1.5\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= 0 + 0 + (3\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{k}) \times (-1.5\mathbf{i})$$

$$= \{-9\mathbf{i} - 0.75\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{3\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20\cos 30°\mathbf{j} + 20\sin 30° = \{17.32\mathbf{j} + 10\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$

$$= 0 + (3\mathbf{i}) \times (17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\Omega}_{xyz} \times \mathbf{r}_{B/A}] + [\Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = 0 + 0 + 0 + [(3\mathbf{i}) \times (-30\mathbf{j} + 51.96\mathbf{k})] = \{-155.88\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + [(0.5\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

$$= \{-10.2\mathbf{i} - 30\mathbf{j} + 52.0\mathbf{k}\} \text{ m/s} \qquad \text{Ans}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-9\mathbf{i} - 0.75\mathbf{j}) + [(3\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k} \times [0.5\mathbf{k} \times (17.32\mathbf{j} + 10\mathbf{k})]$$
$$+ [2(0.5\mathbf{k}) \times (-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-31.0\mathbf{i} - 161\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2 \qquad \text{Ans}$$

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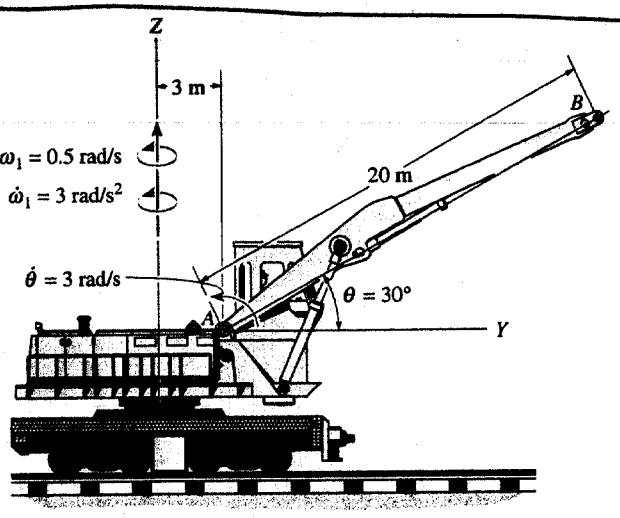
20-39. The locomotive crane is traveling to the right at 2 m/s and has an acceleration of 1.5 m/s^2, while the boom is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, $\theta = 30°$ and the boom is rotating upward at a constant rate $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



$\Omega = \{0.5\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 $\quad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = 2\mathbf{j} + (0.5\mathbf{k}) \times (3\mathbf{j}) = \{-1.5\mathbf{i} + 2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= 1.5\mathbf{j} + (0.5\mathbf{k}) \times (2\mathbf{j}) + (3\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{k}) \times (-1.5\mathbf{i} + 2\mathbf{j})$$

$$= \{-11\mathbf{i} + 0.75\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{3\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20\cos 30°\mathbf{j} + 20\sin 30° = \{17.32\mathbf{j} + 10\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$

$$= 0 + (3\mathbf{i}) \times (17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\Omega}_{xyz} \times \mathbf{r}_{B/A}] + [\Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = 0 + 0 + 0 + [(3\mathbf{i}) \times (-30\mathbf{j} + 51.96\mathbf{k})] = \{-155.88\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + 2\mathbf{j} + [(0.5\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

$$= \{-10.2\mathbf{i} - 28\mathbf{j} + 52.0\mathbf{k}\} \text{ m/s} \qquad \text{Ans}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-11\mathbf{i} + 0.75\mathbf{j}) + [(3\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k} \times [0.5\mathbf{k} \times (17.32\mathbf{j} + 10\mathbf{k})]$$
$$+ [2(0.5\mathbf{k}) \times (-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-33.0\mathbf{i} - 159\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2 \qquad \text{Ans}$$

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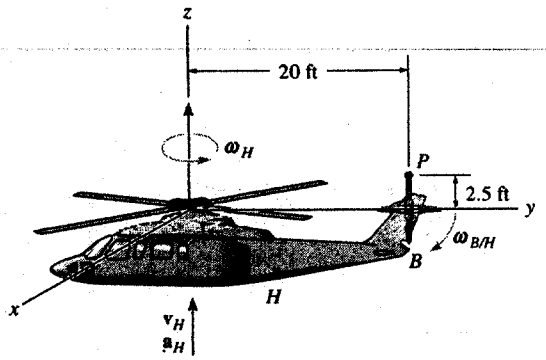
***20-40.** At the instant shown, the helicopter is moving upwards with a velocity $v_H = 4$ ft/s and has an acceleration $a_H = 2$ ft/s^2. At the same instant the frame H, *not* the horizontal blade, is rotating about a vertical axis with a constant angular velocity $\omega_H = 0.9$ rad/s. If the tail blade B is rotating with a constant angular velocity $\omega_{B/H} = 180$ rad/s, measured relative to H, determine the velocity and acceleration of point P, located on the tip of the blade, at the instant the blade is in the vertical position.



Relative to XYZ, let xyz have

$\Omega = \{0.9\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{20\mathbf{j}\}$ ft ($\mathbf{r}_B$ changes direction relative to XYZ.)

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = 4\mathbf{k} + (0.9\mathbf{k}) \times (20\mathbf{j}) = \{-18\mathbf{i} + 4\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times \left(\mathbf{r}_B\right)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= \left[2\mathbf{k} + \mathbf{0}\right] + \mathbf{0} + \left[(0.9\mathbf{k}) \times (-18\mathbf{i} + 4\mathbf{k})\right]$$

$$= \{-16.2\mathbf{j} + 2\mathbf{k}\} \text{ ft/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{-180\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/B})_{xyz} = \{2.5\mathbf{k}\}$ ft ($(\mathbf{r}_{P/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/B})_{xyz} = \mathbf{0} + (-180\mathbf{i}) \times (2.5\mathbf{k}) = \{450\mathbf{j}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/B})_{xyz} = (\ddot{\mathbf{r}}_{P/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times (\mathbf{r}_{P/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{xyz}$$

$$(\mathbf{a}_{B/A})_{xyz} = \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (-180\mathbf{i}) \times (450\mathbf{j}) = \{-81\,000\mathbf{k}\} \text{ ft/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_B + \Omega \times \mathbf{r}_{P/B} + (\mathbf{v}_{P/B})_{xyz}$$

$$= (-18\mathbf{i} + 4\mathbf{k}) + \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + (450\mathbf{j})$$

$$= \{-18\mathbf{i} + 450\mathbf{j} + 4\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_P = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{P/B} + \Omega \times (\Omega \times \mathbf{r}_{P/B}) + 2\Omega \times (\mathbf{v}_{P/B})_{xyz} + (\mathbf{a}_{P/B})_{xyz}$$

$$= (-16.2\mathbf{j} + 2\mathbf{k}) + \mathbf{0} + (0.9\mathbf{k}) \times \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + \left[2(0.9\mathbf{k}) \times (450\mathbf{j})\right] + (-81\,000\mathbf{k})$$

$$= \{-810\mathbf{i} - 16.2\mathbf{j} - 81\,000\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

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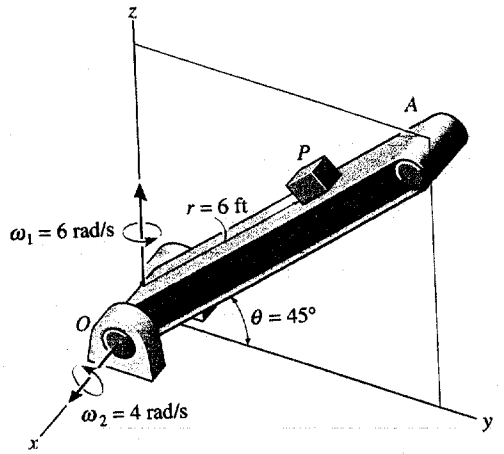
20-41. At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a constant rate $\dot{r} = 5$ ft/s, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = 0$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = 0$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = 0$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$

$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$

$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}$ ft/s

$(\mathbf{a}_{P/O}) = (\ddot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times (\dot{\mathbf{r}}_{P/O})_{xyz} + \dot{\Omega}_{P/O} \times \mathbf{r}_{P/O} + \Omega_{P/O} \times \dot{\mathbf{r}}_{P/O}$

$= 0 + (4\mathbf{i}) \times (3.536\mathbf{j} + 3.536\mathbf{k}) + 0 + (4\mathbf{i}) \times (-13.44\mathbf{j} + 20.51\mathbf{k})$

$= \{-96.18\mathbf{j} - 39.60\mathbf{k}\}$ ft/s^2

$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$

$= 0 + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$

$= 0 + 0 + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-96.18\mathbf{j} - 39.60\mathbf{k})$

$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2 **Ans**

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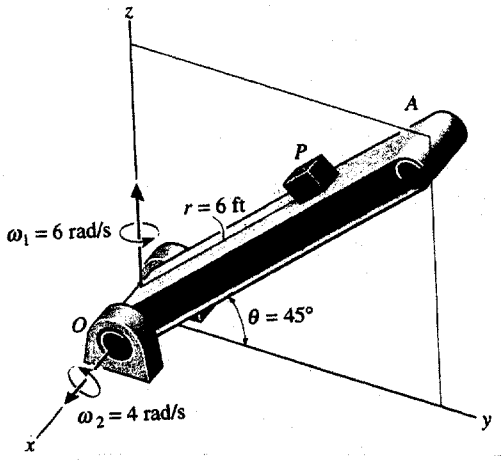
20-42. At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a rate $\dot{r} = 5$ ft/s, which is increasing at $\ddot{r} = 8$ ft/s^2, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$

$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$

$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = 8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k} - 96.18\mathbf{j} - 39.60\mathbf{k}$$

$$= \{-90.52\mathbf{j} - 33.945\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans**

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-90.52\mathbf{j} - 33.945\mathbf{k})$$

$$= -152.75\mathbf{j} + 161.23\mathbf{i} - 90.52\mathbf{j} - 33.945\mathbf{k}$$

$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}$ ft/s^2 **Ans**

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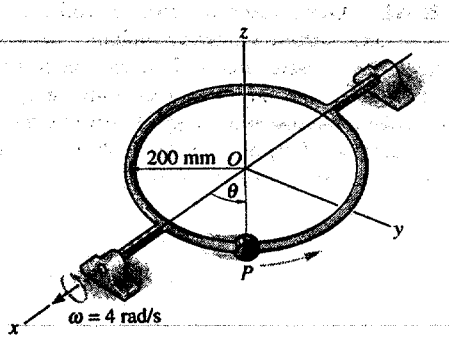
20-43. The particle P slides around the circular hoop with a constant angular velocity of $\dot{\theta} = 6$ rad/s, while the hoop rotates about the x axis at a constant rate of $\omega = 4$ rad/s. If at the instant shown the hoop is in the x–y plane and the angle $\theta = 45°$, determine the velocity and acceleration of the particle at this instant.



Relative to XYZ, let xyz have

$\Omega = \omega = \{4\mathbf{i}\}$ rad/s, $\quad \dot{\Omega} = \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}$; $\quad \mathbf{v}_O = \mathbf{0}$; $\quad \mathbf{a}_O = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have

$\Omega_{xyz} = \{6\mathbf{k}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to XYZ.)

$(\mathbf{r}_{P/O})_{xyz} = 0.2\cos 45°\mathbf{i} + 0.2\sin 45°\mathbf{j} = \{0.1414\mathbf{i} + 0.1414\mathbf{j}\}$ m ($(\mathbf{r}_{P/O})_{xyz}$ changes direction relative to XYZ.)

$$(\mathbf{v}_{P/O})_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{P/O}\right)_{xyz} = \mathbf{0} + (6\mathbf{k}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})$$

$$= \{-0.8485\mathbf{i} + 0.8485\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = \left(\ddot{\mathbf{r}}_{P/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'}\right] + \dot{\Omega} \times \left(\mathbf{r}_{P/O}\right)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (6\mathbf{k}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) = \{-5.0912\mathbf{i} - 5.0912\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz} = \mathbf{0} + (4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j}) - 0.8485\mathbf{i} + 0.8485\mathbf{j}$$

$$= \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_P = \mathbf{a}_O + \Omega \times \mathbf{r}_{P/O} + \Omega \times \left(\Omega \times \mathbf{r}_{P/O}\right) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (4\mathbf{i}) \times \left[(4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})\right] + 2(4\mathbf{i}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) - 5.0912\mathbf{i} - 5.0912\mathbf{j}$$

$$= \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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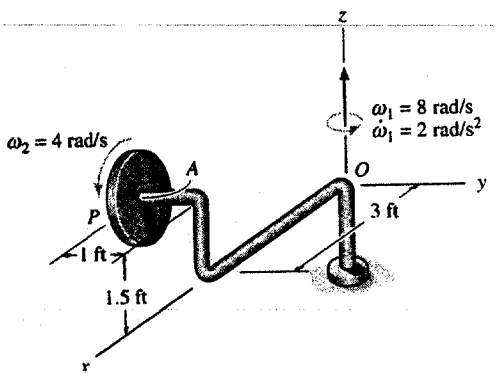
***20-44.** At the given instant, the rod is spinning about the z axis with an angular velocity $\omega_1 = 8$ rad/s and angular acceleration $\dot{\omega}_1 = 2$ rad/s^2. At this same instant, the disk is spinning at a constant rate $\omega_2 = 4$ rad/s, measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.



Coordinate Axes : The rotating x, y, z frame and fixed X, Y, Z frame are set with the origins at point A and O respectively.

Motion of A : Here, $\mathbf{r}_A$ changes direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_A$ can be found by setting another set of coordinate axis x', y', z' coincident with X, Y, Z rotating at $\Omega = \omega_1 = \{8\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{2\mathbf{k}\}$ rad/s^2. Here, $\mathbf{r}_A = \{3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}\}$ ft.

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{x'y'z'} + \Omega \times \mathbf{r}_A = \mathbf{0} + 8\mathbf{k} \times (3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}) = \{8\mathbf{i} + 24\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = \left[(\ddot{\mathbf{r}}_A)_{x'y'z'} + \Omega \times (\dot{\mathbf{r}}_A)_{x'y'z'}\right] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$
$$= (\mathbf{0} + \mathbf{0}) + 2\mathbf{k} \times (3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}) + 8\mathbf{k} \times (8\mathbf{i} + 24\mathbf{j}) = \{-190\mathbf{i} + 70\mathbf{j}\} \text{ ft/s}^2$$

Motion of P with Respect to A : Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{4\mathbf{j}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = 0$. Here, $\mathbf{r}_{P/A} = \{1\mathbf{i}\}$ ft.

$$(\mathbf{v}_{P/A})_{xyz} = \dot{\mathbf{r}}_{P/A} = (\dot{\mathbf{r}}_{P/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{P/A} = \mathbf{0} + (4\mathbf{j}) \times (1\mathbf{i}) = \{-4\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = \ddot{\mathbf{r}}_{P/A} = \left[(\ddot{\mathbf{r}}_{P/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{P/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{P/A}$$
$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (4\mathbf{j}) \times (-4\mathbf{k}) = \{-16\mathbf{i}\} \text{ ft/s}^2$$

Motion of Point A : Here, $\Omega = \omega_1 = \{8\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{2\mathbf{k}\}$ rad/s^2. Applying Eqs. 20 – 11 and 20 – 12, we have

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz} = (8\mathbf{i} + 24\mathbf{j}) + 8\mathbf{k} \times 1\mathbf{i} + (-4\mathbf{k}) = \{8.00\mathbf{i} + 32.0\mathbf{j} - 4.00\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$
$$= (-190\mathbf{i} + 70\mathbf{j}) + 2\mathbf{k} \times 1\mathbf{i} + 8\mathbf{k} \times (8\mathbf{k} \times 1\mathbf{i}) + 2(8\mathbf{k}) \times (-4\mathbf{k}) + (-16\mathbf{i}) = \{-270\mathbf{i} + 72.0\mathbf{j}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

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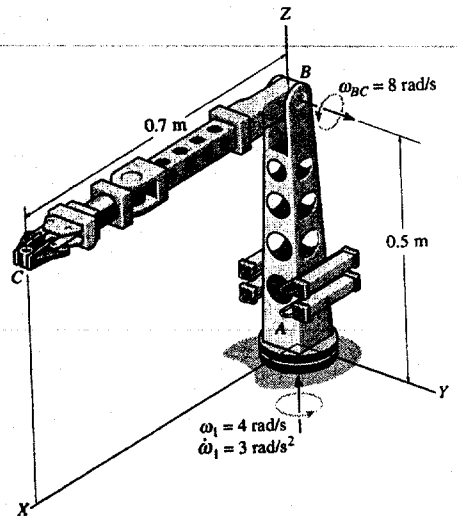
20-45. At the instant shown, the base of the robotic arm is turning about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom segment BC is rotating at a constant rate of $\omega_{BC} = 8$ rad/s. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz have origin at B and have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m ($(\mathbf{r}_{C/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\ddot{\mathbf{r}}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \dot{\Omega}_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i}\}\ \text{m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$+2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

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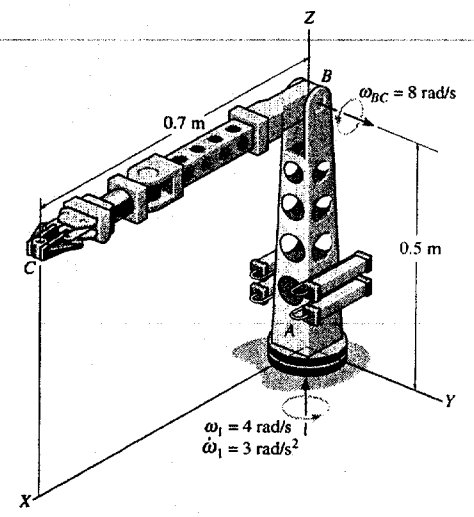
20-46. At the instant shown, the base of the robotic arm is turning about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom segment BC is rotating at $\omega_{BC} = 8$ rad/s, which is increasing at $\dot{\omega}_{BC} = 2$ rad/s^2. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz with origin at B have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\dot{\Omega}_{xyz} = \{2\mathbf{j}\}$ rad/s^2 (Ω does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m (Ω does not change direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\ddot{\mathbf{r}}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (2\mathbf{j}) \times (0.7\mathbf{i}) + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i} - 1.40\mathbf{k}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$+ 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i} - 1.40\mathbf{k}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j} - 1.40\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

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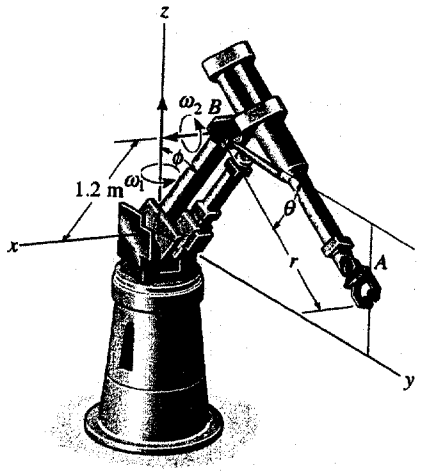
20–47. At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and about joint B at $\omega_2 = 2$ rad/s. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



$\Omega = \{5\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = 0$

$\mathbf{r}_B = 1.2\sin 30°\mathbf{j} + 1.2\cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\}$ m

$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = 0 + (5\mathbf{k}) \times (0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\}$ m/s

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= [0 + 0] + 0 + [(5\mathbf{k}) \times (-3\mathbf{i})]$$

$$= \{-15\mathbf{j}\}\ \text{m/s}^2$$

$\Omega_{xyz} = \{2\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = 0$

$\mathbf{r}_{A/B} = 1.6\cos 45°\mathbf{j} - 1.6\sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\}$ m

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B}$$

$$= 0 + (2\mathbf{i}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})$$

$$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\}\ \text{m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\Omega}_{xyz} \times \mathbf{r}_{A/B}\right] + \left[\Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}\right]$$

$$(\mathbf{a}_{A/B})_{xyz} = [0 + 0] + 0 + [(2\mathbf{i}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})]$$

$$= \{-4.5255\mathbf{j} + 4.5255\mathbf{k}\}\ \text{m/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$= (-3\mathbf{i}) + [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$$

$$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-15\mathbf{j}) + 0 + (5\mathbf{k}) \times [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})] + [2(5\mathbf{k}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-4.5255\mathbf{j} + 4.5255\mathbf{k})$$

$$= \{-22.6\mathbf{i} - 47.8\mathbf{j} + 4.53\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

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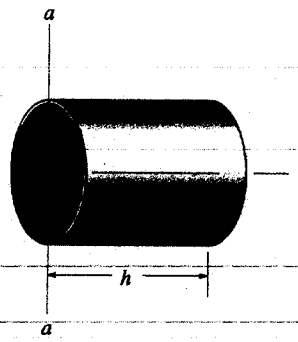
21-1. Show that the sum of the moments of inertia of a body, $I_{xx} + I_{yy} + I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of the origin.

$$I_{xx} + I_{yy} + I_{zz} = \int_m (y^2 + z^2)\,dm + \int_m (x^2 + z^2)\,dm + \int_m (x^2 + y^2)\,dm$$
$$= 2\int_m (x^2 + y^2 + z^2)\,dm$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. Since $|r|$ is constant, it does not depend on the orientation of the x, y, z axis. Consequently, $I_{xx} + I_{yy} + I_{zz}$ is also indepenent of the orientation of the x, y, z axis.

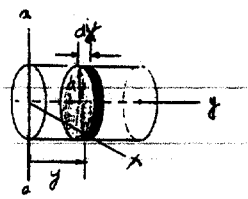
Q.E.D.

21-2. Determine the moment of inertia of the cylinder with respect to the a–a axis of the cylinder. The cylinder has a mass m.



The mass of the differential element is $dm = \rho dV = \rho(\pi a^2)\,dy$.

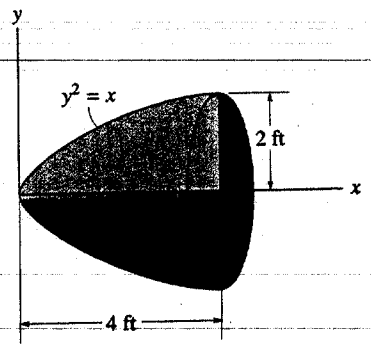
$$dI_{zz} = \tfrac{1}{4}dm a^2 + dm(y^2)$$
$$= \tfrac{1}{4}[\rho(\pi a^2)\,dy]a^2 + [\rho(\pi a^2)\,dy]y^2$$
$$= (\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2)\,dy$$
$$I_{zz} = \int dI_{zz} = \int_0^h (\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2)\,dy$$
$$= \frac{\rho\pi a^2 h}{12}(3a^2 + 4h^2)$$



However, $m = \int_m dm = \int_0^h \rho(\pi a^2)\,dy = \rho\pi a^2 h$

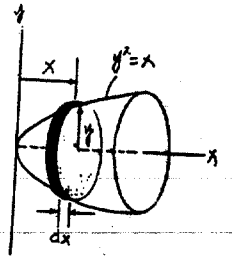
Hence, $I_{zz} = \frac{m}{12}(3a^2 + 4h^2)$ Ans

21-3. Determine moment of inertia I_y of the solid formed by revolving the shaded area around the x axis. The density of the material is $\rho = 12$ slug/ft^3.



The mass of the differential element is $dm = \rho dV = \rho(\pi y^2)\,dx = \rho\pi x\,dx$.

$$dI_y = \tfrac{1}{2}dm y^2 + dm x^2$$
$$= \tfrac{1}{4}[\rho\pi x\,dx](x) + (\rho\pi x\,dx)x^2$$
$$= \rho\pi(\tfrac{1}{4}x^2 + x^3)\,dx$$
$$I_y = \int dI_y = \rho\pi\int_0^4 (\tfrac{1}{4}x^2 + x^3)dx = 69.33\,\pi\rho$$
$$= 69.33(\pi)(12) = 2614\ \text{slug}\cdot\text{ft}^2$$



Ans

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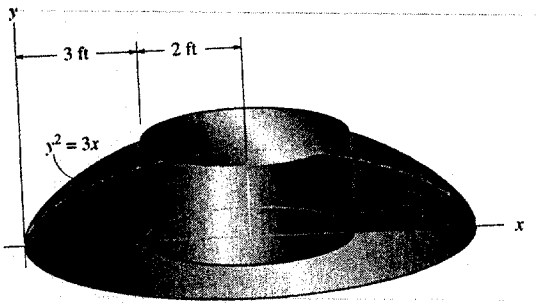
***21-4.** Determine the product of inertia I_{xy} of the body formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



$$\int_0^3 dm = \rho 2\pi \int_0^3 (5-x)y\,dx = \rho 2\pi \int_0^3 (5-x)\sqrt{3x}\,dx = 38.4\rho\pi$$

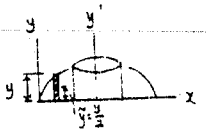
$$\int_0^3 \tilde{y}\,dm = \rho 2\pi \int_0^3 \frac{y}{2}(5-x)y\,dx$$

$$= \rho\pi \int_0^3 (5-x)(3x)\,dx$$

$$= 40.5\rho\pi$$

Thus, $\quad \bar{y} = \dfrac{40.5\rho\pi}{38.4\rho\pi} = 1.055$ ft

The solid is symmetric about y', thus



$I_{xy'} = 0$

$I_{xy} = I_{xy'} + \bar{x}\bar{y}m$

$= 0 + 5(1.055)(38.4\rho\pi)$

$I_{xy} = 636\rho$ **Ans**

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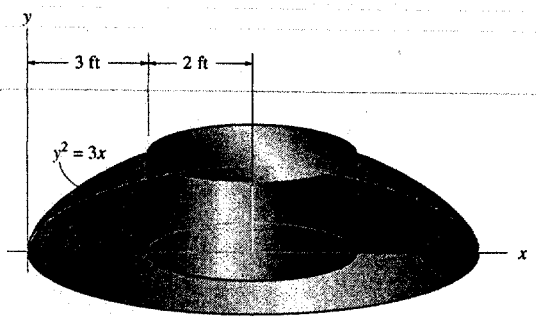
21-5. Determine the moment of inertia I_y of the body formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



$$I_y = \int_0^3 \frac{1}{2} dm\, r^2 - \frac{1}{2}(m')(2)^2$$

$$\int_0^3 \frac{1}{2} dm\, r^2 = \frac{1}{2}\int_0^3 \rho\,\pi\,(5-x)^4\,dy$$

$$= \frac{1}{2}\rho\,\pi \int_0^3 \left(5 - \frac{y^2}{3}\right)^4 dy$$

$$= 490.29\,\rho\,\pi$$

$$m' = \rho\,\pi\,(2)^2(3) = 12\,\rho\,\pi$$

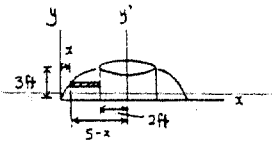
$$I_y = 490.29\,\rho\,\pi - \frac{1}{2}(12\,\rho\,\pi)(2)^2 = 466.29\,\rho\,\pi$$

Mass of body;

$$m = \int_0^3 \rho\,\pi\,(5-x)^2 dy - m'$$

$$= \int_0^3 \rho\,\pi\,(5 - \frac{y^2}{3})^2 dy - 12\,\rho\,\pi$$

$$= 38.4\,\rho\,\pi$$



$$I_y = 466.29\,\rho\,\pi + (38.4\,\rho\,\pi)(5)^2$$

$$= 1426.29\,\rho\,\pi$$

$$I_y = 4.48(10^3)\,\rho \qquad \textbf{Ans}$$

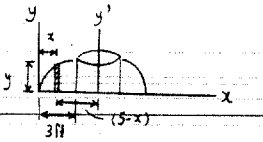
Also,

$$I_y = \int_0^3 r^2 dm$$

$$= \int_0^3 (5-x)^2 \rho(2\pi)(5-x)y\,dx$$

$$= 2\,\rho\,\pi \int_0^3 (5-x)^3 (3x)^{1/2} dx$$

$$= 466.29\,\rho\,\pi$$



$$m = \int_0^3 dm$$

$$= 2\,\rho\,\pi \int_0^3 (5-x)y\,dx$$

$$= 2\,\rho\,\pi \int_0^3 (5-x)(3x)^{1/2} dx$$

$$= 38.4\,\rho\,\pi$$

$$I_y = 466.29\,\rho\,\pi + 38.4\,\rho\,\pi\,(5)^2 = 4.48(10^3)\rho \qquad \textbf{Ans}$$

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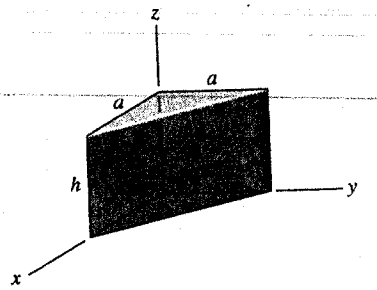
21-6. Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hx dy = \rho h(a-y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a-y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem :

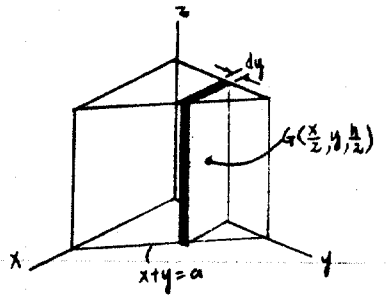
$$dI_{yz} = (dI_{y'z'})_G + dm y_G z_G$$

$$= 0 + (\rho hx dy)(y)\left(\frac{h}{2}\right)$$

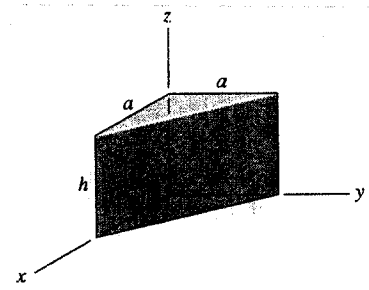
$$= \frac{\rho h^2}{2} xy dy$$

$$= \frac{\rho h^2}{2}(ay - y^2)\,dy$$

$$I_{yz} = \frac{\rho h^2}{2}\int_0^a (ay - y^2)\,dy = \frac{\rho a^3 h^2}{12} = \frac{1}{6}\left(\frac{\rho a^2 h}{2}\right)(ah) = \frac{m}{6}ah$$ **Ans**



21-7. Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hx dy = \rho h(a-y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a-y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem :

$$dI_{xy} = (dI_{x'y'})_G + dm x_G y_G$$

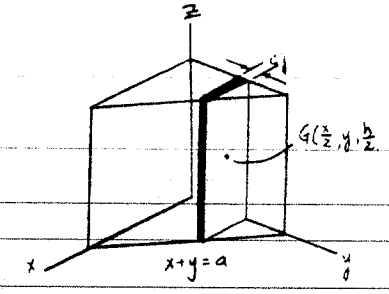
$$= 0 + (\rho hx dy)\left(\frac{x}{2}\right)(y)$$

$$= \frac{\rho h}{2} x^2 y dy$$

$$= \frac{\rho h}{2}(y^3 - 2ay^2 + a^2 y)\,dy$$

$$I_{xy} = \frac{\rho h}{2}\int_0^a (y^3 - 2ay^2 + a^2 y)\,dy$$

$$= \frac{\rho a^4 h}{24} = \frac{1}{12}\left(\frac{\rho a^2 h}{2}\right)a^2 = \frac{m}{12}a^2$$ **Ans**



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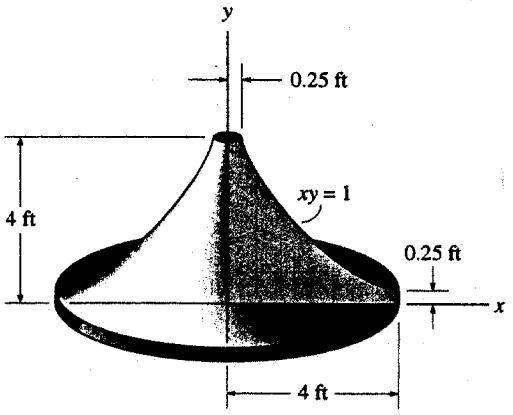
***21-8.** Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



For k_y : The mass of the differential element is $dm = \rho dV = \rho(\pi x^2)\,dy = \rho\pi\frac{dy}{y^2}$.

$$dI_y = \tfrac{1}{2}dmx^2 = \tfrac{1}{2}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) = \tfrac{1}{2}\rho\pi\tfrac{dy}{y^6}$$

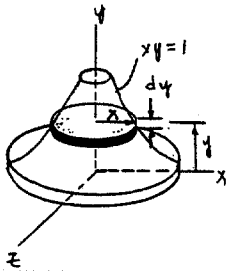
$$I_y = \int dI_y = \tfrac{1}{2}\rho\pi\int_{0.25}^{4}\tfrac{dy}{y^6} + \tfrac{1}{2}\left[\rho(\pi)(4)^2(0.25)\right](4)^2$$

$$= 134.03\rho$$

However, $$m = \int_m dm = \rho\pi\int_{0.25}^{4}\tfrac{dy}{y^2} + \rho\left[\pi(4)^2(0.25)\right] = 24.35\rho$$

Hence, $$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{134.03\rho}{24.35\rho}} = 2.35 \text{ ft}$$ **Ans**

For k_x : $0.25 \text{ ft} < y \le 4 \text{ ft}$



$$dI_x' = \tfrac{1}{4}dmx^2 + dmy^2$$

$$= \tfrac{1}{4}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) + \left(\rho\pi\tfrac{dy}{y^2}\right)y^2$$

$$= \rho\pi\left(\tfrac{1}{4y^4} + 1\right)dy$$

$$I_x' = \int dI_x' = \rho\pi\int_{0.25}^{4}\left(\tfrac{1}{4y^4} + 1\right)dy = 28.53\rho$$

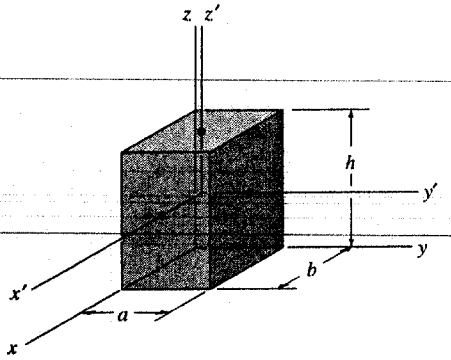
$$I_x'' = \tfrac{1}{4}\left[\rho\pi(4)^2(0.25)\right](4)^2 + \left[\rho\pi(4)^2(0.25)\right](0.125)^2$$

$$= 50.46\rho$$

$$I_x = I_x' + I_x'' = 28.53\rho + 50.46\rho = 78.99\rho$$

Hence, $$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{78.99\rho}{24.35\rho}} = 1.80 \text{ ft}$$ **Ans**

21-9. Determine the mass moment of inertia of the homogeneous block with respect to its centroidal x' axis. The mass of the block is m.



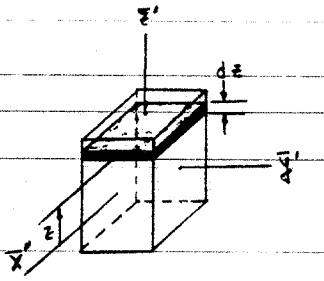
The mass of the differential element is $dm = \rho dV = \rho abdz$.

$$dI_{x'} = \frac{1}{12}dma^2 + dmz^2$$

$$= \frac{1}{12}(\rho abdz)a^2 + (\rho abdz)z^2$$

$$= \frac{\rho ab}{12}(a^2 + 12z^2)\,dz$$

$$I_{x'} = \int dI_{x'} = \frac{\rho ab}{12}\int_{-\frac{h}{2}}^{\frac{h}{2}}(a^2 + 12z^2)\,dz = \frac{\rho abh}{12}(a^2 + h^2)$$



However, $$m = \int_m dm = \rho ab\int_{-\frac{h}{2}}^{\frac{h}{2}}dz = \rho abh$$

Hence, $$I_{x'} = \frac{m}{12}(a^2 + h^2)$$ **Ans**

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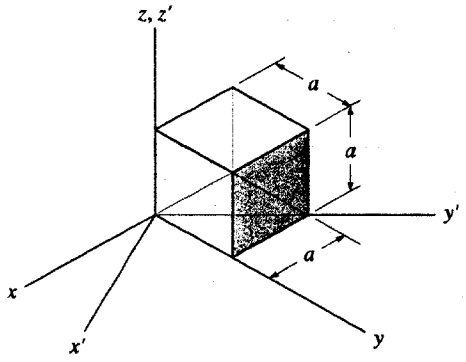
21-10. Determine the elements of the inertia tensor for the cube with respect to the x,y,z coordinate system. The mass of the cube is m.

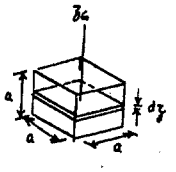


$$dI_{zG} = \frac{1}{12}(dm)(a^2 + a^2)$$

$$I_{zG} = \frac{1}{6}a^2\int_0^a \rho(a^2)\,dz$$

$$= \frac{1}{6}a^2(\rho a^3)$$

$$= \frac{1}{6}ma^2$$

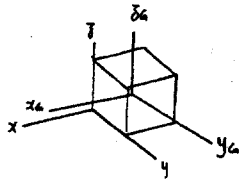


$$I_x = I_y = I_z = \frac{1}{6}ma^2 + m[(\tfrac{a}{2})^2 + (\tfrac{a}{2})^2] = \frac{2}{3}ma^2$$

$$I_{xy} = 0 + m(-\tfrac{a}{2})(\tfrac{a}{2}) = -\frac{1}{4}ma^2$$

$$I_{yz} = 0 + m(\tfrac{a}{2})(\tfrac{a}{2}) = \frac{1}{4}ma^2$$

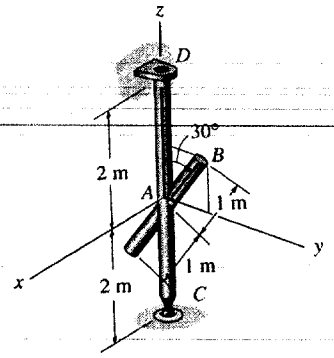
$$I_{xz} = 0 + m(-\tfrac{a}{2})(\tfrac{a}{2}) = -\frac{1}{4}ma^2$$



Changing the signs of the products of inertia as represented by the equation in the text, we have

$$\begin{pmatrix} \frac{2}{3}ma^2 & \frac{1}{4}ma^2 & \frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & \frac{2}{3}ma^2 & -\frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & -\frac{1}{4}ma^2 & \frac{2}{3}ma^2 \end{pmatrix}$$ **Ans**

21-11. Determine the moments of inertia about the x, y, z axes of the rod assembly. The rods have a mass of 0.75 kg/m.



$$I_x = \frac{1}{12}\big[0.75(4)\big](4)^2 + \frac{1}{12}\big[0.75(2)\big](2)^2 = 4.50\ \text{kg}\cdot\text{m}^2$$ **Ans**

$$I_y = \frac{1}{12}\big[0.75(4)\big](4)^2 + \frac{1}{12}\big[0.75(2)\big](2\cos 30°)^2 = 4.38\ \text{kg}\cdot\text{m}^2$$ **Ans**

$$I_z = 0 + \frac{1}{12}\big[0.75(2)\big](2\sin 30°)^2 = 0.125\ \text{kg}\cdot\text{m}^2$$ **Ans**

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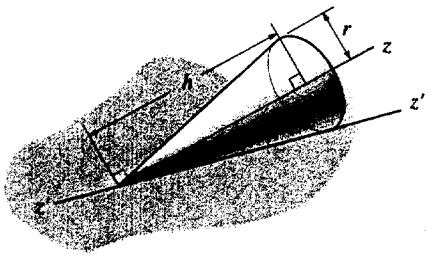
***21-12.** Determine the moment of inertia of the cone about the z' axis. The weight of the cone is 15 lb, the *height* is $h = 1.5$ ft, and the radius is $r = 0.5$ ft.



$$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = 18.43°$$

$$I_{xx} = I_{yy} = \left[\frac{3}{80}m\{4(0.5)^2 + (1.5)^2\}\right] + m\left[1.5 - \left(\frac{1.5}{4}\right)\right]^2$$

$$I_{xx} = I_{yy} = 1.3875\,m$$

$$I_z = \frac{3}{10}m(0.5)^2 = 0.075m$$

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Eq. 21–5.

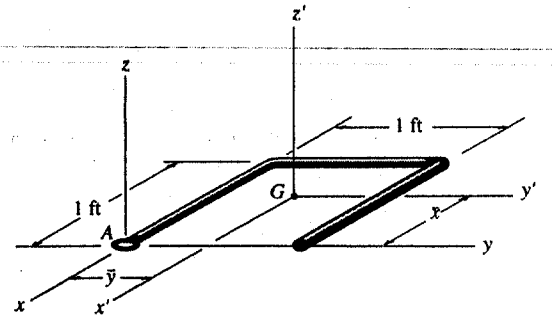
$$I_{z'z'} = u_{z'x}^2 I_{xx} + u_{z'y}^2 I_{yy} + u_{z'z}^2 I_{zz}$$

$$= 0 + [\cos(108.43°)]^2(1.3875m) + [\cos(18.43°)]^2(0.075m)$$

$$= 0.2062m$$

$$I_{z'z'} = 0.2062\left(\frac{15}{32.2}\right) = 0.0961 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

21-13. The bent rod has a weight of 1.5 lb/ft. Locate the center of gravity $G(\bar{x}, \bar{y})$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.



Due to symmetry $\bar{y} = 0.5$ ft **Ans**

$$\bar{x} = \frac{\Sigma \tilde{x}W}{\Sigma w} = \frac{(-1)(1.5)(1) + 2[(-0.5)(1.5)(1)]}{3[1.5(1)]} = -0.667 \text{ ft} \qquad \textbf{Ans}$$

$$I_{x'} = 2\left[\left(\frac{1.5}{32.2}\right)(0.5)^2\right] + \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2$$

$$= 0.0272 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

$$I_{y'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.667 - 0.5)^2\right]$$

$$+\left(\frac{1.5}{32.2}\right)(1 - 0.667)^2$$

$$= 0.0155 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

$$I_{z'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.5^2 + 0.1667^2)\right]$$

$$+\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.3333)^2$$

$$= 0.0427 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

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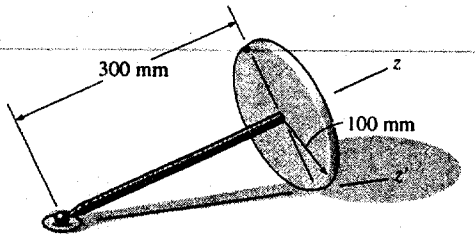
21-14. Determine the moment of inertia of both the 1.5-kg rod and 4-kg disk about the z' axis.



Due to symmetry $I_{xy} = I_{yz} = I_{zx} = 0$

$$I_y = I_x = \left[\frac{1}{4}(4)(0.1)^2 + 4(0.3)^2\right] + \frac{1}{3}(1.5)(0.3)^2$$

$$= 0.415 \text{ kg} \cdot \text{m}^2$$

$$I_z = \frac{1}{2}(4)(0.1)^2 = 0.02 \text{ kg} \cdot \text{m}^2$$

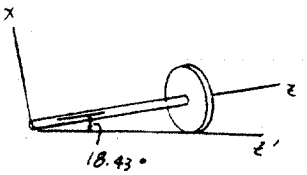
$u_z = \cos(18.43°) = 0.9487, \quad u_y = \cos 90° = 0,$

$u_x = \cos(90° + 18.43°) = -0.3162$

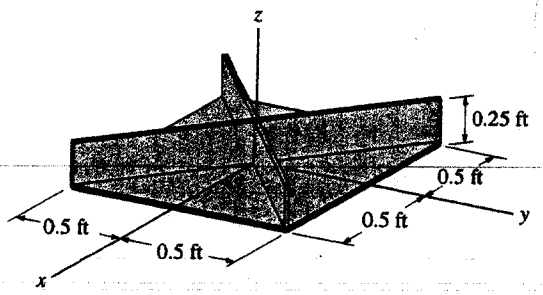
$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.415(-0.3162)^2 + 0 + 0.02(0.9487)^2 - 0 - 0 - 0$$

$$= 0.0595 \text{ kg} \cdot \text{m}^2 \quad \textbf{Ans}$$



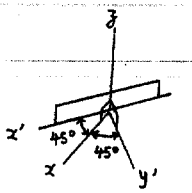
21-15. Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight of 6 lb/ft^2.



Horizontal plate:

$$I_{xx} = \frac{1}{12}\left(\frac{6(1)(1)}{32.2}\right)(1)^2 = 0.0155$$

Vertical plates:



$I_{x'} = 0.707, \quad I_{y'} = 0.707, \quad I_{z'} = 0$

$$I_{x'x'} = \frac{1}{3}\left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)(\tfrac{1}{4})^2 = 0.001372$$

$$I_{y'y'} = \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{12}\right)\left[(\tfrac{1}{4})^2 + (1\sqrt{2})^2\right] + \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)(\tfrac{1}{8})^2$$

$$= 0.01235$$

Using Eq. 21–5,

$$I_{xx} = (0.707)^2(0.001372) + (0.707)^2(0.01235)$$

$$= 0.00686$$

Thus,

$$I_{xx} = 0.0155 + 2(0.00686) = 0.0292 \text{ slug} \cdot \text{ft}^2 \quad \textbf{Ans}$$

***21-16.** Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a weight of 6 lb/ft^2.

Due to symmetry,

$I_{yz} = 0$ **Ans**

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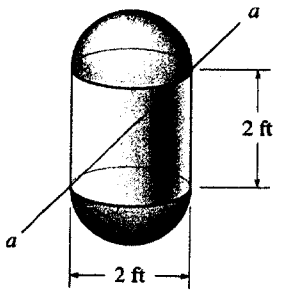
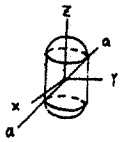
21-17. Determine the moment of inertia of the composite body about the *aa* axis. The cylinder weighs 20 lb, and each hemisphere weighs 10 lb.





$u_{ax} = 0.707$

$u_{az} = 0$

$u_{ay} = 0.707$

$$I_{zz} = \frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 + 2\left[\frac{2}{5}\left(\frac{10}{32.2}\right)(1)^2\right]$$

$$= 0.5590 \text{ slug}\cdot\text{ft}^2$$

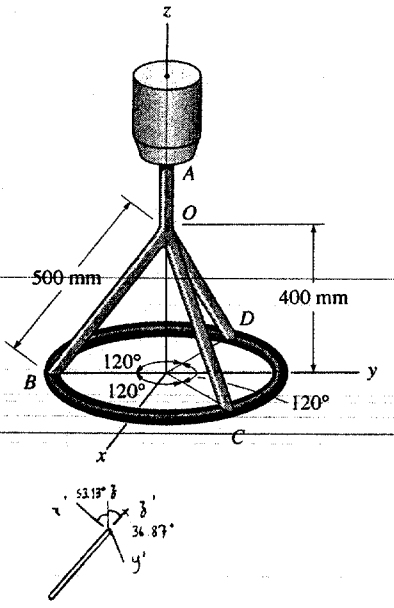
$$I_{xx} = I_{yy} = \frac{1}{12}\left(\frac{20}{32.2}\right)[3(1)^2 + (2)^2] + 2\left[0.259\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}\left(\frac{11}{8}\right)^2\right]$$

$$I_{xx} = I_{yy} = 1.6975 \text{ slug}\cdot\text{ft}^2$$

$$I_{aa} = 0 + (0.707)^2(1.6975) + (0.707)^2(0.559)$$

$$= 1.13 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

21-18. Determine the moment of inertia of the rod-and-thin-ring assembly about the z axis. The rods and ring have a mass of 2 kg/m.



For the rod,

$u_{x'} = 0.6, \quad u_{y'} = 0, \quad u_{z'} = 0.8$

$$I_{x'} = I_{y'} = \frac{1}{3}[(0.5)(2)](0.5)^2 = 0.08333 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = 0$$

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

From Eq. 21–5,

$$I_z = 0.08333(0.6)^2 + 0 + 0 - 0 - 0 - 0$$

$$I_z = 0.03 \text{ kg}\cdot\text{m}^2$$

For the ring,

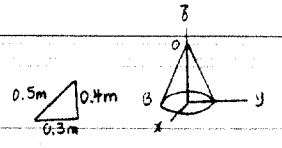
The radius is $r = 0.3$ m

Thus,

$$I_z = mR^2 = [2(2\pi)(0.3)](0.3)^2 = 0.3393 \text{ kg}\cdot\text{m}^2$$

Thus the moment of inertia of the assembly is

$$I_z = 3(0.03) + 0.339 = 0.429 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$



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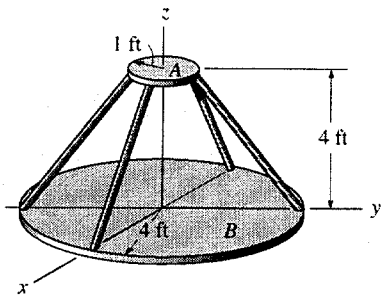
21-19. The assembly consists of a 15-lb plate A, 40-lb plate B, and four 7-lb rods. Determine the moments of inertia of the assembly with respect to the principal x, y, z axes.



Due to symmetry

$$\bar{y} = \bar{x} = 0$$

$$\bar{z} = \frac{\sum \tilde{z}w}{\sum w} = \frac{4(15) + 0(40) + 2(28)}{15 + 40}$$

$$= 1.3976 \text{ ft}$$

$$I_z = (I_z)_{\text{upper}} + (I_z)_{\text{lower}} + (I_z)_{\text{rods}}$$

$$= \frac{1}{2}\left(\frac{15}{32.2}\right)(1)^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)(4)^2$$

$$+ 4\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(3)^2 + \left(\frac{7}{32.2}\right)(2.5)^2\right]$$

$$I_z = 16.3 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans}$$

$$I_x = (I_x)_{\text{upper}} + (I_x)_{\text{lower}} + (I_x)_{\text{rods-1}} + (I_x)_{\text{rods-2}}$$

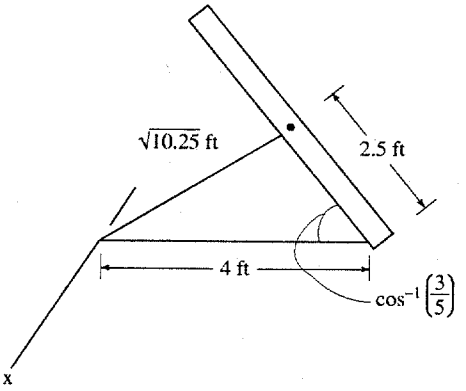
$$= \left[\frac{1}{4}\left(\frac{15}{32.2}\right)(1)^2 + \left(\frac{15}{32.2}\right)(4)^2\right] + \frac{1}{4}\left(\frac{40}{32.2}\right)(4)^2$$

$$+ 2\left[\frac{1}{3}\left(\frac{7}{32.2}\right)(4)^2\right] + 2\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(5)^2 + \left(\frac{7}{32.2}\right)(10.25)\right]$$

$$I_x = 20.2 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans}$$

And by symmetry,

$$I_y = 20.2 \text{ slug} \cdot \text{ft}^2 \quad \textbf{Ans}$$



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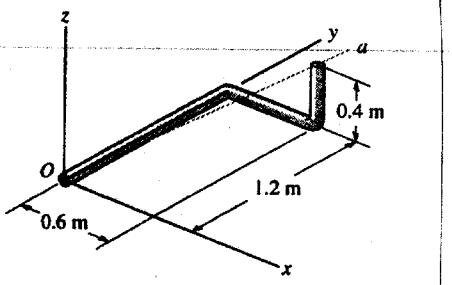
***21-20.** The bent rod has a mass of 4 kg/m. Determine the moment of inertia of the rod about the *Oa* axis.



$$I_{xy} = [4(1.2)](0)(0.6) + [4(0.6)](0.3)(1.2) + [4(0.4)](0.6)(1.2) = 2.016\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = [4(1.2)](0.6)(0) + [4(0.6)](1.2)(0) + [4(0.4)](1.2)(0.2) = 0.384\ \text{kg}\cdot\text{m}^2$$

$$I_{zx} = [4(1.2)](0)(0) + [4(0.6)](0)(0.3) + [4(0.4)](0.2)(0.6) = 0.192\ \text{kg}\cdot\text{m}^2$$

$$I_x = \frac{1}{3}[4(1.2)](1.2)^2 + [4(0.6)](1.2)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)](1.2^2 + 0.2^2)\right]$$

$$= 8.1493\ \text{kg}\cdot\text{m}^2$$

$$I_y = 0 + \frac{1}{3}[4(0.6)](0.6)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)](0.6^2 + 0.2^2)\right]$$

$$= 0.9493\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{3}[4(1.2)](1.2)^2 + \left[\frac{1}{12}[4(0.6)](0.6)^2 + [4(0.6)](0.3^2 + 1.2^2)\right] + [4(0.4)](1.2^2 + 0.6^2)$$

$$= 8.9280\ \text{kg}\cdot\text{m}^2$$

$$\mathbf{u}_{Oa} = \frac{0.6\mathbf{i} + 1.2\mathbf{j} + 0.4\mathbf{k}}{\sqrt{0.6^2 + 1.2^2 + 0.4^2}} = \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$I_{Oa} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 8.1493\left(\frac{3}{7}\right)^2 + 0.9493\left(\frac{6}{7}\right)^2 + 8.9280\left(\frac{2}{7}\right)^2 - 2(2.016)\left(\frac{3}{7}\right)\left(\frac{6}{7}\right)$$

$$- 2(0.384)\left(\frac{6}{7}\right)\left(\frac{2}{7}\right) - 2(0.192)\left(\frac{2}{7}\right)\left(\frac{3}{7}\right)$$

$$= 1.21\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

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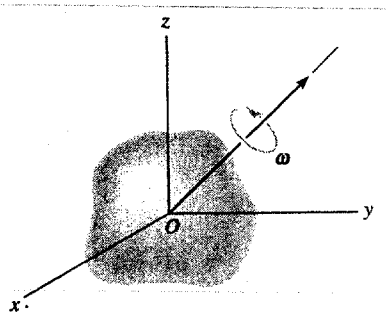
21-21. If a body contains *no planes of symmetry*, the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity $\boldsymbol{\omega}$, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21–10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation



$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.

$\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$

Equating the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components to the scalar equations (Eq. 21-10) yields

$$(I_{xx} - I)\omega_x - I_{xy}\omega_y - I_{xz}\omega_z = 0$$

$$-I_{yx}\omega_x + (I_{yy} - I)\omega_y - I_{yz}\omega_z = 0$$

$$-I_{zx}\omega_x - I_{zy}\omega_y + (I_{zz} - I)\omega_z = 0$$

Solution for ω_x, ω_y, and ω_z requires

$$\begin{vmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -I_{yx} & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{vmatrix} = 0$$

Expanding

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0 \quad \textbf{QED}$$

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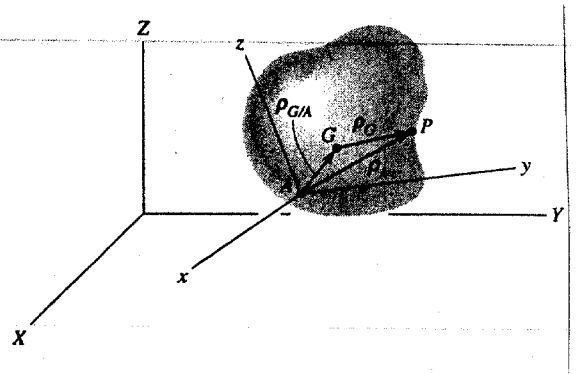
21-22. Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H}_A$ can be expressed by Eq. 21–9. This requires substituting $\boldsymbol{\rho}_A = \boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}$ into Eq. 21–6 and expanding, noting that $\int \boldsymbol{\rho}_G\, dm = \mathbf{0}$ by definition of the mass center and $\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}$.



$$\mathbf{H}_A = \left(\int_m \boldsymbol{\rho}_A\, dm\right) \times \mathbf{v}_A + \int_m \boldsymbol{\rho}_A \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_A)\, dm$$

$$= \left(\int_m (\boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A})\, dm\right) \times \mathbf{v}_A + \int_m (\boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}) \times \left[\boldsymbol{\omega} \times (\boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A})\right] dm$$

$$= \left(\int_m \boldsymbol{\rho}_G dm\right) \times \mathbf{v}_A + (\boldsymbol{\rho}_{G/A} \times \mathbf{v}_A) \int_m dm + \int_m \boldsymbol{\rho}_G \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_G)\, dm$$
$$+ \left(\int_m \boldsymbol{\rho}_G dm\right) \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}) + \boldsymbol{\rho}_{G/A} \times \left(\boldsymbol{\omega} \times \int_m \boldsymbol{\rho}_G dm\right) + \boldsymbol{\rho}_{G/A} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}) \int_m dm$$

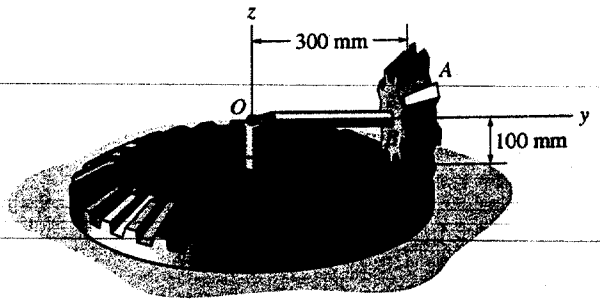
Since $\int_m \boldsymbol{\rho}_G dm = 0$ and from Eq. 21-8 $\mathbf{H}_G = \int_m \boldsymbol{\rho}_G \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_G)\, dm$

$$\mathbf{H}_A = (\boldsymbol{\rho}_{G/A} \times \mathbf{v}_A)\, m + \mathbf{H}_G + \boldsymbol{\rho}_{G/A} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A})\, m$$

$$= \boldsymbol{\rho}_{G/A} \times (\mathbf{v}_A + (\boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}))\, m + \mathbf{H}_G$$

$$= (\boldsymbol{\rho}_{G/A} \times m\mathbf{v}_G) + \mathbf{H}_G \qquad \textbf{Q.E.D.}$$

21-23. The 2-kg gear A rolls on the fixed plate gear C. Determine the angular velocity of rod OB about the z axis after it rotates one revolution about the z axis, starting from rest. The rod is acted upon by the constant moment $M = 5\ \text{N} \cdot \text{m}$. Neglect the mass of rod OB. Assume that gear A is a uniform disk having a radius of 100 mm.

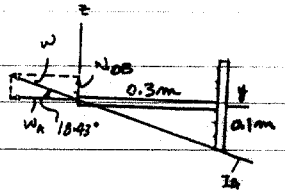


)

$\omega_{OB} = \omega_A \tan 18.43° = 0.3333\omega_A$ [1]

$I_y = \frac{1}{2}(2)(0.1)^2 = 0.01\ \text{kg} \cdot \text{m}^2$

$I_z = \frac{1}{4}(2)(0.1)^2 + 2(0.3)^2 = 0.185\ \text{kg} \cdot \text{m}^2$



$T_1 + \Sigma U_{1-2} = T_2$

$0 + 5(2\pi) = \frac{1}{2}(0.01)\,\omega_A^2 + \frac{1}{2}(0.185)\,\omega_{OB}^2$ [2]

Solving Eqs.[1] and [2] yields :

$\omega_{OB} = 15.1\ \text{rad/s}$ **Ans**

$\omega_A = 45.35\ \text{rad/s}$

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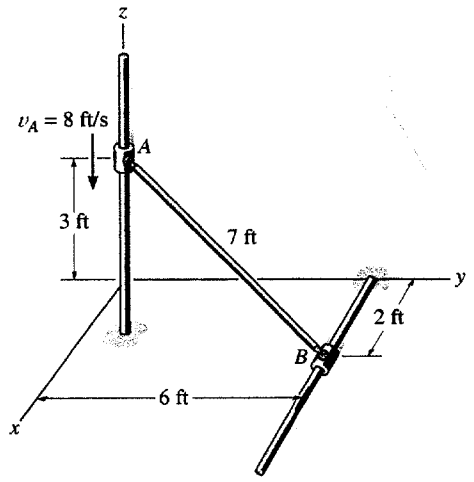
***21-24.** Rod AB has a weight of 6 lb and is attached to two smooth collars at its end points by ball-and-socket joints. If collar A is moving downward at a speed of 8 ft/s, determine the kinetic energy of the rod at the instant shown. Assume that at this instant the angular velocity of the rod is directed perpendicular to the rod's axis.



$\mathbf{v}_A = \{-8\mathbf{k}\}$ m/s $\quad \mathbf{v}_B = v_B\mathbf{i}$ $\quad \boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ m $\quad \mathbf{r}_{G/A} = \{1\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k}\}$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

Equating i, j and k components

$$-3\omega_y - 6\omega_z = v_B \qquad [1]$$

$$3\omega_x + 2\omega_z = 0 \qquad [2]$$

$$6\omega_x - 2\omega_y - 8 = 0 \qquad [3]$$

Since $\boldsymbol{\omega}_{AB}$

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad [4]$$

Solving Eqs. [1] to [4] yields :

$\omega_x = 0.9796$ rad/s $\quad \omega_y = -1.0612$ rad/s $\quad \omega_z = -1.4694$ rad/s

$v_B = 12.0$ ft/s

Hence $\boldsymbol{\omega}_{AB} = \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\}$ rad/s $\quad \mathbf{v}_B = \{12.0\mathbf{i}\}$ ft/s

$$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{G/A}$$

$$= -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9796 & -1.0612 & -1.4694 \\ 1 & 3 & -1.5 \end{vmatrix}$$

$$= \{6.0\mathbf{i} - 4.0\mathbf{k}\} \text{ ft/s}$$

$$\omega_{AB}^2 = 0.9796^2 + (-1.0612)^2 + (-1.4694)^2 = 4.2449$$

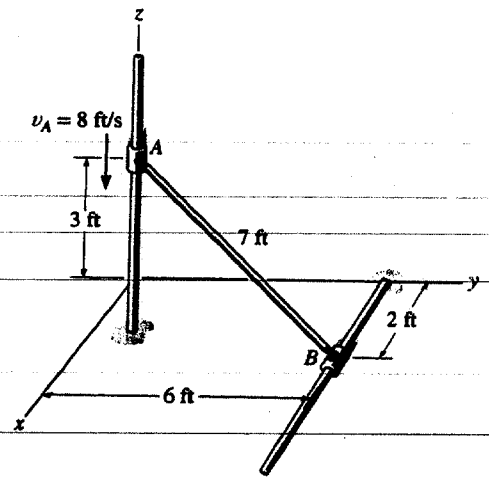
$$v_G^2 = 6^2 + 4^2 = 52$$

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega_{AB}^2$$

$$= \tfrac{1}{2}\left(\tfrac{6}{32.2}\right)(52) + \tfrac{1}{2}\left[\tfrac{1}{12}\left(\tfrac{6}{32.2}\right)\left(\sqrt{2^2 + 6^2 + (-3)^2}\right)^2\right](4.2449)$$

$$= 6.4596 = 6.46 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans}$$

21-25. At the instant shown the collar at A on the 6-lb rod AB has a velocity of $v_A = 8$ ft/s. Determine the kinetic energy of the rod after the collar has descended 3 ft. Neglect friction and the thickness of the rod. Neglect the mass of the collar and the collar are attached to the rod using ball-and-socket joints.



From Prob. 21–32.

$T_1 = 6.4596$ ft·lb

$T_1 + V_1 = T_2 + V_2$

$6.4596 + 6(1.5) = T_2 + 0$

$T_2 = 15.5$ ft·lb $\qquad$ **Ans**

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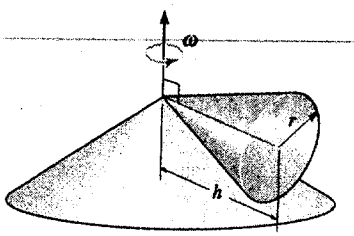
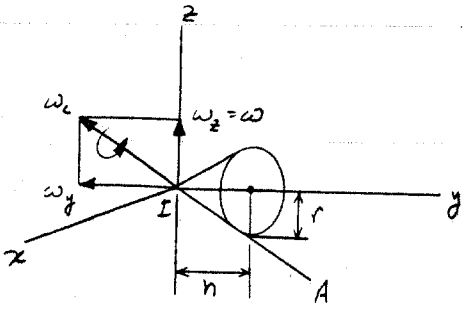
21-26. The cone has a mass m and rolls without slipping on the conical surface so that it has an angular velocity about the vertical axis of ω. Determine the kinetic energy of the cone due to this motion.



$$\frac{\omega_s}{r} = \frac{\omega_c}{h}$$

$$\omega_s = \left(\frac{h}{r}\right)\omega_c = \left(\frac{h}{r}\right)\omega$$

$$I_z = \left(\frac{3}{80}\right)m\left(4r^2 + h^2\right) + m\left(\frac{3}{4}h\right)^2 = \left(\frac{3}{20}\right)mr^2 + \left(\frac{3}{5}\right)mh^2 = \left(\frac{3}{20}\right)m\left(r^2 + 4h^2\right)$$

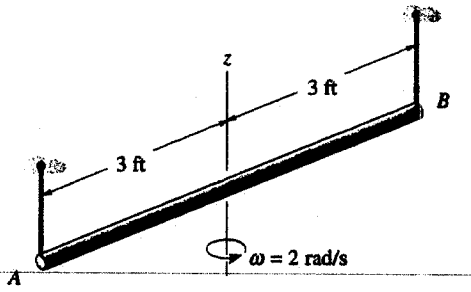


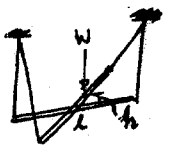
$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$= 0 + \frac{1}{2}\left(\frac{3}{10}mr^2\right)\left(\frac{h}{r}\omega\right)^2 + \frac{1}{2}\left[\left(\frac{3}{20}\right)m\left(r^2 + 4h^2\right)\right]\omega^2 = \frac{m\omega^2}{20}\left[3h^2 + \frac{3}{2}r^2 + 6h^2\right]$$

$$T = \frac{9mh^2}{20}\left[1 + \frac{r^2}{6h^2}\right]\omega^2 \qquad \textbf{Ans}$$

21-27. The rod weighs 3 lb/ft and is suspended from parallel cords at A and B. If the rod has an angular velocity of 2 rad/s about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.





$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{12}\frac{W}{g}l^2\right]\omega^2 + 0 = 0 + Wh$$

$$h = \frac{1}{24}\frac{l^2\omega^2}{g} = \frac{1}{24}\frac{(6)^2(2)^2}{(32.2)}$$

$$h = 0.1863 \text{ ft} = 2.24 \text{ in.} \qquad \textbf{Ans}$$

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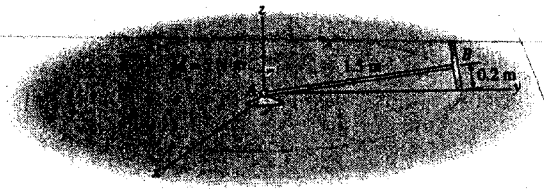
***21-28.** The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N·m couple moment is applied, determine the angular velocity of the rod about the z axis after the assembly has made two revolutions about the z axis starting from rest. The disk rolls without slipping.

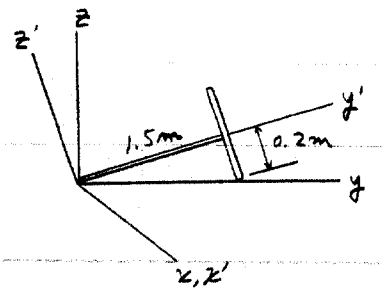


$$I_{x'} = I_{z'} = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$

$$I_{y'} = \frac{1}{2}(5)(0.2)^2 = 0.100$$

$$\omega = -\omega_{y'}\mathbf{j}' + \omega_z\mathbf{k} = -\omega_{y'}\mathbf{j}' + \omega_z \sin 7.595°\mathbf{j}' + \omega_z \cos 7.595°\mathbf{k}'$$

$$= (0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$



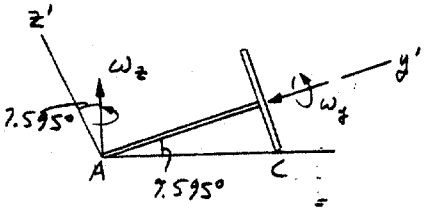
Since points A and C have zero velocity,

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$0 = 0 + \left[(0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

$$0 = -1.48684\omega_z - 0.026433\omega_z + 0.2\omega_{y'}$$

$$\omega_{y'} = 7.5664\omega_z$$



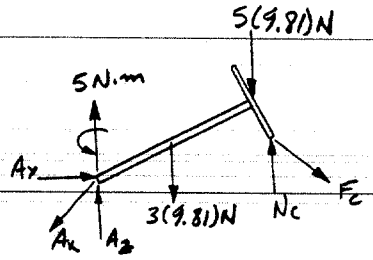
Thus,

$$\omega = -7.4342\omega_z\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 5(2\pi)(2) = 0 + \frac{1}{2}(0.100)(-7.4342\omega_z)^2 + \frac{1}{2}(13.55)(0.99123\omega_z)^2$$

$$\omega_z = 2.58 \text{ rad/s} \qquad \textbf{Ans}$$



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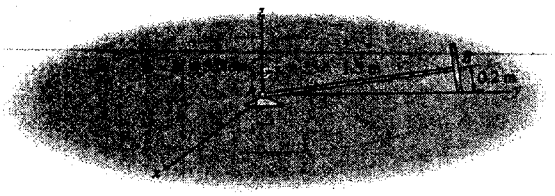
21-29. The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N·m couple moment gives it an angular velocity about the z axis of $\omega_z = 2$ rad/s, determine the magnitude of the angular momentum of the assembly about A.



$$I_{x'} = I_{z'} = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$

$$I_{y'} = \frac{1}{2}(5)(0.2)^2 = 0.100$$

$$\omega = -\omega_{y'}\mathbf{j}' + \omega_z\mathbf{k} = -\omega_{y'}\mathbf{j}' + \omega_z \sin 7.595°\mathbf{j}' + \omega_z \cos 7.595°\mathbf{k}'$$

$$= (0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

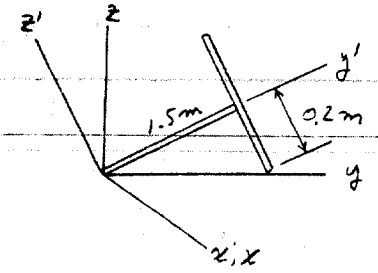
Since points A and C have zero velocity,

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$\mathbf{0} = \mathbf{0} + \left[(0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

$$0 = -1.48684\omega_z - 0.026433\omega_z + 0.2\omega_{y'}$$

$$\omega_{y'} = -7.5664\omega_z$$

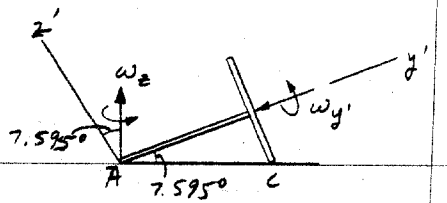


Thus,

$$\omega = -7.4342\omega_z\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

Since $\omega_z = 2$ rad/s

$$\omega = -14.868\mathbf{j}' + 1.9825\mathbf{k}'$$



So that,

$$\mathbf{H}_A = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j} + I_{z'}\omega_{z'}\mathbf{k}' = \mathbf{0} + 0.100(-14.868)\mathbf{j}' + 13.55(1.9825)\mathbf{k}'$$

$$= -1.4868\mathbf{j}' + 26.862\mathbf{k}'$$

$$H_A = \sqrt{(-1.4868)^2 + (26.862)^2} = 26.9 \text{ kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans}$$

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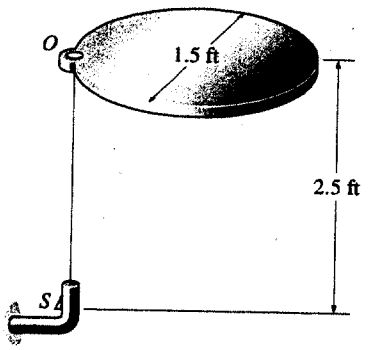
21-30. The circular plate has a weight of 19 lb and a diameter of 1.5 ft. If it is released from rest and falls horizontally 2.5 ft onto the hook at S, which provides a permanent connection, determine the velocity of the mass center of the plate just after the connection with the hook is made.



Conservation of energy :

$T_1 + V_1 = T_2 + V_2$

$0 + 19(2.5) = \frac{1}{2}\left(\frac{19}{32.2}\right)(v_G)_2^2 + 0$

$(v_G)_2 = 12.69 \text{ ft/s}$

Conservation of momentum about point O :

$(\mathbf{H}_O)_1 = \left[-\left(\frac{19}{32.2}\right)(12.69)(0.75)\right]\mathbf{i} = \{-5.6153\mathbf{i}\} \text{ slug} \cdot \text{ft}^2/\text{s}$

$I_x = \left[\frac{1}{4}\left(\frac{19}{32.2}\right)(0.75)^2 + \left(\frac{19}{32.2}\right)(0.75)^2\right] = 0.4149 \text{ slug} \cdot \text{ft}^2$

$I_y = \frac{1}{4}\left(\frac{19}{32.2}\right)(0.75)^2 = 0.08298 \text{ slug} \cdot \text{ft}^2$

$I_z = \left[\frac{1}{2}\left(\frac{19}{32.2}\right)(0.75)^2 + \left(\frac{19}{32.2}\right)(0.75)^2\right] = 0.4979 \text{ slug} \cdot \text{ft}^2$

$(\mathbf{H}_O)_3 = I_x \omega_x \mathbf{i} + I_y \omega_y \mathbf{j} + I_z \omega_z \mathbf{k}$

$= 0.4149\omega_x \mathbf{i} + 0.08298\omega_y \mathbf{j} + 0.4979\omega_z \mathbf{k}$

$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_3$

$-5.6153\mathbf{i} = 0.4149\omega_x \mathbf{i} + 0.08298\omega_y \mathbf{j} + 0.4979\omega_z \mathbf{k}$

Equating i, j and k components

$-5.6153 = 0.4149\omega_x \qquad \omega_x = -13.54 \text{ rad/s}$

$0 = 0.08298\omega_y \qquad \omega_y = 0$

$0 = 0.4979\omega_z \qquad \omega_z = 0$

Hence $\omega = \{-13.54\mathbf{i}\} \text{ rad/s}$

$\mathbf{v}_G = \omega \times \mathbf{r}_{G/O}$

$= (-13.54\mathbf{i}) \times (0.75\mathbf{j})$

$= \{-10.2\mathbf{k}\} \text{ ft/s}$ Ans

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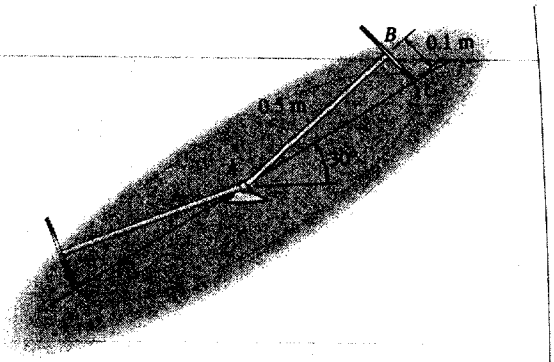
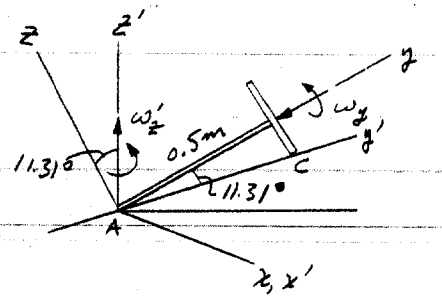
21-31. The 2-kg thin disk is connected to the slender rod which is fixed to the ball-and-socket joint at A. If it is released from rest in the position shown, determine the spin of the disk about the rod when the disk reaches its lowest position. Neglect the mass of the rod. The disk rolls without slipping.





$I_x = I_z = \frac{1}{4}(2)(0.1)^2 + 2(0.5)^2 = 0.505\ \text{kg}\cdot\text{m}^2$

$I_y = \frac{1}{2}(2)(0.1)^2 = 0.01\ \text{kg}\cdot\text{m}^2$

$\omega = \omega_y + \omega_z = -\omega_y\mathbf{j} + \omega_z \sin 11.31°\mathbf{j} + \omega_z \cos 11.31°\mathbf{k}$

$= (0.19612\omega_z - \omega_y)\mathbf{j} + (0.98058\omega_z)\mathbf{k}$

Since $\mathbf{v}_A = \mathbf{v}_C = \mathbf{0}$, then

$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$

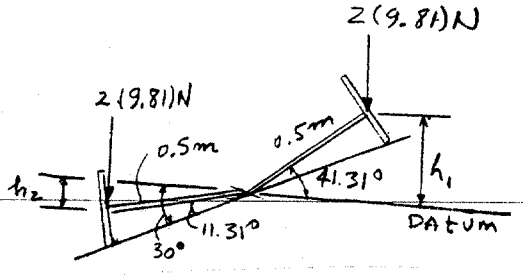
$\mathbf{0} = \mathbf{0} + \left[(0.19612\omega_z - \omega_y)\mathbf{j} + (0.98058\omega_z)\mathbf{k}\right] \times (0.5\mathbf{j} - 0.1\mathbf{k})$

$0 = -0.019612\omega_z + 0.1\omega_y - 0.49029\omega_z$

$\omega_z = 0.19612\omega_y$

Thus,

$\omega = -0.96154\omega_y\mathbf{j} + 0.19231\omega_y\mathbf{k}$



$h_1 = 0.5 \sin 41.31° = 0.3301\ \text{m}, \quad h_2 = 0.5 \sin 18.69° = 0.1602\ \text{m}$

$T_1 + V_1 = T_2 + V_2$

$0 + 2(9.81)(0.3301) = \left[0 + \frac{1}{2}(0.01)(-0.96154\omega_y)^2 + \frac{1}{2}(0.505)(0.19231\omega_y)^2\right] - 2(9.81)(0.1602$

$\omega_y = 26.2\ \text{rad/s}$ **Ans**

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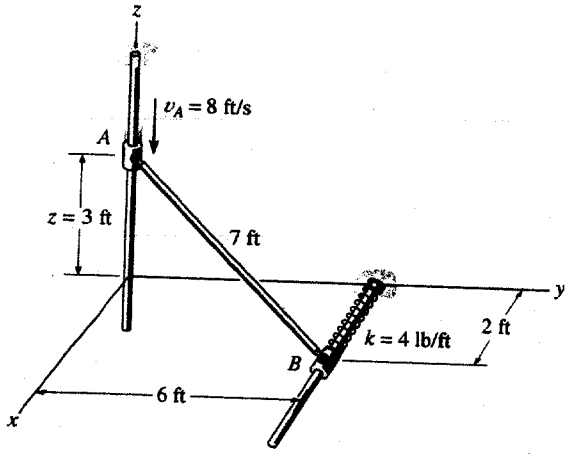
***21-32.** Rod AB has a weight of 6 lb and is attached to two smooth collars at its ends by ball-and-socket joints. If collar A is moving downward with a speed of 8 ft/s when $z = 3$ ft, determine the speed of A at the instant $z = 0$. The spring has an unstretched length of 2 ft. Neglect the mass of the collars. Assume the angular velocity of rod AB is perpendicular to its axis.



$\mathbf{v}_A = \{-8\mathbf{k}\}$ ft/s

$\mathbf{v}_B = v_B\mathbf{i}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ ft

$\boldsymbol{\omega} = \{\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\}$ rad/s

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

expanding and equating components yields,

$v_B = -3\omega_y - 6\omega_z$ (1)

$0 = 3\omega_x + 2\omega_z$ (2)

$0 = -8 + 6\omega_x - 2\omega_y$ (3)

Also, $\boldsymbol{\omega} \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$

$2\omega_x + 6\omega_y - 3\omega_z = 0$ (4)

Solving Eqs. (1) – (4) yields:

$\omega_x = 0.9796$ rad/s

$\omega_y = -1.061$ rad/s

$\omega_z = -1.469$ rad/s

Thus,

$\omega = \sqrt{(0.9796)^2 + (-1.061)^2 + (-1.469)^2} = 2.06$ rad/s

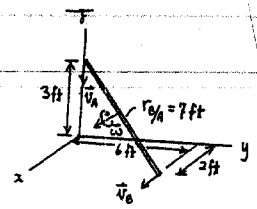
$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{G/A}$

where $\mathbf{r}_{G/A} = \frac{1}{2}\mathbf{r}_{B/A} = \frac{1}{2}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$ ft

$$\mathbf{v}_G = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9796 & -1.061 & -1.469 \\ 1 & 3 & -1.5 \end{vmatrix}$$

$\mathbf{v}_G = \{5.9985\mathbf{i} - 4\mathbf{k}\}$ ft/s

$v_G = \sqrt{(5.9985)^2 + (-4)^2} = 7.2097$ ft/s



Hence, since $\boldsymbol{\omega}$ is directed perpendicular to the axis of the rod,

$T_1 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_0^2$

$= \frac{1}{2}[\frac{1}{12}(\frac{6}{32.2})(7)^2](2.06)^2 + \frac{1}{2}(\frac{6}{32.2})(7.2097)^2$

$= 6.46$ lb·ft

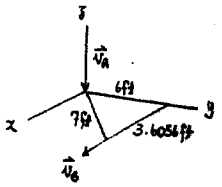
$T_1 + V_1 = T_2 + V_2$

$6.46 + 1.5(6) = T_2 + \frac{1}{2}(4)(3.6056 - 2)^2$

$T_2 = 10.304$ lb·ft

$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$

$$-v_A\mathbf{k} = v_B\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 3.6056 & 6 & 0 \end{vmatrix}$$



$0 = v_B - 6\omega_z$

$0 = \omega_z(0.36056)$

$-v_A = 6\omega_x - 3.6056\omega_y$

$\boldsymbol{\omega} \cdot \mathbf{r}_{A/B} = 0$

$3.6056\omega_x + 6\omega_y = 0$

Solving,

$\omega_z = 0$

$v_B = 0$ (location of IC)

$v_A = 13.590\omega_y$

$\omega_x = -1.664\omega_y$

$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$

$$\mathbf{v}_G = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.664 & 1 & 0 \\ -1.803 & -3 & 0 \end{vmatrix}\omega_y = \{6.795\omega_y\mathbf{k}\}\ \text{ft/s}$$

$T_2 = \frac{1}{2}(\frac{6}{32.2})(6.795\omega_y)^2 + \frac{1}{2}[\frac{1}{12}(\frac{6}{32.2})(7)^2][(-1.664\omega_y)^2 + \omega_y^2]$

$= 10.304$

$\omega_y = -1.34$ rad/s

$v_A = 13.590(-1.34) = -18.2\mathbf{j}$ ft/s Ans

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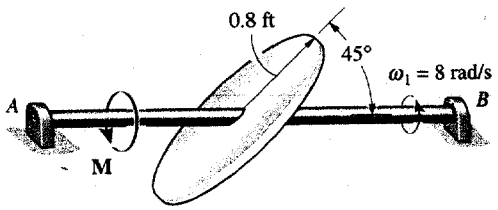
21-33. The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 3$ s if a constant torque $M = 2$ lb · ft is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



Due to symmetry

$I_{xy} = I_{yz} = I_{zx} = 0$

$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$

$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$

For x' axis

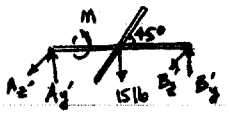
$u_x = \cos 45° = 0.7071 \quad u_y = \cos 45° = 0.7071$

$u_z = \cos 90° = 0$

$I_{x'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$

$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$

$= 0.1118 \text{ slug} \cdot \text{ft}^2$



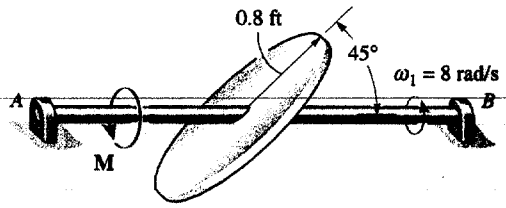
Principle of impulse and momentum

$(H_{x'})_1 + \Sigma \int M_{x'} \, dt = (H_{x'})_2$

$0.1118(8) + 2(3) = 0.1118\,\omega_2$

$\omega_2 = 61.7 \text{ rad/s}$ Ans

21-34. The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 2$ s if a torque $M = (4e^{0.1t})$ lb · ft, where t is in seconds, is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



Due to symmetry

$I_{xy} = I_{yz} = I_{zx} = 0$

$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$

$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$

For x' axis

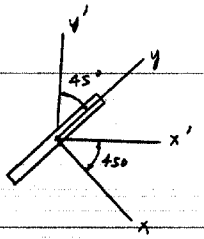
$u_x = \cos 45° = 0.7071 \quad u_y = \cos 45° = 0.7071$

$u_z = \cos 90° = 0$

$I_{x'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$

$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$

$= 0.1118 \text{ slug} \cdot \text{ft}^2$



Principle of impulse and momentum

$(H_{x'})_1 + \Sigma \int M_{x'} \, dt = (H_{x'})_2$

$0.1118(8) + \int_0^2 4e^{0.1t} dt = 0.1118\omega_2$

$\omega_2 = 87.2 \text{ rad/s}$ Ans

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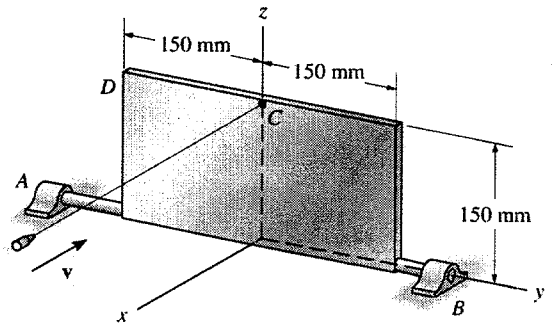
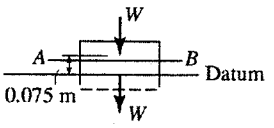
21-35. The 15-kg rectangular plate is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a 3-g bullet is fired into it, perpendicular to its surface, with a velocity $\mathbf{v} = \{-2000\mathbf{i}\}$ m/s. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity $\mathbf{v}$, instead of at C, does the angular velocity remain the same? Why or why not?



Consider the projectile and plate as an entire system.



Angular momentum is conserved about the AB axis.

$$(\mathbf{H}_{AB})_1 = -(0.003)(2000)(0.15)\mathbf{j} = \{-0.9\mathbf{j}\}$$

$$(\mathbf{H}_{AB})_1 = (\mathbf{H}_{AB})_2$$

$$-0.9\mathbf{j} = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

Equating components,

$$\omega_x = 0$$

$$\omega_z = 0$$

$$\omega_y = \frac{-0.9}{\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]} = -8 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right](8)^2 + 15(9.81)(0.15)$$

$$= \frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]\omega_{AB}^2$$

$$\omega_{AB} = 21.4 \text{ rad/s} \qquad \textbf{Ans}$$

If the projectile strikes the plate at D, the angular velocity is the same, only the impulsive reactions at the bearing supports A and B will be different.

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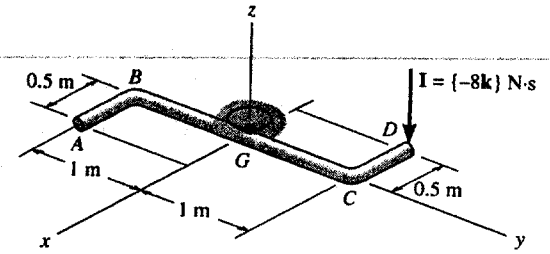
***21-36.** The rod assembly is supported at G by a ball-and-socket joint. Each segment has a mass of 0.5 kg/m. If the assembly is originally at rest and an impulse of $\mathbf{I} = \{-8\mathbf{k}\}$ N·s is applied at D, determine the angular velocity of the assembly just after the impact.



Moments and products of inertia :

$$I_{xx} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[0.5(0.5)\right](1)^2 = 0.8333\ \text{kg}\cdot\text{m}^2$$

$$I_{yy} = \frac{1}{12}\left[1(0.5)\right](1)^2 = 0.04166\ \text{kg}\cdot\text{m}^2$$

$$I_{zz} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[\frac{1}{12}\left[0.5(0.5)\right](0.5)^2 + \left[0.5(0.5)\right](1^2 + 0.25^2)\right]$$

$$= 0.875\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = \left[0.5(0.5)\right](-0.25)(1) + \left[0.5(0.5)\right](0.25)(-1) = -0.125\ \text{kg.m}^2$$

$$I_{yz} = I_{xz} = 0$$

From Eq. 21-10

$$H_x = 0.8333\omega_x + 0.125\omega_y$$

$$H_y = 0.125\omega_x + 0.04166\omega_y$$

$$H_z = 0.875\omega_z$$

$$(\mathbf{H}_G)_1 + \Sigma\int_{t_1}^{t_2}\mathbf{M}_G\,dt = (\mathbf{H}_G)_2$$

$$0 + (-0.5\mathbf{i} + 1\mathbf{j}) \times (-8\mathbf{k}) = (0.8333\omega_x + 0.125\omega_y)\mathbf{i} + (0.125\omega_x + 0.04166\omega_y)\mathbf{j} + 0.875\omega_z\mathbf{k}$$

Equating **i**, **j** and **k** components

$$-8 = 0.8333\omega_x + 0.125\omega_y \qquad (1)$$

$$-4 = 0.125\omega_x + 0.04166\omega_y \qquad (2)$$

$$0 = 0.875\omega_z \qquad (3)$$

Solving Eqs.(1) to (3) yields :

$$\omega_x = 8.73\ \text{rad/s} \qquad \omega_y = -122\ \text{rad/s} \qquad \omega_z = 0$$

Then $\omega = \{8.73\mathbf{i} - 122\mathbf{j}\}$ rad/s **Ans**

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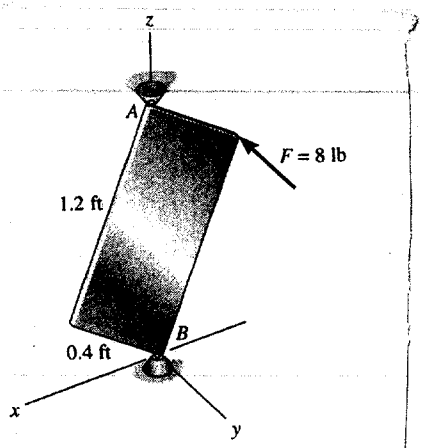
21-37. The 15-lb plate is subjected to a force $F = 8$ lb which is always directed perpendicular to the face of the plate. If the plate is originally at rest, determine its angular velocity after it has rotated one revolution (360°). The plate is supported by ball-and-socket joints at A and B.



Due to symmetry $I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$

$$I_{x'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2)^2 = 0.05590 \text{ slug} \cdot \text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2^2 + 0.4^2) = 0.06211 \text{ slug} \cdot \text{ft}^2$$

$$I_{z'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(0.4)^2 = 0.006211 \text{ slug} \cdot \text{ft}^2$$

For z axis

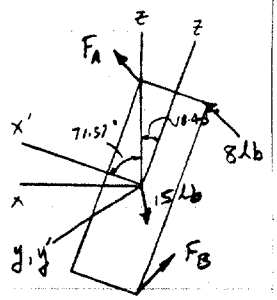
$u_{x'} = \cos 71.57° = 0.3162 \qquad u_{y'} = \cos 90° = 0$

$u_{z'} = \cos 18.43° = 0.9487$

$$I_z = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 - 2I_{x'y'}u_{x'}u_{y'} - 2I_{y'z'}u_{y'}u_{z'} - 2I_{z'x'}u_{z'}u_{x'}$$

$$= 0.05590(0.3162)^2 + 0 + 0.006211(0.9487)^2 - 0 - 0 - 0$$

$$= 0.01118 \text{ slug} \cdot \text{ft}^2$$



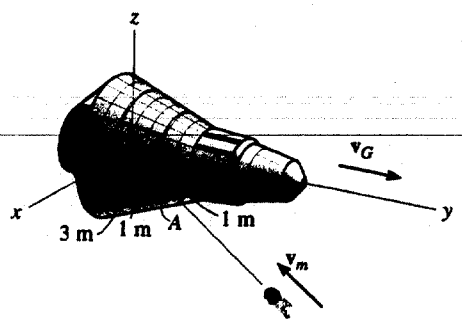
Principle of work and energy :

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 8(1.2\sin 18.43°)(2\pi) = \frac{1}{2}(0.01118)\omega^2$$

$\omega = 58.4$ rad/s **Ans**

21-38. The space capsule has a mass of 3.5 Mg and the radii of gyration are $k_x = k_z = 0.8$ m and $k_y = 0.5$ m. If it is traveling with a velocity $\mathbf{v}_G = \{600\mathbf{j}\}$ m/s, compute its angular velocity just after it is struck by a meteoroid having a mass of 0.60 kg and a velocity $\mathbf{V}_m = \{-200\mathbf{i} - 400\mathbf{j} + 200\mathbf{k}\}$ m/s. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.



Angular momentum for meteoriod and space capsule is conserved about G.

$(H_G)_1 = (H_G)_2$

$$(1\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}) \times 0.6(-200\mathbf{i} - 400\mathbf{j} + 200\mathbf{k}) = 3500(0.8)^2\omega_x\mathbf{i} + 3500(0.5)^2\omega_y\mathbf{j} + 3500(0.8)^2\omega_z\mathbf{k}$$

$$120\mathbf{i} + 120\mathbf{k} = 2240\omega_x\mathbf{i} + 875\omega_y\mathbf{j} + 2240\omega_z\mathbf{k}$$

$$\omega_x = \frac{120}{2240} = 0.0536 \text{ rad/s}$$

$$\omega_y = 0$$

$$\omega_z = \frac{120}{2240} = 0.0536 \text{ rad/s}$$

$\omega = \{0.0536\mathbf{i} + 0.0536\mathbf{k}\}$ rad/s **Ans**

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21-39. Derive the scalar form of the rotational equation of motion along the x axis when $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= \left(\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k}\right)_{xyz} + \boldsymbol{\Omega} \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\boldsymbol{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Subsitute H_x , H_y and H_z using Eq. 21-10. For the **i** component

$$\Sigma M_x = \frac{d}{dt}(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y) \quad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

***21-40.** Derive the scalar form of the rotational equation of motion along the x axis when $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= \left(\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k}\right)_{xyz} + \boldsymbol{\Omega} \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\boldsymbol{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x , H_y and H_z using Eq. 21-10. For the **i** component

$$\Sigma M_x = \frac{d}{dt}(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y)$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = (I_x\dot{\omega}_x - I_{xy}\dot{\omega}_y - I_{xz}\dot{\omega}_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y) \qquad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

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21-41. Derive the Euler equations of motion for $\Omega \neq \omega$, i.e., Eqs. 21–26.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= (\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k})_{xyz} + \Omega \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\Omega = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left((\dot{H}_x)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left((\dot{H}_y)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left((\dot{H}_z)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21-10. For the **i** component

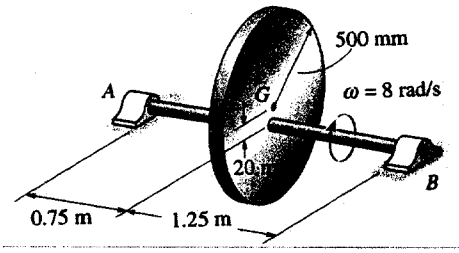
$$\Sigma M_x = \frac{d}{dt}(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y)$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

$$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z \qquad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

21-42. The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the maximum reactions exerted on the journal bearings at A and B.



$\omega_x = 0$

$\omega_y = -8$ rad/s

$\omega_z = 0$

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$;

$B_z(1.25) - A_z(0.75) = 0 - 0$

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$;

$-B_x(1.25) + A_x(0.75) = 0 - 0$

$\Sigma F_x = ma_x$; $A_x + B_x = 0$

$\Sigma F_z = ma_z$; $A_z + B_z - 40(9.81) = 40(8)^2(0.020)$

Solving,

$A_x = 0$

$B_x = 0$

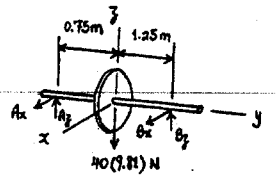
$A_z = 277$ N

$B_z = 166$ N

Thus,

$F_A = 277$ N **Ans**

$F_B = 166$ N **Ans**



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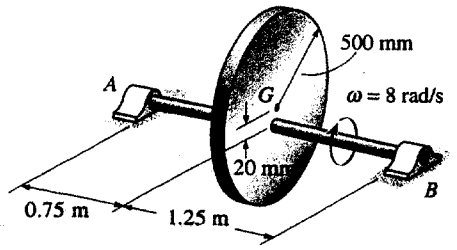
21-43. The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the minimum reactions exerted on the journal bearings at A and B during the motion.



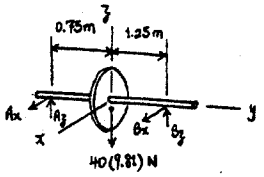
$\omega_x = 0$

$\omega_y = -8$ rad/s

$\omega_z = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_z(1.25) - A_z(0.75) = 0 - 0$



$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y;$

$-B_x(1.25) + A_x(0.75) = 0 - 0$

$\Sigma F_x = ma_x; \quad A_x + B_x = 0$

$\Sigma F_z = ma_z; \quad A_z + B_z - 40(9.81) = -40(8)^2(0.020)$

Solving,

$A_x = 0$

$B_x = 0$

$A_z = 213.25$ N

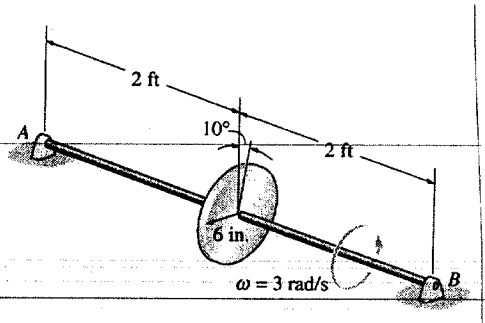
$B_z = 127.95$ N

Thus,

$F_A = 213$ N **Ans**

$F_B = 128$ N **Ans**

***21-44.** The 20-lb disk is mounted on the horizontal shaft AB such that its plane forms an angle of 10° with the vertical. If the shaft rotates with an angular velocity of 3 rad/s, determine the vertical reactions developed at the bearings when the disk is in the position shown.



$I_x = \frac{1}{2}\left(\frac{20}{32.2}\right)(0.5)^2 = 0.07764$ slug · ft^2

$I_y = I_z = \frac{1}{4}\left(\frac{20}{32.2}\right)(0.5)^2 = 0.03882$ slug · ft^2

$\omega = 3\cos 10°\mathbf{i} + 3\sin 10°\mathbf{j} = \{2.9544\mathbf{i} + 0.5209\mathbf{j}\}$ rad/s

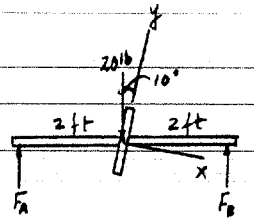
Applying the third of Eqs. 21-25 with $\omega_x = 2.9544$ rad/s $\omega_y = 0.5209$ rad/s $\dot{\omega}_z = 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y;$

$F_B(2) - F_A(2) = 0 - (0.07764 - 0.03882)(2.9544)(0.5209)$ (1)

Also,

$\Sigma F_z = m(a_G)_z; \quad F_A + F_B - 20 = 0$ (2)



Solving,

$F_A = 10.0$ lb **Ans**

$F_B = 9.99$ lb **Ans**

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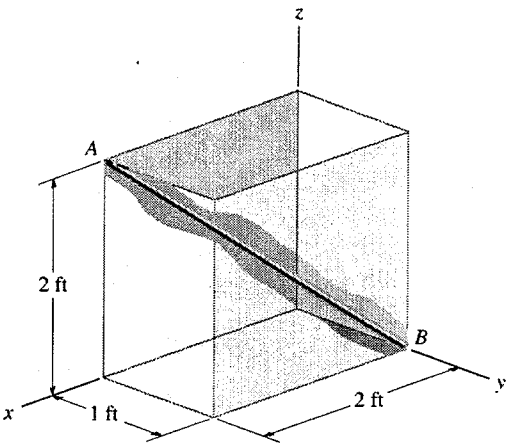
21-45. The 4-lb bar rests along the smooth corners of an open box. At the instant shown, the box has a velocity $\mathbf{v} = \{3\mathbf{j}\}$ ft/s and an acceleration $\mathbf{a} = \{-6\mathbf{j}\}$ ft/s^2. Determine the x, y, z components of force which the corners exert on the bar.



$\sum F_x = m(a_G)_x; \quad A_x + B_x = 0 \qquad [1]$

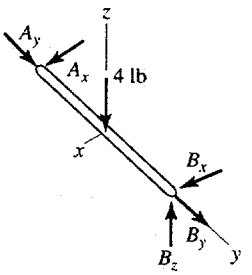
$\sum F_y = m(a_G)_y; \quad A_y + B_y = \left(\frac{4}{32.2}\right)(-6) \qquad [2]$

$\sum F_z = m(a_G)_z; \quad B_z - 4 = 0 \quad B_z = 4 \text{ lb}$ **Ans**

Applying Eq. 21-25 with $\omega_x = \omega_y = \omega_z = 0 \; \dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$\sum (M_G)_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \quad B_y(1) - A_y(1) + 4(0.5) = 0 \qquad [3]$

$\sum (M_G)_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \quad A_x(1) - B_x(1) + 4(1) = 0 \qquad [4]$



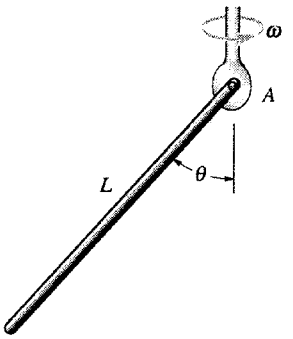
Solving Eqs. [1] to [4] yields:

$A_x = -2.00 \text{ lb} \quad A_y = 0.627 \text{ lb} \quad B_x = 2.00 \text{ lb} \quad B_y = -1.37 \text{ lb}$ **Ans**

$\sum (M_G)_z = I_z\dot{\omega}_z - (I_x - I_y)\,\omega_x\omega_y;$

$(-2.00)(0.5) - (2.00)(0.5) - (-1.37)(1) + (0.627)(1) = 0$ **(O.K!)**

21-46. The conical pendulum consists of a bar of mass m and length L that is supported by the pin at its end A. If the pin is subjected to a rotation ω, determine the angle θ that the bar makes with the vertical as it rotates. Also, determine the components of reaction at the pin.



$I_x = I_z = \frac{1}{3}mL^2, \quad I_y = 0$

$\omega_x = 0, \quad \omega_y = -\omega\cos\theta, \quad \omega_z = \omega\sin\theta$

$\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = 0$

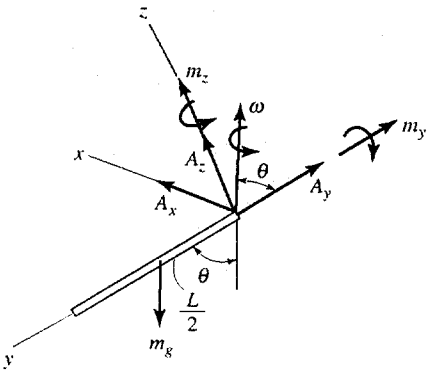
$\sum M_x = I_x\omega_x - (I_y - I_z)\omega_y\omega_z$

$-mg\left(\frac{L}{2}\sin\theta\right) = 0 - \left(0 - \frac{1}{3}mL^2\right)(-\omega\cos\theta)(\omega\sin\theta)$

$\frac{g}{2} = \frac{1}{3}L\omega^2\cos\theta$

$\cos\theta = \frac{3g}{2L\omega^2}$

$\theta = \cos^{-1}\left(\frac{3g}{2L\omega^2}\right)$ **Ans**



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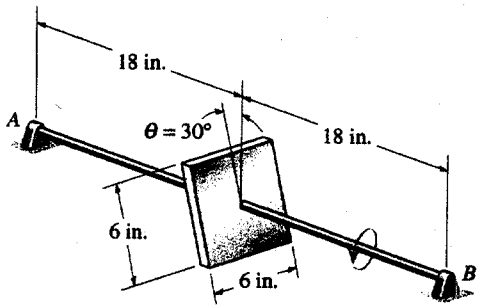
21-47. The 20-lb plate is mounted on the shaft AB so that the plane of the plate makes an angle $\theta = 30°$ with the vertical. If the shaft is turning in the direction shown with an angular velocity of 25 rad/s, determine the vertical reactions at the bearing supports A and B when the plate is in the position shown.

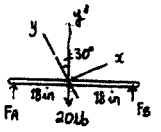


$\omega = 25\cos 30°\mathbf{i} - 25\sin 30°\mathbf{j}$

$\dot{\omega} = \dot{\omega}\cos 30°\mathbf{i} - \dot{\omega}\sin 30°\mathbf{j}$

$I_x = \frac{20}{32.2}\left(\frac{1}{12}\right)\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] = 0.02588 \text{ slug}\cdot\text{ft}^2$

$I_y = \frac{20}{32.2}\left(\frac{1}{12}\right)\left(\frac{1}{2}\right)^2 = 0.01294 \text{ slug}\cdot\text{ft}^2$



Using the third Eq. 21–25,

$\Sigma M_z = F_B\left(\frac{18}{12}\right) - F_A\left(\frac{18}{12}\right)$

$= 0 - (0.02588 - 0.01294)(25\cos 30°)(-25\sin 30°)$

Thus,

$F_B - F_A = 2.3346$

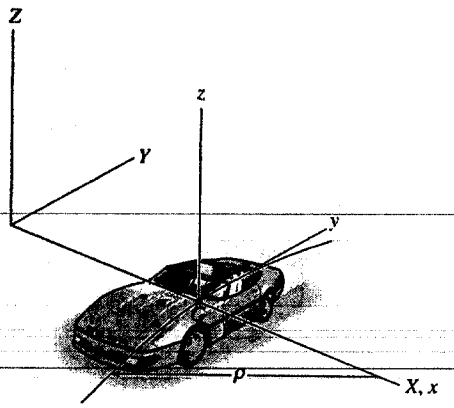
Also,

$\Sigma F_y = m(a_G)_y; \quad F_A + F_B - 20 = 0$

$F_A = 8.83$ lb **Ans**

$F_B = 11.2$ lb **Ans**

***21-48.** The car is traveling around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.



Applying Eq. 21–24 with $\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = \frac{v_G}{\rho}$,

$\omega_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$\Sigma M_x = -I_{yz}\left[0 - \left(\frac{v_G}{\rho}\right)^2\right] = \frac{I_{yz}}{\rho^2}v_G^2$ **Ans**

$\Sigma M_y = -I_{zx}\left[\left(\frac{v_G}{\rho}\right)^2 - 0\right] = -\frac{I_{zx}}{\rho^2}v_G^2$ **Ans**

$\Sigma M_z = 0$ **Ans**

Note: This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia I_{yz} and I_{zx}. (See Example 13–6)

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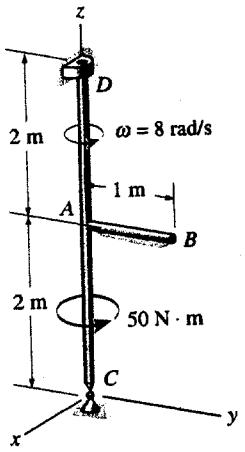
21-49. The rod assembly is supported by a ball-and-socket joint at C and a journal bearing at D, which develops only x and y force reactions. The rods have a mass of 0.75 kg/m. Determine the angular acceleration of the rods and the components of reaction at the supports at the instant $\omega = 8$ rad/s as shown.



$\Omega = \omega = 8\mathbf{k}$

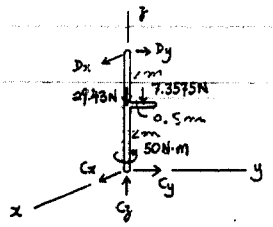
$\omega_x = \omega_y = 0, \quad \omega_z = 8$ rad/s

$\dot{\omega}_x = \dot{\omega}_y = 0, \quad \dot{\omega}_z = \dot{\omega}_z$

$I_{xz} = I_{zy} = 0$

$I_y = 0.75(1)(2)(0.5) = 0.75$ kg·m^2

$I_{zz} = \frac{1}{3}(0.75)(1)(1)^2 = 0.25$ kg·m^2



Eqs. 21–24 become

$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$

Thus,

$-D_y(4) - 7.3575(0.5) = 0.75(8)^2$

$D_y = -12.9$ N Ans

$D_x(4) = -0.75\omega_z$

$50 = 0.25\dot{\omega}_z$

$\dot{\omega}_z = 200$ rad/s^2 Ans

$D_x = -37.5$ N Ans

$\Sigma F_x = m(a_G)_x; \quad C_x - 37.5 = -1(0.75)(200)(0.5)$

$C_x = -37.5$ N Ans

$\Sigma F_y = m(a_G)_y; \quad C_y - 12.9 = -(1)(0.75)(8)^2(0.5)$

$C_y = -11.1$ N Ans

$\Sigma F_z = m(a_G)_z; \quad C_z - 7.3575 - 29.43 = 0$

$C_z = 36.8$ N Ans

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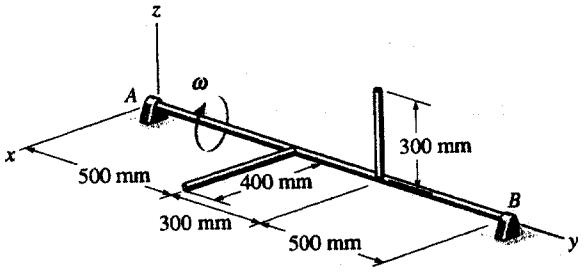
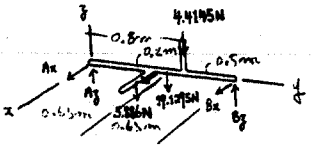
21-50. The rod assembly is supported by journal bearings at A and B, which develops only x and y force reactions on the shaft. If the shaft AB is rotating in the direction shown at $\boldsymbol{\omega} = \{-5\mathbf{j}\}$ rad/s, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass of each rod is 1.5 kg/m.



$\omega_x = \omega_z = 0, \quad \omega_y = -5$ rad/s

$\dot{\omega}_x = \dot{\omega}_z = 0$



Eqs. 21–24 become

$$\Sigma M_x = -I_{xy}\dot{\omega}_y - I_{yz}\omega_y^2$$

$$\Sigma M_y = I_y\dot{\omega}_y$$

$$\Sigma M_z = I_{xy}\omega_y^2 - I_{yz}\dot{\omega}_y$$

$$I_y = \frac{1}{3}(0.4)(1.5)(0.4)^2 + \frac{1}{3}(0.3)(1.5)(0.3)^2 = 0.0455 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = [0 + (1.5)(0.3)(0.15)(0.8)] = 0.0540 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = [0 + (1.5)(0.4)(0.2)(0.5)] = 0.0600 \text{ kg}\cdot\text{m}^2$$

Thus,

$$-5.886(0.5) - 19.1295(0.65) - 4.4145(0.8) + B_z(1.3) = -0.0600\dot{\omega}_y - 0.0540(-5)^2$$

$$5.886(0.2) = 0.0455\dot{\omega}_y$$

$$-B_x(1.3) = 0.0600(-5)^2 - (0.0540)\dot{\omega}_y$$

$\dot{\omega}_y = 25.9$ rad/s^2 **Ans**

$B_x = -0.0791$ N **Ans**

$B_z = 12.3$ N **Ans**

$$\Sigma F_x = m(a_G)_x; \quad A_x - 0.0791 = -0.4(1.5)(5)^2(0.2) + 0.3(1.5)(25.9)(0.15)$$

$A_x = -1.17$ N **Ans**

$$\Sigma F_z = m(a_G)_z; \quad A_z + 12.31 - 5.886 - 19.1295 - 4.4145 = -0.4(1.5)(25.9)(0.2) - 0.3(1.5)(5)^2(0.15)$$

$A_z = 12.3$ N **Ans**

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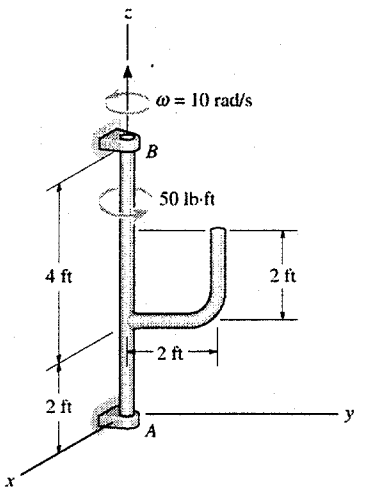
21-51. The rod assembly has a weight of 5 lb/ft. It is supported at B by a smooth journal bearing, which develops x and y force reactions, and at A by a smooth thrust bearing, which develops x, y, and z force reactions. If a 50-lb · ft torque is applied along rod AB, determine the components of reaction at the bearings when the assembly has an angular velocity $\omega = 10$ rad/s at the instant shown.



$$I_y = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](3)^2 + \left[\frac{2(5)}{32.2}\right](2)^2$$

$$= 15.3209 \text{ slug} \cdot \text{ft}^2$$

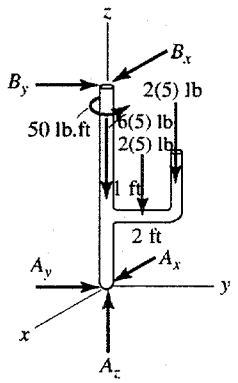
$$I_x = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2^2 + 3^2)$$

$$+ \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](1^2 + 2^2)$$

$$I_x = 16.9772 \text{ slug} \cdot \text{ft}^2$$

$$I_z = \frac{1}{3}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2)^2 = 1.6563 \text{ slug} \cdot \text{ft}^2$$

$$I_{yz} = \left[\frac{2(5)}{32.2}\right](1)(2) + \left[\frac{2(5)}{32.2}\right](2)(3) = 2.4845 \text{ slug} \cdot \text{ft}^2 \quad I_{xy} = I_{zx} = 0$$



Applying Eq. 21-24 with $\omega_x = \omega_y = 0$ $\omega_z = 10$ rad/s $\dot{\omega}_x = \dot{\omega}_y = 0$

$$-B_y(6) = 0 - 0 - 0 - 2.4845(0 - 10^2) - 0 \quad B_y = -41.4 \text{ lb} \quad \textbf{Ans}$$

$$B_x(6) = 0 - 0 - 2.4845\dot{\omega}_z - 0 - 0 \qquad (1)$$

$$50 = 1.6563\dot{\omega}_z \qquad (2)$$

Solving Eqs. (1) and (2) yields:

$$\dot{\omega}_z = 30.19 \text{ rad/s}^2$$

$$B_x = -12.5 \text{ lb} \quad \textbf{Ans}$$

$$\sum F_x = m(a_G)_x; \quad A_x + (-12.50) = -\left[\frac{2(5)}{32.2}\right](1)(30.19) - \left[\frac{2(5)}{32.2}\right](2)(30.19)$$

$$A_x = -15.6 \text{ lb} \quad \textbf{Ans}$$

$$\sum F_y = m(a_G)_y; \quad A_y + (-41.4) = -\left[\frac{2(5)}{32.2}\right](1)(10)^2 - \left[\frac{2(5)}{32.2}\right](2)(10)^2$$

$$A_y = -51.8 \text{ lb} \quad \textbf{Ans}$$

$$\sum F_z = m(a_G)_z; \quad A_z - 2(5) - 2(5) - 6(5) = 0 \quad A_z = 50 \text{ lb} \quad \textbf{Ans}$$

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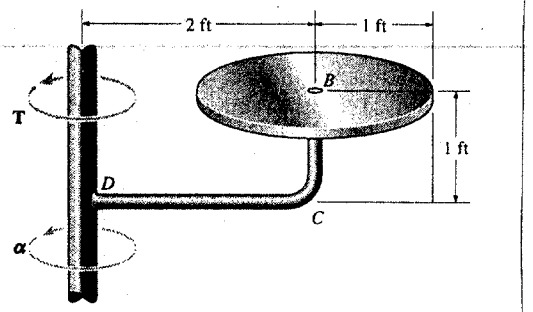
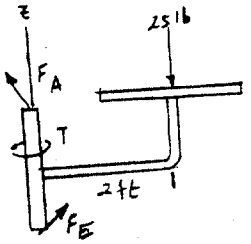
***21-52.** The 25-lb disk is *fixed* to rod *BCD*, which has negligible mass. Determine the torque T which must be applied to the vertical shaft so that the shaft has an angular acceleration of $\alpha = 6$ rad/s^2. The shaft is free to turn in its bearings.



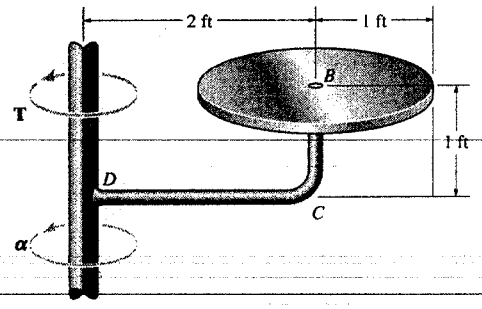


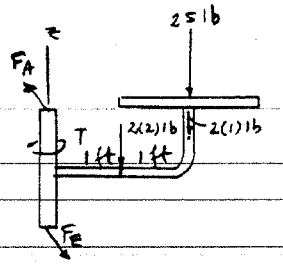
$$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 = 3.4938 \text{ slug} \cdot \text{ft}^2$$

Applying the third of Eq. 21-25 with $I_x = I_y$, $\omega_x = \omega_y = 0$, $\dot{\omega}_z = 6$ rad/s^2

$$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y; \quad T = 3.4938(6) = 21.0 \text{ lb} \cdot \text{ft} \quad \textbf{Ans}$$

21-53. Solve Prob. 21-52, assuming rod *BCD* has a weight of 2 lb/ft.





$$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 + \frac{1}{3}\left(\frac{2(2)}{32.2}\right)(2)^2 + \left(\frac{1(2)}{32.2}\right)(2)^2 = 3.9079 \text{ slug} \cdot \text{ft}^2$$

Applying the third of Eq. 21-25 with $I_x = I_y$, $\omega_x = \omega_y = 0$, $\dot{\omega}_z = 6$ rad/s^2

$$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y; \quad T = 3.9079(6) = 23.4 \text{ lb} \cdot \text{ft} \quad \textbf{Ans}$$

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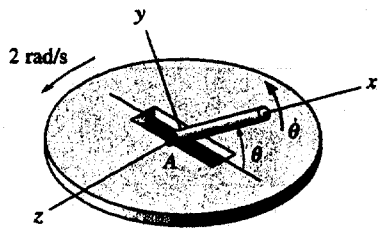
21-54. The *thin rod* has a mass of 0.8 kg and a total length of 150 mm. It is rotating about its midpoint at a constant rate $\dot{\theta} = 6$ rad/s, while the table to which its axle A is fastened is rotating at 2 rad/s. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in any position θ.



The x,y,z *axes* are fixed as shown.

$\omega_x = 2\sin\theta$

$\omega_y = 2\cos\theta$

$\omega_z = \dot{\theta} = 6$

$\dot{\omega}_x = 2\dot{\theta}\cos\theta = 12\cos\theta$

$\dot{\omega}_y = -2\dot{\theta}\sin\theta = -12\sin\theta$

$\dot{\omega}_z = 0$

$I_x = 0$

$I_y = I_z = \frac{1}{12}(0.8)(0.15)^2 = 1.5(10^{-3})$

Using Eqs. 21–25:

$\Sigma M_x = 0 - 0 = 0$ Ans

$\Sigma M_y = 1.5(10^{-3})(-12\sin\theta) - [1.5(10^{-3}) - 0](6)(2\sin\theta)$

$\Sigma M_y = (-0.036\sin\theta)\ \text{N}\cdot\text{m}$ Ans

$\Sigma M_z = 0 - [0 - 1.5(10^{-3})](2\sin\theta)(2\cos\theta)$

$\Sigma M_z = 0.006\sin\theta\cos\theta = (0.003\sin 2\theta)\ \text{N}\cdot\text{m}$ Ans

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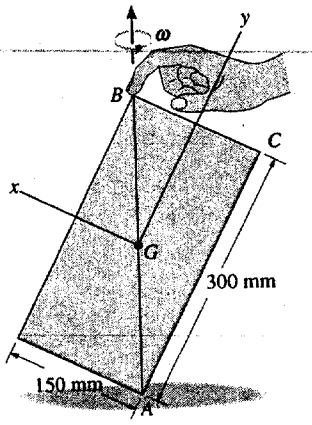
21-55. A thin uniform plate having a mass of 0.4 kg is spinning with a constant angular velocity ω about its diagonal AB. If the person holding the corner of the plate at B releases his finger, the plate will fall downward on its side AC. Determine the necessary couple moment M which if applied to the plate would prevent this from happening.



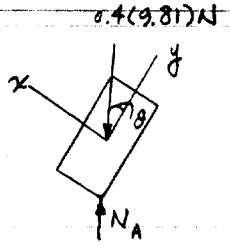
Using the principal axis shown,

$$I_x = \frac{1}{12}(0.4)(0.3)^2 = 3\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{12}(0.4)(0.15)^2 = 0.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{12}(0.4)\left[(0.3)^2 + (0.15)^2\right] = 3.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$\theta = \tan^{-1}\left(\frac{75}{150}\right) = 26.57°$$



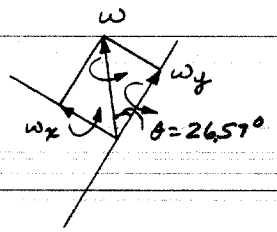
$\omega_x = \omega\sin 26.57°, \quad \dot{\omega}_x = 0$

$\omega_y = \omega\cos 26.57°, \quad \dot{\omega}_y = 0$

$\omega_z = 0, \quad \dot{\omega}_z = 0$

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$

$M_x = 0$



$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$

$M_y = 0$

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$

$M_z = 0 - \left[3\left(10^{-3}\right) - 0.75\left(10^{-3}\right)\right]\omega^2\sin 26.57°\cos 26.57°$

$M_z = -0.9\left(10^{-3}\right)\omega^2\ \text{N}\cdot\text{m} = -0.9\omega^2\ \text{mN}\cdot\text{m}$ **Ans**

The couple acts outward, perpendicular to the face of the plate.

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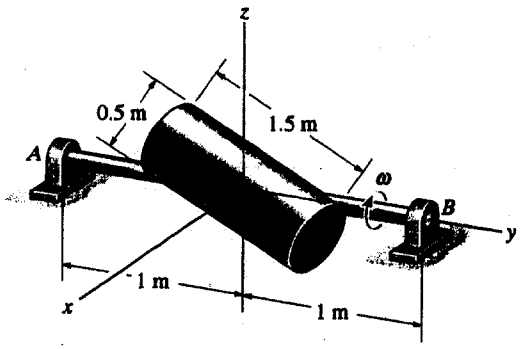
***21-56.** The cylinder has a mass of 30 kg and is mounted on an axle that is supported by bearings at A and B. If the axle is turning at $\boldsymbol{\omega} = \{-40\mathbf{j}\}$ rad/s, determine the vertical components of force acting at the bearings at this instant.



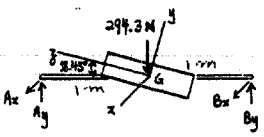
$\omega_x = 0$

$\omega_y = -40\sin 18.43° = -12.65$ rad/s

$\omega_z = 40\cos 18.43° = 37.95$ rad/s

$\dot{\omega}_x = 0, \quad \dot{\omega}_y = 0, \quad \dot{\omega}_z = 0$

$(a_G)_X = (a_G)_Y = (a_G)_Z = 0$



$I_z = I_y = \frac{1}{12}(30)[3(0.25)^2 + (1.5)^2] = 6.09375\ \text{kg}\cdot\text{m}^2$

$I_x = \frac{1}{2}(30)(0.25)^2 = 0.9375\ \text{kg}\cdot\text{m}^2$

Using the first of Eqs. 21–25,

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$

$B_Y(1) - A_Y(1) = 0 - (6.09375 - 0.9375)(-12.65)(37.95)$

$B_Y - A_Y = 2475.348$

$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$

$A_X(1\cos 18.43°) - B_X(1\cos 18.43°) = 0 - 0$

$A_X = B_X$

$\Sigma F_X = m(a_G)_X; \quad A_X + B_X = 0$

$\Sigma F_Y = m(a_G)_Y; \quad A_Y + B_Y = 0$

$A_X = -B_X$

Also, summing forces in the vertical direction

$\Sigma F_Y = m(a_G)_Y; \quad A_Y + B_Y - 294.3 = 0$

Solving,

$A_X = B_X = 0$ Ans

$A_Y = -1.09$ kN Ans

$B_Y = 1.38$ kN Ans

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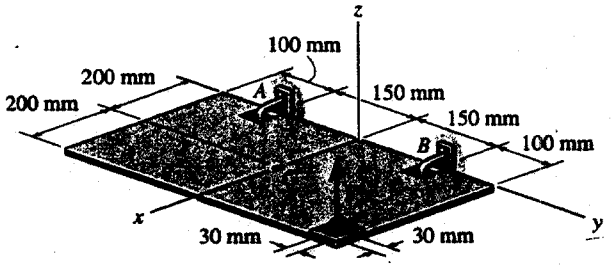
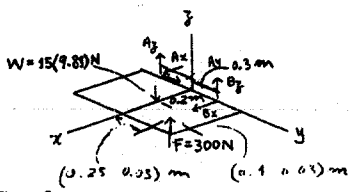
21-57. The uniform hatch door, having a mass of 15 kg and a mass center at G, is supported in the horizontal plane by bearings at A and B. If a vertical force $F = 300$ N is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.



$\omega_x = \omega_y = \omega_z = 0$

$\dot{\omega}_x = \dot{\omega}_z = 0$



Eqs. 21–25 reduce to

$\Sigma M_x = 0; \quad 300(0.25-0.03)+B_z(0.15)-A_z(0.15) = 0$

$B_z - A_z = -440 \qquad (1)$

$\Sigma M_y = I_y \dot{\omega}_y; \quad 15(9.81)(0.2) - (300)(0.4-0.03) = [\frac{1}{12}(15)(0.4)^2 + 15(0.2)^2]\dot{\omega}_y$

$\dot{\omega}_y = -102 \text{ rad/s}^2$ **Ans**

$\Sigma M_z = 0; \quad -B_x(0.15) + A_x(0.15) = 0$

$\Sigma F_x = m(a_G)_x; \quad -A_x + B_x = 0$

$A_x = B_x = 0$ **Ans**

$\Sigma F_y = m(a_G)_y; \quad A_y = 0$ **Ans**

$\Sigma F_z = m(a_G)_z; \quad 300-15(9.81) + B_z + A_z = 15(101.96)(0.2)$

$B_z + A_z = 153.03 \qquad (2)$

Solving Eqs. (1) and (2) yields

$A_z = 297$ N **Ans**

$B_z = -143$ N **Ans**

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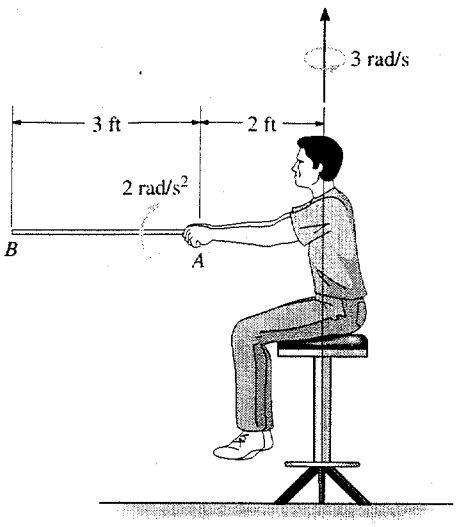
21-58. The man sits on a swivel chair which is rotating with a constant angular velocity of 3 rad/s. He holds the uniform 5-lb rod AB horizontal. He suddenly gives it an angular acceleration of 2 rad/s^2, measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



$$I_x = I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(3)^2 = 0.1165 \text{ ft}^4$$

$$I_y = 0$$

$$\Omega = \omega = 3\mathbf{k}$$

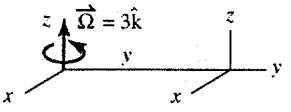
$$\omega_x = \omega_y = 0$$

$$\omega_z = 3 \text{ rad/s}$$

$$\dot{\Omega} = (\dot{\omega}_{xyz}) + \Omega \times \omega = -2\mathbf{i} + 0$$

$$\dot{\omega}_x = -2 \text{ rad/s}^2$$

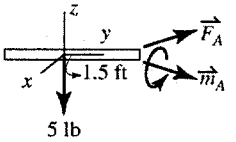
$$\dot{\omega}_y = \dot{\omega}_z = 0$$



$$(A_G)_x = 0$$

$$(A_G)_y = (3.5)(3)^2 = 31.5 \text{ ft/s}^2$$

$$(A_G)_z = 2(1.5) = 3 \text{ ft/s}^2$$



$$\Sigma F_x = m(a_G)_x; \quad A_x = 0 \qquad \textbf{Ans}$$

$$\Sigma F_y = m(a_G)_y; \quad A_y = \frac{5}{32.2}(31.5) = 4.89 \text{ lb} \qquad \textbf{Ans}$$

$$\Sigma F_z = m(a_G)_z; \quad -5 + A_z = \frac{5}{32.2}(3)$$

$$A_z = 5.47 \text{ lb} \qquad \textbf{Ans}$$

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z;$$

$$M_x + 5.47(1.5) = 0.1165(-2) - 0$$

$$M_x = -8.43 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$\Sigma M_y = I_y\dot{\omega}_x - (I_z - I_x)\omega_z\omega_x;$$

$$0 + M_y = 0 - 0$$

$$M_y = 0 \qquad \textbf{Ans}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;$$

$$M_y = 0 - 0$$

$$M_z = 0 \qquad \textbf{Ans}$$

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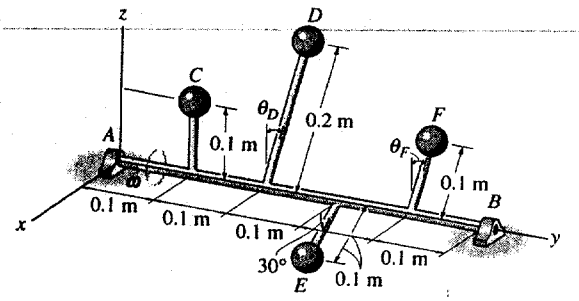
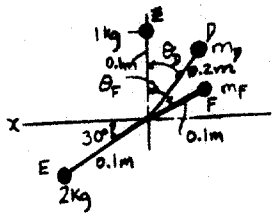
21-59. Four spheres are connected to shaft AB. If $m_C = 1$ kg and $m_E = 2$ kg, determine the mass of D and F and the angles of the rods, θ_D and θ_F, so that the shaft is dynamically balanced, that is, so that the bearings at A and B exert only vertical reactions on the shaft as it rotates. Neglect the mass of the rods.





For $\bar{x} = 0$; $\Sigma \tilde{x}_i m_i = 0$

$(0.1\cos 30°)(2) - (0.1\sin\theta_F)m_F - (0.2\sin\theta_D)m_D = 0$ (1)

For $\bar{z} = 0$; $\Sigma \tilde{z}_i m_i = 0$

$(0.1)(1) - (0.1\sin 30°)(2) + (0.2\cos\theta_D)m_D + (0.1\cos\theta_F)m_F = 0$ (2)

For $I_{xy} = 0$; $\Sigma \tilde{x}_i \tilde{y}_i m_i = 0$

$-(0.2)(0.2\sin\theta_D)m_D + (0.3)(0.1\cos 30°)(2) - (0.4)(0.1\sin\theta_F)m_F = 0$ (3)

For $I_{zy} = 0$; $\Sigma \tilde{z}_i \tilde{y}_i m_i = 0$

$(0.1)(0.1)(1) + (0.2)(0.2\cos\theta_D)m_D - (0.3)(0.1\sin 30°)(2) + (0.1\cos\theta_F)(0.4)(m_F) = 0$ (4)

Solving,

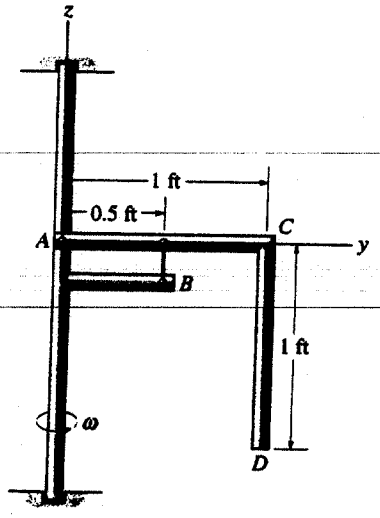
$\theta_D = 139°$ **Ans**

$m_D = 0.661$ kg **Ans**

$\theta_F = 40.9°$ **Ans**

$m_F = 1.32$ kg **Ans**

***21-60.** The bent uniform rod ACD has a weight of 5 lb/ft and is supported at A by a pin and at B by a cord. If the vertical shaft rotates with a constant angular velocity $\omega = 20$ rad/s, determine the x, y, z components of force and moment developed at A and the tension in the cord.



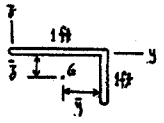
$\omega_x = \omega_y = 0$

$\omega_z = 20$ rad/s

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

The center of mass is located at

$\bar{z} = \frac{5(1)(\frac{1}{2})}{5(2)} = 0.25$ ft



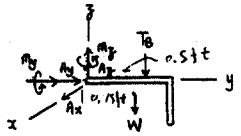
$\bar{y} = 0.25$ ft (symmetry)

$I_{yz} = \frac{5}{32.2}(-0.5)(1) = -0.0776$ slug·ft^2

$I_{zx} = 0$

Eqs. 21–24 reduce to

$\Sigma M_x = I_{yz}(\omega_z)^2$;



$-T_B(0.5) - 10(0.75) = -0.0776(20)^2$

$T_B = 47.1$ lb **Ans**

$\Sigma M_y = 0$; $M_y = 0$ **Ans**

$\Sigma M_z = 0$; $M_z = 0$ **Ans**

$\Sigma F_x = ma_x$; $A_x = 0$ **Ans**

$\Sigma F_y = ma_y$; $A_y = -(\frac{10}{32.2})(20)^2(1 - 0.25)$

$A_y = -93.2$ lb **Ans**

$\Sigma F_z = ma_z$; $A_z - 47.1 - 10 = 0$

$A_z = 57.1$ lb **Ans**

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21-61. Show that the angular velocity of a body, in terms of Euler angles ϕ, θ, and ψ, may be expressed as $\omega = (\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j} + (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k}$, where **i**, **j**, and **k** are directed along the x, y, z axes as shown in Fig. 21-15*d*.

From Fig. 21-15*b*, due to rotation ϕ, the x, y, z components of $\dot{\phi}$ are simply $\dot{\phi}$ along z axis.

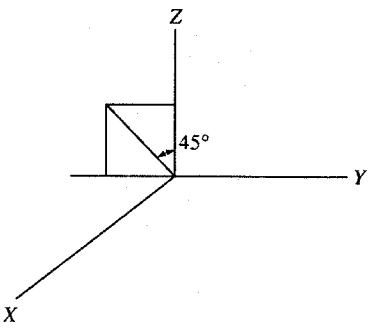
From Fig. 21-15*c*, due to rotation θ, the x, y, z components of $\dot{\phi}$ and $\dot{\theta}$ are $\dot{\phi}\sin\theta$ in the y direction, $\dot{\phi}\cos\theta$ in the z direction, and $\dot{\theta}$ in the x direction.

Lastly, rotation ψ, Fig. 21-15*d*, produces the final components which yields

$$\omega = (\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j} + (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k} \qquad \textbf{Q.E.D.}$$

21-62. A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles $\phi = 30°, \theta = 45°$, and $\psi = 60°$. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the X, Y, and Z axes. Are these directions the same for any order of the rotations? Why?

The rotations $\phi = 30°$ and $\psi = 60°$ do not change the orientation of the rod since it causes the rod to rotate about its axis only.



$$\mathbf{u} = \cos 90°\mathbf{i} + \cos 135°\mathbf{j} + \cos 45°\mathbf{k}$$

$$= -0.707\mathbf{j} + 0.707\mathbf{k}$$

Coordinate direction angles:

$\alpha = \cos^{-1}(0) = 90°$ **Ans**

$\beta = \cos^{-1}(-0.707) = 135°$ **Ans**

$\gamma = \cos^{-1}(0.707) = 45°$ **Ans**

The orientation of the rod will not be the same for any order of rotation because finite rotations are not vectors.

No **Ans**

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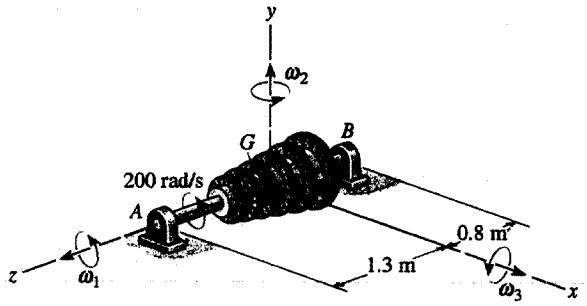
21-63. The turbine on a ship has a mass of 400 kg and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is $k_z = 0.3$ m, and $k_x = k_y = 0.5$ m. If it is spinning at 200 rad/s, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling, $\omega_1 = 0.2$ rad/s, (b) turning, $\omega_2 = 0.8$ rad/s, (c) pitching, $\omega_3 = 1.4$ rad/s.

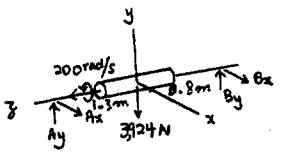


a) $\omega_1 = 0.2 + 200 = 200.2$ rad/s

$\Sigma F_y = m(a_G)_y$; $A_y + B_y - 3924 = 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_y - I_x)\omega_y \omega_x$;

$B_y(0.8) - A_y(1.3) = 0 - 0$



Thus,

$A_y = 1.49$ kN Ans

$B_y = 2.43$ kN Ans

b) $\Omega = 0.8\mathbf{j}$

$\omega = 0.8\mathbf{j} + 200\mathbf{k}$

$\dot{\omega} = 0 + 0.8\mathbf{j} \times (0.8\mathbf{j} + 200\mathbf{k}) = 160\mathbf{i}$

$\Sigma F_y = m(a_G)_y$; $A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z$;

$B_y(0.8) - A_y(1.3) = 400(0.5)^2(160) - [400(0.5)^2 - 400(0.3)^2](0.8)(200)$

$B_y(0.8) - A_y(1.3) = 5760$

Thus,

$A_y = -1.24$ kN Ans

$B_y = 5.17$ kN Ans

c) $\Omega = 1.4\mathbf{i}$

$\omega = 1.4\mathbf{i} + 200\mathbf{k}$

$\dot{\omega} = 1.4\mathbf{i} \times (1.4\mathbf{i} + 200\mathbf{k}) = -280\mathbf{j}$

$\Sigma F_y = m(a_G)_y$; $A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z$;

$B_y(0.8) - A_y(1.3) = 0 - 0$

Thus,

$A_y = 1.49$ kN Ans

$B_y = 2.43$ kN Ans

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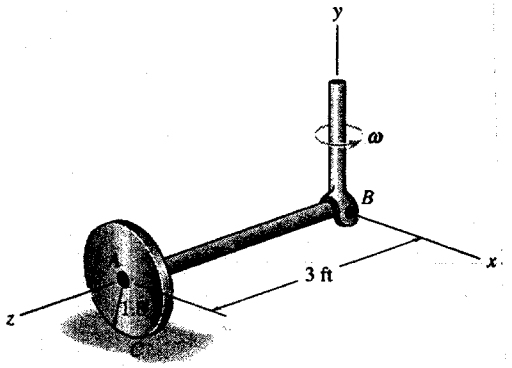
***21-64.** The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, and the vertical drive shaft is turning at 8 rad/s, determine the normal reaction the wheel exerts on the ground at C.

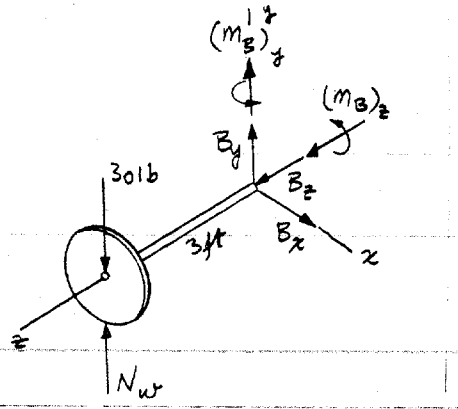


$\Omega_z = \omega = 8$ rad/s

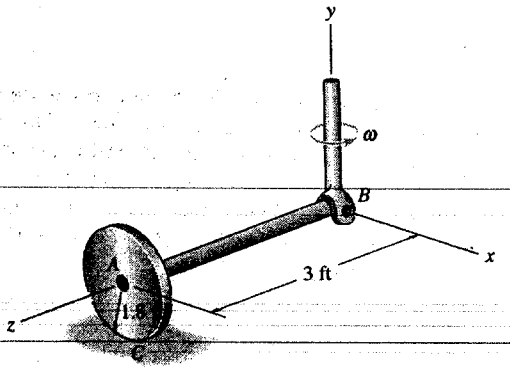
$\omega_z = -\dfrac{3(8)}{1.8} = -13.33$ rad/s

$\Sigma M_z = I_z \Omega_z \omega_z; \quad 30(3) - N_w(3) = \left[\left(\dfrac{30}{32.2}\right)(1.2)^2\right](8)(-13.33)$

$N_w = 77.7$ lb **Ans**

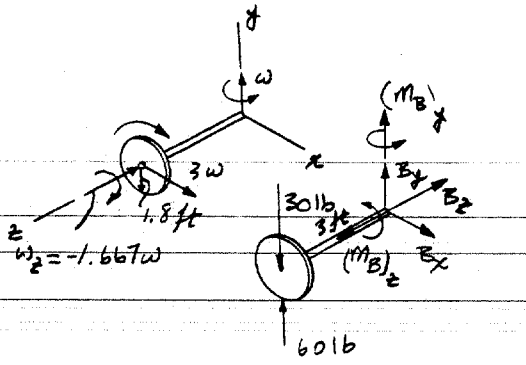


21-65. The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, determine its angular velocity ω so that the normal reaction at C becomes 60 lb.



$\Sigma M_z = I_z \Omega_z \omega_z; \quad 30(3) - 60(3) = \left[\dfrac{30}{32.2}(1.2)^2\right]\omega(-1.667\omega)$

$\omega = 6.34$ rad/s **Ans**



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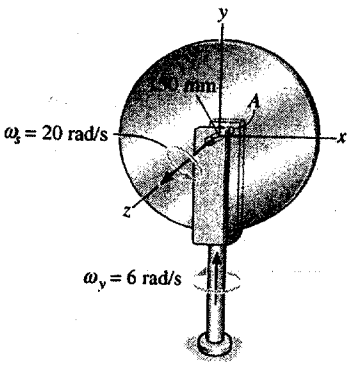
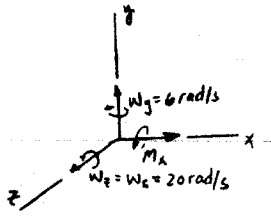
21-66. The 20-kg disk is spinning about its center at $\omega_s = 20$ rad/s while the supporting axle is rotating at $\omega_y = 6$ rad/s. Determine the gyroscopic moment caused by the force reactions which the pin A exerts on the disk due to the motion.



$\Sigma M_x = I_z \Omega_y \omega_z$; $M_x = \left[\frac{1}{2}(20)(0.15)^2\right](6)(20) = 27.0\ \text{N} \cdot \text{m}$ **Ans**



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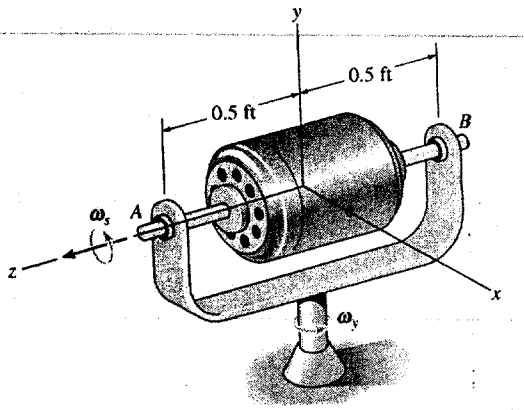
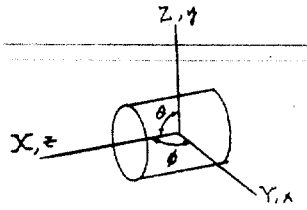
21-67. The motor weighs 50 lb and has a radius of gyration of 0.2 ft about the z axis. The shaft of the motor is supported by bearings at A and B, and is turning at a constant rate of $\omega_s = \{100\mathbf{k}\}$ rad/s, while the frame has an angular velocity of $\omega_y = \{2\mathbf{j}\}$ rad/s. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.



Applying Eq. 21-30 : For the coordinate system shown $\theta = 90°$ $\phi = 90°$
$\dot{\theta} = 0$ $\dot{\phi} = 2$ rad/s $\dot{\psi} = 100$ rad/s .

$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\psi})$ reduces to

$\Sigma M_x = I_z\dot{\phi}\dot{\psi}$; $M_x = \left[\left(\frac{50}{32.2}\right)(0.2)^2\right](2)(100) = 12.4$ lb · ft **Ans**

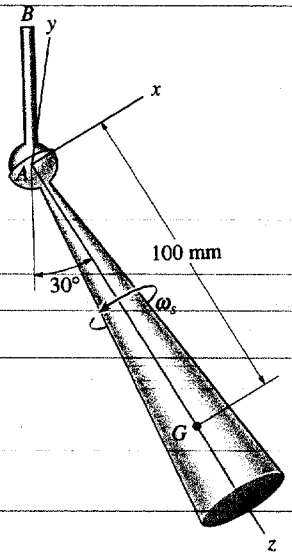


Since $\omega_x = 0$

$\Sigma M_y = 0$; $M_y = 0$ **Ans**

$\Sigma M_z = 0$; $M_z = 0$ **Ans**

***21-68.** The conical top has a mass of 0.8 kg, and the moments of inertia are $I_x = I_y = 3.5(10^{-3})$ kg · m^2 and $I_z = 0.8(10^{-3})$ kg · m^2. If it spins freely in the ball-and-socket joint at A with an angular velocity $\omega_s = 750$ rad/s, compute the precession of the top about the axis of the shaft AB.



$\omega_s = 750$ rad/s

Using Eq. 21 – 30.

$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\phi})$

$0.1(0.8)(9.81)\sin 30° = -3.5(10^{-3})\dot{\phi}^2 \sin 30°\cos 30°$
$+ 0.8(10^{-3})\dot{\phi}\sin 30°(\dot{\phi}\cos 30° + 750)$

Thus,

$1.160(10^{-3})\dot{\phi}^2 - 300(10^{-3})\dot{\phi} + 0.3924 = 0$

$\dot{\phi} = 1.31$ rad/s (low precession) **Ans**

$\dot{\phi} = 255$ rad/s (high precession) **Ans**

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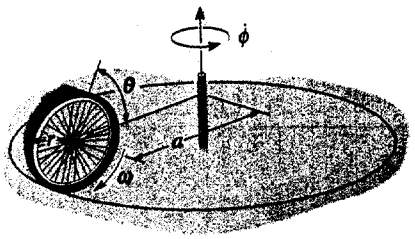
21-69. A wheel of mass m and radius r rolls with constant spin ω about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.

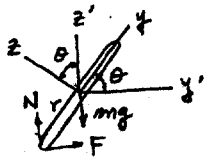


Since no slipping occurs,

$$(r)\dot\psi = (a + r\cos\theta)\dot\phi$$

or

$$\dot\psi = \left(\frac{a + r\cos\theta}{r}\right)\dot\phi \qquad (1)$$



Also,

$$\omega = \dot\phi + \dot\psi$$

$$\Sigma F_{y'} = m(a_G)_{y'}; \quad F = m(a\dot\phi^2) \qquad (2)$$

$$\Sigma F_{z'} = m(a_G)_{z'}; \quad N - mg = 0 \qquad (3)$$

$$I_x = I_y = \frac{mr^2}{2}, \quad I_z = mr^2$$

$$\omega = \dot\phi\sin\theta\,\mathbf{j} + (-\dot\psi + \dot\phi\cos\theta)\mathbf{k}$$

Thus,

$$\omega_x = 0, \quad \omega_y = \dot\phi\sin\theta, \quad \omega_z = -\dot\psi + \dot\phi\cos\theta$$

$$\dot\omega = \dot\phi \times \dot\psi = -\dot\phi\dot\psi\sin\theta$$

$$\dot\omega_x = -\dot\phi\dot\psi\sin\theta, \quad \dot\omega_y = \dot\omega_z = 0$$

Applying

$$\Sigma M_x = I_x\dot\omega_x + (I_z - I_y)\omega_z\omega_y$$

$$F r\sin\theta - N r\cos\theta = \frac{m r^2}{2}(-\dot\phi\dot\psi\sin\theta) + (m r^2 - \frac{m r^2}{2})(-\dot\psi + \dot\phi\cos\theta)(\dot\phi\sin\theta)$$

Using Eqs. (1), (2) and (3), and eliminating $\dot\psi$, we have

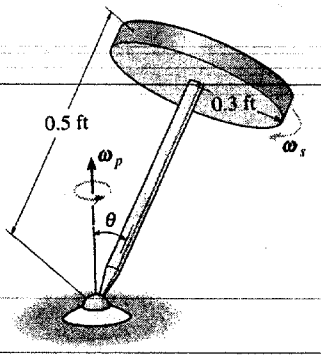
$$m a\dot\phi^2 r\sin\theta - m g r\cos\theta = \frac{m r^2}{2}(-\dot\phi)\sin\theta\left(\frac{a + r\cos\theta}{r}\right)\dot\phi + \frac{m r^2}{2}\left(\frac{-\dot\phi a}{r}\right)\dot\phi\sin\theta$$

$$m a\dot\phi^2\sin\theta r - m g r\cos\theta = \frac{m r^2}{2}\left(\frac{-\dot\phi^2 a}{r}\right)\sin\theta - \frac{m r^2}{2}(\dot\phi^2\sin\theta\cos\theta)$$

$$2 g\cos\theta = a\dot\phi^2\sin\theta + r\dot\phi^2\sin\theta\cos\theta$$

$$\dot\phi = \left(\frac{2 g\,\mathrm{ctn}\,\theta}{a + r\cos\theta}\right)^{1/2} \qquad \text{Ans}$$

21-70. The top consists of a thin disk that has a weight of 8 lb and a radius of 0.3 ft. The rod has a negligible mass and a length of 0.5 ft. If the top is spinning with an angular velocity $\omega_s = 300$ rad/s, determine the steady-state precessional angular velocity ω_p of the rod when $\theta = 40°$.



$$\Sigma M_x = -I\dot\phi^2\sin\theta\cos\theta + I_z\dot\phi\sin\theta\left(\dot\phi\cos\theta + \dot\psi\right)$$

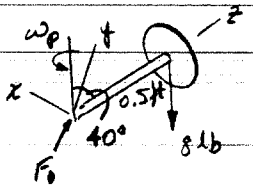
$$8(0.5\sin40°) = -\left[\frac{1}{4}\left(\frac{8}{32.2}\right)(0.3)^2 + \left(\frac{8}{32.2}\right)(0.5)^2\right]\omega_p^2\sin40°\cos40°$$

$$+\left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p\sin40°\,(\omega_p\cos40° + 300)$$

$$0.02783\omega_p^2 - 2.1559\omega_p + 2.571 = 0$$

$\omega_p = 1.21$ rad/s Ans (Low precession)

~~$\omega_p = 76.3$ rad/s~~ Ans (High precession)



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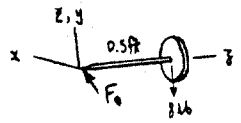
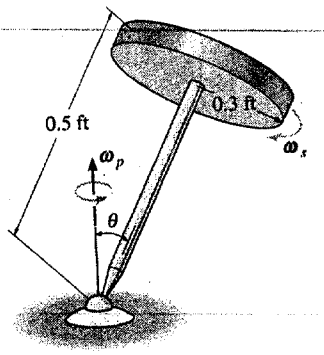
21-71. Solve Prob. 21-70 when $\theta = 90°$.



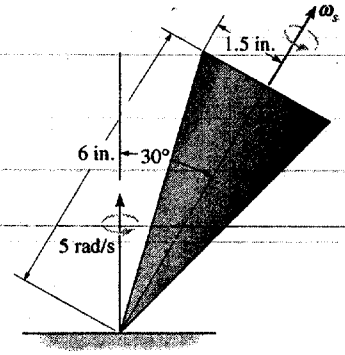


$\Sigma M_z = I_z \Omega_z \omega_z$

$$8(0.5) = \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p\,(300)$$

$\omega_p = 1.19$ rad/s **Ans**

***21-72.** The top has a mass of 3 lb and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of 5 rad/s, determine its spin.



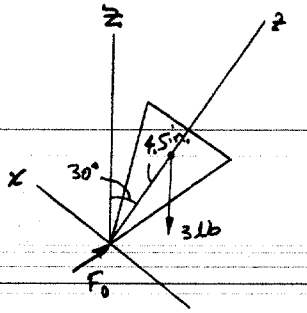
$$I = \frac{3}{80}\left(\frac{3}{32.2}\right)\left[4\left(\frac{1.5}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] + \frac{3}{32.2}\left(\frac{4.5}{12}\right)^2 = 0.01419 \text{ slug} \cdot \text{ft}^2$$

$$I_z = \frac{3}{10}\left(\frac{3}{32.2}\right)\left(\frac{1.5}{12}\right)^2 = 0.43672\left(10^{-3}\right) \text{ slug} \cdot \text{ft}^2$$

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z \dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$(3)\left(\frac{4.5}{12}\right)(\sin 30°) = -(0.01419)(5)^2 \sin 30°\cos 30° + 0.43672\left(10^{-3}\right)(5)\sin 30°\left(5\cos 30° + \dot{\psi}\right)$$

$\dot{\psi} = 652$ rad/s **Ans**



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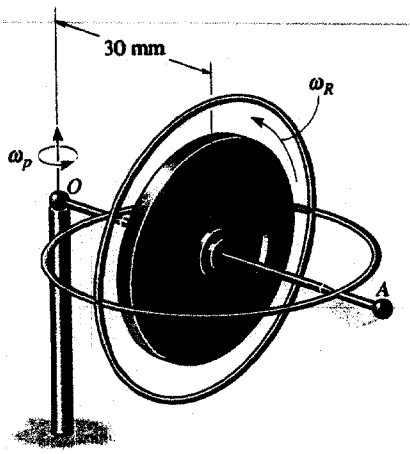
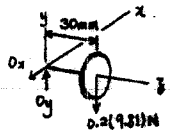
21-73. The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at $\omega_p = 2$ rad/s, determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has a mass of 200 g and a radius of gyration $k_{OA} = 20$ mm about OA.



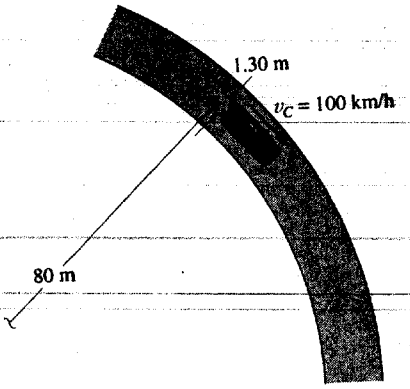


$\Sigma M_z = I_z \Omega_z \omega_z$

$(0.2)(9.81)(0.03) = [0.2(0.02)^2](2)(\omega_R)$

$\omega_R = 368$ rad/s **Ans**

21-74. The car is traveling at $v_C = 100$ km/h around the horizontal curve having a radius of 80 m. If each wheel has a mass of 16 kg, a radius of gyration $k_G = 300$ mm about its spinning axis, and a radius of 400 mm, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is 1.30 m.



$I = 2[16(0.3)^2] = 2.88$ kg · m^2

$\omega_s = \dfrac{100(1000)}{3600(0.4)} = 69.44$ rad/s

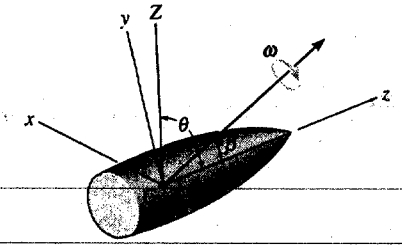
$\omega_p = \dfrac{100(1000)}{80(3600)} = 0.347$ rad/s

$M = I \omega_s \omega_p$

$\Delta F(1.30) = 2.88(69.44)(0.347)$

$\Delta F = 53.4$ N **Ans**

21-75. The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z, respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity ω and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



From Eq. 21-34 $\omega_y = \dfrac{H_G \sin\theta}{I}$ and $\omega_z = \dfrac{H_G \cos\theta}{I_z}$ Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan\theta$

However, $\omega_y = \omega\sin\beta$ and $\omega_z = \omega\cos\beta$

$\dfrac{\omega_y}{\omega_z} = \tan\beta = \dfrac{I_z}{I}\tan\theta$

$\tan\theta = \dfrac{I}{I_z}\tan\beta$ **Q.E.D.**

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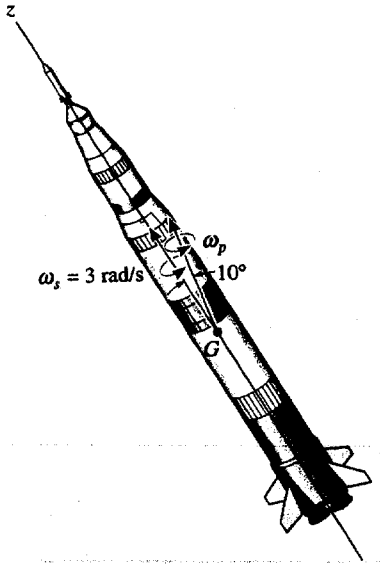
***21-76.** While the rocket is in free flight, it has a spin of 3 rad/s and precesses about an axis measured 10° from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is 1/15, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?

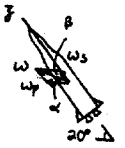


Determine the angle β from the result of Prob. 21-75.

$$\tan\theta = \frac{I}{I_z}\tan\beta$$

$$\tan 10° = \frac{15}{1}\tan\beta$$

$$\beta = 0.673° \qquad \textbf{Ans}$$



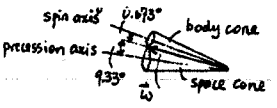
Thus,

$$\alpha = 10° - 0.673° = 9.33°$$

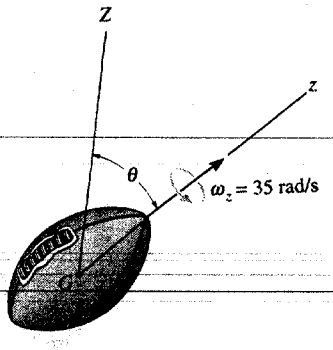
Hence,

Regular Precession **Ans**

Since $I_z < I$.



21-77. The 0.2-kg football is thrown with a spin ω_z = 35 rad/s. If the angle θ is measured as 60°, determine the precession about the Z axis



Gyroscopic Motion : Here, the spinning angular velocity $\psi = \omega_z = 35$ rad/s. The moment inertia of the football about the z axis is $I_z = 0.2(0.05^2) = 0.5(10^{-3})$ kg · m^2 and the moment inertia of the satelite about its transverse axis is $I = 0.2(0.1^2)$ $= 2(10^{-3})$ kg · m^2. Applying the third of Eq. 21 – 36 with $\theta = 60°$, we have

$$\psi = \frac{I - I_z}{I I_z} H_G \cos\theta$$

$$35 = \left[\frac{2(10^{-3}) - 0.5(10^{-3})}{2(10^{-3})0.5(10^{-3})}\right] H_G \cos 60°$$

$$H_G = 0.04667 \text{ kg} \cdot \text{m}^2/\text{s}$$

Applying the second of Eq. 21 – 36, we have

$$\dot{\phi} = \frac{H_G}{I} = \frac{004667}{2(10^{-3})} = 23.3 \text{ rad/s} \qquad \textbf{Ans}$$

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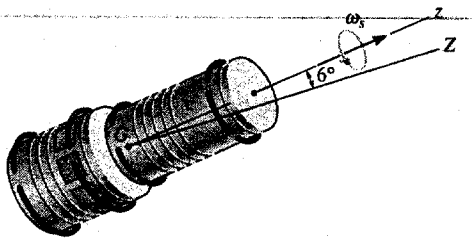
21-78. The space capsule has a mass of 3.2 Mg, and about axes passing through the mass center G the axial and transverse radii of gyration are $k_z = 0.90$ m and $k_t = 1.85$ m, respectively. If it is spinning at $\omega_s = 0.8$ rev/s, determine its angular momentum. Precession occurs about the Z axis.



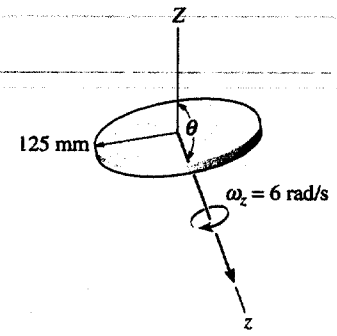
Gyroscopic Motion : Here, the spinning angular velocity $\dot{\psi} = \omega_s = 0.8(2\pi)$ $= 1.6\pi$ rad/s. The moment of inertia of the satelite about the z axis is $I_z = 3200(0.9^2)$ $= 2592\ \text{kg}\cdot\text{m}^2$ and the moment of inertia of the satelite about its transverse axis is $I = 3200(1.85^2) = 10\,952\ \text{kg}\cdot\text{m}^2$. Applying the third of Eq. 21 – 36 with $\theta = 6°$, we have

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

$$1.6\pi = \left[\frac{10\,952 - 2592}{10\,952(2592)}\right] H_G \cos 6°$$

$$H_G = 17.16(10^3)\ \text{kg}\cdot\text{m}^2/\text{s} = 17.2\ \text{Mg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

21-79. The 4-kg disk is thrown with a spin $\omega_z = 6$ rad/s. If the angle θ is measured as 160°, determine the precision about the Z axis.



$I = \frac{1}{4}(4)(0.125)^2 = 0.015625\ \text{kg}\cdot\text{m}^2 \qquad I_z = \frac{1}{2}(4)(0.125)^2 = 0.03125\ \text{kg}\cdot\text{m}^2$

Applying Eq. 21-36 with $\theta = 160°$ $\dot{\psi} = 6$ rad/s

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

$$6 = \frac{0.015625 - 0.03125}{0.015625(0.03125)} H_G \cos 160°$$

$$H_G = 0.1995\ \text{kg.m}^2/\text{s}$$

$$\phi = \frac{H_G}{I} = \frac{0.1995}{0.015625} = 12.8\ \text{rad/s} \qquad \textbf{Ans}$$

Note that this is a case of retrograde precession since $I_z > I$.

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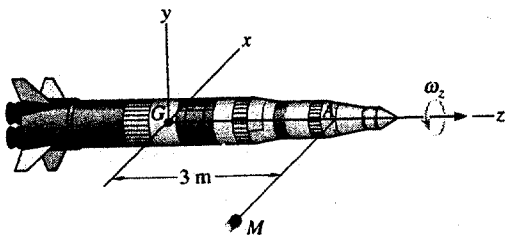
***21-80.** The rocket has a mass of 4 Mg and radii of gyration $k_z = 0.85$ m and $k_y = 2.3$ m. It is initially spinning about the z axis at $\omega_z = 0.05$ rad/s when a meteoroid M strikes it at A and creates an impulse $\mathbf{I} = \{300\mathbf{i}\}$ N·s. Determine the axis of precession after the impact.



The impulse creates an angular momentum about the y axis of

$H_y = 300(3) = 900\ \text{kg}\cdot\text{m}^2/\text{s}$

Since

$\omega_z = 0.05$ rad/s,

then

$\mathbf{H}_G = 900\mathbf{j} + \left[4000(0.85)^2\right](0.05)\mathbf{k} = 900\mathbf{j} + 144.5\mathbf{k}$

The axis of precession is defined by $\mathbf{H}_G$.

$$\mathbf{u}_{H_G} = \frac{900\mathbf{j} + 144.5\mathbf{k}}{911.53} = 0.9874\mathbf{j} + 0.159\mathbf{k}$$

Thus,

$\alpha = \cos^{-1}(0) = 90°$ **Ans**

$\beta = \cos^{-1}(0.9874) = 9.12°$ **Ans**

$\gamma = \cos^{-1}(0.159) = 80.9°$ **Ans**

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22-1. When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \qquad \textbf{Ans}$$

$$f = \frac{1}{\tau} = 2.21 \text{ Hz} \qquad \textbf{Ans}$$

22-2. A spring has a stiffness of 600 N/m. If a 4-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation which describes the block's motion. Assume that positive displacement is measured downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25 \text{ rad/s}$$

$v = 0, \quad x = -0.05$ m at $t = 0$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.05 = 0 + B$$

$$B = -0.05$$

$$v = A\omega_n \cos \omega_n t - Bp \sin \omega_n t$$

$$0 = A(12.25) - 0$$

$$A = 0$$

Thus, $x = -0.05 \cos(12.2t)$ m **Ans**

22-3. When a 3-kg block is suspended from a spring, the spring is stretched a distance of 60 mm. Determine the natural frequency and the period of vibration for a 0.2-kg block attached to the same spring.

$$k = \frac{F}{\Delta x} = \frac{3(9.81)}{0.060} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.2}} = 49.52 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = \frac{49.52}{2\pi} = 7.88 \text{ Hz} \qquad \textbf{Ans}$$

$$\tau = \frac{1}{f} = \frac{1}{7.88} = 0.127 \text{ s} \qquad \textbf{Ans}$$

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***22-4.** An 8-kg block is suspended from a spring having a stiffness $k = 80$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 90 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is measured downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{8}} = 3.162 \text{ rad/s}$$

$v = -0.4$ m/s, $x = -0.09$ m at $t = 0$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.09 = 0 + B$$

$$B = -0.09$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$-0.4 = A(3.162) - 0$$

$$A = -0.126$$

Thus, $x = -0.126 \sin(3.16t) - 0.09 \cos(3.16t)$ m **Ans**

$$C = \sqrt{A^2 + B^2} = \sqrt{(-0.126)^2 + (-0.09)^2} = 0.155 \text{ m}$$ **Ans**

22-5. A 2-lb weight is suspended from a spring having a stiffness $k = 2$ lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

$$k = 2(12) = 24 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = 3.13 \text{ Hz}$$ **Ans**

$$y = -\frac{1}{12}, \quad v = 0 \text{ at } t = 0$$

From Eqs. 22-3 and 22-4,

$$-\frac{1}{12} = 0 + B$$

$$B = -0.0833$$

$$0 = A\omega_n + 0$$

$$A = 0$$

$$C = \sqrt{A^2 + B^2} = 0.0833 \text{ ft} = 1 \text{ in.}$$ **Ans**

Position equation,

$$y = (0.0833 \cos 19.7t) \text{ ft}$$ **Ans**

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22-6. A 6-lb weight is suspended from a spring having a stiffness $k = 3$ lb/in. If the weight is given an upward velocity of 20 ft/s when it is 2 in. above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

$k = 3(12) = 36$ lb/ft

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36}{\frac{6}{32.2}}} = 13.90 \text{ rad/s}$$

$t = 0, \quad v = -20$ ft/s, $\quad y = -\frac{1}{6}$ ft

From Eq. 22-3,

$$-\frac{1}{6} = 0 + B$$

$$B = -0.167$$

From Eq. 22-4,

$$-20 = A(13.90) + 0$$

$$A = -1.44$$

Thus,

$y = [-1.44 \sin(13.9t) - 0.167 \cos(13.9t)]$ ft **Ans**

From Eq. 22-10,

$C = \sqrt{A^2 + B^2} = \sqrt{(1.44)^2 + (-0.167)^2} = 1.45$ ft **Ans**

22-7. A 6-kg block is suspended from a spring having stiffness of $k = 200$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 75 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{6}} = 5.774$$

$x = A \sin \omega_n t + B \cos \omega_n t$

$x = -0.075$ m when $t = 0$,

$-0.075 = 0 + B; \quad B = -0.075$

$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$

$v = -0.4$ m/s when $t = 0$,

$-0.4 = A(5.774) - 0; \quad A = -0.0693$

Thus,

$x = -0.0693 \sin(5.77t) - 0.075 \cos(5.77t)$ **Ans**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.0693)^2 + (-0.075)^2} = 0.102$ m **Ans**

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***22-8.** A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the natural frequency of the vibration? Assume that positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.165$$

$$f = \frac{\omega_n}{2\pi} = \frac{8.165}{2\pi} = 1.299 = 1.30 \text{ Hz} \quad \textbf{Ans}$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$x = -0.05$ m when $t = 0$,

$$-0.05 = 0 + B; \quad B = -0.05$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$v = 0$ when $t = 0$,

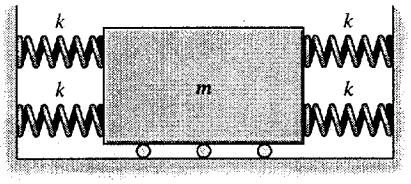
$$0 = A(8.165) - 0; \quad A = 0$$

Hence,

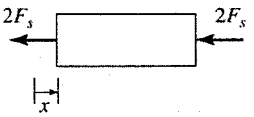
$$x = -0.05 \cos(8.16t) \quad \textbf{Ans}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)^2} = 0.05 \text{ m} = 50 \text{ mm} \quad \textbf{Ans}$$

22-9. Determine the frequency of vibration for the block. The springs are originally compressed Δ.



$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad -4kx = m\ddot{x}$$



$$\ddot{x} + \frac{4k}{m}x = 0$$

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}} \quad \textbf{Ans}$$

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22-10. A pendulum has a 0.4-m-long cord and is given a tangential velocity of 0.2 m/s toward the vertical from a position $\theta = 0.3$ rad. Determine the equation which describes the angular motion.

See Example 22-1.

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.4}} = 4.95$$

$\theta = A \sin \omega_n t + B \cos \omega_n t$

$\theta = 0.3$ rad when $t = 0$,

$0.3 = 0 + B; \quad B = 0.3$

$\dot{\theta} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$

Since $s = \theta l$, $\dot{v} = \dot{\theta} l$. Hence,

$-0.2 = \dot{\theta}(0.4)$, $\dot{\theta} = -0.5$ when $t = 0$,

$-0.5 = A(4.95); \quad A = -0.101$

Thus,

$\theta = -0.101 \sin(4.95t) + 0.3 \cos(4.95t)$ **Ans**

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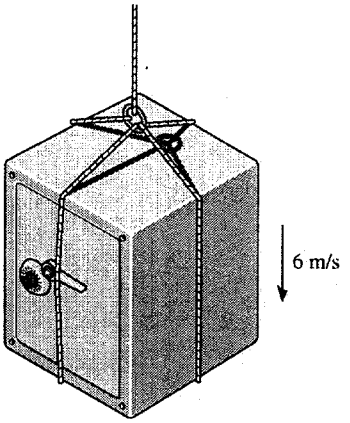
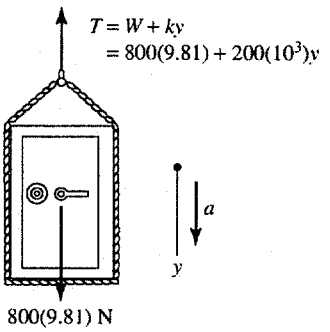
22-11. A cable is used to suspend the 800-lb safe. If the safe is being lowered at 6 m/s when the motor controlling the cable suddenly jams (stops), determine the maximum tension in the cable and the frequency of vibration of the safe. Neglect the mass of the cable and assume it is elastic such that it stretches 20 mm when subjected to a tension of 4 kN.



Freebody Diagram: Here the stiffness of the cable is $k = \frac{4000}{0.02} = 200(10^3)$ N/m. When the safe is being displaced by an amount y downward vertically from its equilibrium position, the *restoring force* that developed in the cable $T = W + ky = 800(9.81) + 200(10^3)y$.



Equation of Motion:

$$+\uparrow \sum F_x = 0; \quad 800(9.81) + 200(10^3)y - 800(9.81) = -800a \quad [1]$$

Kinematics: Since $a = \frac{d^2y}{dt^2} = \ddot{y}$, then substituting this value into Eq. [1], we have

$$200(10^3)y = -800\ddot{y}$$

$$\ddot{y} + 250x = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = 250$, thus, $\omega_n = 15.81$ rad/s. Applying Eq. 22-14, we have

$$f = \frac{\omega_n}{2\pi} = \frac{15.81}{2\pi} = 2.52 \text{ Hz} \quad \textbf{Ans}$$

The solution of the above differential equation (Eq. [2]) is in the form of

$$y = C\sin(15.81t + \phi) \quad [3]$$

Taking the time derivative of Eq. [3], we have

$$\dot{y} = 15.81C\cos(15.81t + \phi) \quad [4]$$

Applying the initial condition of $y = 0$ and $\dot{y} = 6$ m/s at $t = 0$ to Eqs. [3] and [4] yields

$$0 = C\sin\phi \qquad [5]$$

$$6 = 15.81C\cos\phi \quad [6]$$

Solving Eqs. [5] and [6] yields

$$\phi = 0^\circ \quad C = 0.3795 \text{ m}$$

Since $y_{max} = C = 0.3795$ m, the maximum cable tension is given by

$$T_{max} = W + ky_{max} = 800(9.81) + 200(10^3)(0.3795) = 83.7 \text{ kN} \quad \textbf{Ans}$$

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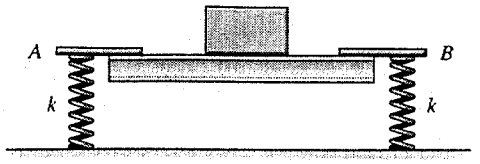
***22-12.** The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{\tau^2}{(2\pi)^2} = \frac{m}{k}$$

$$\frac{(0.83)^2}{(2\pi)^2} = \frac{m_B}{2k} \qquad (1)$$

$$\frac{(1.52)^2}{(2\pi)^2} = \frac{m_B + 50}{2k} \qquad (2)$$

Eqs. (1) and (2) become

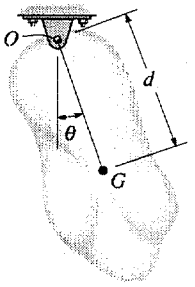
$$m_B = 0.03490k$$

$$m_B + 50 = 0.1170k$$

$$m_B = 21.2 \text{ kg} \quad \textbf{Ans}$$

$$k = 609 \text{ N/m} \quad \textbf{Ans}$$

22-13. The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



$$\curvearrowleft + \sum M_O = I_O\alpha; \qquad -mgd\sin\theta = [mk_G^2 + md^2]\ddot{\theta}$$

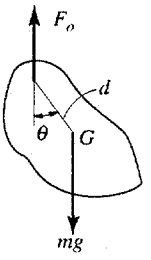
$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$

However, for small rotation $\sin\theta = \theta$. Hence

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{g^d}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{g^d}} \quad \textbf{Ans}$$



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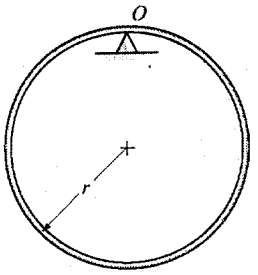
22-14. The thin hoop of mass m is supported by a knife-edge. Determine the natural period of vibration for small amplitudes of swing.

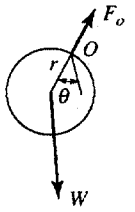


$I_O = mr^2 + mr^2 = 2mr^2$

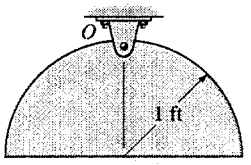
$\curvearrowleft + \sum M_O = I_O\alpha; \quad -mgr\theta = (2mr^2)\ddot{\theta}$

$$\ddot{\theta} + \left(\frac{8}{2r}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2r}{g}} \quad \textbf{Ans}$$



22-15. The semicircular disk weighs 20 lb. Determine the natural period of vibration if it is displaced a small amount and released.



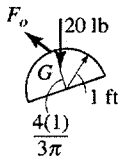
Moment of inertia about O:

$I_A = I_G + md^2$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 = I_G + \left(\frac{20}{32.2}\right)\left[\frac{4(1)}{3\pi}\right]^2 \quad I_A = 0.1987 \text{ slug} \cdot \text{ft}^2$$

$I_O = I_G + md^2$

$$= 0.1987 + \left(\frac{20}{32.2}\right)\left[1 - \frac{4(1)}{3\pi}\right]^2 = 0.4045 \text{ slug} \cdot \text{ft}^2$$



Equation of motion:

$$\Sigma M_O = I_{Oxy}\alpha; \quad 20\left[1 - \frac{4(1)}{3k}\right]\sin\theta = -0.4045\ddot{\theta}$$

$\ddot{\theta} + 28.462 \sin\theta = 0$

However, for small rotation $\sin\theta = \theta$. Hence

$\ddot{\theta} + 28.462\theta = 0$

From the above differential equation, $\omega_n = \sqrt{28.462} = 5.335$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.335} = 1.18 \text{ s} \quad \textbf{Ans}$$

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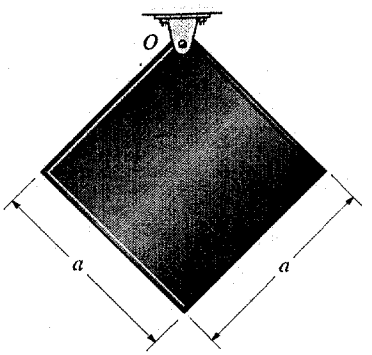
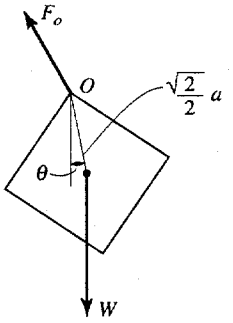
***22-16.** The square plate has a mass m and is suspended at its corner by the pin O. Determine the natural period of vibration if it is displaced a small amount and released.



$$I_O = \frac{1}{12}m(a^2 + a^2) + m\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$$



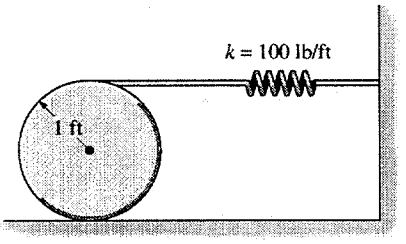
$$\curvearrowleft + \sum M_O = I_O\alpha; \qquad -mg\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3}ma^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

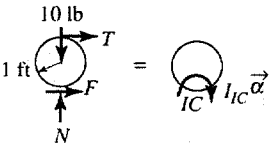
$$\omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}}$$

$$\tau = \frac{2\pi}{\omega_n} = 6.10\sqrt{\frac{a}{g}} \qquad \textbf{Ans}$$

22-17. The disk has a weight of 10 lb and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.4 rad, determine the equation which describes its oscillatory motion when it is released.



$$I_{IC} = \frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(1)^2 = 0.4658 \text{ slug} \cdot \text{ft}^2$$



$$\curvearrowleft \downarrow + \sum M_{IC} = I_{IC}\alpha; \quad -100(2\theta) = 0.4658\ddot{\theta}$$

$$\ddot{\theta} + 429.3\theta = 0$$

$$\omega_n = \sqrt{429.3} = 20.72 \text{ rad/s}$$

$$\theta = A\sin\omega_n t + B\cos\omega_n t$$

$$\omega = 0, \quad \theta = 0.4, \quad t = 0$$

$$0.4 = 0 + B$$

$$B = 0.4$$

$$\omega = \dot{\theta} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$$

$$0 = A\omega_n - 0$$

$$A = 0$$

Thus,

$$\theta = 0.4\cos(20.7t) \qquad \textbf{Ans}$$

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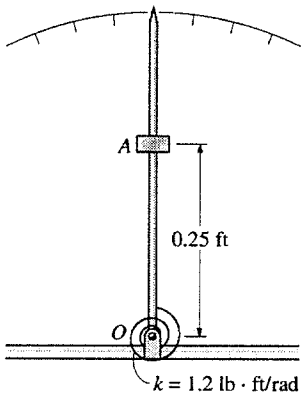
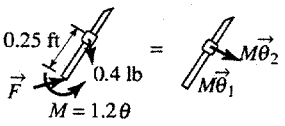
22-18. The pointer on a metronome supports a 0.4-lb slider A, which is positioned at a fixed distance from the pivot O of the pointer. When the pointer is displaced, a torsional spring at O exerts a restoring torque on the pointer having a magnitude $M = (1.2\theta)$ lb · ft, where θ represents the angle of displacement from the vertical, measured in radians. Determine the natural period of vibration when the pointer is displaced a small amount θ and released. Neglect the mass of the pointer.



$$I_O = \frac{0.4}{32.2}(0.25)^2 = 0.7764(10^{-3})\ \text{slug} \cdot \text{ft}^2$$



$$\curvearrowleft + \sum M_O = I_O\alpha; \quad -1.2\theta + 0.4(0.25)\sin\theta = 0.7764(10^{-3})\ddot{\theta}$$

For small θ, $\sin\theta = \theta$

So that

$$\ddot{\theta} + 1417.5\theta = 0$$

$$\omega_n = \sqrt{1417.5} = 37.64 \text{ rad/s}$$

$$\tau = \frac{2\pi}{37.64} = 0.167 \text{ s} \qquad \textbf{Ans}$$

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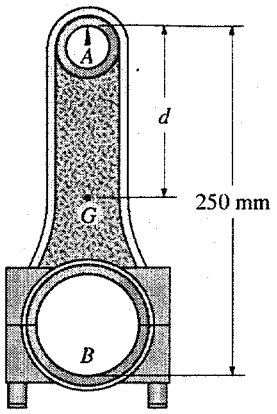
22-19. The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the period of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



Freebody Diagram: When an object of arbitary shape having a mass m is pinned at O and is displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y,

$$\curvearrowleft + \sum M_O = I_O\alpha; \qquad -mg\sin\theta(l) = I_O\alpha \qquad [1]$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\ddot{\theta} \quad \text{or} \quad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22-12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad [3]$$

When the rod is rotating about A, $\tau = \tau_A = 3.38$ s and $l = d$. Substitute these values into Eq. [3], we have

$$3.38 = 2\pi\sqrt{\frac{I_A}{mgd}} \quad I_A = 0.2894mgd$$

When the rod is rotating about B, $\tau = \tau_B = 3.96$ s and $l = 0.25 - d$. Substitute these values into Eq. [3], we have

$$3.96 = 2\pi\sqrt{\frac{I_B}{mg(0.25-d)}} \quad I_B = 0.3972mg(0.25-d)$$

However, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = I_B - m(0.25-d)^2$$

Then,

$$0.2894mgd - md^2 = 0.3972mg(0.25-d) - m(0.25-d)^2$$

$$d = 0.1462 \text{ m} = 146 \text{ mm} \qquad \textbf{Ans}$$

Thus, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = 0.2894m(9.81)(0.1462) - m(0.1462^2) = 0.3937m$$

The radius of gyration is

$$k_G = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.3937m}{m}} = 0.627 \text{ m} \qquad \textbf{Ans}$$

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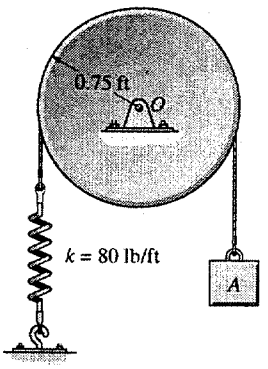
***22-20.** The disk, having a weight of 15 lb, is pinned at its center O and supports the block A that has a weight of 3 lb. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.



For equilibrium:

$T_{st} = 3$ lb

$\curvearrowleft + \sum M_O = I_O\alpha + ma(0.75)$

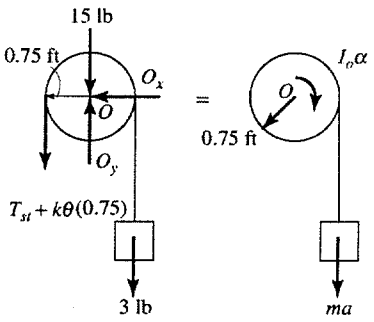
$a = 0.75\alpha$

$$-T_{st}(0.75) - (80)(\theta)(0.75)(0.75) + (3)(0.75) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\ddot{\theta} + \left(\frac{3}{32.2}\right)(0.75)\ddot{\theta}(0.75)$$

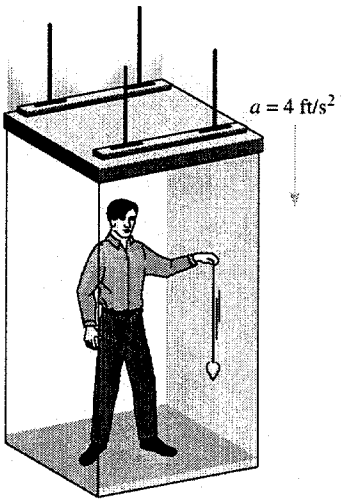
$$-2.25 - 45\theta + 2.25 = 0.131\ddot{\theta} + 0.05241\ddot{\theta}$$

$$\ddot{\theta} + 245.3\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s} \qquad \textbf{Ans}$$



22-21. While standing in an elevator, the man holds a pendulum which consists of an 18-in. cord and a 0.5-lb bob. If the elevator is descending with an acceleration $a = 4$ ft/s^2, determine the natural period of vibration for small amplitudes of swing.



Since the acceleration of the pendulum is $(32.2 + 4) = 36.2$ ft/s^2

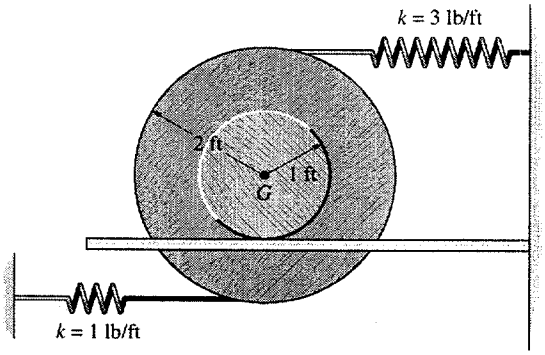
Using the result of Example 22-1,

We have

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{36.2}{18/12}} = 4.91 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.91} = 1.28 \text{ s} \qquad \textbf{Ans}$$

22-22. The 50-lb spool is attached to two springs. If the spool is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is $k_G = 1.5$ ft. The spool rolls without slipping.



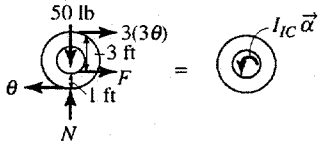
$$I_{IC} = \frac{50}{32.2}(1.5)^2 + \frac{50}{32.2}(1)^2 = 5.047 \text{ slug} \cdot \text{ft}^2$$

$$\curvearrowleft + \sum M_{IC} = I_{IC}\alpha; \quad -3(3\theta)(3) - 1(\theta) = 5.047\ddot{\theta}$$

$$\ddot{\theta} + 5.5483\theta = 0$$

$$\omega_n = \sqrt{5.5483}$$

$$\tau = \frac{2\pi}{\sqrt{5.5483}} = 2.67 \text{ s} \qquad \textbf{Ans}$$



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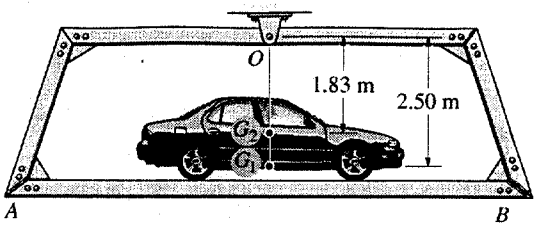
22-23. The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



Freebody Diagram: When an object of arbitary shape having a mass m is pinned at O and being displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\curvearrowleft + \sum M_O = I_O\alpha: \quad -mg\sin\theta(l) = I_O\alpha \qquad [1]$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\ddot{\theta} \quad \text{or} \quad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22-12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad [3]$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s, $m = 400$ kg and $l = 2.50$ m. Substitute these values into Eq. [3], we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}}\quad (I_O)_p = 1407.55 \text{ kg}\cdot\text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400$ kg $+ 1200$ kg $= 1600$ kg, $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substitute these values into Eq. [3], we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}}\quad (I_O)_C = 6522.76 \text{ kg}\cdot\text{m}^2$$

Thus, the mass moment inertia of the car about its mass center is

$$(I_G)_C = (I_O)_C - m_C d^2$$

$$= 6522.76 - 1200(1.83^2) = 2.50(10^3) \text{ kg}\cdot\text{m}^2 \quad \textbf{Ans}$$

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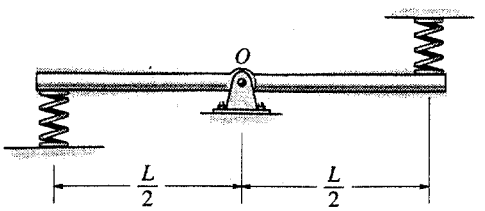
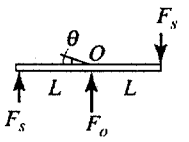
***22-24.** The uniform rod has a mass m and is supported by the pin O. If the rod is given a small displacement and released, determine the natural period of vibration. The springs are unstretched when the rod is in the position shown.



$$F_s = k\left(\frac{L}{2}\right)\theta$$



$$\curvearrowleft + \sum M_O = I_O\alpha; \quad -F_s(L) = I_O\ddot{\theta}$$

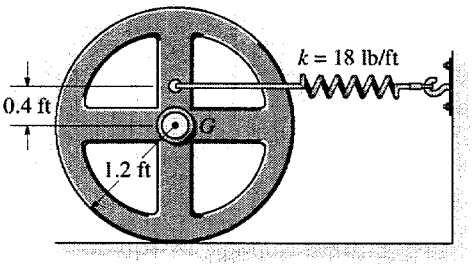
$$-k\left(\frac{L}{2}\right)(L)(\theta) = \frac{1}{12}mL^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{6k}{m}\theta = 0$$

$$\omega_n = \sqrt{\frac{6k}{m}}$$

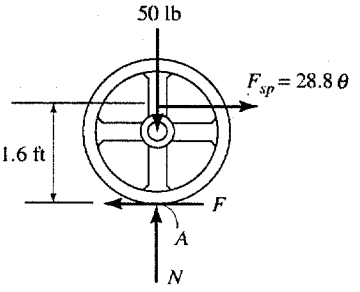
$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{2m}{3k}} \qquad \textbf{Ans}$$

***22-25.** The 50-lb wheel has a radius of gyration about its mass center G of $k_G = 0.7$ ft. Determine the frequency of vibration if it is displaced slightly from the equilibrium position and released. Assume no slipping.



Kinematics: Since the wheel rolls without slipping, then $a_G = \alpha r = 1.2\alpha$. Also when the wheel undergoes a small angular displacement θ about point A, the spring is stretched by $x = 1.6\sin\theta$. Since θ is small, then $\sin\theta = \theta$. Thus, $x = 1.6\theta$.

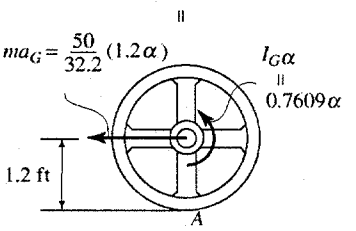
Freebody Diagram: The spring force $F_{sp} = kx = 18(1.6\theta) = 28.8\theta$ will create the *restoring moment* about point A.



Equation of Motion: The mass moment inertia of the wheel about its mass center is $I_G = mk_G^2 = \frac{50}{32.2}(0.7^2) = 0.7609$ slug · ft^2.

$$\curvearrowleft + \sum M_A = (M_k)_A; -28.8\theta(1.6) = \frac{50}{32.2}(1.2\alpha)(1.2) + 0.7609\alpha$$

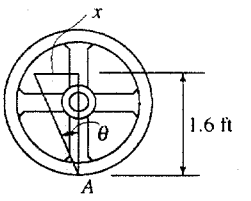
$$\alpha + 15.376\theta = 0 \qquad [1]$$



Since $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$, then substitute this values into Eq. [1], we have

$$\ddot{\theta} + 15.376\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = 15.376$, thus, $\omega_n = 3.921$ rad/s. Applying Eq. 22-14, we have



$$f = \frac{\omega_n}{2\pi} = \frac{3.921}{2\pi} = 0.624 \text{ Hz} \qquad \textbf{Ans}$$

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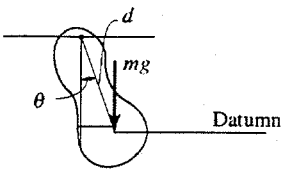
22-26. Solve Prob. 22-13 using energy methods.

$$T + V = \frac{1}{2}[mk_G^2 + md^2]\dot{\theta}^2 + mg(d)(1 - \cos\theta)$$



$$(k_G^2 + d^2)\dot{\theta}\ddot{\theta} + gd(\sin\theta)\dot{\theta} = 0$$

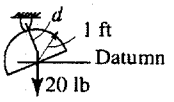
$$\sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{gd}{(k_G^2 + d^2)}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{(k_G^2 + d^2)}{gd}} \qquad \textbf{Ans}$$

22-27. Solve Prob. 22-15 using energy methods.

$$I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 = 0.3106 \text{ slug} \cdot \text{ft}^2$$



$$I_O = I_G + md^2$$

$$= (I_A - m\bar{r}^2) + md^2$$

$$= 0.3106 - \frac{20}{32.2}\left[\frac{4(1)}{3\pi}\right]^2 + \frac{20}{32.2}\left[1 - \frac{4(1)}{3\pi}\right]^2$$

$$= 0.4045 \text{ slug} \cdot \text{ft}^2$$

$$T_{max} = \frac{1}{2}I_O\omega_n^2\theta_{max}^2$$

$$V_{max} = mgd(1 - \cos\theta_{max})$$

For small θ_{max}, $\cos\theta_{max} \approx 1 - \frac{\theta_{max}^2}{2}$

$$V_{max} = mgd\left(\frac{\theta_{max}^2}{2}\right)$$

$$T_{max} = V_{max}$$

$$\frac{1}{2}(0.4045)\omega_n^2\theta_{max}^2 = 20\left[1 - \frac{4(1)}{3\pi}\right]\left(\frac{\theta_{max}^2}{2}\right)$$

$$\omega_n = 5.33 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.33} = 1.18 \text{ s} \qquad \textbf{Ans}$$

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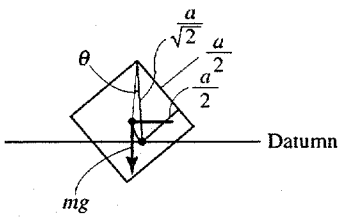
***22-28.** Solve Prob. 22-16 using energy methods.

$$T + V = \frac{1}{2}\left[\frac{1}{12}m(a^2 + a^2) + m\left(\frac{a}{\sqrt{2}}\right)^2\right]\dot{\theta}^2 + mg\left(\frac{a}{\sqrt{2}}\right)(1 - \cos\theta)$$

$$\frac{2}{3}ma^2\dot{\theta}\ddot{\theta} + mg\left(\frac{a}{\sqrt{2}}\right)(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta = \theta\ddot{\theta} + \frac{3g}{2\sqrt{2}a}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{1.0299}\left(\sqrt{\frac{a}{g}}\right) = 6.10\sqrt{\frac{a}{g}} \qquad \textbf{Ans}$$



22-29. Solve Prob. 22-20 using energy methods.

$s = 0.75\theta, \quad s = 0.75\theta$

$$T + V = \frac{1}{2}\left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\dot{\theta}^2 + \frac{1}{2}\left(\frac{3}{32.2}\right)(0.75\dot{\theta})^2$$

$$+ \frac{1}{2}(80)(s_{eq} + 0.75\theta)^2 - 3(0.75\theta)$$

$$0 = 0.1834\dot{\theta}\ddot{\theta} + 80(s_{eq} + 0.75\theta)(0.75\dot{\theta}) - 2.25\dot{\theta}$$

$F_{eq} = 80s_{eq} = 3$

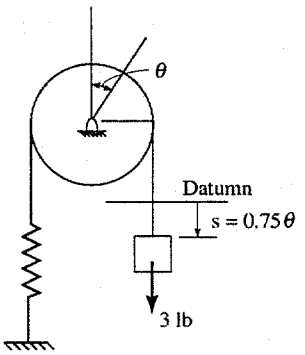
$s_{eq} = 0.0375$ ft

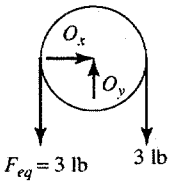
Thus,

$0.1834\ddot{\theta} + 45\theta = 0$

$\ddot{\theta} + 245.3 = 0$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s} \qquad \textbf{Ans}$$





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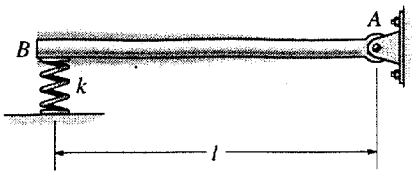
22-30. The uniform rod of mass m is supported by a pin at A and a spring at B. If the end B is given a small downward displacement and released, determine the natural period of vibration.



$$T = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\dot{\theta}^2$$

$$V = \frac{1}{2}k(y_{eq} + y_2)^2 - mgy_1$$

$$= \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{1}{2}\right)\theta$$

$$T + V = \frac{1}{6}ml^2\dot{\theta}^2 + \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{l\theta}{2}\right)$$

Time derivative

$$0 = \frac{1}{3}ml^2\dot{\theta}\ddot{\theta} + kl(\theta_{eq} + \theta)\dot{\theta} - mgl\frac{\dot{\theta}}{2}$$

For equilibrium

$$k(l\theta_{eq}) = mgl/2, \theta_{eq} = \frac{mg}{2k}$$

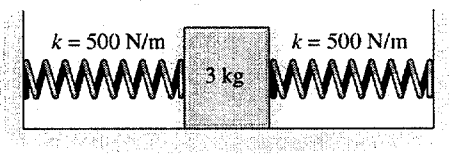
Thus,

$$0 = \frac{1}{3}ml\ddot{\theta} + k\theta$$

$$\ddot{\theta} + (3k/m)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{3k}} \quad \textbf{Ans}$$

22-31. Determine the differential equation of motion of the 3-kg block when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.



$T + V = \text{const.}$

$$T = \frac{1}{2}(3)\dot{x}^2$$

$$V = \frac{1}{2}(500)x^2 + \frac{1}{2}(500)x^2$$

$$T + V = 1.5\dot{x}^2 + 500x^2$$

$$1.5\,(2\dot{x})\,\ddot{x} + 1000x\dot{x} = 0$$

$$3\ddot{x} + 1000x = 0$$

$$\ddot{x} + 333x = 0 \quad \textbf{Ans}$$

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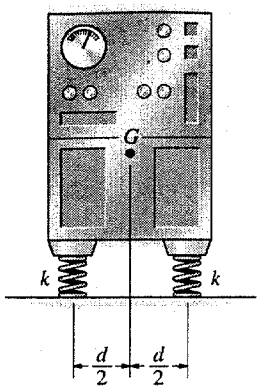
***22-32.** The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



$T + V = \text{const.}$

$T = \frac{1}{2}m(\dot{y})^2$

$V = mgy + \frac{1}{2}(4k)(\Delta s - y)^2$

$T + V = \frac{1}{2}m(\dot{y})^2 + mgy + \frac{1}{2}(4k)(\Delta s - y)^2$

$m\dot{y}\ddot{y} + mg\dot{y} - 4k(\Delta s - y)\dot{y} = 0$

$m\ddot{y} + mg + 4ky - 4k\Delta s = 0$

Since $\Delta s = \frac{mg}{4k}$

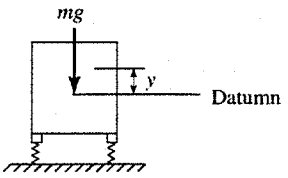
Then,

$m\ddot{y} + 4ky = 0$

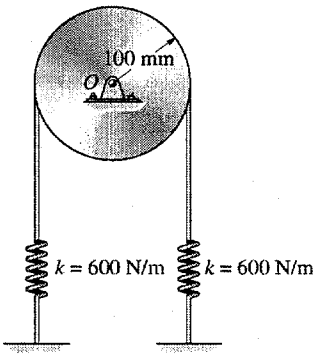
$\ddot{y} + \frac{4k}{m}y = 0$

$\omega_n = \sqrt{\frac{4k}{m}}$

$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{k}}$ **Ans**



22-33. The 7-kg disk is pin-connected at its midpoint. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_O. This term will cancel out after taking the time derivative of the energy equation.



$E = T + V$

$= \frac{1}{2}k(\theta r + \delta_O)^2 + \frac{1}{2}k(\theta r - \delta_O)^2 + \frac{1}{2}I_O(\dot{\theta})^2$

$E = k(\theta r + \delta_O)\dot{\theta}r + k(\theta r - \delta_O)\dot{\theta}r + I_O\dot{\theta}\ddot{\theta} = 0$

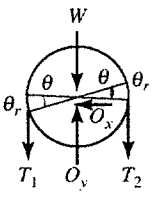
Thus,

$\ddot{\theta} + \frac{2kr^2}{I_O}\theta = 0$

$\omega_n = \sqrt{\frac{2kr^2}{I_O}}$

$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{2kr^2}}$

$\tau = 2\pi\sqrt{\frac{\frac{1}{2}(7)(0.1)^2}{2(600)(0.1)^2}} = 0.339 \text{ s}$ **Ans**



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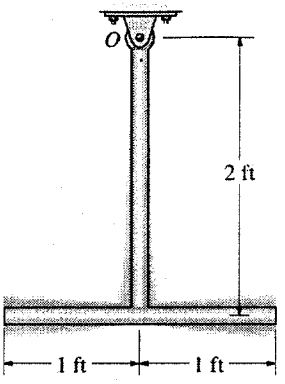
22-34. Determine the natural period of vibration of the pendulum. Consider the two rods to be slender, each having a weight of 8 lb/ft.

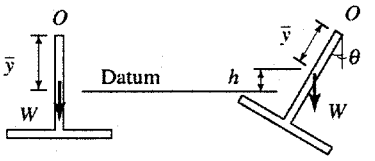


$$\bar{y} = \frac{1(8)(2) + 2(8)(2)}{8(2) + 8(2)} = 1.5 \text{ ft}$$

$$I_O = \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(1)^2\right]$$

$$+ \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(2)^2\right] = 2.8157 \text{ slug} \cdot \text{ft}^2$$

$h = \bar{y}(1 - \cos\theta)$



$T + V = \text{const.}$

$$T = \frac{1}{2}(2.8157)\left(\dot{\theta}\right)^2 = 1.4079\dot{\theta}^2$$

$$V = 8(4)(1.5)(1 - \cos\theta) = 48(1 - \cos\theta)$$

$$T + V = 1.4079\dot{\theta}^2 + 48(1 - \cos\theta)$$

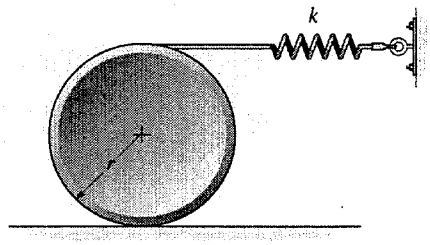
$$1.4079\left(2\dot{\theta}\right)\ddot{\theta} + 48(\sin\theta)\dot{\theta} = 0$$

For small θ, $\sin\theta = \theta$, then

$$\ddot{\theta} + 17.047\theta = 0$$

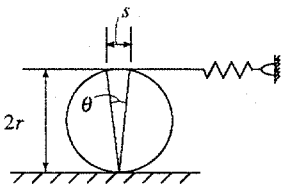
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{17.047}} = 1.52 \text{ s} \quad \textbf{Ans}$$

22-35. Determine the natural period of vibration of the disk having a mass m and radius r. Assume the disk does not slip on the surface of contact as it oscillates.



$T + V = \text{const.}$

$s = (2r)\theta$



$$T + V = \frac{1}{2}\left[\frac{1}{2}mr^2 + mr^2\right]\dot{\theta}^2 + \frac{1}{2}k(2r\theta)^2$$

$$0 = \frac{3}{2}mr^2\dot{\theta}\ddot{\theta} + 4kr^2\theta\dot{\theta}$$

$$\ddot{\theta} + \frac{8k}{3m}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{8k}{3m}}} = 3.85\sqrt{\frac{m}{k}} \quad \textbf{Ans}$$

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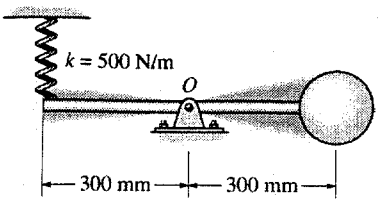
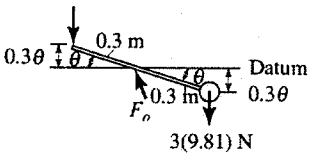
***22-36.** Determine the natural period of vibration of the 3-kg sphere. Neglect the mass of the rod and the size of the sphere.



$$E = T + V$$

$$= \frac{1}{2}(3)(0.3\dot{\theta})^2 + \frac{1}{2}(500)(\delta_{st} + 0.3\theta)^2 - 3(9.81)(0.3\theta)$$

$$E = \dot{\theta}[3(0.3)^2\ddot{\theta} + 500(\delta_{st} + 0.3\theta)(0.3) - 3(9.81)(0.3)] = 0$$



By statics,

$$T(0.3) = 3(9.81)(0.3)$$

$$T = 3(9.81)\ \text{N}$$

$$\delta_{st} = \frac{3(9.81)}{500}$$

Thus,

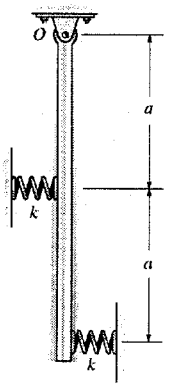
$$3(0.3)^2\ddot{\theta} + 500(0.3)^2\theta = 0$$

$$\ddot{\theta} + 166.67\theta = 0$$

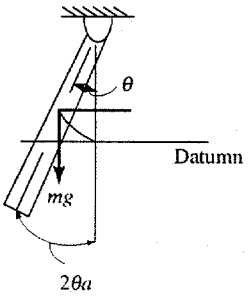
$$\omega_n = \sqrt{166.67} = 12.91\ \text{rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.91} = 0.487\ \text{s} \quad \textbf{Ans}$$

22-37. The slender rod has a mass m and is pinned at its end O. When it is vertical, the springs are unstretched. Determine the natural period of vibration.



$$T + V = \frac{1}{2}\left[\frac{1}{3}m(2a)^2\right]\dot{\theta}^2 + \frac{1}{2}k(2\theta a)^2 + \frac{1}{2}k(\theta a)^2 + mga(1 - \cos\theta)$$



$$0 = \frac{4}{3}ma^2\dot{\theta}\ddot{\theta} + 4ka^2\theta\dot{\theta} + ka^2\theta\dot{\theta} + mga\sin\theta\dot{\theta}$$

$$\sin\theta = \theta$$

$$\frac{4}{3}ma^2\ddot{\theta} + 5ka^2\theta + mga\theta = 0$$

$$\ddot{\theta} + \left(\frac{15ka + 3mg}{4ma}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{4\pi}{\sqrt{3}}\left(\frac{ma}{5ka + mg}\right)^{\frac{1}{2}} \quad \textbf{Ans}$$

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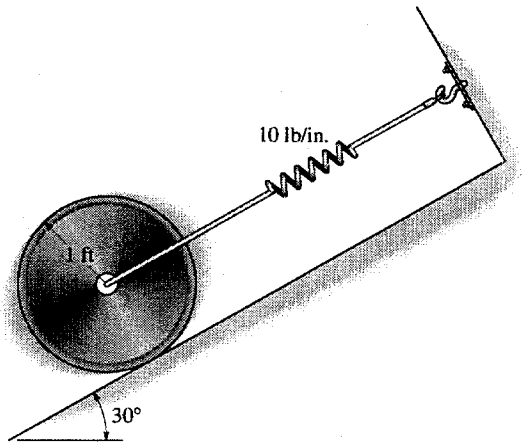
22-38. Determine the natural frequency of vibration of the 20-lb disk. Assume the disk does not slip on the inclined surface.



θ is the displacement of the disk.

The disk rolls a distance $s = r\theta$

$\Delta k = r\theta \sin 30°$

$E = T + V$

$= \frac{1}{2} I_{IC}(\dot{\theta})^2 + \frac{1}{2} k[\delta_{st} + \theta r]^2 - W(r\theta \sin 30°)$

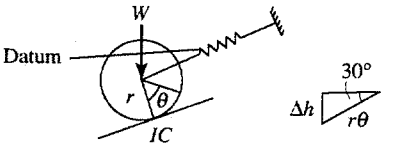
$E = \dot{\theta}(I_{IC}\ddot{\theta} + k\delta_{st} r + k\theta r^2 - Wr \sin 30°) = 0$

Since $k\delta_{st} = Wr \sin 30°$

$\ddot{\theta} + \frac{kr^2}{I_{IC}}\theta = 0$

$I_{IC} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

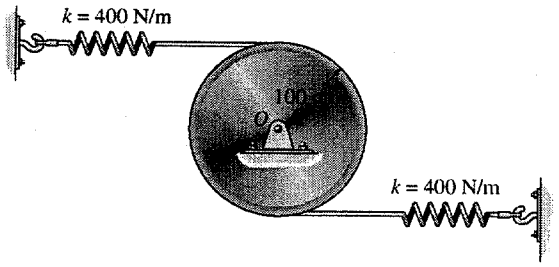
Thus,



$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kr^2}{I_{IC}}}$

$f = \frac{1}{2\pi}\sqrt{\frac{10(12)(1)^2}{\frac{3}{2}\left(\frac{30}{32.2}\right)(1)^2}} = 1.81 \text{ Hz}$ **Ans**

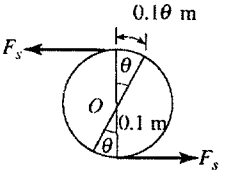
22-39. If the disk has a mass of 8 kg, determine the natural frequency of vibration. The springs are originally unstretched.



$I_O = \frac{1}{2}(8)(0.1)^2 = 0.04$

$T_{max} = V_{max}$

$\frac{1}{2} I_O (p\theta_{max})^2 = 2\left[\frac{1}{2}k(r\theta_{max})^2\right]$

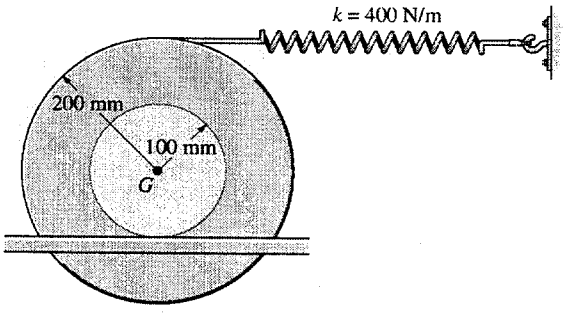


Thus,

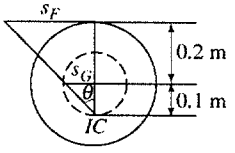
$\omega_n = \sqrt{\frac{2kr^2}{I_O}}$

$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2(400)(0.1)^2}{0.04}} = 2.25 \text{ Hz}$ **Ans**

***22-40.** Determine the differential equation of motion of the 3-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm.



Kinematics: Since no slipping occurs, $s_G = 0.1\theta$ hence $s_F = \frac{0.3}{0.1} S_G = 0.3\theta$. Also,



$v_G = 0.1\dot{\theta}$.

$E = T + V$

$E = \frac{1}{2}[(3)(0.125)^2]\dot{\theta}^2 + \frac{1}{2}(3)(0.1\theta)^2 + \frac{1}{2}(400)(0.3\theta)^2 = \text{const.}$

$= 0.03844\dot{\theta}^2 + 18\theta^2$

$0.076875\dot{\theta}\ddot{\theta} + 36\theta\dot{\theta} = 0$

$0.076875\dot{\theta}(\ddot{\theta} + 468.29\theta) = 0$ Since $0.076875\theta \neq 0$

$\ddot{\theta} + 468\theta = 0$ **Ans**

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22-41. The block shown in Fig. 22-16 has a mass of 20 kg, and the spring has a stiffness $k = 600$ N/m. When the block is displaced and released, two successive amplitudes are measured as $x_1 = 150$ mm and $x_2 = 87$ mm. Determine the coefficient of viscous damping, c.

Assuming that the system is underdamped.

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1} \quad (1)$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2} \quad (2)$$

Divide Eq. (1) by Eq. (2) $\quad \dfrac{x_1}{x_2} = \dfrac{e^{-\left(\frac{c}{2m}\right)t_1}}{e^{-\left(\frac{c}{2m}\right)t_2}}$

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)(t_2 - t_1) \quad (3)$$

However, $\quad t_2 - t_1 = \tau_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$ and $\omega_n = \dfrac{C_c}{2m}$

$$t_2 - t_1 = \frac{4m\pi}{C_c\sqrt{1 - \left(\frac{c}{C_c}\right)^2}} \quad (4)$$

Substitute Eq. (4) into Eq. (3) yields:

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)\frac{4m\pi}{C_c\sqrt{1 - \left(\frac{C}{C_c}\right)^2}}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1 - \left(\frac{C}{C_c}\right)^2}} \quad (5)$$

From Eq. (5)

$x_1 = 0.15$ m $\quad x_2 = 0.087$ m $\quad \omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{600}{20}} = 5.477$ rad/s

$C_c = 2m\omega_n = 2(20)(5.477) = 219.09$ N · s/m

$$\ln\left(\frac{0.15}{0.087}\right) = \frac{2\pi\left(\frac{c}{219.09}\right)}{\sqrt{1 - \left(\frac{c}{219.09}\right)^2}}$$

$c = 18.9$ N · s/m **Ans**

Since $C < C_c$, the system is underdamped. Therefore, the assumption is OK!

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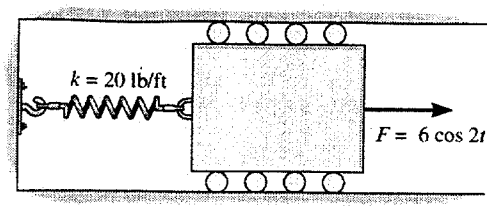
22-42. The 20-lb block is attached to a spring having a stiffness of 20 lb/ft. A force $F = (6\cos 2t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{\frac{20}{32.2}}} = 5.6745 \text{ rad/s}$$

$$C = \frac{\frac{6}{20}}{1 - \left(\frac{2}{5.6745}\right)^2} = 0.343 \text{ ft}$$

$x_p = C\cos 2t$

$\dot{x}_p = -C(2)\sin 2t$

Maximum velocity is

$v_{max} = C(2) = 0.343(2) = 0.685$ ft/s **Ans**

22-43. A 4-kg block is suspended from a spring that has a stiffness of $k = 600$ N/m. The block is drawn downward 50 mm from the equilibrium position and released from rest when $t = 0$. If the support moves with an impressed displacement of $\delta = (10\sin 4t)$ mm, where t is in seconds, determine the equation that describes the vertical motion of the block. Assume positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25$$

The general solution is defined by Eq. 22-23 with $k\delta_0$ substituted for F_0.

$$y = A\sin\omega_n t + B\cos\omega_n t + \left(\frac{\delta_0}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]}\right)\sin\omega t$$

$\delta = (0.01\sin 4t)$m, hence $\delta_0 = 0.01$, $\omega = 4$, so that

$y = A\sin 12.25t + B\cos 12.25t + 0.0112\sin 4t$

$y = 0.05$ when $t = 0$,

$0.05 = 0 + B + 0; \quad B = 0.05$ m

$\dot{y} = A(12.25)\cos 12.25t - B(12.25)\sin 12.25t + 0.0112(4)\cos 4t$

$v = y = 0$ when $t = 0$.

$0 = A(12.25) - 0 + 0.0112(4); \quad A = -0.00366$ m

Expressing the result in mm, we have

$y = (-3.66\sin 12.25t + 50\cos 12.25t + 11.2\sin 4t)$ mm **Ans**

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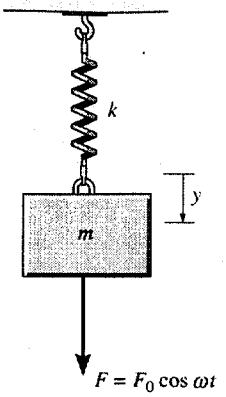
***22-44.** If the block is subjected to the impressed force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m)\cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



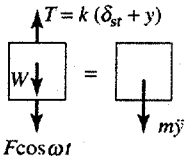
$+\downarrow \sum F_y = ma_y; \quad F_0 \cos \omega t + W - k\delta_{st} - ky = m\ddot{y}$

Since $W = k\delta_{st}$,

$$\ddot{y} + \left(\frac{k}{m}\right) y = \frac{F_0}{m} \cos \omega t \qquad (1) \quad \textbf{Q.E.D.}$$

$y_c = A \sin \omega_n y + B \cos \omega_n y$ (complementary solution)

$y_p = C \cos \omega t$ (particular solution)



Substitute y_p into Eq. (1),

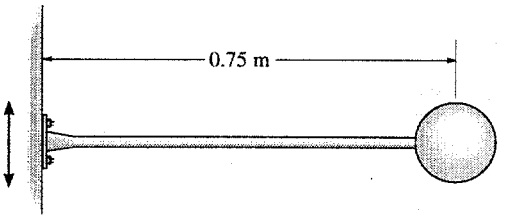
$$C\left(-\omega^2 + \frac{k}{m}\right)\cos \omega t = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{\left(\frac{k}{m} - \omega^2\right)}$$

$y = y_c + y_p$

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{F_0}{(k - m\omega^2)}\right) \cos \omega t \quad \textbf{Ans}$$

22-45. The light elastic rod supports a 4-kg sphere. When an 18-N vertical force is applied to the sphere, the rod deflects 14 mm. If the wall oscillates with harmonic frequency of 2 Hz and has an amplitude of 15 mm, determine the amplitude of vibration for the sphere.



$$k = \frac{F}{\Delta y} = \frac{18}{0.014} = 1285.71 \text{ N/m}$$

$\omega = 2 \text{ Hz} = 2(2\pi) = 12.57 \text{ rad/s}$

$\delta_0 = 0.015 \text{ m}$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1285.71}{4}} = 17.93$$

Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| = \left| \frac{0.015}{1 - \left(\frac{12.57}{17.93}\right)^2} \right|$$

$(x_p)_{max} = 0.0295 \text{ m} = 29.5 \text{ mm}$ **Ans**

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22-46. A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m/s, determine the period of free vibration.

$$F = cv \quad c = \frac{F}{v} = \frac{2.5}{0.2} = 12.5 \text{ N} \cdot \text{s/m}$$

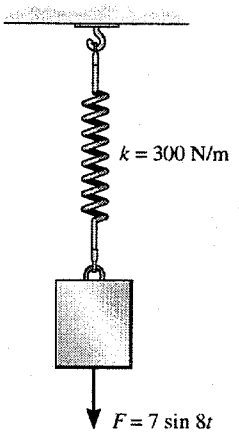
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{120}{0.8}} = 12.247 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(0.8)(12.247) = 19.60 \text{ N} \cdot \text{s/m}$$

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 12.247\sqrt{1 - \left(\frac{12.5}{19.6}\right)^2} = 9.432 \text{ rad/s}$$

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{9.432} = 0.666 \text{ s} \qquad \textbf{Ans}$$

22-47. A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.



The general solution is defined by:

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right) \sin \omega t$$

Since

$F = 7 \sin 8t$, $\quad F_0 = 7$ N, $\quad \omega = 8$ rad/s, $\quad k = 300$ N/m

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2}\right) \sin 8t$$

$y = 0.1$ m when $t = 0$,

$0.1 = 0 + B - 0; \quad B = 0.1$ m

$$\dot{y} = A(7.746) \cos 7.746t - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$$

$y = \dot{y} = 0$ when $t = 0$,

$\dot{y} = A(7.746) - 2.8 = 0; \quad A = 0.361$

Expressing the results in mm, we have

$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t)$ mm $\qquad$ **Ans**

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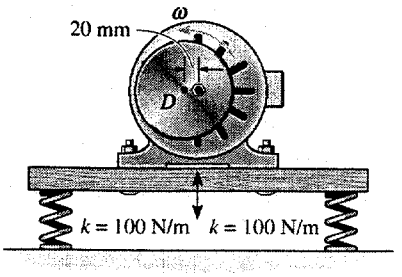
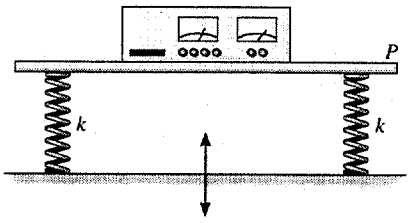
***22-48.** The electric motor has a mass of 50 kg and is supported by *four springs*, each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular rotation ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



Resonance occurs when $\omega = \omega_n$.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

22-49. The instrument is centered uniformly on a platform P, which in turn is supported by *four* springs, each spring having a stiffness $k = 130$ N/m. If the floor is subjected to a vibration $\omega = 7$ Hz, having a vertical displacement amplitude $\delta_0 = 0.17$ ft, determine the vertical displacement amplitude of the platform and instrument. The instrument and the platform have a total weight of 18 lb.



$k = 4(130) = 520$ lb/ft

$\delta_0 = 0.17$ ft

$\omega = 7 \text{ Hz} = 7(2\pi) = 43.98$ rad/s

Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

Since $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{520}{\frac{18}{32.2}}} = 30.50$ rad/s

Then,

$$(x_p)_{max} = \left| \frac{0.17}{1 - \left(\frac{43.98}{30.50}\right)^2} \right| = 0.157 \text{ ft}$$

$(x_p)_{max} = 1.89$ in. **Ans**

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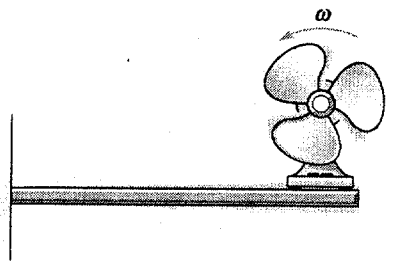
22-50. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the amplitude of steady-state vibration of the fan when the angular velocity of the fan is 10 rad/s. *Hint:* See the first part of Example 22-8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

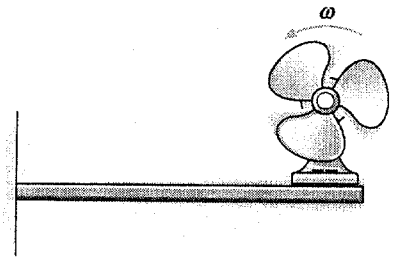
Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

$$(x_p)_{max} = \left| \frac{\frac{35}{4905}}{1 - \left(\frac{10}{14.01}\right)^2} \right| = 0.0146 \text{ m}$$

$(x_p)_{max} = 14.6$ mm **Ans**

22-51. What will be the amplitude of steady-state vibration of the fan in Prob. 22-50 if the angular velocity of the fan is 18 rad/s? *Hint:* See the first part of Example 22-8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

$$(x_p)_{max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$(x_p)_{max} = 35.5$ mm **Ans**

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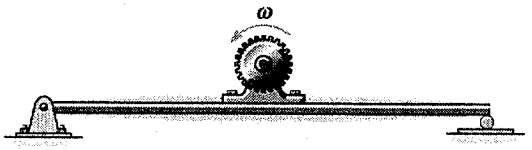
***22-52.** The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. because of the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weighs 150 lb. Neglect the mass of the beam.



$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\omega = \omega_n = 19.7$ rad/s **Ans**

22-53. What will be the amplitude of steady-state vibration of the motor in Prob. 22-52 if the angular velocity of the flywheel is 20 rad/s?

The constant value F_o of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_o = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22-21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1-\left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.} \quad \textbf{Ans}$$

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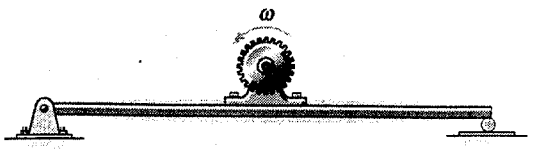
22-54. Determine the angular velocity of the flywheel in Prob. 22-52 which will produce an amplitude of vibration of 0.25 in.



The constant value F_o of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_o = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

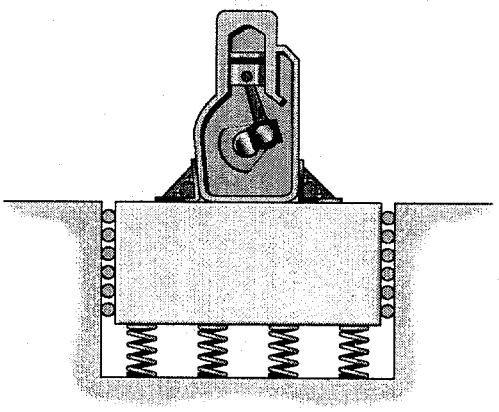
From Eq. 22.21, the amplitude of the steady state motion is

$$C = \left| \frac{F_O/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

$$\frac{0.25}{12} = \left| \frac{0.006470\left(\frac{\omega^2}{1800}\right)}{1 - \left(\frac{\omega}{19.657}\right)^2} \right|$$

$$\omega = 19.0 \text{ rad/s} \qquad \textbf{Ans}$$

22-55. The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have a total weight of 1500 lb and the engine, when running, creates an impressed force $F = (50 \sin 2t)$ lb, where t is in seconds. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as $k = 2000$ lb/ft.



The steady-state vibration is defined by Eq. 22-22.

$$x_p = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \sin \omega t$$

Since $F = 50 \sin 2t$

Then $F_0 = 50$ lb, $\omega = 2$ rad/s

$k = 2000$ lb/ft

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{\frac{1500}{32.2}}} = 6.55 \text{ rad/s}$$

$$\text{Hence, } x_p = \frac{\frac{50}{2000}}{1 - \left(\frac{2}{6.55}\right)^2} \sin 2t$$

$$x_p = (0.0276 \sin 2t) \text{ ft} \qquad \textbf{Ans}$$

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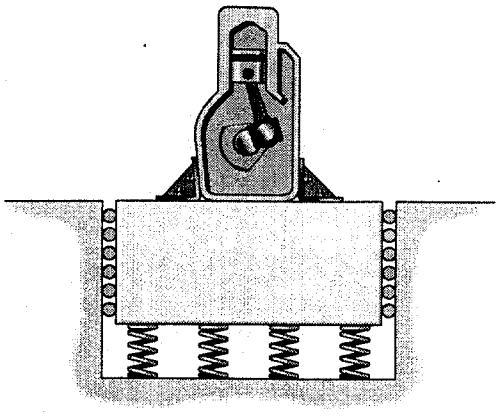
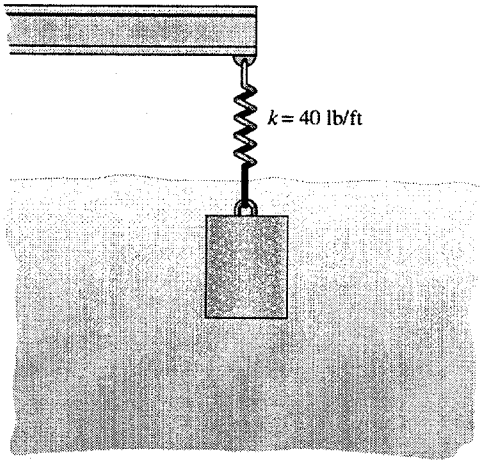
***22-56.** Determine the rotational speed ω of the engine in Prob. 22-55 which will cause resonance.



Resonance occurs when

$$\omega = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{2000}{1500}}{32.2}} = 6.55 \text{ rad/s} \quad \textbf{Ans}$$

22-57. The block, having a weight of 1.5 lb is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Assume that positive displacement is downward.



Viscous Damped Free Vibration: Here $c = 0.8$ lb · s/ft, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{15/32.2}} = 9.266$ rad/s and $c_c = 2m\omega_n = 2\left(\frac{15}{32.2}\right)(9.266) = 8.633$ lb · s/ft. Since $c < c_c$, the system is underdamped and the solution of the differential equation is in the form of

$$y = D[e^{-(c/2m)t}\sin(\omega_d t + \phi)] \qquad [1]$$

Taking the time derivative of Eq. [1], we have

$$v = \dot{y} = D\left[-\left(\frac{c}{2m}\right)e^{-(c/2m)t}\sin(\omega_d t + \phi) + \omega_d e^{-(c/2m)t}\cos(\omega_d t + \phi)\right]$$

$$= De^{-(c/2m)t}\left[-\left(\frac{c}{2m}\right)\sin(\omega_d t + \phi) + \omega_d \cos(\omega_d t + \phi)\right] \qquad [2]$$

Applying the initial condition $v = 0$ at $t = 0$ to Eq. [2], we have

$$0 = De^{-0}\left[-\left(\frac{c}{2m}\right)\sin(0 + \phi) + \omega_d \cos(0 + \phi)\right]$$

$$0 = D\left[-\left(\frac{c}{2m}\right)\sin\phi + \omega_d \cos\phi\right] \qquad [3]$$

Here, $\frac{c}{2m} = \frac{0.8}{2(15/32.2)} = 0.8587$ and $\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)} = 9.266\sqrt{1 - \left(\frac{0.8}{8.633}\right)^2} = 9.227$ rad/s. Substituting these values into Eq. [1] yields

$$0 = D[-0.8587\sin\phi + 9.227\cos\phi] \qquad [4]$$

Applying the initial condition $y = 0.8$ ft at $t = 0$ to Eq. [1], we have

$$0.8 = D[e^{-0}\sin(0 + \phi)]$$

$$0.8 = D\sin\phi \qquad [5]$$

Solving Eqs. [4] and [5] yields

$$\phi = 85.74° = 1.50 \text{ rad} \qquad D = 0.8022 \text{ ft}$$

Substituting these values into Eq. [1] yields

$$y = 0.802[e^{-0.859t}\sin(9.23t + 1.50)] \quad \textbf{Ans}$$

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22-58. A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$\delta = 0.15 \sin 2t$

$\delta_0 = 0.15, \omega = 2$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1-\left(\frac{2}{18.57}\right)^2}\right)$$

$\phi' = 9.89°$ **Ans**

22-59. Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22-58.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$\delta = 0.15 \sin 2t$

$\delta_0 = 0.15, \quad \omega = 2$

$$\text{MF} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}} = \frac{1}{\sqrt{\left[1-\left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

MF = 0.997 **Ans**

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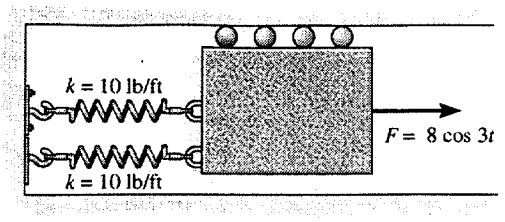
***22-60.** The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A force $F = (8\cos 3t)$lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



Freebody Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.

Equation of Motion:

$$\stackrel{+}{\rightarrow}\sum F_x = 0; \qquad -2(10x) + 8\cos 3t = \frac{30}{32.2}a$$

$$a + 21.47x = 8.587\cos 3t \qquad [1]$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587\cos 3t \qquad [2]$$

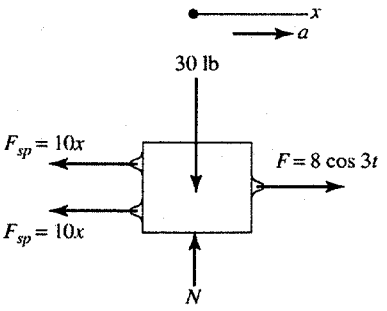
Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C\cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C\cos 3t + 21.47C\cos 3t = 8.587\cos 3t$$

$$C = 0.6888 \text{ ft}$$



Thus, $x_p = 0.6888\cos 3t$ [3]

Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663\sin 3t$$

Thus, $(v_p)_{max} = 2.07$ ft/s **Ans**

22-61. A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$N, where v is in m/s.

$c = 50$ N · s/m $\quad k = 600$ N/m $\quad m = 7$ kg

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6 \text{ N} \cdot \text{s/m}$$

Since $c < c_c$ the system is underdamped,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258\sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.571$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\omega_d\cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d\cos(\omega_d t + \phi) - \frac{c}{2m}\sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0}\sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

$$-0.6 = De^{-0}[8.542\cos 0° - 0]$$

$$D = -0.0702 \text{ m}$$

$$y = \{-0.0702[e^{-3.57t}\sin(8.54t)]\} \text{ m} \qquad \textbf{Ans}$$

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22-62. The damping factor, c/c_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22-17, show that the ratio $\ln x_1/x_2 = 2\pi(c/c_c)/\sqrt{1-(c/c_e)^2}$. The quantity $\ln x_1/x_2$ is called the *logarithmic decrement.*

Using Eq. 22-32,

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

The maximum displacement is

$$x_{max} = De^{-\left(\frac{c}{2m}\right)t}$$

At $t = t_1$, and $t = t_2$

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2}$$

Hence,

$$\frac{x_1}{x_2} = \frac{De^{-\left(\frac{c}{2m}\right)t_1}}{De^{-\left(\frac{c}{2m}\right)t_2}} = e^{-\left(\frac{c}{2m}\right)(t_1 - t_2)}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$

then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{c\pi}{m\omega_d}$

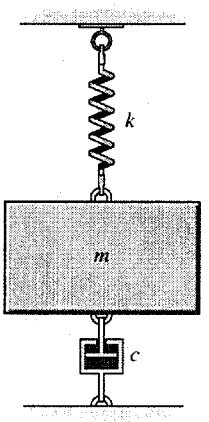
Using Eq. 22-33, $c_c = 2m\omega_n$

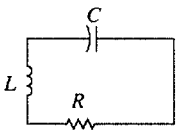
$$\omega_d = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1-\left(\frac{c}{c_c}\right)^2}$$

So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\dfrac{c}{c_c}\right)}{\sqrt{1-\left(\dfrac{c}{c_c}\right)^2}} \qquad \textbf{Q.E.D.}$$

22-63. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.





For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22-1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \quad \textbf{Ans}$$

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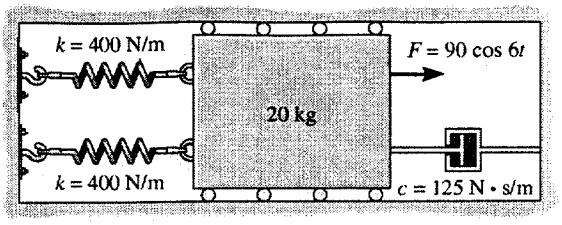
***22-64.** The 20-kg block is subjected to the action of the harmonic force $F = (90\cos 6t)$ N, where t is in seconds. Write the equation which describes the steady-state motion.



$F = 90\cos 6t$

$F_O = 90$ N, $\quad \omega = 6$ rad/s

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{20}} = 6.32 \text{ rad/s}$$

From Eq. 22-29,

$$c_c = 2m\omega_n = 2(20)(6.32) = 253.0$$

Using Eqs. 22-39,

$$x = C'\cos(\omega t - \phi)$$

$$C' = \frac{\frac{F_0}{k}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{\frac{90}{800}}{\sqrt{[1-\left(\frac{6}{6.32}\right)^2]^2 + \left[2\left(\frac{125}{253.0}\right)\left(\frac{6}{6.32}\right)\right]^2}}$$

$$= 0.119$$

$$\phi = \tan^{-1}\left[\frac{\frac{c\omega}{k}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

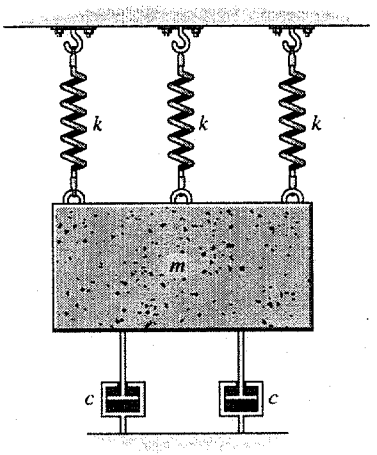
$$= \tan^{-1}\left[\frac{\frac{125(6)}{800}}{1-\left(\frac{6}{6.32}\right)^2}\right]$$

$$\phi = 83.9°$$

Thus,

$$x = 0.119\cos(6t - 83.9°) \text{ m} \quad \textbf{Ans}$$

22-65. Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N · s/m, $m = 25$ kg.



Freebody Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached to the block.

Equation of Motion:

$$+\uparrow \sum F_x = 0; \qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$$

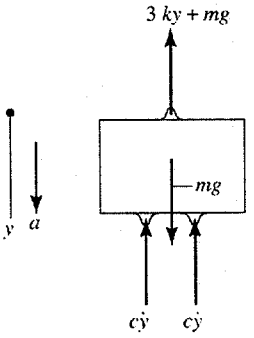
$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad [1]$$

Here, $m = 25$ kg, $c = 200$ N · s/m and $k = 100$ N/m. Substituting these values into Eq. [1] yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \quad \textbf{Ans}$$

Comparing the above differential equation with Eq. 22-27, we have $m = 1$ kg, $c = 16$ N · s/m and $k = 12$ N/m. Thus, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{1}} = 3.464$ rad/s.



$$c_c = 2m\omega_n = 2(1)(3.464) = 6.928 \text{ N} \cdot \text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped**. **Ans**

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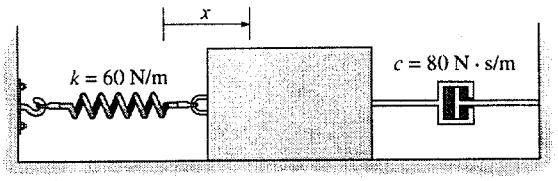
22-66. The 10-kg block-spring-damper system is continually damped. If the block is displaced to $x = 50$ mm and released from rest, determine the time required for it to return to the position $x = 2$ mm.



$m = 10$ kg, $c = 80$, $k = 60$

$c_c = 2\sqrt{km} = 2\sqrt{600} = 49.0 < c$ (Overdamped)

$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

$$\lambda_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} = \frac{-80}{20} \pm \sqrt{\left(\frac{80}{20}\right)^2 - \frac{60}{10}}$$

$\lambda_1 = -0.8377$, $\lambda_2 = -7.1623$

At $t = 0$, $x = 0.05$, $\dot{x} = 0$

$0.05 = A + B$

$A = 0.05 - B$

$\dot{x} = A\lambda_1 e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t}$

$0 = A\lambda_1 + B\lambda_2$

$0 = (0.05 - B)\lambda_1 + B\lambda_2$

$$B = \frac{0.05\lambda_1}{\lambda_1 - \lambda_2} = -6.6228(10^{-3})$$

$A = 0.056623$

$x = 0.056623e^{-0.8377t} - 6.6228(10^{-3})e^{-7.1623t}$

Set $x = 0.0002$ m and solve,

$t = 3.99$ s **Ans**

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